Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

6-Hyperbolic-functions/6.3-Hyperbolic-tangent/173-6.3.7-d-hyper-ˆm-a+b-c-tanh-ˆn-ˆp

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CHAPTER 1

Introduction

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [263]. This is test number [173].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (263)	0.00 (0)
Mathematica	100.00 (263)	0.00 (0)
Fricas	100.00 (263)	0.00 (0)
Maple	94.68 (249)	5.32 (14)
Giac	76.05 (200)	23.95 (63)
Mupad	70.34 (185)	29.66 (78)
Maxima	67.30 (177)	32.70 (86)
Sympy	15.21 (40)	84.79 (223)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

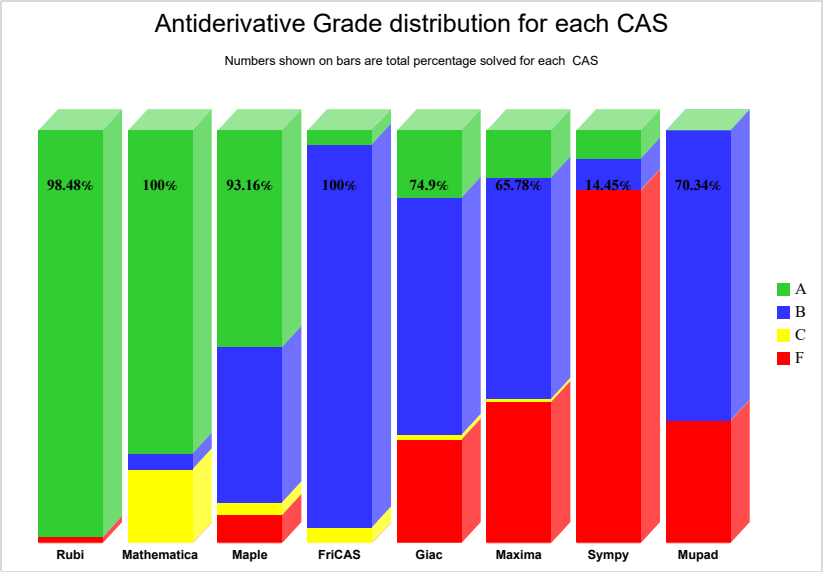
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

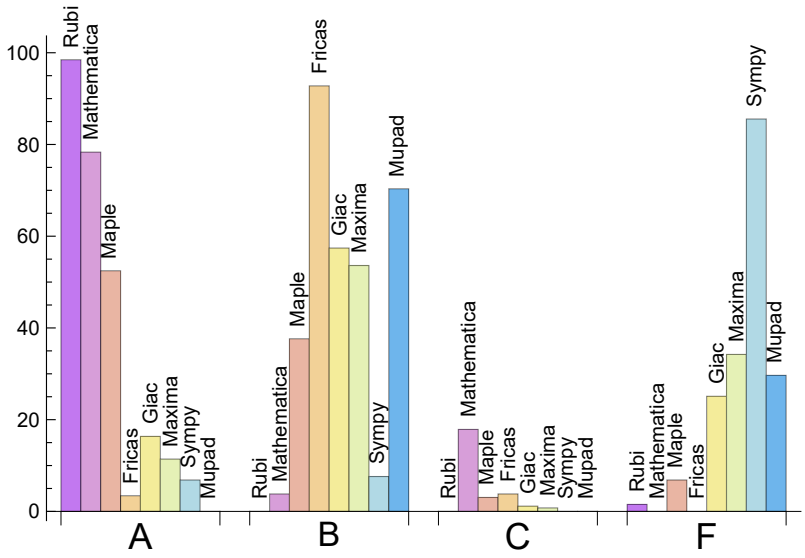
System	% A grade	% B grade	% C grade	% F grade
Rubi	92.015	0.000	6.464	1.521
Mathematica	78.327	3.802	17.871	0.000
Maple	52.471	37.643	3.042	6.844
Giac	16.350	57.414	1.141	25.095
Maxima	11.407	53.612	0.760	34.221
Sympy	6.844	7.605	0.000	85.551
Fricas	3.422	92.776	3.802	0.000
Mupad	0.000	70.342	0.000	29.658

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima

and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Fricas	0	0.00	0.00	0.00
Rubi	0	0.00	0.00	0.00
Maple	14	100.00	0.00	0.00
Giac	63	98.41	0.00	1.59
Mupad	78	0.00	100.00	0.00
Maxima	86	98.84	0.00	1.16
Sympy	223	87.44	12.56	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.30
Rubi	0.32
Fricas	0.39
Giac	0.48
Mathematica	1.71
Mupad	2.47
Sympy	6.36
Maple	9.51

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	96.23	1.05	82.00	1.03
Mathematica	126.30	1.64	86.00	1.00
Sympy	147.45	2.45	115.50	1.91
Maple	178.94	2.13	142.00	1.55
Giac	275.22	3.32	204.50	2.52
Maxima	417.18	4.13	235.00	2.92
Mupad	434.32	6.79	177.00	2.67
Fricas	3902.31	52.85	1942.00	23.42

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

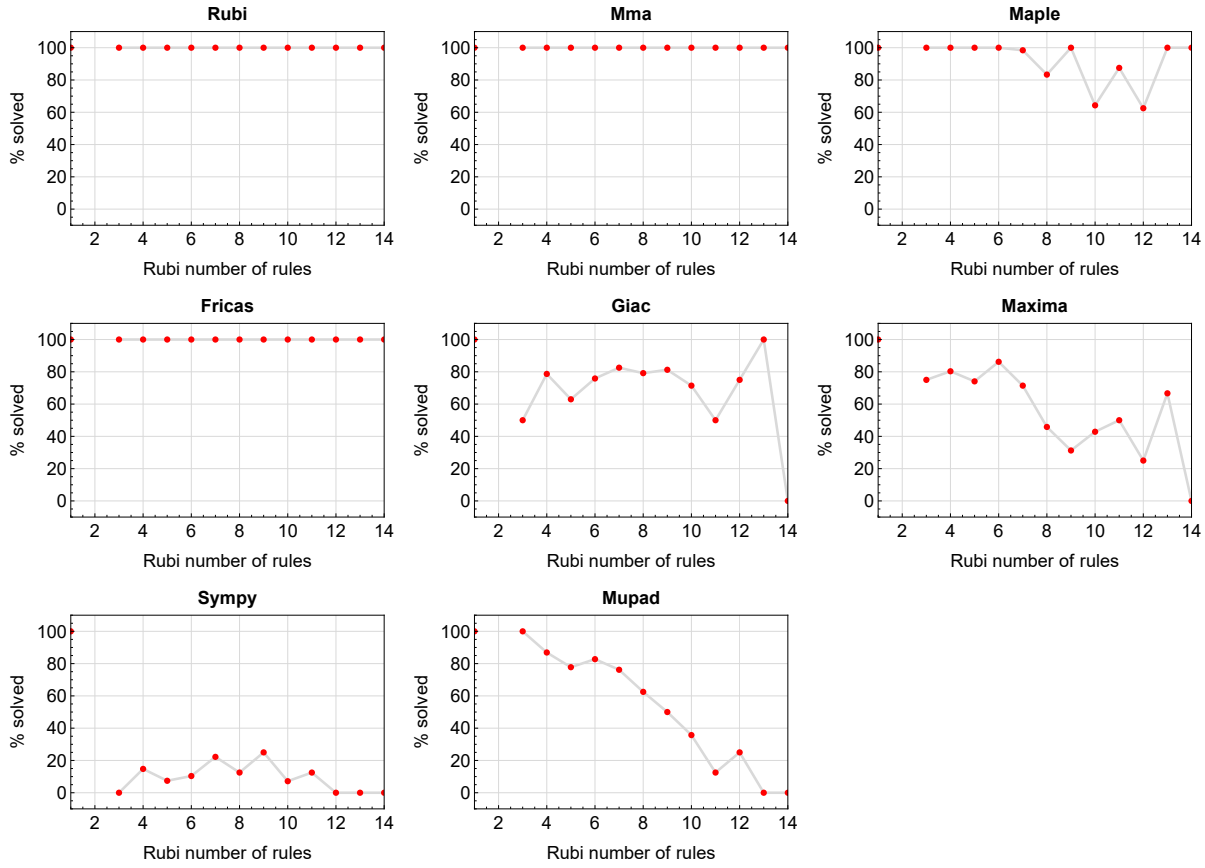


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

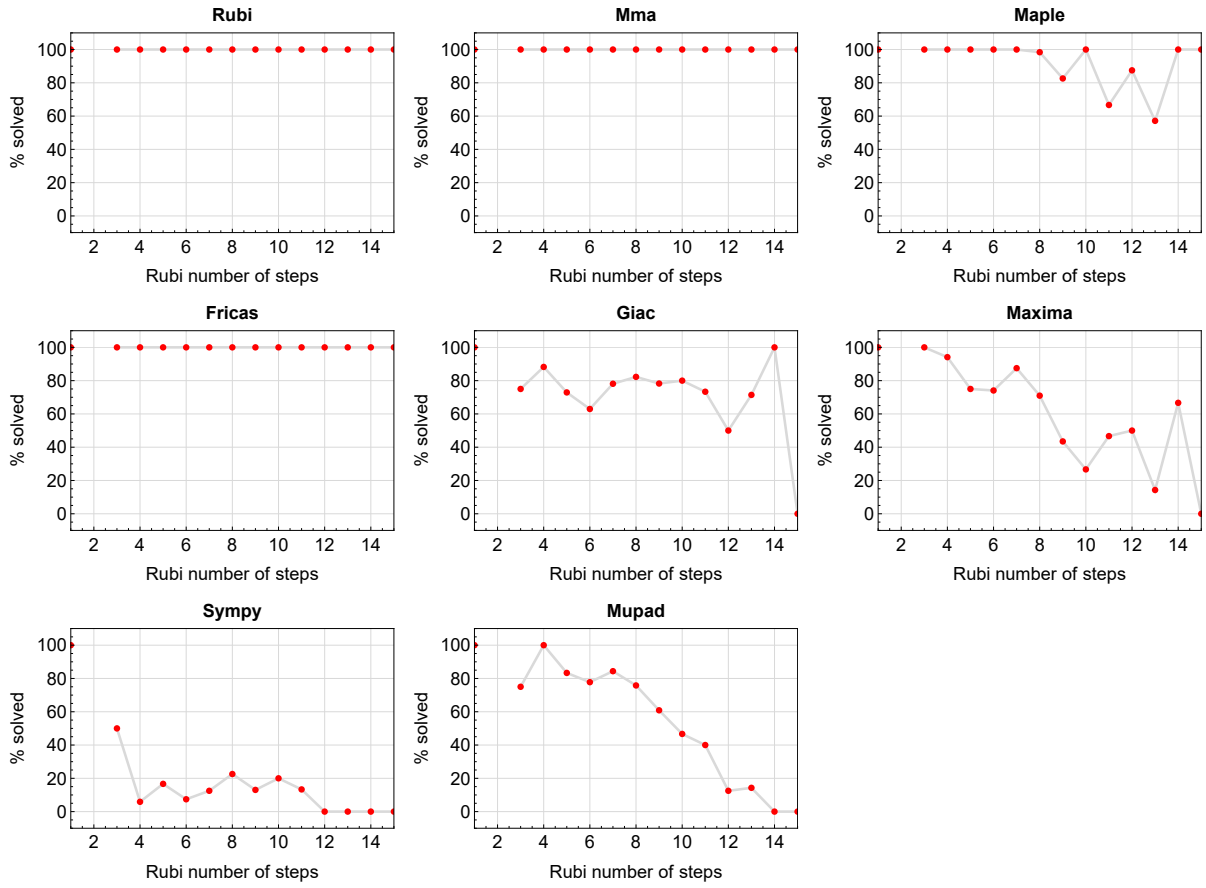


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

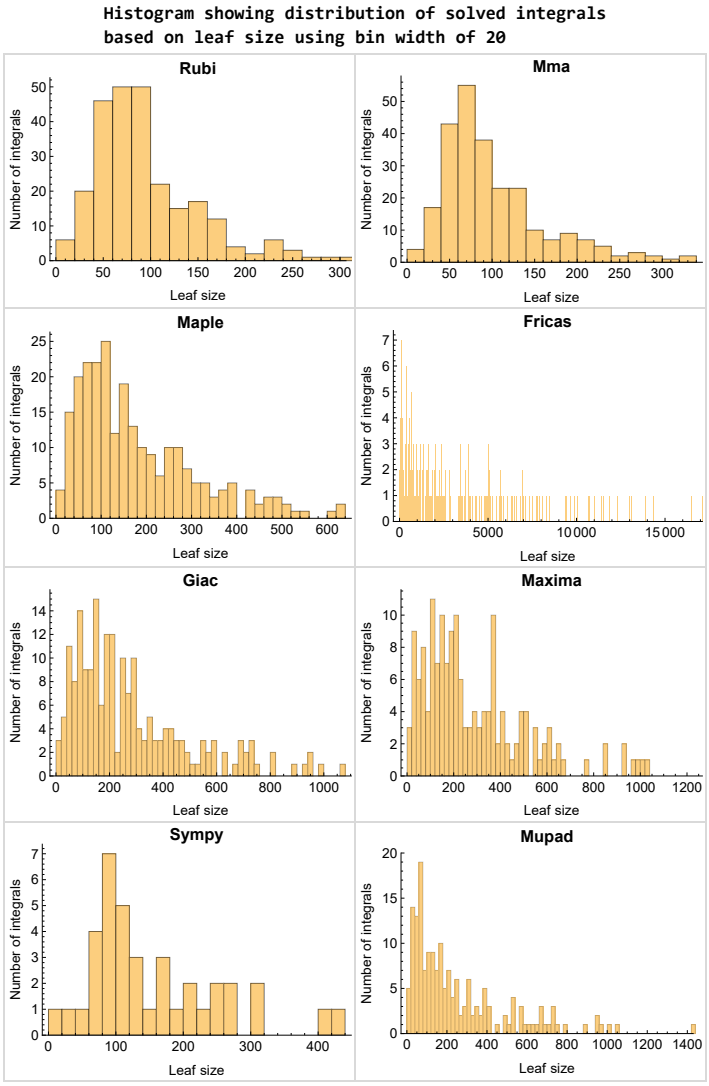


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

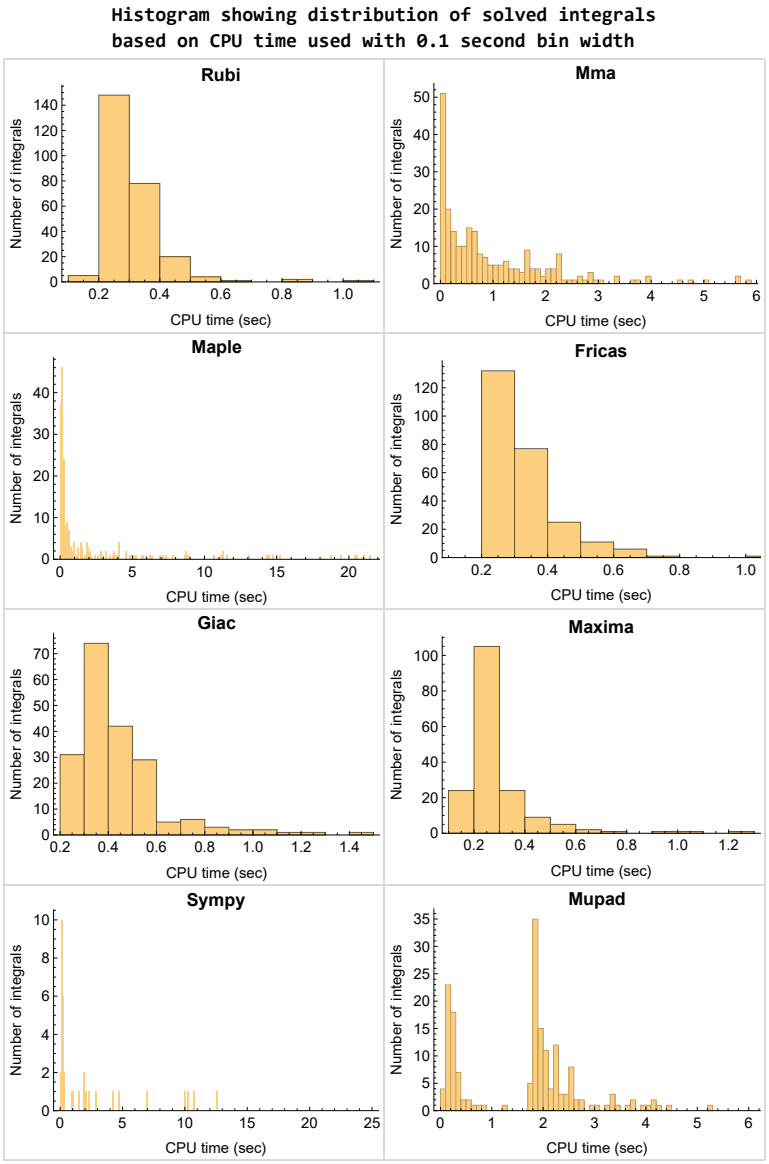


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

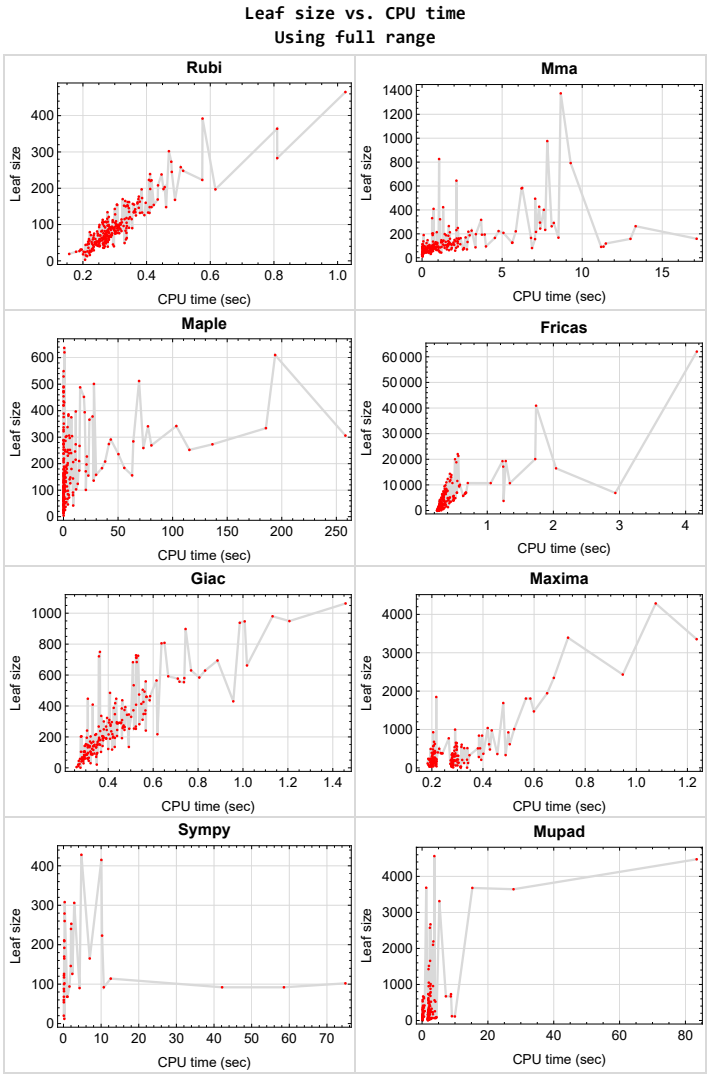


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{74, 76, 77, 79}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {74, 76, 77, 79}

Maple {}

Maxima {}

Fricas {74, 76, 77, 79}

Sympy {}

Giac {}

Mupad {76, 77, 79}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {151, 153, 155, 163, 165, 167, 178, 188, 199, 216, 218, 237}

Mathematica {93, 95, 98, 217, 236, 243, 245, 250, 252, 254}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

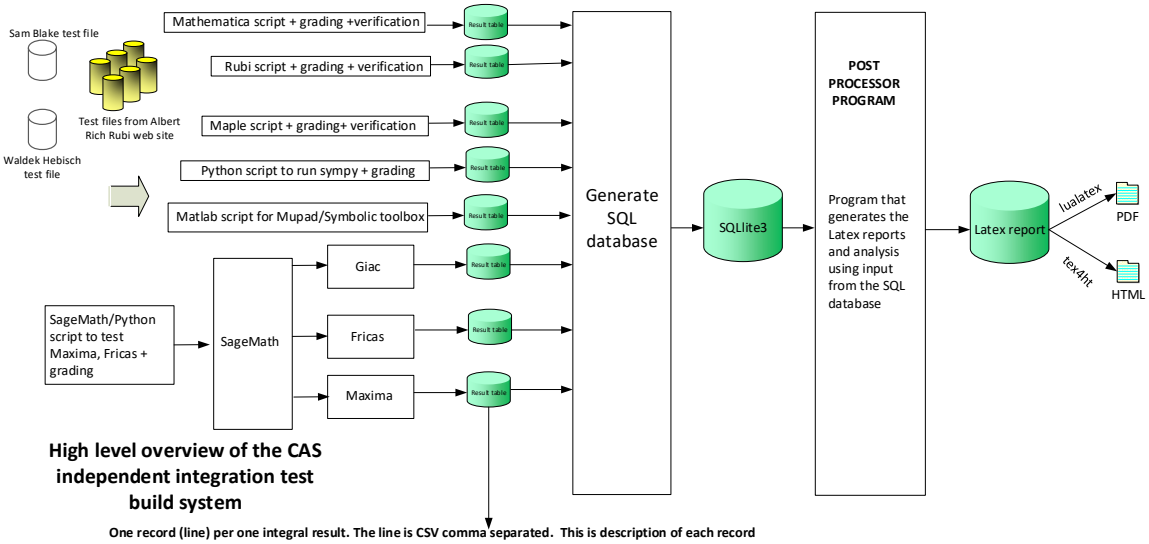
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

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2.1 List of integrals sorted by grade for each CAS

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2.1.2 Mma 22
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2.1.4 Fricas 23
2.1.5 Maxima 23
2.1.6 Giac 24
2.1.7 Mupad 24
2.1.8 Sympy 25

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 54, 56, 57, 59, 62, 64, 65, 67, 70, 72, 73, 75, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 138, 140, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263 }

B grade { }

C grade { 50, 52, 53, 55, 58, 60, 61, 63, 66, 68, 69, 71, 135, 137, 139, 141, 143 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 30, 32, 33, 35, 38, 40, 41, 43, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 96, 97, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 141, 143, 145, 147, 148, 149, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 210, 212, 213, 214, 216, 218, 219, 221, 222, 223, 225, 226, 227, 228, 229, 231, 233, 234, 235, 237, 240, 248, 255, 256, 257, 258, 259, 260, 261, 262, 263 }

B grade { 7, 74, 76, 77, 79, 104, 144, 146, 202, 241 }

C grade { 26, 28, 29, 31, 34, 36, 37, 39, 42, 44, 45, 47, 73, 75, 78, 80, 93, 95, 98, 133, 140, 142, 150, 154, 209, 211, 215, 217, 220, 224, 230, 232, 236, 238, 239, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 14, 15, 16, 17, 19, 21, 23, 24, 29, 31, 37, 39, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 71, 72, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 93, 95, 98, 101, 103, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 257, 258, 260, 261 }

B grade { 10, 12, 18, 20, 22, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 38, 40, 41, 42, 43, 44, 45, 46, 48, 62, 70, 91, 94, 96, 97, 99, 100, 102, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 156, 157, 208, 209, 210, 211, 212, 213, 219, 220, 221, 222, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 238, 239, 240, 241, 242, 243, 246, 247, 248, 249, 250, 251, 252, 255, 256 }

C grade { 73, 75, 78, 80, 111, 259, 262, 263 }

F normal fail { 214, 215, 216, 217, 218, 223, 224, 235, 236, 237, 244, 245, 253, 254 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 3, 4, 81, 82, 83, 89, 138, 206 }

B grade { 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 78, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 261, 262, 263 }

C grade { 73, 74, 75, 76, 77, 79, 80, 226, 228, 256 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 5, 6, 30, 49, 50, 51, 52, 53, 54, 66, 68, 81, 86, 94, 102, 110, 112, 121, 138, 139, 140, 150, 172, 174, 175, 176, 178, 202, 204, 206 }

B grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 32, 33, 35, 38, 40, 41, 43, 46, 48, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 69, 70, 71, 72, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 107, 114, 116, 119, 123, 125, 128, 130, 132, 134, 135, 136, 137, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 207, 257, 258 }

C grade { 203, 205 }

F normal fail { 26, 28, 29, 31, 34, 36, 37, 39, 44, 45, 47, 73, 75, 78, 80, 106, 108, 109, 111, 113, 115, 117, 118, 120, 122, 124, 126, 127, 129, 131, 133, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236,

237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 259, 260, 261, 262, 263 }

F(-1) timedout fail { }

F(-2) exception fail { 42 }

2.1.6 Giac

A grade { 6, 8, 14, 23, 49, 50, 51, 52, 53, 54, 58, 60, 61, 63, 66, 68, 78, 80, 81, 84, 93, 137, 138, 139, 140, 141, 150, 153, 171, 173, 174, 175, 176, 177, 178, 189, 202, 203, 205, 206, 207, 257, 258 }

B grade { 1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 24, 55, 56, 57, 59, 62, 64, 65, 67, 69, 70, 71, 72, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 134, 135, 136, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 204, 208, 209, 210, 211, 212, 213, 215, 216, 217, 218, 219, 220, 221, 222, 224, 225, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255 }

C grade { 226, 228, 256 }

F normal fail { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 73, 75, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 214, 223, 259, 260, 261, 262, 263 }

F(-1) timedout fail { }

F(-2) exception fail { 79 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 190, 191, 192, 193, 194, 195, 196, 201, 202, 203, 204, 205, 206, 207, 208, 210, 212, 219, 221, 225, 226, 227, 229, 231, 233, 234, 238, 240, 242, 247, 249, 251, 255, 256, 257, 258 }

C grade { }

F normal fail { }

F(-1) timedout fail { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 74, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 186, 187, 188, 189, 197, 198, 199, 200, 209, 211, 213, 214, 215, 216, 217, 218, 220, 222, 223, 224, 228, 230, 232, 235, 236, 237, 239, 241, 243, 244, 245, 246, 248, 250, 252, 253, 254, 259, 260, 261, 262, 263 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 134, 136, 138, 148, 160, 180, 182, 184, 207, 208, 210, 212, 219, 221, 233, 242, 251, 257 }

B grade { 135, 137, 140, 144, 145, 146, 147, 156, 157, 158, 159, 168, 169, 170, 171, 172, 173, 174, 175, 258 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 72, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 127, 128, 129, 130, 131, 132, 139, 141, 142, 143, 149, 150, 151, 152, 153, 154, 161, 162, 163, 164, 176, 177, 178, 179, 185, 186, 187, 188, 189, 197, 198, 199, 200, 202, 203, 204, 205, 206, 209, 211, 213, 214, 215, 216, 217, 218, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 256, 259, 260, 261, 262, 263 }

F(-1) timedout fail { 35, 41, 42, 43, 44, 65, 66, 73, 74, 75, 76, 125, 126, 133, 155, 165, 166, 167, 181, 183, 190, 191, 192, 193, 194, 195, 196, 201 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	96	56	96	154	120	0	142	101
N.S.	1	1.32	0.77	1.32	2.11	1.64	0.00	1.95	1.38
time (sec)	N/A	0.286	0.685	1.469	0.196	0.267	0.000	0.316	0.248

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	42	73	75	136	91	0	105	99
N.S.	1	0.89	1.55	1.60	2.89	1.94	0.00	2.23	2.11
time (sec)	N/A	0.258	0.300	0.634	0.213	0.258	0.000	0.320	1.845

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	60	41	66	101	71	0	107	64
N.S.	1	1.36	0.93	1.50	2.30	1.61	0.00	2.43	1.45
time (sec)	N/A	0.246	0.433	0.319	0.218	0.246	0.000	0.295	1.786

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	26	45	44	67	42	0	63	27
N.S.	1	1.04	1.80	1.76	2.68	1.68	0.00	2.52	1.08
time (sec)	N/A	0.230	0.219	0.212	0.214	0.246	0.000	0.287	1.735

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	24	52	27	40	167	0	68	64
N.S.	1	0.92	2.00	1.04	1.54	6.42	0.00	2.62	2.46
time (sec)	N/A	0.221	0.039	0.201	0.206	0.251	0.000	0.284	0.137

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	22	24	25	39	88	0	45	43
N.S.	1	0.92	1.00	1.04	1.62	3.67	0.00	1.88	1.79
time (sec)	N/A	0.229	0.019	0.425	0.218	0.242	0.000	0.289	0.102

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	59	123	50	152	924	0	142	156
N.S.	1	1.16	2.41	0.98	2.98	18.12	0.00	2.78	3.06
time (sec)	N/A	0.251	0.278	1.258	0.205	0.271	0.000	0.295	1.856

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	39	61	55	113	244	0	80	173
N.S.	1	0.89	1.39	1.25	2.57	5.55	0.00	1.82	3.93
time (sec)	N/A	0.241	0.346	2.637	0.206	0.257	0.000	0.296	1.809

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	137	94	166	295	394	0	293	293
N.S.	1	1.16	0.80	1.41	2.50	3.34	0.00	2.48	2.48
time (sec)	N/A	0.381	2.232	3.167	0.200	0.267	0.000	0.403	1.982

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	69	71	148	265	259	0	205	215
N.S.	1	0.90	0.92	1.92	3.44	3.36	0.00	2.66	2.79
time (sec)	N/A	0.290	1.827	1.966	0.211	0.247	0.000	0.373	0.305

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	82	70	118	217	291	0	213	248
N.S.	1	1.04	0.89	1.49	2.75	3.68	0.00	2.70	3.14
time (sec)	N/A	0.297	1.546	0.876	0.218	0.258	0.000	0.342	1.872

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	46	46	98	171	167	0	139	154
N.S.	1	0.94	0.94	2.00	3.49	3.41	0.00	2.84	3.14
time (sec)	N/A	0.252	1.109	0.527	0.197	0.271	0.000	0.332	1.828

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	46	64	63	196	890	0	123	160
N.S.	1	0.90	1.25	1.24	3.84	17.45	0.00	2.41	3.14
time (sec)	N/A	0.265	0.979	0.451	0.200	0.258	0.000	0.327	0.169

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	41	43	68	136	264	0	86	209
N.S.	1	0.89	0.93	1.48	2.96	5.74	0.00	1.87	4.54
time (sec)	N/A	0.258	1.199	1.838	0.200	0.252	0.000	0.348	1.866

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	79	127	67	181	2462	0	153	261
N.S.	1	0.96	1.55	0.82	2.21	30.02	0.00	1.87	3.18
time (sec)	N/A	0.292	5.638	3.711	0.202	0.287	0.000	0.360	0.177

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	64	59	81	210	393	0	143	143
N.S.	1	0.89	0.82	1.12	2.92	5.46	0.00	1.99	1.99
time (sec)	N/A	0.282	2.067	8.706	0.202	0.260	0.000	0.336	1.930

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	198	125	246	480	879	0	505	730
N.S.	1	1.09	0.69	1.35	2.64	4.83	0.00	2.77	4.01
time (sec)	N/A	0.419	5.624	10.637	0.204	0.266	0.000	0.554	2.120

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	94	91	239	439	540	0	330	361
N.S.	1	0.90	0.87	2.28	4.18	5.14	0.00	3.14	3.44
time (sec)	N/A	0.326	11.183	5.004	0.212	0.255	0.000	0.502	2.033

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	155	95	180	377	725	0	393	668
N.S.	1	1.27	0.78	1.48	3.09	5.94	0.00	3.22	5.48
time (sec)	N/A	0.386	3.990	2.975	0.211	0.274	0.000	0.476	0.328

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	63	62	170	321	383	0	236	308
N.S.	1	0.90	0.89	2.43	4.59	5.47	0.00	3.37	4.40
time (sec)	N/A	0.263	0.546	1.809	0.202	0.280	0.000	0.413	1.870

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	76	93	118	560	2277	0	196	317
N.S.	1	0.90	1.11	1.40	6.67	27.11	0.00	2.33	3.77
time (sec)	N/A	0.291	11.331	1.218	0.205	0.276	0.000	0.420	0.266

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	56	70	141	348	572	0	202	590
N.S.	1	0.88	1.09	2.20	5.44	8.94	0.00	3.16	9.22
time (sec)	N/A	0.272	2.803	5.620	0.203	0.257	0.000	0.422	1.885

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	161	159	103	403	5037	0	206	412
N.S.	1	1.06	1.05	0.68	2.65	33.14	0.00	1.36	2.71
time (sec)	N/A	0.394	17.146	11.550	0.214	0.305	0.000	0.439	0.299

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	87	87	136	493	925	0	257	622
N.S.	1	0.89	0.89	1.39	5.03	9.44	0.00	2.62	6.35
time (sec)	N/A	0.305	3.337	27.663	0.195	0.253	0.000	0.464	1.932

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	151	93	215	514	2024	0	0	250
N.S.	1	1.28	0.79	1.82	4.36	17.15	0.00	0.00	2.12
time (sec)	N/A	0.360	0.885	11.143	0.339	0.298	0.000	0.000	2.316

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	76	135	202	0	1367	0	0	955
N.S.	1	1.01	1.80	2.69	0.00	18.23	0.00	0.00	12.73
time (sec)	N/A	0.292	1.228	3.896	0.000	0.285	0.000	0.000	3.336

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	97	67	151	316	916	0	0	198
N.S.	1	1.24	0.86	1.94	4.05	11.74	0.00	0.00	2.54
time (sec)	N/A	0.301	0.668	1.190	0.303	0.291	0.000	0.000	2.207

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	52	107	104	0	666	0	0	520
N.S.	1	0.98	2.02	1.96	0.00	12.57	0.00	0.00	9.81
time (sec)	N/A	0.245	0.658	0.475	0.000	0.308	0.000	0.000	2.694

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	53	135	67	0	587	0	0	284
N.S.	1	0.96	2.45	1.22	0.00	10.67	0.00	0.00	5.16
time (sec)	N/A	0.271	0.686	0.357	0.000	0.290	0.000	0.000	2.598

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	46	48	112	62	618	0	0	136
N.S.	1	0.96	1.00	2.33	1.29	12.88	0.00	0.00	2.83
time (sec)	N/A	0.256	0.586	0.488	0.293	0.279	0.000	0.000	1.976

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	94	198	111	0	1790	0	0	787
N.S.	1	1.11	2.33	1.31	0.00	21.06	0.00	0.00	9.26
time (sec)	N/A	0.306	1.414	0.737	0.000	0.305	0.000	0.000	2.599

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	71	246	134	1628	0	0	254
N.S.	1	1.00	1.01	3.51	1.91	23.26	0.00	0.00	3.63
time (sec)	N/A	0.266	0.690	1.570	0.305	0.276	0.000	0.000	2.281

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	223	132	512	1690	7366	0	0	0
N.S.	1	1.16	0.69	2.67	8.80	38.36	0.00	0.00	0.00
time (sec)	N/A	0.435	1.679	69.258	0.479	0.376	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	126	160	267	0	5025	0	0	0
N.S.	1	1.02	1.29	2.15	0.00	40.52	0.00	0.00	0.00
time (sec)	N/A	0.427	2.283	14.742	0.000	0.343	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	152	105	381	840	3918	0	0	0
N.S.	1	1.15	0.80	2.89	6.36	29.68	0.00	0.00	0.00
time (sec)	N/A	0.350	1.496	4.502	0.396	0.336	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	93	133	167	0	2252	0	0	0
N.S.	1	1.01	1.45	1.82	0.00	24.48	0.00	0.00	0.00
time (sec)	N/A	0.269	1.474	1.719	0.000	0.308	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	112	188	161	0	2614	0	0	0
N.S.	1	1.09	1.83	1.56	0.00	25.38	0.00	0.00	0.00
time (sec)	N/A	0.318	1.501	0.951	0.000	0.348	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	81	86	252	212	2562	0	0	0
N.S.	1	0.99	1.05	3.07	2.59	31.24	0.00	0.00	0.00
time (sec)	N/A	0.269	1.134	1.546	0.338	0.311	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	150	221	187	0	6335	0	0	0
N.S.	1	1.06	1.57	1.33	0.00	44.93	0.00	0.00	0.00
time (sec)	N/A	0.355	5.852	2.454	0.000	0.348	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	115	114	338	282	5062	0	0	0
N.S.	1	1.02	1.01	2.99	2.50	44.80	0.00	0.00	0.00
time (sec)	N/A	0.372	1.625	3.720	0.384	0.330	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	273	184	610	3392	18818	0	0	0
N.S.	1	1.14	0.77	2.54	14.13	78.41	0.00	0.00	0.00
time (sec)	N/A	0.507	1.643	193.922	0.733	0.533	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	168	227	341	0	13095	0	0	0
N.S.	1	1.01	1.37	2.05	0.00	78.89	0.00	0.00	0.00
time (sec)	N/A	0.499	3.094	77.406	0.000	0.449	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	220	158	488	1806	12965	0	0	0
N.S.	1	1.19	0.85	2.64	9.76	70.08	0.00	0.00	0.00
time (sec)	N/A	0.429	2.286	15.259	0.585	0.453	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	134	157	252	0	7119	0	0	0
N.S.	1	1.06	1.25	2.00	0.00	56.50	0.00	0.00	0.00
time (sec)	N/A	0.296	2.294	5.912	0.000	0.370	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	177	249	304	0	10716	0	0	0
N.S.	1	1.13	1.60	1.95	0.00	68.69	0.00	0.00	0.00
time (sec)	N/A	0.410	2.234	2.878	0.000	0.473	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	116	109	306	478	8312	0	0	0
N.S.	1	1.04	0.97	2.73	4.27	74.21	0.00	0.00	0.00
time (sec)	N/A	0.288	1.838	5.215	0.427	0.379	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	222	401	304	0	21301	0	0	0
N.S.	1	1.13	2.05	1.55	0.00	108.68	0.00	0.00	0.00
time (sec)	N/A	0.442	7.605	7.892	0.000	0.563	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	154	149	397	615	14334	0	0	0
N.S.	1	1.02	0.99	2.63	4.07	94.93	0.00	0.00	0.00
time (sec)	N/A	0.444	2.026	11.279	0.505	0.435	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	133	92	100	194	1530	0	203	156
N.S.	1	1.01	0.70	0.76	1.47	11.59	0.00	1.54	1.18
time (sec)	N/A	0.430	0.206	1.491	0.277	0.282	0.000	0.321	0.288

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	114	112	104	174	1070	0	132	171
N.S.	1	1.16	1.14	1.06	1.78	10.92	0.00	1.35	1.74
time (sec)	N/A	0.342	0.040	0.975	0.282	0.276	0.000	0.309	0.247

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	90	69	68	141	924	0	140	115
N.S.	1	0.90	0.69	0.68	1.41	9.24	0.00	1.40	1.15
time (sec)	N/A	0.382	0.107	0.426	0.280	0.323	0.000	0.317	2.008

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	76	77	73	105	528	0	86	128
N.S.	1	1.21	1.22	1.16	1.67	8.38	0.00	1.37	2.03
time (sec)	N/A	0.276	0.031	0.410	0.283	0.334	0.000	0.300	0.150

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	59	75	56	83	522	0	74	233
N.S.	1	1.20	1.53	1.14	1.69	10.65	0.00	1.51	4.76
time (sec)	N/A	0.274	0.018	0.440	0.279	0.278	0.000	0.293	3.242

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	27	29	28	44	141	0	45	79
N.S.	1	0.93	1.00	0.97	1.52	4.86	0.00	1.55	2.72
time (sec)	N/A	0.249	0.024	0.672	0.202	0.251	0.000	0.313	0.171

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	83	114	54	156	1188	0	143	173
N.S.	1	1.17	1.61	0.76	2.20	16.73	0.00	2.01	2.44
time (sec)	N/A	0.291	0.027	1.961	0.278	0.269	0.000	0.310	3.304

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	48	74	46	184	1739	0	149	162
N.S.	1	0.86	1.32	0.82	3.29	31.05	0.00	2.66	2.89
time (sec)	N/A	0.289	0.214	4.062	0.294	0.268	0.000	0.308	1.991

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	197	156	180	379	5034	0	374	359
N.S.	1	1.16	0.92	1.06	2.23	29.61	0.00	2.20	2.11
time (sec)	N/A	0.638	7.056	7.300	0.281	0.313	0.000	0.472	0.480

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	208	121	195	348	3341	0	289	397
N.S.	1	1.14	0.66	1.07	1.91	18.36	0.00	1.59	2.18
time (sec)	N/A	0.468	2.228	3.698	0.290	0.293	0.000	0.456	0.411

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	148	137	130	301	3649	0	296	306
N.S.	1	1.15	1.06	1.01	2.33	28.29	0.00	2.29	2.37
time (sec)	N/A	0.487	2.204	2.022	0.284	0.280	0.000	0.416	2.054

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	145	90	145	253	2230	0	203	338
N.S.	1	1.18	0.73	1.18	2.06	18.13	0.00	1.65	2.75
time (sec)	N/A	0.361	1.564	1.427	0.279	0.278	0.000	0.378	0.311

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	115	125	111	447	2498	0	178	522
N.S.	1	1.17	1.28	1.13	4.56	25.49	0.00	1.82	5.33
time (sec)	N/A	0.355	0.955	1.036	0.288	0.286	0.000	0.377	4.052

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	42	94	98	256	518	0	122	483
N.S.	1	0.89	2.00	2.09	5.45	11.02	0.00	2.60	10.28
time (sec)	N/A	0.261	0.975	4.037	0.206	0.252	0.000	0.400	2.003

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	125	159	91	378	4642	0	192	561
N.S.	1	1.17	1.49	0.85	3.53	43.38	0.00	1.79	5.24
time (sec)	N/A	0.380	1.741	8.677	0.292	0.318	0.000	0.391	3.771

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	83	147	101	468	4125	0	249	344
N.S.	1	0.86	1.52	1.04	4.82	42.53	0.00	2.57	3.55
time (sec)	N/A	0.344	1.207	20.507	0.297	0.282	0.000	0.380	0.276

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	283	294	274	647	12323	0	694	682
N.S.	1	1.03	1.07	1.00	2.35	44.81	0.00	2.52	2.48
time (sec)	N/A	0.876	7.365	41.602	0.298	0.403	0.000	0.887	2.550

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	392	291	366	604	8462	0	580	757
N.S.	1	1.12	0.83	1.04	1.72	24.11	0.00	1.65	2.16
time (sec)	N/A	0.643	8.227	23.771	0.317	0.358	0.000	0.739	2.458

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	223	244	206	544	9862	0	577	617
N.S.	1	1.01	1.11	0.94	2.47	44.83	0.00	2.62	2.80
time (sec)	N/A	0.611	7.376	8.875	0.316	0.369	0.000	0.710	0.582

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	302	233	297	484	6410	0	463	707
N.S.	1	1.12	0.87	1.10	1.80	23.83	0.00	1.72	2.63
time (sec)	N/A	0.538	7.689	5.718	0.297	0.317	0.000	0.586	2.211

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	245	168	245	654	7127	0	414	671
N.S.	1	1.12	0.77	1.12	2.99	32.54	0.00	1.89	3.06
time (sec)	N/A	0.510	8.517	3.445	0.294	0.345	0.000	0.547	7.234

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	63	113	171	679	1192	0	311	1515
N.S.	1	0.89	1.59	2.41	9.56	16.79	0.00	4.38	21.34
time (sec)	N/A	0.272	2.387	20.423	0.208	0.273	0.000	0.553	2.085

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	258	264	208	586	10985	0	426	731
N.S.	1	1.11	1.14	0.90	2.53	47.35	0.00	1.84	3.15
time (sec)	N/A	0.567	13.337	38.158	0.294	0.382	0.000	0.557	8.786

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	118	213	156	997	9459	0	437	646
N.S.	1	0.86	1.54	1.13	7.22	68.54	0.00	3.17	4.68
time (sec)	N/A	0.350	1.737	62.879	0.291	0.353	0.000	0.584	0.538

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	491	465	645	452	0	17123	0	0	3313
N.S.	1	0.95	1.31	0.92	0.00	34.87	0.00	0.00	6.75
time (sec)	N/A	1.085	2.145	18.727	0.000	1.241	0.000	0.000	5.254

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	C	F(-1)	N/A	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	826	289	533	62017	0	3	0
N.S.	1	1.00	35.91	12.57	23.17	2696.39	0.00	0.13	0.00
time (sec)	N/A	0.235	1.064	7.246	0.844	4.169	0.000	3.880	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	384	364	423	301	0	10695	0	0	2100
N.S.	1	0.95	1.10	0.78	0.00	27.85	0.00	0.00	5.47
time (sec)	N/A	0.892	1.315	2.074	0.000	1.052	0.000	0.000	3.332

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	C	F(-1)	N/A	B
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	409	159	250	40923	0	3	4474
N.S.	1	1.00	19.48	7.57	11.90	1948.71	0.00	0.14	213.05
time (sec)	N/A	0.211	0.719	1.616	0.506	1.738	0.000	3.415	83.407

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	C	N/A	N/A	B
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	331	96	160	20085	19	3	3679
N.S.	1	1.00	15.76	4.57	7.62	956.43	0.90	0.14	175.19
time (sec)	N/A	0.223	0.628	0.632	0.397	1.726	0.436	2.970	15.272

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	158	190	116	0	640	0	21	669
N.S.	1	1.01	1.21	0.74	0.00	4.08	0.00	0.13	4.26
time (sec)	N/A	0.398	0.663	0.668	0.000	0.281	0.000	0.349	8.731

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	C	N/A	F(-2)	B
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	214	136	188	6846	20	0	3643
N.S.	1	1.00	9.30	5.91	8.17	297.65	0.87	0.00	158.39
time (sec)	N/A	0.242	0.919	1.233	0.448	2.937	0.351	0.000	27.782

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	198	322	173	0	3726	0	180	4563
N.S.	1	0.92	1.50	0.80	0.00	17.33	0.00	0.84	21.22
time (sec)	N/A	0.478	1.122	1.599	0.000	1.247	0.000	0.365	3.728

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	79	44	82	104	63	0	104	74
N.S.	1	1.25	0.70	1.30	1.65	1.00	0.00	1.65	1.17
time (sec)	N/A	0.243	0.132	4.873	0.203	0.248	0.000	0.321	0.188

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	28	44	37	83	45	0	84	74
N.S.	1	0.93	1.47	1.23	2.77	1.50	0.00	2.80	2.47
time (sec)	N/A	0.233	0.013	1.898	0.198	0.259	0.000	0.303	0.196

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	48	32	54	69	30	0	78	27
N.S.	1	1.45	0.97	1.64	2.09	0.91	0.00	2.36	0.82
time (sec)	N/A	0.244	0.050	0.623	0.194	0.262	0.000	0.293	1.852

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	25	47	32	55	102	0	47	66
N.S.	1	0.93	1.74	1.19	2.04	3.78	0.00	1.74	2.44
time (sec)	N/A	0.217	0.018	0.328	0.281	0.268	0.000	0.283	0.126

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	44	48	56	80	323	0	84	125
N.S.	1	1.10	1.20	1.40	2.00	8.08	0.00	2.10	3.12
time (sec)	N/A	0.223	0.014	0.615	0.275	0.255	0.000	0.293	0.121

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	26	28	25	34	159	0	59	59
N.S.	1	0.93	1.00	0.89	1.21	5.68	0.00	2.11	2.11
time (sec)	N/A	0.227	0.013	1.415	0.193	0.257	0.000	0.296	1.872

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	71	93	85	181	1046	0	153	280
N.S.	1	1.08	1.41	1.29	2.74	15.85	0.00	2.32	4.24
time (sec)	N/A	0.238	0.010	4.028	0.281	0.265	0.000	0.300	1.896

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	43	86	42	371	345	0	95	304
N.S.	1	0.90	1.79	0.88	7.73	7.19	0.00	1.98	6.33
time (sec)	N/A	0.245	0.021	8.915	0.205	0.257	0.000	0.309	0.154

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	108	63	124	171	95	0	186	102
N.S.	1	1.27	0.74	1.46	2.01	1.12	0.00	2.19	1.20
time (sec)	N/A	0.296	1.683	13.067	0.204	0.264	0.000	0.359	0.252

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	49	72	72	161	519	0	152	130
N.S.	1	0.91	1.33	1.33	2.98	9.61	0.00	2.81	2.41
time (sec)	N/A	0.270	0.622	5.132	0.282	0.264	0.000	0.351	1.982

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	63	54	96	140	105	0	167	77
N.S.	1	1.24	1.06	1.88	2.75	2.06	0.00	3.27	1.51
time (sec)	N/A	0.282	1.080	1.943	0.193	0.255	0.000	0.353	1.930

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	61	54	97	152	774	0	160	182
N.S.	1	1.02	0.90	1.62	2.53	12.90	0.00	2.67	3.03
time (sec)	N/A	0.278	0.212	0.961	0.279	0.279	0.000	0.325	0.210

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	91	101	427	134	199	1373	0	170	303
N.S.	1	1.11	4.69	1.47	2.19	15.09	0.00	1.87	3.33
time (sec)	N/A	0.279	7.314	2.113	0.289	0.262	0.000	0.319	0.157

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	44	49	126	53	391	0	169	482
N.S.	1	0.90	1.00	2.57	1.08	7.98	0.00	3.45	9.84
time (sec)	N/A	0.246	0.216	6.236	0.201	0.272	0.000	0.333	1.957

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	125	128	792	174	345	2824	0	267	572
N.S.	1	1.02	6.34	1.39	2.76	22.59	0.00	2.14	4.58
time (sec)	N/A	0.309	9.279	14.466	0.278	0.277	0.000	0.358	0.169

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	68	83	158	928	677	0	238	732
N.S.	1	0.89	1.09	2.08	12.21	8.91	0.00	3.13	9.63
time (sec)	N/A	0.279	1.855	29.886	0.205	0.254	0.000	0.364	1.869

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	106	81	184	267	227	0	282	133
N.S.	1	1.16	0.89	2.02	2.93	2.49	0.00	3.10	1.46
time (sec)	N/A	0.335	6.865	55.729	0.205	0.259	0.000	0.539	2.039

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	87	85	494	155	284	1840	0	264	232
N.S.	1	0.98	5.68	1.78	3.26	21.15	0.00	3.03	2.67
time (sec)	N/A	0.307	7.062	22.770	0.280	0.282	0.000	0.473	0.333

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	87	69	148	256	369	0	265	243
N.S.	1	1.12	0.88	1.90	3.28	4.73	0.00	3.40	3.12
time (sec)	N/A	0.300	2.558	6.308	0.206	0.257	0.000	0.466	2.014

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	101	89	193	295	2411	0	272	355
N.S.	1	1.02	0.90	1.95	2.98	24.35	0.00	2.75	3.59
time (sec)	N/A	0.317	0.437	2.813	0.289	0.277	0.000	0.410	0.287

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	165	119	240	362	3465	0	310	535
N.S.	1	1.11	0.80	1.61	2.43	23.26	0.00	2.08	3.59
time (sec)	N/A	0.350	11.476	6.984	0.280	0.282	0.000	0.405	1.996

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	59	67	227	71	786	0	347	1050
N.S.	1	0.88	1.00	3.39	1.06	11.73	0.00	5.18	15.67
time (sec)	N/A	0.263	0.253	21.438	0.187	0.271	0.000	0.430	2.036

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	197	158	291	553	6114	0	485	951
N.S.	1	0.99	0.80	1.47	2.79	30.88	0.00	2.45	4.80
time (sec)	N/A	0.408	13.015	43.363	0.298	0.318	0.000	0.408	1.988

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	91	218	269	1847	1185	0	447	1424
N.S.	1	0.89	2.14	2.64	18.11	11.62	0.00	4.38	13.96
time (sec)	N/A	0.304	2.891	80.580	0.217	0.264	0.000	0.436	1.956

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	153	115	273	514	2180	0	0	967
N.S.	1	1.28	0.96	2.28	4.28	18.17	0.00	0.00	8.06
time (sec)	N/A	0.363	0.948	11.204	0.327	0.303	0.000	0.000	2.503

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	75	79	230	0	1850	0	0	2194
N.S.	1	0.94	0.99	2.88	0.00	23.12	0.00	0.00	27.42
time (sec)	N/A	0.318	0.564	3.968	0.000	0.298	0.000	0.000	3.439

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	97	77	159	316	948	0	0	880
N.S.	1	1.26	1.00	2.06	4.10	12.31	0.00	0.00	11.43
time (sec)	N/A	0.305	0.745	1.238	0.298	0.288	0.000	0.000	2.556

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	51	53	149	0	766	0	0	154
N.S.	1	0.96	1.00	2.81	0.00	14.45	0.00	0.00	2.91
time (sec)	N/A	0.246	0.136	0.576	0.000	0.286	0.000	0.000	2.240

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	102	0	511	0	0	147
N.S.	1	1.00	1.00	2.83	0.00	14.19	0.00	0.00	4.08
time (sec)	N/A	0.228	0.046	0.904	0.000	0.268	0.000	0.000	0.279

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	114	36	455	0	0	81
N.S.	1	1.00	1.00	3.56	1.12	14.22	0.00	0.00	2.53
time (sec)	N/A	0.242	0.550	3.195	0.276	0.286	0.000	0.000	2.056

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	53	55	144	0	540	0	0	449
N.S.	1	0.96	1.00	2.62	0.00	9.82	0.00	0.00	8.16
time (sec)	N/A	0.255	0.312	8.707	0.000	0.293	0.000	0.000	2.298

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	48	50	197	63	649	0	0	176
N.S.	1	0.96	1.00	3.94	1.26	12.98	0.00	0.00	3.52
time (sec)	N/A	0.250	0.567	21.085	0.300	0.284	0.000	0.000	2.221

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	94	79	236	0	1584	0	0	1012
N.S.	1	1.09	0.92	2.74	0.00	18.42	0.00	0.00	11.77
time (sec)	N/A	0.303	0.769	50.330	0.000	0.294	0.000	0.000	2.597

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	70	71	252	140	2032	0	0	252
N.S.	1	0.93	0.95	3.36	1.87	27.09	0.00	0.00	3.36
time (sec)	N/A	0.281	0.868	115.364	0.301	0.298	0.000	0.000	2.238

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	120	111	378	0	6934	0	0	0
N.S.	1	0.94	0.87	2.95	0.00	54.17	0.00	0.00	0.00
time (sec)	N/A	0.367	1.304	26.642	0.000	0.344	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	162	110	386	840	4324	0	0	0
N.S.	1	1.16	0.79	2.76	6.00	30.89	0.00	0.00	0.00
time (sec)	N/A	0.358	1.606	4.574	0.384	0.344	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	96	84	286	0	3502	0	0	0
N.S.	1	0.95	0.83	2.83	0.00	34.67	0.00	0.00	0.00
time (sec)	N/A	0.327	0.853	1.817	0.000	0.355	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	81	78	252	0	2041	0	0	0
N.S.	1	0.98	0.94	3.04	0.00	24.59	0.00	0.00	0.00
time (sec)	N/A	0.256	0.340	4.077	0.000	0.295	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	64	63	210	125	1515	0	0	0
N.S.	1	0.97	0.95	3.18	1.89	22.95	0.00	0.00	0.00
time (sec)	N/A	0.248	1.028	14.326	0.326	0.296	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	70	69	183	0	1555	0	0	0
N.S.	1	0.97	0.96	2.54	0.00	21.60	0.00	0.00	0.00
time (sec)	N/A	0.261	0.193	35.269	0.000	0.286	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	75	83	259	127	1443	0	0	0
N.S.	1	0.97	1.08	3.36	1.65	18.74	0.00	0.00	0.00
time (sec)	N/A	0.258	1.020	73.200	0.342	0.284	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	107	203	273	0	2140	0	0	0
N.S.	1	1.05	1.99	2.68	0.00	20.98	0.00	0.00	0.00
time (sec)	N/A	0.306	0.641	136.479	0.000	0.345	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	92	102	306	209	2869	0	0	0
N.S.	1	0.95	1.05	3.15	2.15	29.58	0.00	0.00	0.00
time (sec)	N/A	0.314	1.387	258.305	0.393	0.315	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	163	265	351	0	6396	0	0	0
N.S.	1	1.05	1.71	2.26	0.00	41.26	0.00	0.00	0.00
time (sec)	N/A	0.399	1.672	0.304	0.000	0.373	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	238	164	501	1806	13887	0	0	0
N.S.	1	1.20	0.83	2.53	9.12	70.14	0.00	0.00	0.00
time (sec)	N/A	0.470	2.408	27.890	0.568	0.458	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	146	136	375	0	11392	0	0	0
N.S.	1	0.95	0.88	2.44	0.00	73.97	0.00	0.00	0.00
time (sec)	N/A	0.393	2.223	7.195	0.000	0.433	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	154	134	394	0	7909	0	0	0
N.S.	1	1.07	0.93	2.74	0.00	54.92	0.00	0.00	0.00
time (sec)	N/A	0.321	1.223	19.445	0.000	0.370	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	99	77	284	366	5840	0	0	0
N.S.	1	1.03	0.80	2.96	3.81	60.83	0.00	0.00	0.00
time (sec)	N/A	0.265	2.613	63.990	0.401	0.344	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	121	123	342	0	6614	0	0	0
N.S.	1	0.94	0.95	2.65	0.00	51.27	0.00	0.00	0.00
time (sec)	N/A	0.313	0.832	103.406	0.000	0.364	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	113	115	334	360	5659	0	0	0
N.S.	1	0.98	1.00	2.90	3.13	49.21	0.00	0.00	0.00
time (sec)	N/A	0.283	1.962	185.475	0.456	0.336	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	107	87	276	0	5077	0	0	0
N.S.	1	1.03	0.84	2.65	0.00	48.82	0.00	0.00	0.00
time (sec)	N/A	0.279	0.466	0.221	0.000	0.323	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	143	128	376	332	5233	0	0	0
N.S.	1	1.09	0.98	2.87	2.53	39.95	0.00	0.00	0.00
time (sec)	N/A	0.319	1.960	0.189	0.489	0.340	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	170	317	389	0	8070	0	0	0
N.S.	1	1.09	2.03	2.49	0.00	51.73	0.00	0.00	0.00
time (sec)	N/A	0.400	3.688	0.207	0.000	0.377	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	49	97	68	199	339	82	134	50
N.S.	1	0.91	1.80	1.26	3.69	6.28	1.52	2.48	0.93
time (sec)	N/A	0.351	0.055	0.079	0.209	0.272	0.142	0.316	1.863

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	59	43	71	168	1205	88	92	53
N.S.	1	1.20	0.88	1.45	3.43	24.59	1.80	1.88	1.08
time (sec)	N/A	0.362	0.290	0.073	0.286	0.268	0.116	0.327	0.110

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	65	45	105	160	54	86	34
N.S.	1	1.00	1.81	1.25	2.92	4.44	1.50	2.39	0.94
time (sec)	N/A	0.274	0.036	0.064	0.201	0.271	0.104	0.305	1.803

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	40	41	49	76	399	60	57	37
N.S.	1	1.29	1.32	1.58	2.45	12.87	1.94	1.84	1.19
time (sec)	N/A	0.273	0.038	0.084	0.276	0.270	0.091	0.287	1.795

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	28	22	31	37	20	29	18
N.S.	1	1.00	1.47	1.16	1.63	1.95	1.05	1.53	0.95
time (sec)	N/A	0.168	0.001	0.040	0.190	0.255	0.089	0.290	0.069

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	39	33	24	35	70	0	46	228
N.S.	1	1.56	1.32	0.96	1.40	2.80	0.00	1.84	9.12
time (sec)	N/A	0.274	0.055	0.232	0.198	0.264	0.000	0.286	1.854

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	32	27	31	38	90	30	25
N.S.	1	1.00	1.78	1.50	1.72	2.11	5.00	1.67	1.39
time (sec)	N/A	0.214	0.035	0.115	0.199	0.271	4.275	0.321	1.784

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	44	39	35	106	407	0	58	76
N.S.	1	1.42	1.26	1.13	3.42	13.13	0.00	1.87	2.45
time (sec)	N/A	0.280	0.131	0.245	0.197	0.272	0.000	0.310	1.818

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	61	39	105	156	0	86	162
N.S.	1	1.00	1.69	1.08	2.92	4.33	0.00	2.39	4.50
time (sec)	N/A	0.277	0.050	0.212	0.200	0.254	0.000	0.330	1.814

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	63	51	56	206	1216	0	93	177
N.S.	1	1.29	1.04	1.14	4.20	24.82	0.00	1.90	3.61
time (sec)	N/A	0.366	0.377	0.273	0.190	0.269	0.000	0.338	1.827

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	81	190	135	369	796	165	300	91
N.S.	1	0.98	2.29	1.63	4.45	9.59	1.99	3.61	1.10
time (sec)	N/A	0.301	0.101	0.118	0.205	0.259	0.211	0.380	0.167

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	66	130	333	3441	170	191	100
N.S.	1	1.00	0.87	1.71	4.38	45.28	2.24	2.51	1.32
time (sec)	N/A	0.303	0.500	0.096	0.279	0.302	0.191	0.376	0.214

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	65	137	97	231	483	117	218	67
N.S.	1	1.03	2.17	1.54	3.67	7.67	1.86	3.46	1.06
time (sec)	N/A	0.281	0.082	0.093	0.198	0.259	0.155	0.346	1.829

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	60	50	92	186	1638	122	116	76
N.S.	1	1.05	0.88	1.61	3.26	28.74	2.14	2.04	1.33
time (sec)	N/A	0.262	0.370	0.103	0.280	0.285	0.127	0.334	1.835

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	47	65	60	114	201	68	103	47
N.S.	1	1.09	1.51	1.40	2.65	4.67	1.58	2.40	1.09
time (sec)	N/A	0.230	0.894	0.061	0.184	0.274	0.112	0.296	1.841

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	54	48	59	104	668	0	141	210
N.S.	1	1.10	0.98	1.20	2.12	13.63	0.00	2.88	4.29
time (sec)	N/A	0.275	0.162	0.176	0.272	0.266	0.000	0.337	1.961

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	40	64	36	64	97	0	71	59
N.S.	1	1.11	1.78	1.00	1.78	2.69	0.00	1.97	1.64
time (sec)	N/A	0.270	0.122	0.254	0.197	0.269	0.000	0.346	1.826

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	55	50	62	134	677	0	141	211
N.S.	1	1.06	0.96	1.19	2.58	13.02	0.00	2.71	4.06
time (sec)	N/A	0.287	0.190	0.196	0.197	0.263	0.000	0.375	0.236

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	47	65	43	114	197	0	103	175
N.S.	1	1.09	1.51	1.00	2.65	4.58	0.00	2.40	4.07
time (sec)	N/A	0.276	0.543	0.166	0.209	0.244	0.000	0.368	0.155

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	58	77	236	1649	0	118	197
N.S.	1	1.00	0.81	1.07	3.28	22.90	0.00	1.64	2.74
time (sec)	N/A	0.307	0.472	0.219	0.186	0.274	0.000	0.417	1.919

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	65	98	58	231	473	0	218	529
N.S.	1	1.03	1.56	0.92	3.67	7.51	0.00	3.46	8.40
time (sec)	N/A	0.293	0.128	0.168	0.191	0.255	0.000	0.418	0.184

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	89	74	92	390	3454	0	192	362
N.S.	1	0.97	0.80	1.00	4.24	37.54	0.00	2.09	3.93
time (sec)	N/A	0.317	0.566	0.244	0.199	0.305	0.000	0.461	0.244

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	109	123	218	583	1563	260	534	138
N.S.	1	0.96	1.08	1.91	5.11	13.71	2.28	4.68	1.21
time (sec)	N/A	0.315	1.723	0.169	0.208	0.273	0.304	0.513	0.229

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	104	98	205	540	7502	279	309	155
N.S.	1	0.97	0.92	1.92	5.05	70.11	2.61	2.89	1.45
time (sec)	N/A	0.314	0.354	0.144	0.289	0.336	0.275	0.462	1.881

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	93	108	164	400	1036	192	418	106
N.S.	1	0.99	1.15	1.74	4.26	11.02	2.04	4.45	1.13
time (sec)	N/A	0.302	2.004	0.130	0.207	0.273	0.224	0.431	1.865

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	76	151	351	4298	211	216	123
N.S.	1	1.00	0.92	1.82	4.23	51.78	2.54	2.60	1.48
time (sec)	N/A	0.282	0.308	0.130	0.282	0.296	0.186	0.401	1.854

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	75	95	112	239	567	126	241	86
N.S.	1	1.01	1.28	1.51	3.23	7.66	1.70	3.26	1.16
time (sec)	N/A	0.253	0.717	0.094	0.196	0.273	0.158	0.306	0.159

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	74	67	80	214	2381	0	267	380
N.S.	1	1.03	0.93	1.11	2.97	33.07	0.00	3.71	5.28
time (sec)	N/A	0.288	0.620	0.208	0.276	0.295	0.000	0.390	0.364

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	60	81	59	147	341	0	135	218
N.S.	1	1.02	1.37	1.00	2.49	5.78	0.00	2.29	3.69
time (sec)	N/A	0.280	2.613	0.199	0.197	0.262	0.000	0.429	1.868

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	70	63	77	203	1686	0	274	327
N.S.	1	0.97	0.88	1.07	2.82	23.42	0.00	3.81	4.54
time (sec)	N/A	0.293	0.487	0.246	0.285	0.268	0.000	0.468	2.913

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	60	82	56	147	341	0	135	219
N.S.	1	1.02	1.39	0.95	2.49	5.78	0.00	2.29	3.71
time (sec)	N/A	0.278	1.412	0.244	0.209	0.268	0.000	0.492	1.839

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	67	90	264	2393	0	267	381
N.S.	1	1.00	0.81	1.08	3.18	28.83	0.00	3.22	4.59
time (sec)	N/A	0.310	0.648	0.245	0.212	0.294	0.000	0.523	0.360

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	76	100	72	239	557	0	241	568
N.S.	1	1.03	1.35	0.97	3.23	7.53	0.00	3.26	7.68
time (sec)	N/A	0.302	1.620	0.216	0.205	0.265	0.000	0.567	1.893

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	100	76	106	420	4305	0	217	380
N.S.	1	0.97	0.74	1.03	4.08	41.80	0.00	2.11	3.69
time (sec)	N/A	0.326	0.266	0.270	0.208	0.311	0.000	0.619	0.274

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	108	128	179	410	1176	209	447	133
N.S.	1	0.98	1.16	1.63	3.73	10.69	1.90	4.06	1.21
time (sec)	N/A	0.295	1.195	0.134	0.207	0.279	0.228	0.309	0.184

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	155	170	262	624	2133	308	721	188
N.S.	1	0.97	1.06	1.64	3.90	13.33	1.92	4.51	1.18
time (sec)	N/A	0.326	2.720	0.206	0.212	0.297	0.319	0.359	1.950

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	60	79	133	742	415	132	72
N.S.	1	1.00	0.91	1.20	2.02	11.24	6.29	2.00	1.09
time (sec)	N/A	0.312	0.212	0.109	0.277	0.301	10.068	0.346	0.245

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	69	66	87	509	777	428	87	56
N.S.	1	1.17	1.12	1.47	8.63	13.17	7.25	1.47	0.95
time (sec)	N/A	0.300	0.242	0.136	0.389	0.289	4.762	0.325	1.864

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	52	44	49	82	118	306	96	46
N.S.	1	1.13	0.96	1.07	1.78	2.57	6.65	2.09	1.00
time (sec)	N/A	0.311	0.052	0.076	0.278	0.298	2.859	0.351	1.830

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	54	47	72	215	486	253	65	38
N.S.	1	1.17	1.02	1.57	4.67	10.57	5.50	1.41	0.83
time (sec)	N/A	0.268	0.039	0.120	0.302	0.288	2.081	0.313	0.099

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	47	35	44	58	82	146	61	43
N.S.	1	1.12	0.83	1.05	1.38	1.95	3.48	1.45	1.02
time (sec)	N/A	0.254	0.033	0.061	0.200	0.265	1.932	0.307	1.815

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	65	71	57	484	240	63	37
N.S.	1	1.00	1.44	1.58	1.27	10.76	5.33	1.40	0.82
time (sec)	N/A	0.320	0.108	0.104	0.278	0.283	1.999	0.296	0.085

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	65	55	65	101	118	0	97	194
N.S.	1	1.08	0.92	1.08	1.68	1.97	0.00	1.62	3.23
time (sec)	N/A	0.302	0.082	0.162	0.200	0.307	0.000	0.310	2.085

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	71	67	89	329	784	0	89	402
N.S.	1	1.18	1.12	1.48	5.48	13.07	0.00	1.48	6.70
time (sec)	N/A	0.298	0.234	0.196	0.347	0.307	0.000	0.323	2.209

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	82	60	88	159	747	0	133	313
N.S.	1	0.96	0.71	1.04	1.87	8.79	0.00	1.56	3.68
time (sec)	N/A	0.329	0.212	0.204	0.217	0.340	0.000	0.346	2.244

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	96	91	109	1038	2368	0	147	519
N.S.	1	1.17	1.11	1.33	12.66	28.88	0.00	1.79	6.33
time (sec)	N/A	0.363	0.748	0.228	0.419	0.308	0.000	0.352	2.206

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	84	69	91	217	1141	102	194	170
N.S.	1	1.01	0.83	1.10	2.61	13.75	1.23	2.34	2.05
time (sec)	N/A	0.331	0.557	0.114	0.299	0.329	74.908	0.430	2.317

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	105	90	104	1010	1950	0	195	1655
N.S.	1	1.18	1.01	1.17	11.35	21.91	0.00	2.19	18.60
time (sec)	N/A	0.305	0.616	0.192	0.522	0.316	0.000	0.380	2.440

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	71	57	84	170	629	92	149	210
N.S.	1	0.99	0.79	1.17	2.36	8.74	1.28	2.07	2.92
time (sec)	N/A	0.312	0.500	0.106	0.216	0.291	42.147	0.364	0.399

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	99	86	100	614	2025	0	177	106
N.S.	1	1.16	1.01	1.18	7.22	23.82	0.00	2.08	1.25
time (sec)	N/A	0.314	0.445	0.183	0.424	0.311	0.000	0.357	0.606

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	55	86	170	623	92	149	129
N.S.	1	1.00	0.81	1.26	2.50	9.16	1.35	2.19	1.90
time (sec)	N/A	0.290	0.447	0.102	0.218	0.284	58.576	0.339	2.048

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	106	97	103	206	1942	0	195	110
N.S.	1	1.19	1.09	1.16	2.31	21.82	0.00	2.19	1.24
time (sec)	N/A	0.283	0.587	0.183	0.319	0.316	0.000	0.304	2.144

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	94	83	110	235	1148	0	195	0
N.S.	1	0.99	0.87	1.16	2.47	12.08	0.00	2.05	0.00
time (sec)	N/A	0.335	2.130	0.254	0.217	0.356	0.000	0.359	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	134	111	120	976	3725	0	336	0
N.S.	1	1.13	0.93	1.01	8.20	31.30	0.00	2.82	0.00
time (sec)	N/A	0.382	1.851	0.272	0.434	0.337	0.000	0.369	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	116	93	131	402	3468	0	323	0
N.S.	1	0.94	0.75	1.06	3.24	27.97	0.00	2.60	0.00
time (sec)	N/A	0.358	1.023	0.332	0.223	0.451	0.000	0.395	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	169	139	142	2345	8482	0	281	0
N.S.	1	1.06	0.87	0.89	14.75	53.35	0.00	1.77	0.00
time (sec)	N/A	0.460	1.637	0.344	0.677	0.379	0.000	0.410	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	173	144	157	3354	7528	0	408	2669
N.S.	1	1.20	1.00	1.09	23.29	52.28	0.00	2.83	18.53
time (sec)	N/A	0.405	1.341	0.304	1.236	0.395	0.000	0.524	2.553

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	105	91	127	376	2584	0	245	416
N.S.	1	0.96	0.83	1.17	3.45	23.71	0.00	2.25	3.82
time (sec)	N/A	0.352	1.220	0.154	0.238	0.304	0.000	0.472	0.745

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	157	135	148	2432	7757	0	384	2574
N.S.	1	1.15	0.99	1.08	17.75	56.62	0.00	2.80	18.79
time (sec)	N/A	0.371	1.270	0.299	0.947	0.399	0.000	0.460	2.373

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	95	80	115	384	2611	0	245	397
N.S.	1	0.97	0.82	1.17	3.92	26.64	0.00	2.50	4.05
time (sec)	N/A	0.332	0.514	0.168	0.235	0.310	0.000	0.440	2.454

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	162	137	152	1472	7791	0	388	255
N.S.	1	1.18	1.00	1.11	10.74	56.87	0.00	2.83	1.86
time (sec)	N/A	0.349	1.257	0.302	0.599	0.374	0.000	0.424	3.907

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	91	77	116	378	2554	0	245	235
N.S.	1	0.97	0.82	1.23	4.02	27.17	0.00	2.61	2.50
time (sec)	N/A	0.305	0.602	0.146	0.242	0.315	0.000	0.393	2.754

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	170	147	156	507	7496	0	409	260
N.S.	1	1.20	1.04	1.10	3.57	52.79	0.00	2.88	1.83
time (sec)	N/A	0.349	0.345	0.264	0.380	0.366	0.000	0.331	0.809

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	134	117	161	498	4800	0	295	0
N.S.	1	0.97	0.85	1.17	3.61	34.78	0.00	2.14	0.00
time (sec)	N/A	0.372	1.773	0.526	0.229	0.518	0.000	0.438	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	203	166	167	1944	11865	0	437	0
N.S.	1	1.14	0.93	0.94	10.92	66.66	0.00	2.46	0.00
time (sec)	N/A	0.485	6.803	0.417	0.651	0.443	0.000	0.462	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	160	138	184	770	10720	0	474	0
N.S.	1	0.94	0.81	1.08	4.50	62.69	0.00	2.77	0.00
time (sec)	N/A	0.418	2.083	0.688	0.266	0.706	0.000	0.538	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	248	194	189	4285	22038	0	493	0
N.S.	1	1.09	0.85	0.83	18.79	96.66	0.00	2.16	0.00
time (sec)	N/A	0.556	3.911	0.527	1.076	0.554	0.000	0.562	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	239	203	219	925	20020	0	750	3685
N.S.	1	1.19	1.01	1.09	4.60	99.60	0.00	3.73	18.33
time (sec)	N/A	0.436	0.684	0.447	0.499	0.513	0.000	0.363	1.258

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	14	4	5	8	0	5	3
N.S.	1	1.00	4.67	1.33	1.67	2.67	0.00	1.67	1.00
time (sec)	N/A	0.233	0.010	0.136	0.338	0.238	0.000	0.259	1.773

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	5	96	0	1	14
N.S.	1	1.00	1.00	0.94	0.31	6.00	0.00	0.06	0.88
time (sec)	N/A	0.244	0.009	0.086	0.313	0.255	0.000	0.276	0.222

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	26	23	21	28	140	0	45	20
N.S.	1	1.18	1.05	0.95	1.27	6.36	0.00	2.05	0.91
time (sec)	N/A	0.232	0.023	0.062	0.312	0.258	0.000	0.269	0.086

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	23	28	32	385	0	41	27
N.S.	1	1.00	0.66	0.80	0.91	11.00	0.00	1.17	0.77
time (sec)	N/A	0.247	0.023	0.083	0.314	0.303	0.000	0.281	1.819

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	15	11	14	11	2	0	11	2
N.S.	1	1.36	1.00	1.27	1.00	0.18	0.00	1.00	0.18
time (sec)	N/A	0.238	0.037	0.078	0.283	0.262	0.000	0.267	0.132

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	25	36	12	21	14
N.S.	1	1.00	1.00	0.92	1.92	2.77	0.92	1.62	1.08
time (sec)	N/A	0.227	0.035	0.044	0.279	0.251	0.224	0.267	1.848

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	92	85	288	0	4529	126	980	119
N.S.	1	1.06	0.98	3.31	0.00	52.06	1.45	11.26	1.37
time (sec)	N/A	0.331	0.579	0.175	0.000	0.493	2.355	1.132	9.060

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	133	580	337	0	9360	0	938	0
N.S.	1	1.10	4.79	2.79	0.00	77.36	0.00	7.75	0.00
time (sec)	N/A	0.395	6.212	0.104	0.000	0.577	0.000	0.986	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	68	60	253	0	2329	94	630	66
N.S.	1	1.08	0.95	4.02	0.00	36.97	1.49	10.00	1.05
time (sec)	N/A	0.289	0.189	0.087	0.000	0.343	1.587	0.769	3.682

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	89	193	276	0	4825	0	554	0
N.S.	1	1.05	2.27	3.25	0.00	56.76	0.00	6.52	0.00
time (sec)	N/A	0.318	3.739	0.088	0.000	0.402	0.000	0.737	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	49	44	238	0	1543	68	349	51
N.S.	1	1.11	1.00	5.41	0.00	35.07	1.55	7.93	1.16
time (sec)	N/A	0.261	0.038	0.072	0.000	0.298	0.985	0.565	2.227

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	81	238	0	3443	0	253	0
N.S.	1	1.00	1.35	3.97	0.00	57.38	0.00	4.22	0.00
time (sec)	N/A	0.234	0.370	0.109	0.000	0.357	0.000	0.494	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	61	56	0	0	3467	0	0	0
N.S.	1	1.09	1.00	0.00	0.00	61.91	0.00	0.00	0.00
time (sec)	N/A	0.301	0.038	0.000	0.000	0.358	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	42	0	0	1539	0	348	0
N.S.	1	1.00	0.88	0.00	0.00	32.06	0.00	7.25	0.00
time (sec)	N/A	0.283	0.141	0.000	0.000	0.303	0.000	0.557	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	87	83	0	0	4891	0	557	0
N.S.	1	1.05	1.00	0.00	0.00	58.93	0.00	6.71	0.00
time (sec)	N/A	0.325	0.243	0.000	0.000	0.406	0.000	0.717	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	B	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	78	82	161	0	0	2355	0	629	0
N.S.	1	1.05	2.06	0.00	0.00	30.19	0.00	8.06	0.00
time (sec)	N/A	0.332	2.162	0.000	0.000	0.354	0.000	0.831	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	134	111	0	0	9642	0	947	0
N.S.	1	1.11	0.92	0.00	0.00	79.69	0.00	7.83	0.00
time (sec)	N/A	0.372	0.737	0.000	0.000	0.573	0.000	1.008	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	89	86	488	0	4941	223	1063	112
N.S.	1	1.09	1.05	5.95	0.00	60.26	2.72	12.96	1.37
time (sec)	N/A	0.321	0.472	0.079	0.000	0.493	10.257	1.457	9.969

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	132	584	529	0	10046	0	949	0
N.S.	1	1.07	4.75	4.30	0.00	81.67	0.00	7.72	0.00
time (sec)	N/A	0.425	6.252	0.083	0.000	0.587	0.000	1.207	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	70	59	473	0	2385	165	662	64
N.S.	1	1.11	0.94	7.51	0.00	37.86	2.62	10.51	1.02
time (sec)	N/A	0.275	0.190	0.058	0.000	0.372	6.974	1.018	4.103

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	92	110	473	0	4841	0	584	0
N.S.	1	1.05	1.25	5.38	0.00	55.01	0.00	6.64	0.00
time (sec)	N/A	0.295	0.729	0.101	0.000	0.428	0.000	0.806	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	76	71	0	0	4039	0	0	0
N.S.	1	1.07	1.00	0.00	0.00	56.89	0.00	0.00	0.00
time (sec)	N/A	0.326	0.081	0.000	0.000	0.416	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	197	0	0	3913	0	430	0
N.S.	1	1.00	2.56	0.00	0.00	50.82	0.00	5.58	0.00
time (sec)	N/A	0.331	3.360	0.000	0.000	0.403	0.000	0.957	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	51	97	0	679	0	104	68
N.S.	1	1.00	1.65	3.13	0.00	21.90	0.00	3.35	2.19
time (sec)	N/A	0.209	0.096	0.194	0.000	0.268	0.000	0.279	0.194

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	53	142	0	226	0	104	43
N.S.	1	1.00	1.18	3.16	0.00	5.02	0.00	2.31	0.96
time (sec)	N/A	0.220	0.057	0.160	0.000	0.245	0.000	0.290	1.938

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	53	74	158	0	1027	0	202	78
N.S.	1	1.06	1.48	3.16	0.00	20.54	0.00	4.04	1.56
time (sec)	N/A	0.226	0.180	0.081	0.000	0.272	0.000	0.278	0.255

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	70	76	211	0	361	0	204	0
N.S.	1	1.04	1.13	3.15	0.00	5.39	0.00	3.04	0.00
time (sec)	N/A	0.241	0.092	0.075	0.000	0.263	0.000	0.281	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	76	68	164	0	2827	0	592	65
N.S.	1	1.09	0.97	2.34	0.00	40.39	0.00	8.46	0.93
time (sec)	N/A	0.316	0.573	0.117	0.000	0.426	0.000	0.667	2.676

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	95	208	178	0	5494	0	559	0
N.S.	1	1.08	2.36	2.02	0.00	62.43	0.00	6.35	0.00
time (sec)	N/A	0.313	5.081	0.113	0.000	0.484	0.000	0.567	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	52	47	129	0	1625	0	345	39
N.S.	1	1.11	1.00	2.74	0.00	34.57	0.00	7.34	0.83
time (sec)	N/A	0.277	0.127	0.091	0.000	0.382	0.000	0.492	2.208

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	101	137	0	3361	0	252	0
N.S.	1	1.00	1.68	2.28	0.00	56.02	0.00	4.20	0.00
time (sec)	N/A	0.287	0.448	0.100	0.000	0.439	0.000	0.523	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	114	0	1361	68	188	23
N.S.	1	1.00	1.00	3.93	0.00	46.93	2.34	6.48	0.79
time (sec)	N/A	0.244	0.021	0.129	0.000	0.307	1.044	0.408	2.182

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	114	0	1287	0	188	25
N.S.	1	1.00	1.00	3.68	0.00	41.52	0.00	6.06	0.81
time (sec)	N/A	0.204	0.029	0.115	0.000	0.301	0.000	0.391	2.122

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	61	56	0	0	3527	0	254	0
N.S.	1	1.09	1.00	0.00	0.00	62.98	0.00	4.54	0.00
time (sec)	N/A	0.285	0.056	0.000	0.000	0.382	0.000	0.515	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	B	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	130	0	0	1565	0	343	0
N.S.	1	1.00	2.55	0.00	0.00	30.69	0.00	6.73	0.00
time (sec)	N/A	0.285	0.854	0.000	0.000	0.321	0.000	0.484	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	96	107	0	0	5711	0	565	0
N.S.	1	1.09	1.22	0.00	0.00	64.90	0.00	6.42	0.00
time (sec)	N/A	0.336	0.465	0.000	0.000	0.494	0.000	0.615	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	77	67	322	0	3991	0	460	70
N.S.	1	1.07	0.93	4.47	0.00	55.43	0.00	6.39	0.97
time (sec)	N/A	0.364	0.130	0.101	0.000	0.493	0.000	0.571	3.042

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	99	188	328	0	6973	0	371	0
N.S.	1	1.18	2.24	3.90	0.00	83.01	0.00	4.42	0.00
time (sec)	N/A	0.350	2.971	0.096	0.000	0.540	0.000	0.566	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	58	52	287	0	2525	0	287	45
N.S.	1	1.12	1.00	5.52	0.00	48.56	0.00	5.52	0.87
time (sec)	N/A	0.295	0.147	0.079	0.000	0.348	0.000	0.443	2.778

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	112	289	0	2281	0	293	0
N.S.	1	1.00	2.11	5.45	0.00	43.04	0.00	5.53	0.00
time (sec)	N/A	0.295	1.690	0.078	0.000	0.351	0.000	0.458	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	54	41	273	0	2277	92	292	41
N.S.	1	1.10	0.84	5.57	0.00	46.47	1.88	5.96	0.84
time (sec)	N/A	0.269	0.041	0.068	0.000	0.356	10.708	0.449	2.509

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	223	272	0	2509	0	288	0
N.S.	1	1.00	3.98	4.86	0.00	44.80	0.00	5.14	0.00
time (sec)	N/A	0.244	4.778	0.090	0.000	0.357	0.000	0.438	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	98	70	0	0	6955	0	372	0
N.S.	1	1.26	0.90	0.00	0.00	89.17	0.00	4.77	0.00
time (sec)	N/A	0.337	0.098	0.000	0.000	0.543	0.000	0.512	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	B	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	85	91	263	0	0	3929	0	459	0
N.S.	1	1.07	3.09	0.00	0.00	46.22	0.00	5.40	0.00
time (sec)	N/A	0.361	8.093	0.000	0.000	0.492	0.000	0.569	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	148	231	549	0	19265	0	805	0
N.S.	1	1.25	1.96	4.65	0.00	163.26	0.00	6.82	0.00
time (sec)	N/A	0.422	2.145	0.110	0.000	1.230	0.000	0.637	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	90	68	469	0	7033	0	711	92
N.S.	1	1.07	0.81	5.58	0.00	83.73	0.00	8.46	1.10
time (sec)	N/A	0.357	0.125	0.098	0.000	0.675	0.000	0.526	4.406

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	99	132	491	0	5719	0	684	0
N.S.	1	1.10	1.47	5.46	0.00	63.54	0.00	7.60	0.00
time (sec)	N/A	0.330	2.840	0.103	0.000	0.635	0.000	0.527	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	86	63	435	0	6621	0	725	82
N.S.	1	1.16	0.85	5.88	0.00	89.47	0.00	9.80	1.11
time (sec)	N/A	0.313	0.096	0.085	0.000	0.665	0.000	0.533	4.162

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	88	97	216	454	0	6507	0	728	0
N.S.	1	1.10	2.45	5.16	0.00	73.94	0.00	8.27	0.00
time (sec)	N/A	0.336	7.114	0.077	0.000	0.664	0.000	0.524	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	82	43	420	0	5779	114	683	76
N.S.	1	1.17	0.61	6.00	0.00	82.56	1.63	9.76	1.09
time (sec)	N/A	0.284	0.052	0.054	0.000	0.637	12.557	0.510	4.280

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	93	106	976	420	0	6933	0	714	0
N.S.	1	1.14	10.49	4.52	0.00	74.55	0.00	7.68	0.00
time (sec)	N/A	0.284	7.819	0.098	0.000	0.681	0.000	0.524	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	142	73	0	0	19305	0	808	0
N.S.	1	1.31	0.68	0.00	0.00	178.75	0.00	7.48	0.00
time (sec)	N/A	0.386	0.101	0.000	0.000	1.280	0.000	0.650	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	B	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	131	148	1375	0	0	10671	0	898	0
N.S.	1	1.13	10.50	0.00	0.00	81.46	0.00	6.85	0.00
time (sec)	N/A	0.441	8.660	0.000	0.000	1.342	0.000	0.744	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	35	62	0	543	0	58	63
N.S.	1	1.00	1.40	2.48	0.00	21.72	0.00	2.32	2.52
time (sec)	N/A	0.195	0.027	0.169	0.000	0.259	0.000	0.274	0.185

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	37	66	0	175	0	58	22
N.S.	1	1.00	1.37	2.44	0.00	6.48	0.00	2.15	0.81
time (sec)	N/A	0.202	0.025	0.167	0.000	0.250	0.000	0.282	1.867

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	90	95	91	194	2074	100	142	91
N.S.	1	1.01	1.07	1.02	2.18	23.30	1.12	1.60	1.02
time (sec)	N/A	0.292	0.750	0.088	0.273	0.289	0.147	0.298	1.869

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	40	40	41	73	95	102	25	38
N.S.	1	1.05	1.05	1.08	1.92	2.50	2.68	0.66	1.00
time (sec)	N/A	0.317	0.085	0.084	0.285	0.265	0.254	0.272	0.107

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	132	166	620	0	11528	0	0	0
N.S.	1	1.06	1.34	5.00	0.00	92.97	0.00	0.00	0.00
time (sec)	N/A	0.399	4.547	0.898	0.000	0.547	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	90	86	116	0	5136	0	0	0
N.S.	1	1.01	0.97	1.30	0.00	57.71	0.00	0.00	0.00
time (sec)	N/A	0.328	0.078	0.714	0.000	0.435	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	43	40	37	0	1286	0	0	0
N.S.	1	1.08	1.00	0.92	0.00	32.15	0.00	0.00	0.00
time (sec)	N/A	0.261	0.022	0.562	0.000	0.406	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	77	73	431	0	3914	0	0	0
N.S.	1	1.04	0.99	5.82	0.00	52.89	0.00	0.00	0.00
time (sec)	N/A	0.304	0.543	0.681	0.000	0.482	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	135	113	637	0	16463	0	0	0
N.S.	1	1.14	0.96	5.40	0.00	139.52	0.00	0.00	0.00
time (sec)	N/A	0.375	0.877	0.711	0.000	2.040	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [259] had the largest ratio of [.933332999999999968]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	7	1.32	21	0.333
2	A	7	6	0.89	21	0.286
3	A	7	6	1.36	21	0.286
4	A	6	5	1.04	19	0.263
5	A	7	6	0.92	19	0.316
6	A	6	5	0.92	21	0.238
7	A	7	6	1.16	21	0.286
8	A	5	4	0.89	21	0.190
9	A	8	7	1.16	23	0.304
10	A	7	6	0.90	23	0.261
11	A	8	7	1.04	23	0.304
12	A	6	5	0.94	21	0.238
13	A	7	6	0.90	21	0.286
14	A	6	5	0.89	23	0.217
15	A	8	7	0.96	23	0.304
16	A	5	4	0.89	23	0.174
17	A	9	8	1.09	23	0.348
18	A	7	6	0.90	23	0.261
19	A	11	10	1.27	23	0.435
20	A	6	5	0.90	21	0.238
21	A	7	6	0.90	21	0.286
22	A	6	5	0.88	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	11	10	1.06	23	0.435
24	A	5	4	0.89	23	0.174
25	A	9	8	1.28	23	0.348
26	A	8	7	1.01	23	0.304
27	A	8	7	1.24	23	0.304
28	A	6	5	0.98	21	0.238
29	A	8	7	0.96	21	0.333
30	A	6	5	0.96	23	0.217
31	A	8	7	1.11	23	0.304
32	A	6	5	1.00	23	0.217
33	A	11	10	1.16	23	0.435
34	A	8	7	1.02	23	0.304
35	A	10	9	1.15	23	0.391
36	A	7	6	1.01	21	0.286
37	A	10	9	1.09	21	0.429
38	A	7	6	0.99	23	0.261
39	A	10	9	1.06	23	0.391
40	A	7	6	1.02	23	0.261
41	A	13	12	1.14	23	0.522
42	A	9	8	1.01	23	0.348
43	A	12	11	1.19	23	0.478
44	A	8	7	1.06	21	0.333
45	A	12	11	1.13	21	0.524
46	A	8	7	1.04	23	0.304
47	A	12	11	1.13	23	0.478
48	A	8	7	1.02	23	0.304
49	A	9	8	1.01	21	0.381
50	C	4	4	1.16	21	0.190
51	A	8	7	0.90	21	0.333
52	C	4	4	1.21	19	0.211
53	C	4	4	1.20	19	0.211
54	A	6	5	0.93	21	0.238
55	C	4	4	1.17	21	0.190
56	A	5	4	0.86	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	9	8	1.16	23	0.348
58	C	4	4	1.14	23	0.174
59	A	8	7	1.15	23	0.304
60	C	4	4	1.18	21	0.190
61	C	4	4	1.17	21	0.190
62	A	6	5	0.89	23	0.217
63	C	4	4	1.17	23	0.174
64	A	5	4	0.86	23	0.174
65	A	9	8	1.03	23	0.348
66	C	4	4	1.12	23	0.174
67	A	8	7	1.01	23	0.304
68	C	4	4	1.12	21	0.190
69	C	4	4	1.12	21	0.190
70	A	6	5	0.89	23	0.217
71	C	4	4	1.11	23	0.174
72	A	5	4	0.86	23	0.174
73	A	5	4	0.95	23	0.174
74	N/A	3	0	1.00	23	0.000
75	A	6	5	0.95	23	0.217
76	N/A	3	0	1.00	21	0.000
77	N/A	3	0	1.00	21	0.000
78	A	13	12	1.01	23	0.522
79	N/A	3	0	1.00	23	0.000
80	A	5	4	0.92	23	0.174
81	A	6	5	1.25	21	0.238
82	A	4	3	0.93	21	0.143
83	A	5	4	1.45	21	0.190
84	A	5	4	0.93	19	0.211
85	A	5	4	1.10	19	0.211
86	A	4	3	0.93	21	0.143
87	A	6	5	1.08	21	0.238
88	A	5	4	0.90	21	0.190
89	A	7	6	1.27	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
90	A	5	4	0.91	23	0.174
91	A	5	4	1.24	23	0.174
92	A	5	4	1.02	21	0.190
93	A	6	5	1.11	21	0.238
94	A	5	4	0.90	23	0.174
95	A	7	6	1.02	23	0.261
96	A	5	4	0.89	23	0.174
97	A	5	4	1.16	23	0.174
98	A	5	4	0.98	23	0.174
99	A	5	4	1.12	23	0.174
100	A	5	4	1.02	21	0.190
101	A	8	7	1.11	21	0.333
102	A	5	4	0.88	23	0.174
103	A	9	8	0.99	23	0.348
104	A	5	4	0.89	23	0.174
105	A	8	7	1.28	23	0.304
106	A	5	4	0.94	23	0.174
107	A	7	6	1.26	23	0.261
108	A	5	4	0.96	21	0.190
109	A	4	3	1.00	21	0.143
110	A	4	3	1.00	23	0.130
111	A	6	5	0.96	23	0.217
112	A	5	4	0.96	23	0.174
113	A	7	6	1.09	23	0.261
114	A	5	4	0.93	23	0.174
115	A	5	4	0.94	23	0.174
116	A	9	8	1.16	23	0.348
117	A	5	4	0.95	21	0.190
118	A	5	4	0.98	21	0.190
119	A	5	4	0.97	23	0.174
120	A	5	4	0.97	23	0.174
121	A	5	4	0.97	23	0.174
122	A	7	6	1.05	23	0.261
123	A	5	4	0.95	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
124	A	9	8	1.05	23	0.348
125	A	11	10	1.20	23	0.435
126	A	5	4	0.95	21	0.190
127	A	6	5	1.07	21	0.238
128	A	6	5	1.03	23	0.217
129	A	6	5	0.94	23	0.217
130	A	6	5	0.98	23	0.217
131	A	6	5	1.03	23	0.217
132	A	7	6	1.09	23	0.261
133	A	8	7	1.09	23	0.304
134	A	9	9	0.91	21	0.429
135	C	11	11	1.20	21	0.524
136	A	8	8	1.00	21	0.381
137	C	7	7	1.29	19	0.368
138	A	1	1	1.00	12	0.083
139	C	7	7	1.56	19	0.368
140	A	4	4	1.00	21	0.190
141	C	8	8	1.42	21	0.381
142	A	7	7	1.00	21	0.333
143	C	12	12	1.29	21	0.571
144	A	5	4	0.98	23	0.174
145	A	8	7	1.00	23	0.304
146	A	7	6	1.03	23	0.261
147	A	8	7	1.05	21	0.333
148	A	5	4	1.09	14	0.286
149	A	8	7	1.10	21	0.333
150	A	7	6	1.11	23	0.261
151	A	8	7	1.06	23	0.304
152	A	5	4	1.09	23	0.174
153	A	8	7	1.00	23	0.304
154	A	7	6	1.03	23	0.261
155	A	8	7	0.97	23	0.304
156	A	5	4	0.96	23	0.174
157	A	8	7	0.97	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
158	A	7	6	0.99	23	0.261
159	A	8	7	1.00	21	0.333
160	A	5	4	1.01	14	0.286
161	A	8	7	1.03	21	0.333
162	A	7	6	1.02	23	0.261
163	A	8	7	0.97	23	0.304
164	A	5	4	1.02	23	0.174
165	A	8	7	1.00	23	0.304
166	A	7	6	1.03	23	0.261
167	A	8	7	0.97	23	0.304
168	A	5	4	0.98	14	0.286
169	A	5	4	0.97	14	0.286
170	A	8	7	1.00	23	0.304
171	A	7	6	1.17	23	0.261
172	A	8	7	1.13	23	0.304
173	A	8	7	1.17	23	0.304
174	A	8	7	1.12	21	0.333
175	A	6	5	1.00	14	0.357
176	A	8	7	1.08	21	0.333
177	A	9	8	1.18	23	0.348
178	A	8	7	0.96	23	0.304
179	A	10	9	1.17	23	0.391
180	A	8	7	1.01	23	0.304
181	A	7	6	1.18	23	0.261
182	A	8	7	0.99	23	0.304
183	A	9	8	1.16	23	0.348
184	A	8	7	1.00	21	0.333
185	A	7	6	1.19	14	0.429
186	A	8	7	0.99	21	0.333
187	A	12	11	1.13	23	0.478
188	A	8	7	0.94	23	0.304
189	A	12	11	1.06	23	0.478
190	A	10	9	1.20	23	0.391
191	A	8	7	0.96	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
192	A	10	9	1.15	23	0.391
193	A	8	7	0.97	23	0.304
194	A	11	10	1.18	23	0.435
195	A	8	7	0.97	21	0.333
196	A	8	7	1.20	14	0.500
197	A	8	7	0.97	21	0.333
198	A	14	13	1.14	23	0.565
199	A	8	7	0.94	23	0.304
200	A	14	13	1.09	23	0.565
201	A	11	10	1.19	14	0.714
202	A	6	5	1.00	12	0.417
203	A	7	6	1.00	10	0.600
204	A	7	6	1.18	12	0.500
205	A	8	7	1.00	10	0.700
206	A	6	5	1.36	12	0.417
207	A	6	5	1.00	10	0.500
208	A	8	7	1.06	17	0.412
209	A	10	9	1.10	17	0.529
210	A	10	9	1.08	17	0.529
211	A	11	10	1.05	17	0.588
212	A	9	8	1.11	15	0.533
213	A	8	7	1.00	12	0.583
214	A	9	8	1.09	15	0.533
215	A	9	8	1.00	17	0.471
216	A	11	10	1.05	17	0.588
217	A	8	7	1.05	17	0.412
218	A	13	12	1.11	17	0.706
219	A	11	10	1.09	17	0.588
220	A	14	13	1.07	17	0.765
221	A	10	9	1.11	15	0.600
222	A	10	9	1.05	12	0.750
223	A	11	10	1.07	15	0.667
224	A	11	10	1.00	17	0.588
225	A	7	6	1.00	10	0.600

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
226	A	8	7	1.00	12	0.583
227	A	9	8	1.06	10	0.800
228	A	10	9	1.04	12	0.750
229	A	8	7	1.09	17	0.412
230	A	9	8	1.08	17	0.471
231	A	9	8	1.11	17	0.471
232	A	10	9	1.00	17	0.529
233	A	8	7	1.00	15	0.467
234	A	5	4	1.00	12	0.333
235	A	9	8	1.09	15	0.533
236	A	9	8	1.00	17	0.471
237	A	11	10	1.09	17	0.588
238	A	8	7	1.07	17	0.412
239	A	9	8	1.18	17	0.471
240	A	9	8	1.12	17	0.471
241	A	8	7	1.00	17	0.412
242	A	9	8	1.10	15	0.533
243	A	6	5	1.00	12	0.417
244	A	11	10	1.26	15	0.667
245	A	12	11	1.07	17	0.647
246	A	13	12	1.25	17	0.706
247	A	8	7	1.07	17	0.412
248	A	8	7	1.10	17	0.412
249	A	10	9	1.16	17	0.529
250	A	10	9	1.10	17	0.529
251	A	10	9	1.17	15	0.600
252	A	8	7	1.14	12	0.583
253	A	13	12	1.31	15	0.800
254	A	13	12	1.13	17	0.706
255	A	5	4	1.00	10	0.400
256	A	5	4	1.00	12	0.333
257	A	5	4	1.01	14	0.286
258	A	5	4	1.05	8	0.500
259	A	15	14	1.06	15	0.933

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
260	A	13	12	1.01	15	0.800
261	A	8	7	1.08	15	0.467
262	A	11	10	1.04	15	0.667
263	A	12	11	1.14	15	0.733

CHAPTER 3

Listing of integrals

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3.251 $\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$ 1850
3.252 $\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$ 1858
3.253 $\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$ 1865
3.254 $\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$ 1873
3.255 $\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx$ 1881
3.256 $\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx$ 1887
3.257 $\int \left(a+b\tanh^3(c+dx)\right)^2\,dx$ 1892
3.258 $\int \frac{1}{1+\tanh^3(x)}\,dx$ 1898
3.259 $\int \tanh(x)\left(a+b\tanh^4(x)\right)^{3/2}\,dx$ 1903
3.260 $\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx$ 1911
3.261 $\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx$ 1918
3.262 $\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}\,dx$ 1924
3.263 $\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx$ 1931

3.1 $\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

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3.1.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{3}{8}(a+5b)x - \frac{(5a+9b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)}{4d} - \frac{b\tanh(c+dx)}{d}$$

output
```
3/8*(a+5*b)*x-1/8*(5*a+9*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*(a+b)*cosh(d*x+c
)^3*sinh(d*x+c)/d-b*tanh(d*x+c)/d
```

3.1.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{12(a+5b)(c+dx) - 8(a+2b)\sinh(2(c+dx)) + (a+b)\sinh(4(c+dx)) - 32b\tanh(c+dx)}{32d}$$

input
```
Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(12*(a + 5*b)*(c + d*x) - 8*(a + 2*b)*Sinh[2*(c + d*x)] + (a + b)*Sinh[4*(
c + d*x)] - 32*b*Tanh[c + d*x])/(32*d)
```

3.1.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4146, 360, 25, 1471, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \sin(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)dx$$

$$\downarrow 4146$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(b\tanh^2(c+dx)+a\right)}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\downarrow 360$$

$$\frac{\frac{1}{4}\int -\frac{4b\tanh^4(c+dx)+4(a+b)\tanh^2(c+dx)+a+b}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)+\frac{(a+b)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{(a+b)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{1}{4}\int \frac{4b\tanh^4(c+dx)+4(a+b)\tanh^2(c+dx)+a+b}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 1471$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{8b\tanh^2(c+dx)+3a+7b}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{(5a+9b)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{(a+b)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 299$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(3(a+5b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)-8b\tanh(c+dx)\right)-\frac{(5a+9b)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{(a+b)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 219$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}(3(a+5b)\operatorname{arctanh}(\tanh(c+dx))-8b\tanh(c+dx))-\frac{(5a+9b)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{(a+b)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

input
```
Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(((a + b)*Tanh[c + d*x])/(4*(1 - Tanh[c + d*x]^2)^2) + ((3*(a + 5*b)*ArcTa
nh[Tanh[c + d*x]] - 8*b*Tanh[c + d*x])/2 - ((5*a + 9*b)*Tanh[c + d*x])/(2*
(1 - Tanh[c + d*x]^2)))/4)/d
```

3.1.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.1.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\sinh(dx+c)^5}{4\cosh(dx+c)}-\frac{5\sinh(dx+c)^3}{8\cosh(dx+c)}+\frac{15dx}{8}+\frac{15c}{8}-\frac{15\tanh(dx+c)}{8}\right)}{d}$
default	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\sinh(dx+c)^5}{4\cosh(dx+c)}-\frac{5\sinh(dx+c)^3}{8\cosh(dx+c)}+\frac{15dx}{8}+\frac{15c}{8}-\frac{15\tanh(dx+c)}{8}\right)}{d}$
risch	$\frac{3ax}{8}+\frac{15bx}{8}+\frac{e^{4dx+4c}a}{64d}+\frac{e^{4dx+4c}b}{64d}-\frac{e^{2dx+2c}a}{8d}-\frac{e^{2dx+2c}b}{4d}+\frac{e^{-2dx-2c}a}{8d}+\frac{e^{-2dx-2c}b}{4d}-\frac{e^{-4dx-4c}a}{64d}-$

input
```
int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+b*(
1/4*sinh(d*x+c)^5/cosh(d*x+c)-5/8*sinh(d*x+c)^3/cosh(d*x+c)+15/8*d*x+15/8*
c-15/8*tanh(d*x+c)))
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.64

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$=\frac{(a+b)\sinh(dx+c)^5+\left(10(a+b)\cosh(dx+c)^2-7a-15b\right)\sinh(dx+c)^3+8\left(3(a+5b)dx+8b\right)\cos$$
$$64d\cosh(dx+$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/64*((a + b)*sinh(d*x + c)^5 + (10*(a + b)*cosh(d*x + c)^2 - 7*a - 15*b)*
sinh(d*x + c)^3 + 8*(3*(a + 5*b)*d*x + 8*b)*cosh(d*x + c) + (5*(a + b)*cos
h(d*x + c)^4 - 3*(7*a + 15*b)*cosh(d*x + c)^2 - 8*a - 80*b)*sinh(d*x + c))
/(d*cosh(d*x + c))
```

3.1.6 Sympy [F]

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)\sinh^4\left(c+dx\right)dx$$

input
```
integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x)**4, x)
```

3.1.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(67) = 134$.

Time $= 0.20$ (sec) , antiderivative size $= 154$, normalized size of antiderivative $= 2.11$

$$\begin{aligned}
&\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx \\
&= \frac{1}{64}\,a\left(24\,x + \frac{e^{(4\,dx+4\,c)}}{d} - \frac{8\,e^{(2\,dx+2\,c)}}{d} + \frac{8\,e^{(-2\,dx-2\,c)}}{d} - \frac{e^{(-4\,dx-4\,c)}}{d}\right) \\
&\quad + \frac{1}{64}\,b\left(\frac{120\,(dx+c)}{d} + \frac{16\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)}}{d} - \frac{15\,e^{(-2\,dx-2\,c)} + 144\,e^{(-4\,dx-4\,c)} - 1}{d(e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)})}\right)
\end{aligned}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c
)/d - e^(-4*d*x - 4*c)/d) + 1/64*b*(120*(d*x + c)/d + (16*e^(-2*d*x - 2*c)
 - e^(-4*d*x - 4*c))/d - (15*e^(-2*d*x - 2*c) + 144*e^(-4*d*x - 4*c) - 1)/
(d*(e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))))
```

3.1.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(67) = 134$.

Time $= 0.32$ (sec) , antiderivative size $= 142$, normalized size of antiderivative $= 1.95$

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{24\,(dx+c)(a+5\,b)+ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}-8\,ae^{(2\,dx+2\,c)}-16\,be^{(2\,dx+2\,c)}-\left(18\,ae^{(4\,dx+4\,c)}+90\,be^{(4\,dx+4}}{64\,d}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/64*(24*(d*x + c)*(a + 5*b) + a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) - 8*a
*e^(2*d*x + 2*c) - 16*b*e^(2*d*x + 2*c) - (18*a*e^(4*d*x + 4*c) + 90*b*e^(
4*d*x + 4*c) - 8*a*e^(2*d*x + 2*c) - 16*b*e^(2*d*x + 2*c) + a + b)*e^(-4*d
*x - 4*c) + 128*b/(e^(2*d*x + 2*c) + 1))/d
```

3.1.9 Mupad [B] (verification not implemented)

Time $= 0.25$ (sec) , antiderivative size $= 101$, normalized size of antiderivative $= 1.38$

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = x\left(\frac{3\,a}{8}+\frac{15\,b}{8}\right)+\frac{2\,b}{d\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$
$$-\frac{\mathrm{e}^{-4\,c-4\,d\,x}\,(a+b)}{64\,d}+\frac{\mathrm{e}^{4\,c+4\,d\,x}\,(a+b)}{64\,d}$$
$$+\frac{\mathrm{e}^{-2\,c-2\,d\,x}\,(a+2\,b)}{8\,d}-\frac{\mathrm{e}^{2\,c+2\,d\,x}\,(a+2\,b)}{8\,d}$$

input
```
int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2),x)
```

output
```
x*((3*a)/8 + (15*b)/8) + (2*b)/(d*(exp(2*c + 2*d*x) + 1)) - (exp(- 4*c - 4
*d*x)*(a + b))/(64*d) + (exp(4*c + 4*d*x)*(a + b))/(64*d) + (exp(- 2*c - 2
*d*x)*(a + 2*b))/(8*d) - (exp(2*c + 2*d*x)*(a + 2*b))/(8*d)
```

3.2 $\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

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3.2.1 Optimal result

Integrand size = 21, antiderivative size = 47

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{(a+2b)\cosh(c+dx)}{d} + \frac{(a+b)\cosh^3(c+dx)}{3d} - \frac{b\operatorname{sech}(c+dx)}{d}$$

output
```
-(a+2*b)*cosh(d*x+c)/d+1/3*(a+b)*cosh(d*x+c)^3/d-b*sech(d*x+c)/d
```

3.2.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{3a\cosh(c+dx)}{4d} - \frac{7b\cosh(c+dx)}{4d} + \frac{a\cosh(3(c+dx))}{12d} + \frac{b\cosh(3(c+dx))}{12d} - \frac{b\operatorname{sech}(c+dx)}{d}$$

input
```
Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-3*a*Cosh[c + d*x])/(4*d) - (7*b*Cosh[c + d*x])/(4*d) + (a*Cosh[3*(c + d*
x)])/(12*d) + (b*Cosh[3*(c + d*x)])/(12*d) - (b*Sech[c + d*x])/d
```

3.2.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 26, 4147, 25, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int i\sin(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)dx$$

$$\downarrow 26$$

$$i\int \sin(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)dx$$

$$\downarrow 4147$$

$$\frac{\int -\cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)\left(-b\text{sech}^2(c+dx)+a+b\right)d\text{sech}(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{\int \cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)\left(-b\text{sech}^2(c+dx)+a+b\right)d\text{sech}(c+dx)}{d}$$

$$\downarrow 355$$

$$-\frac{\int \left((a+b)\cosh^4(c+dx)+(-a-2b)\cosh^2(c+dx)+b\right)d\text{sech}(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{3}(a+b)\cosh^3(c+dx)-(a+2b)\cosh(c+dx)-b\text{sech}(c+dx)}{d}$$

input
```
Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-((a + 2*b)*Cosh[c + d*x]) + ((a + b)*Cosh[c + d*x]^3)/3 - b*Sech[c + d*x
])/d
```

3.2.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.2.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^4}{3\cosh(dx+c)}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)}-\frac{8}{3\cosh(dx+c)}\right)}{d}$
default	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^4}{3\cosh(dx+c)}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)}-\frac{8}{3\cosh(dx+c)}\right)}{d}$
risch	$\frac{e^{3dx+3c}a}{24d}+\frac{e^{3dx+3c}b}{24d}-\frac{3e^{dx+c}a}{8d}-\frac{7e^{dx+c}b}{8d}-\frac{3e^{-dx-c}a}{8d}-\frac{7e^{-dx-c}b}{8d}+\frac{e^{-3dx-3c}a}{24d}+\frac{e^{-3dx-3c}b}{24d}-\frac{2}{d(e^2$

input
```
int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+b*(1/3*sinh(d*x+c)^4/cosh(d*x+
c)-4/3*sinh(d*x+c)^2/cosh(d*x+c)-8/3/cosh(d*x+c)))
```

3.2.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(45) = 90$.

Time $= 0.26$ (sec) , antiderivative size $= 91$, normalized size of antiderivative $= 1.94$

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{(a+b)\cosh(dx+c)^4+(a+b)\sinh(dx+c)^4-4(2a+5b)\cosh(dx+c)^2+2\left(3(a+b)\cosh(dx+c)^2-\right.}{24\,d\cosh(dx+c)}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/24*((a + b)*cosh(d*x + c)^4 + (a + b)*sinh(d*x + c)^4 - 4*(2*a + 5*b)*co
sh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 4*a - 10*b)*sinh(d*x + c)^2
 - 9*a - 45*b)/(d*cosh(d*x + c))
```

3.2.6 Sympy [F]

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2(c+dx)\right)\sinh^3(c+dx)\,dx$$

input
```
integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x)**3, x)
```

3.2.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(45) = 90$.

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.89

$$\begin{aligned}&\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx\\&=-\frac{1}{24}\,b\left(\frac{21\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{20\,e^{(-2\,dx-2\,c)}+69\,e^{(-4\,dx-4\,c)}-1}{d\left(e^{(-3\,dx-3\,c)}+e^{(-5\,dx-5\,c)}\right)}\right)\\&\quad+\frac{1}{24}\,a\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)\end{aligned}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/24*b*((21*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (20*e^(-2*d*x - 2*c) + 6
9*e^(-4*d*x - 4*c) - 1)/(d*(e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c)))) + 1/24*
a*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*
c)/d)
```

3.2.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(45) = 90$.

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.23

$$\begin{aligned}&\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx\\&=\frac{a\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-12\,a\left(e^{(dx+c)}+e^{(-dx-c)}\right)-24\,b\left(e^{(dx+c)}+e^{(-dx-c)}\right)-\frac{}{e^{(dx+}}}{24\,d}\end{aligned}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/24*(a*(e^(d*x + c) + e^(-d*x - c))^3 + b*(e^(d*x + c) + e^(-d*x - c))^3
- 12*a*(e^(d*x + c) + e^(-d*x - c)) - 24*b*(e^(d*x + c) + e^(-d*x - c)) -
48*b/(e^(d*x + c) + e^(-d*x - c)))/d
```

3.2.9 Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.11

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{e^{-3c-3dx}(a+b)}{24d} + \frac{e^{3c+3dx}(a+b)}{24d} - \frac{e^{c+dx}(3a+7b)}{8d} - \frac{e^{-c-dx}(3a+7b)}{8d} - \frac{2be^{c+dx}}{d\left(e^{2c+2dx}+1\right)}$$

input
```
int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2),x)
```

output
```
(exp(- 3*c - 3*d*x)*(a + b))/(24*d) + (exp(3*c + 3*d*x)*(a + b))/(24*d) -
(exp(c + d*x)*(3*a + 7*b))/(8*d) - (exp(- c - d*x)*(3*a + 7*b))/(8*d) - (2
*b*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1))
```

3.3 $\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

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3.3.1 Optimal result

Integrand size = 21, antiderivative size = 44

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{1}{2}(a+3b)x + \frac{(a+b)\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b\tanh(c+dx)}{d}$$

output
```
-1/2*(a+3*b)*x+1/2*(a+b)*cosh(d*x+c)*sinh(d*x+c)/d+b*tanh(d*x+c)/d
```

3.3.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{-2(a+3b)(c+dx)+(a+b)\sinh(2(c+dx))+4b\tanh(c+dx)}{4d}$$

input
```
Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-2*(a + 3*b)*(c + d*x) + (a + b)*Sinh[2*(c + d*x)] + 4*b*Tanh[c + d*x])/(
4*d)
```

3.3.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 25, 4146, 360, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int -\sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)dx$$

$$\downarrow 25$$

$$-\int \sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)dx$$

$$\downarrow 4146$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 360$$

$$\frac{\frac{(a+b)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}-\frac{1}{2}\int \frac{2b\tanh^2(c+dx)+a+b}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 299$$

$$\frac{\frac{1}{2}\left(2b\tanh(c+dx)-(a+3b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)\right)+\frac{(a+b)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 219$$

$$\frac{\frac{1}{2}(2b\tanh(c+dx)-(a+3b)\operatorname{arctanh}(\tanh(c+dx)))+\frac{(a+b)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}}{d}$$

input
```
Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
((-((a + 3*b)*ArcTanh[Tanh[c + d*x]]) + 2*b*Tanh[c + d*x])/2 + ((a + b)*Ta
nh[c + d*x])/(2*(1 - Tanh[c + d*x]^2)))/d
```

3.3.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.3.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.50

method	result	size
derivativedivides	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)}{d}$	66
default	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)}{d}$	66
risch	$-\frac{ax}{2}-\frac{3bx}{2}+\frac{e^{2dx+2c}a}{8d}+\frac{e^{2dx+2c}b}{8d}-\frac{e^{-2dx-2c}a}{8d}-\frac{e^{-2dx-2c}b}{8d}-\frac{2b}{d(e^{2dx+2c}+1)}$	89

input
```
int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b*(1/2*sinh(d*x+c)^3/co
sh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c)))
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.61

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{(a+b)\sinh(dx+c)^3-4\left((a+3\,b)dx+2\,b\right)\cosh(dx+c)+\left(3\,(a+b)\cosh(dx+c)^2+a+9\,b\right)\sinh(dx}{8\,d\cosh(dx+c)}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/8*((a + b)*sinh(d*x + c)^3 - 4*((a + 3*b)*d*x + 2*b)*cosh(d*x + c) + (3*
(a + b)*cosh(d*x + c)^2 + a + 9*b)*sinh(d*x + c))/(d*cosh(d*x + c))
```

3.3.6 Sympy [F]

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)\sinh^2\left(c+dx\right)dx$$

input
```
integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x)**2, x)
```

3.3.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(40) = 80$.

Time $= 0.22$ (sec) , antiderivative size $= 101$, normalized size of antiderivative $= 2.30$

$$\begin{aligned}&\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx \\ &= -\frac{1}{8}a\left(4x - \frac{e^{(2\,dx+2\,c)}}{d} + \frac{e^{(-2\,dx-2\,c)}}{d}\right) \\ &\quad -\frac{1}{8}b\left(\frac{12\,(dx+c)}{d} + \frac{e^{(-2\,dx-2\,c)}}{d} - \frac{17\,e^{(-2\,dx-2\,c)}+1}{d(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)})}\right)\end{aligned}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/8*b*(12*(d*x + c
)/d + e^(-2*d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c)
+ e^(-4*d*x - 4*c))))
```

3.3.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(40) = 80$.

Time $= 0.29$ (sec) , antiderivative size $= 107$, normalized size of antiderivative $= 2.43$

$$\begin{aligned}&\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx \\ &= -\frac{4\,(dx+c)(a+3\,b) - ae^{(2\,dx+2\,c)} - be^{(2\,dx+2\,c)} - \frac{ae^{(4\,dx+4\,c)}+3\,be^{(4\,dx+4\,c)}-14\,be^{(2\,dx+2\,c)}-a-b}{e^{(4\,dx+4\,c)}+e^{(2\,dx+2\,c)}}}{8\,d}\end{aligned}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/8*(4*(d*x + c)*(a + 3*b) - a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) - (a*e
^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) - 14*b*e^(2*d*x + 2*c) - a - b)/(e^(4
*d*x + 4*c) + e^(2*d*x + 2*c)))/d
```

3.3.9 Mupad [B] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.45

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{\mathrm{e}^{2c+2dx}(a+b)}{8d} - \frac{2b}{d\left(\mathrm{e}^{2c+2dx}+1\right)} - \frac{\mathrm{e}^{-2c-2dx}(a+b)}{8d} - x\left(\frac{a}{2}+\frac{3b}{2}\right)$$

input
```
int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2),x)
```

output
```
(exp(2*c + 2*d*x)*(a + b))/(8*d) - (2*b)/(d*(exp(2*c + 2*d*x) + 1)) - (exp
(- 2*c - 2*d*x)*(a + b))/(8*d) - x*(a/2 + (3*b)/2)
```

3.4 $\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

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3.4.1 Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(a+b)\cosh(c+dx)}{d} + \frac{b\operatorname{sech}(c+dx)}{d}$$

output
```
(a+b)*cosh(d*x+c)/d+b*sech(d*x+c)/d
```

3.4.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{a\cosh(c)\cosh(dx)}{d} + \frac{b\cosh(c+dx)}{d} + \frac{b\operatorname{sech}(c+dx)}{d} + \frac{a\sinh(c)\sinh(dx)}{d}$$

input
```
Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a*Cosh[c]*Cosh[d*x])/d + (b*Cosh[c + d*x])/d + (b*Sech[c + d*x])/d + (a*S
inh[c]*Sinh[d*x])/d
```

3.4.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 26, 4147, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int -i\sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)dx$$

$$\downarrow 26$$

$$-i\int \sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)dx$$

$$\downarrow 4147$$

$$-\frac{\int \cosh^2(c+dx)\left(-b\text{sech}^2(c+dx)+a+b\right)d\text{sech}(c+dx)}{d}$$

$$\downarrow 244$$

$$-\frac{\int \left((a+b)\cosh^2(c+dx)-b\right)d\text{sech}(c+dx)}{d}$$

$$\downarrow 2009$$

$$-\frac{-((a+b)\cosh(c+dx))-b\text{sech}(c+dx)}{d}$$

input
```
Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-((-((a + b)*Cosh[c + d*x]) - b*Sech[c + d*x])/d)
```

3.4.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.4.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

method	result	size
derivativedivides	$\frac{a\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^2}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)}{d}$	44
default	$\frac{a\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^2}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)}{d}$	44
risch	$\frac{\mathrm{e}^{dx+c}a}{2d}+\frac{\mathrm{e}^{dx+c}b}{2d}+\frac{\mathrm{e}^{-dx-c}a}{2d}+\frac{\mathrm{e}^{-dx-c}b}{2d}+\frac{2b\mathrm{e}^{dx+c}}{d(\mathrm{e}^{2dx+2c}+1)}$	81

input
```
int(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*cosh(d*x+c)+b*(sinh(d*x+c)^2/cosh(d*x+c)+2/cosh(d*x+c)))
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(a+b)\cosh(dx+c)^2+(a+b)\sinh(dx+c)^2+a+3\,b}{2\,d\cosh(dx+c)}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a + 3*b)/(d*cosh(
d*x + c))
```

3.4.6 Sympy [F]

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2(c+dx)\right)\sinh(c+dx)\,dx$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x), x)
```

3.4.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(25) = 50$.

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.68

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{1}{2}\,b\left(\frac{e^{(-dx-c)}}{d}+\frac{5\,e^{(-2\,dx-2\,c)}+1}{d(e^{(-dx-c)}+e^{(-3\,dx-3\,c)})}\right) + \frac{a\cosh(dx+c)}{d}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e^(-d*x - c) + e^(-3*
d*x - 3*c)))) + a*cosh(d*x + c)/d
```

3.4.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(25) = 50$.

Time $= 0.29$ (sec) , antiderivative size $= 63$, normalized size of antiderivative $= 2.52$

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{a\left(e^{(dx+c)}+e^{(-dx-c)}\right)+b\left(e^{(dx+c)}+e^{(-dx-c)}\right)+\frac{4\,b}{e^{(dx+c)}+e^{(-dx-c)}}}{2\,d}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2*(a*(e^(d*x + c) + e^(-d*x - c)) + b*(e^(d*x + c) + e^(-d*x - c)) + 4*b
/(e^(d*x + c) + e^(-d*x - c)))/d
```

3.4.9 Mupad [B] (verification not implemented)

Time $= 1.74$ (sec) , antiderivative size $= 27$, normalized size of antiderivative $= 1.08$

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{b}{d\cosh\left(c+d\,x\right)}+\frac{\cosh(c+d\,x)\,\left(a+b\right)}{d}$$

input
```
int(sinh(c + d*x)*(a + b*tanh(c + d*x)^2),x)
```

output
```
b/(d*cosh(c + d*x)) + (cosh(c + d*x)*(a + b))/d
```

3.5 $\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

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3.5.1 Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{a\operatorname{arctanh}(\cosh(c+dx))}{d} - \frac{b\operatorname{sech}(c+dx)}{d}$$

output
```
-a*arctanh(cosh(d*x+c))/d-b*sech(d*x+c)/d
```

3.5.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.00

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{a\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{a\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{b\operatorname{sech}(c+dx)}{d}$$

input
```
Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-((a*Log[Cosh[c/2 + (d*x)/2]])/d) + (a*Log[Sinh[c/2 + (d*x)/2]])/d - (b*Se
ch[c + d*x])/d
```

3.5.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 26, 4147, 25, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{i\left(a-b\tan(ic+idx)^2\right)}{\sin(ic+idx)}dx$$

$$\downarrow 26$$

$$i\int \frac{a-b\tan(ic+idx)^2}{\sin(ic+idx)}dx$$

$$\downarrow 4147$$

$$\frac{\int -\frac{-b\operatorname{sech}^2(c+dx)+a+b}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{\int \frac{-b\operatorname{sech}^2(c+dx)+a+b}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)}{d}$$

$$\downarrow 299$$

$$\frac{-a\int \frac{1}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)-b\operatorname{sech}(c+dx)}{d}$$

$$\downarrow 219$$

$$\frac{-a\operatorname{arctanh}(\operatorname{sech}(c+dx))-b\operatorname{sech}(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-(a*ArcTanh[Sech[c + d*x]]) - b*Sech[c + d*x])/d
```

3.5.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.5.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{-2a \operatorname{arctanh}\left(e^{dx+c}\right)-\frac{b}{\cosh(dx+c)}}{d}$	27
default	$\frac{-2a \operatorname{arctanh}\left(e^{dx+c}\right)-\frac{b}{\cosh(dx+c)}}{d}$	27
risch	$-\frac{2b e^{dx+c}}{d\left(e^{2dx+2c}+1\right)}+\frac{a \ln\left(e^{dx+c}-1\right)}{d}-\frac{a \ln\left(e^{dx+c}+1\right)}{d}$	56

input
```
int(csch(d*x+c)*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a*arctanh(exp(d*x+c))-b/cosh(d*x+c))
```

3.5.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(26) = 52$.

Time $= 0.25$ (sec) , antiderivative size $= 167$, normalized size of antiderivative $= 6.42$

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{2\,b\cosh(dx+c)+\left(a\cosh(dx+c)^2+2\,a\cosh(dx+c)\sinh(dx+c)+a\sinh(dx+c)^2+a\right)\log(\cosh($$
$$d\cosh$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-(2*b*cosh(d*x + c) + (a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c)
 + a*sinh(d*x + c)^2 + a)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a*cosh
(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a)*log
(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*b*sinh(d*x + c))/(d*cosh(d*x + c)^
2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 + d)
```

3.5.6 Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2(c+dx)\right)\operatorname{csch}(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x), x)
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.54

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{a\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d} - \frac{2\,b}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
a*log(tanh(1/2*d*x + 1/2*c))/d - 2*b/(d*(e^(d*x + c) + e^(-d*x - c)))
```

3.5.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(26) = 52.

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.62

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{a\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right) - a\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right) + \frac{4\,b}{e^{(dx+c)}+e^{(-dx-c)}}}{2\,d}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/2*(a*log(e^(d*x + c) + e^(-d*x - c) + 2) - a*log(e^(d*x + c) + e^(-d*x
- c) - 2) + 4*b/(e^(d*x + c) + e^(-d*x - c)))/d
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.46

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{2\operatorname{atan}\left(\frac{a\,e^{d\,x}\,e^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2}}\right)\,\sqrt{a^2}}{\sqrt{-d^2}} - \frac{2\,b\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)/sinh(c + d*x),x)
```

output
```
- (2*atan((a*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^2)^(1/2)))*(a^2)^(1/2))/(
-d^2)^(1/2) - (2*b*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1))
```

3.6 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

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3.6.1 Optimal result

Integrand size = 21, antiderivative size = 24

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{a\coth(c+dx)}{d}+\frac{b\tanh(c+dx)}{d}$$

output
```
-a*coth(d*x+c)/d+b*tanh(d*x+c)/d
```

3.6.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{a\coth(c+dx)}{d}+\frac{b\tanh(c+dx)}{d}$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-((a*Coth[c + d*x])/d) + (b*Tanh[c + d*x])/d
```

3.6.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4146, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int -\frac{a-b\tan(ic+idx)^2}{\sin(ic+idx)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{a-b\tan(ic+idx)^2}{\sin(ic+idx)^2}dx$$

$$\downarrow 4146$$

$$\frac{\int \coth^2(c+dx)\left(b\tanh^2(c+dx)+a\right)d\tanh(c+dx)}{d}$$

$$\downarrow 244$$

$$\frac{\int \left(a\coth^2(c+dx)+b\right)d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{b\tanh(c+dx)-a\coth(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-(a*Coth[c + d*x]) + b*Tanh[c + d*x])/d
```

3.6.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.6.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{b\tanh(dx+c)-\frac{a}{\tanh(dx+c)}}{d}$	25
default	$\frac{b\tanh(dx+c)-\frac{a}{\tanh(dx+c)}}{d}$	25
risch	$-\frac{2\left(\mathrm{e}^{2dx+2c}a+b\,\mathrm{e}^{2dx+2c}+a-b\right)}{d\left(\mathrm{e}^{2dx+2c}-1\right)\left(\mathrm{e}^{2dx+2c}+1\right)}$	59

input
```
int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b*tanh(d*x+c)-a/tanh(d*x+c))
```

3.6.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(24) = 48$.

Time $= 0.24$ (sec) , antiderivative size $= 88$, normalized size of antiderivative $= 3.67$

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx =$$

$$-\frac{4\left(a\cosh\left(dx+c\right)+b\sinh\left(dx+c\right)\right)}{d\cosh\left(dx+c\right)^3+3\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+d\sinh\left(dx+c\right)^3-d\cosh\left(dx+c\right)+\left(3\,d\cosh\left(dx+\right.\right.}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-4*(a*cosh(d*x + c) + b*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x +
 c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3 - d*cosh(d*x + c) + (3*d*cosh(d*x
+ c)^2 + d)*sinh(d*x + c))
```

3.6.6 Sympy [F]

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)\operatorname{csch}^2\left(c+dx\right)dx$$

input
```
integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x)**2, x)
```

3.6.7 Maxima [A] (verification not implemented)

Time $= 0.22$ (sec) , antiderivative size $= 39$, normalized size of antiderivative $= 1.62$

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{2\,b}{d(e^{(-2\,dx-2\,c)}+1)} + \frac{2\,a}{d(e^{(-2\,dx-2\,c)}-1)}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
2*b/(d*(e^(-2*d*x - 2*c) + 1)) + 2*a/(d*(e^(-2*d*x - 2*c) - 1))
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.88

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{2\left(ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b\right)}{d(e^{(4\,dx+4\,c)}-1)}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/(d*(e^(4*d*x + 4*c) - 1
))
```

3.6.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{\frac{2\,(a-b)}{d}+\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,(a+b)}{d}}{\mathrm{e}^{4\,c+4\,d\,x}-1}$$

input
```
int((a + b*tanh(c + d*x)^2)/sinh(c + d*x)^2,x)
```

output
```
-((2*(a - b))/d + (2*exp(2*c + 2*d*x)*(a + b))/d)/(exp(4*c + 4*d*x) - 1)
```

3.7 $\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

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3.7.1 Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(a-2b)\operatorname{arctanh}(\cosh(c+dx))}{2d} - \frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d} + \frac{b\operatorname{sech}(c+dx)}{d}$$

output

```
1/2*(a-2*b)*arctanh(cosh(d*x+c))/d-1/2*a*coth(d*x+c)*csch(d*x+c)/d+b*sech(
d*x+c)/d
```

3.7.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 123 vs. 2(51) = 102.

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.41

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{b\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{b\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{b\operatorname{sech}(c+dx)}{d}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-1/8*(a*Csch[(c + d*x)/2]^2)/d + (a*Log[Cosh[(c + d*x)/2]])/(2*d) - (b*Log
[Cosh[(c + d*x)/2]])/d - (a*Log[Sinh[(c + d*x)/2]])/(2*d) + (b*Log[Sinh[(c
 + d*x)/2]])/d - (a*Sech[(c + d*x)/2]^2)/(8*d) + (b*Sech[c + d*x])/d
```

3.7.3 Rubi [A] (verified)

Time $= 0.25$ (sec) , antiderivative size $= 59$, normalized size of antiderivative $= 1.16$, number of steps used $= 7$, number of rules used $= 6$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used $= \{3042, 26, 4147, 360, 299, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\left(a-b\tan(ic+idx)^2\right)}{\sin(ic+idx)^3}dx$$

$$\downarrow 26$$

$$-i\int \frac{a-b\tan(ic+idx)^2}{\sin(ic+idx)^3}dx$$

$$\downarrow 4147$$

$$-\frac{\int \frac{\operatorname{sech}^2(c+dx)\left(-b\operatorname{sech}^2(c+dx)+a+b\right)}{\left(1-\operatorname{sech}^2(c+dx)\right)^2}d\operatorname{sech}(c+dx)}{d}$$

$$\downarrow 360$$

$$-\frac{\frac{a\operatorname{sech}(c+dx)}{2\left(1-\operatorname{sech}^2(c+dx)\right)}-\frac{1}{2}\int \frac{a-2b\operatorname{sech}^2(c+dx)}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)}{d}$$

$$\downarrow 299$$

$$-\frac{\frac{1}{2}\left(-(a-2b)\int \frac{1}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)-2b\operatorname{sech}(c+dx)\right)+\frac{a\operatorname{sech}(c+dx)}{2\left(1-\operatorname{sech}^2(c+dx)\right)}}{d}$$

$$\downarrow 219$$

$$-\frac{\frac{1}{2}(-((a-2b)\text{arctanh}(\text{sech}(c+dx)))-2b\text{sech}(c+dx))+\frac{a\text{sech}(c+dx)}{2\left(1-\text{sech}^2(c+dx)\right)}}{d}$$

input
```
Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-(((-((a - 2*b)*ArcTanh[Sech[c + d*x]]) - 2*b*Sech[c + d*x])/2 + (a*Sech[c
 + d*x])/(2*(1 - Sech[c + d*x]^2)))/d)
```

3.7.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.7.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+b\left(\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)}{d}$
default	$\frac{a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+b\left(\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)}{d}$
risch	$-\frac{e^{dx+c}\left(a\,e^{4dx+4c}-2b\,e^{4dx+4c}+2\,e^{2dx+2c}a+4b\,e^{2dx+2c}+a-2b\right)}{d\left(e^{2dx+2c}-1\right)^2\left(e^{2dx+2c}+1\right)}-\frac{a\ln\left(e^{dx+c}-1\right)}{2d}+\frac{\ln\left(e^{dx+c}-1\right)b}{d}+\frac{a\ln\left(e^{dx+c}+$

input
```
int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b*(1/cosh(d*x+c)
-2*arctanh(exp(d*x+c))))
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(47) = 94$.

Time = 0.27 (sec) , antiderivative size = 924, normalized size of antiderivative = 18.12

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(2*(a - 2*b)*cosh(d*x + c)^5 + 10*(a - 2*b)*cosh(d*x + c)*sinh(d*x +
c)^4 + 2*(a - 2*b)*sinh(d*x + c)^5 + 4*(a + 2*b)*cosh(d*x + c)^3 + 4*(5*(a
 - 2*b)*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^3 + 4*(5*(a - 2*b)*cosh(d
*x + c)^3 + 3*(a + 2*b)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(a - 2*b)*cosh(
d*x + c) - ((a - 2*b)*cosh(d*x + c)^6 + 6*(a - 2*b)*cosh(d*x + c)*sinh(d*x
 + c)^5 + (a - 2*b)*sinh(d*x + c)^6 - (a - 2*b)*cosh(d*x + c)^4 + (15*(a -
 2*b)*cosh(d*x + c)^2 - a + 2*b)*sinh(d*x + c)^4 + 4*(5*(a - 2*b)*cosh(d*x
 + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a - 2*b)*cosh(d*x +
c)^2 + (15*(a - 2*b)*cosh(d*x + c)^4 - 6*(a - 2*b)*cosh(d*x + c)^2 - a + 2
*b)*sinh(d*x + c)^2 + 2*(3*(a - 2*b)*cosh(d*x + c)^5 - 2*(a - 2*b)*cosh(d*
x + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + a - 2*b)*log(cosh(d*x
+ c) + sinh(d*x + c) + 1) + ((a - 2*b)*cosh(d*x + c)^6 + 6*(a - 2*b)*cosh(
d*x + c)*sinh(d*x + c)^5 + (a - 2*b)*sinh(d*x + c)^6 - (a - 2*b)*cosh(d*x
+ c)^4 + (15*(a - 2*b)*cosh(d*x + c)^2 - a + 2*b)*sinh(d*x + c)^4 + 4*(5*(
a - 2*b)*cosh(d*x + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a -
 2*b)*cosh(d*x + c)^2 + (15*(a - 2*b)*cosh(d*x + c)^4 - 6*(a - 2*b)*cosh(d
*x + c)^2 - a + 2*b)*sinh(d*x + c)^2 + 2*(3*(a - 2*b)*cosh(d*x + c)^5 - 2*
(a - 2*b)*cosh(d*x + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + a - 2
*b)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(5*(a - 2*b)*cosh(d*x + c)^
4 + 6*(a + 2*b)*cosh(d*x + c)^2 + a - 2*b)*sinh(d*x + c))/(d*cosh(d*x +...
```

3.7.6 Sympy [F]

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)\operatorname{csch}^3\left(c+dx\right)dx$$

input
```
integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x)**3, x)
```

3.7.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(47) = 94$.

Time $= 0.21$ (sec) , antiderivative size $= 152$, normalized size of antiderivative $= 2.98$

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{1}{2}a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$
$$- b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} - \frac{2\,e^{(-dx-c)}}{d(e^{(-2\,dx-2\,c)}+1)}\right)$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*a*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c)
 + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - b*
(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(d*(e^
(-2*d*x - 2*c) + 1)))
```

3.7.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(47) = 94$.

Time $= 0.29$ (sec) , antiderivative size $= 142$, normalized size of antiderivative $= 2.78$

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{(a-2\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right) - (a-2\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right) - \frac{4\left(a\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-2\,b(e^{(dx+}\right.}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-4\,e^{(dx+}}$$
$$4\,d$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/4*((a - 2*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) - (a - 2*b)*log(e^(d*x
+ c) + e^(-d*x - c) - 2) - 4*(a*(e^(d*x + c) + e^(-d*x - c))^2 - 2*b*(e^(d
*x + c) + e^(-d*x - c))^2 + 8*b)/((e^(d*x + c) + e^(-d*x - c))^3 - 4*e^(d*
x + c) - 4*e^(-d*x - c)))/d
```

3.7.9 Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.06

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(a\sqrt{-d^2}-2b\sqrt{-d^2}\right)}{d\sqrt{a^2-4ab+4b^2}}\right)\sqrt{a^2-4ab+4b^2}}{\sqrt{-d^2}} - \frac{a\,e^{c+dx}}{d\,(e^{2c+2dx}-1)} + \frac{2b\,e^{c+dx}}{d\,(e^{2c+2dx}+1)} - \frac{2a\,e^{c+dx}}{d\,(e^{4c+4dx}-2e^{2c+2dx}+1)}$$

input
```
int((a + b*tanh(c + d*x)^2)/sinh(c + d*x)^3,x)
```

output
```
(atan((exp(d*x)*exp(c)*(a*(-d^2)^(1/2) - 2*b*(-d^2)^(1/2)))/(d*(a^2 - 4*a*
b + 4*b^2)^(1/2)))*(a^2 - 4*a*b + 4*b^2)^(1/2))/(-d^2)^(1/2) - (a*exp(c +
d*x))/(d*(exp(2*c + 2*d*x) - 1)) + (2*b*exp(c + d*x))/(d*(exp(2*c + 2*d*x)
 + 1)) - (2*a*exp(c + d*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)
)
```

3.8 $\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

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3.8.1 Optimal result

Integrand size = 21, antiderivative size = 44

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(a-b)\coth(c+dx)}{d} - \frac{a\coth^3(c+dx)}{3d} - \frac{b\tanh(c+dx)}{d}$$

output
```
(a-b)*coth(d*x+c)/d-1/3*a*coth(d*x+c)^3/d-b*tanh(d*x+c)/d
```

3.8.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{2a\coth(c+dx)}{3d} - \frac{b\coth(c+dx)}{d} - \frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d} - \frac{b\tanh(c+dx)}{d}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(2*a*Coth[c + d*x])/(3*d) - (b*Coth[c + d*x])/d - (a*Coth[c + d*x]*Csch[c
+ d*x]^2)/(3*d) - (b*Tanh[c + d*x])/d
```

3.8.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4146, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a-b\tan(ic+idx)^2}{\sin(ic+idx)^4}dx$$

$$\downarrow 4146$$

$$\frac{\int \coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)d\tanh(c+dx)}{d}$$

$$\downarrow 355$$

$$\frac{\int \left(a\coth^4(c+dx)+(b-a)\coth^2(c+dx)-b\right)d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{(a-b)\coth(c+dx)-\frac{1}{3}a\coth^3(c+dx)-b\tanh(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
((a - b)*Coth[c + d*x] - (a*Coth[c + d*x]^3)/3 - b*Tanh[c + d*x])/d
```

3.8.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.8.4 Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{a\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+b\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)}{d}$	55
default	$\frac{a\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+b\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)}{d}$	55
risch	$-\frac{4\left(3a\,\mathrm{e}^{4dx+4c}+3b\,\mathrm{e}^{4dx+4c}+2\,\mathrm{e}^{2dx+2c}a-6b\,\mathrm{e}^{2dx+2c}-a+3b\right)}{3d\left(\mathrm{e}^{2dx+2c}-1\right)^3\left(\mathrm{e}^{2dx+2c}+1\right)}$	87

input
```
int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+b*(-1/sinh(d*x+c)/cosh(d*x+c)-2
*tanh(d*x+c)))
```

3.8.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(42) = 84$.

Time = 0.26 (sec) , antiderivative size = 244, normalized size of antiderivative = 5.55

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx =$$

$$-\frac{}{3\left(d\cosh(dx+c)^6+6\,d\cosh(dx+c)\sinh(dx+c)^5+d\sinh(dx+c)^6-2\,d\cosh(dx+c)^4+\left(15\,d\cos\right.\right.}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-8/3*((a + 3*b)*cosh(d*x + c)^2 + 4*a*cosh(d*x + c)*sinh(d*x + c) + (a + 3
*b)*sinh(d*x + c)^2 + a - 3*b)/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh
(d*x + c)^5 + d*sinh(d*x + c)^6 - 2*d*cosh(d*x + c)^4 + (15*d*cosh(d*x + c
)^2 - 2*d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 2*d*cosh(d*x + c))*s
inh(d*x + c)^3 - d*cosh(d*x + c)^2 + (15*d*cosh(d*x + c)^4 - 12*d*cosh(d*x
 + c)^2 - d)*sinh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 - 4*d*cosh(d*x + c)^
3 + d*cosh(d*x + c))*sinh(d*x + c) + 2*d)
```

3.8.6 Sympy [F]

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2(c+dx)\right)\operatorname{csch}^4(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x)**4, x)
```

3.8.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(42) = 84$.

Time $= 0.21$ (sec) , antiderivative size $= 113$, normalized size of antiderivative $= 2.57$

$$\begin{aligned}&\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx\\ &= \frac{4}{3}a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)\\ &\quad+\frac{4\,b}{d(e^{(-4\,dx-4\,c)}-1)}\end{aligned}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^
(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) - 1))) + 4*b/(d*(e^(-4*d*x - 4*c) - 1))
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.82

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{2\left(\frac{3b}{e^{(2dx+2c)}+1}-\frac{3be^{(4dx+4c)}+6ae^{(2dx+2c)}-6be^{(2dx+2c)}-2a+3b}{\left(e^{(2dx+2c)}-1\right)^3}\right)}{3d}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
2/3*(3*b/(e^(2*d*x + 2*c) + 1) - (3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c
) - 6*b*e^(2*d*x + 2*c) - 2*a + 3*b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

3.8.9 Mupad [B] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 173, normalized size of antiderivative = 3.93

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{2b}{d\left(\mathrm{e}^{2c+2dx}+1\right)} - \frac{\frac{2(2a-b)}{3d}+\frac{2b\mathrm{e}^{2c+2dx}}{3d}}{\mathrm{e}^{4c+4dx}-2\,\mathrm{e}^{2c+2dx}+1} - \frac{2b}{3d\left(\mathrm{e}^{2c+2dx}-1\right)} - \frac{\frac{2b}{3d}+\frac{2b\mathrm{e}^{4c+4dx}}{3d}+\frac{4\mathrm{e}^{2c+2dx}(2a-b)}{3d}}{3\,\mathrm{e}^{2c+2dx}-3\,\mathrm{e}^{4c+4dx}+\mathrm{e}^{6c+6dx}-1}$$

input
```
int((a + b*tanh(c + d*x)^2)/sinh(c + d*x)^4,x)
```

output
```
(2*b)/(d*(exp(2*c + 2*d*x) + 1)) - ((2*(2*a - b))/(3*d) + (2*b*exp(2*c + 2
*d*x))/(3*d))/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - (2*b)/(3*d*(ex
p(2*c + 2*d*x) - 1)) - ((2*b)/(3*d) + (2*b*exp(4*c + 4*d*x))/(3*d) + (4*ex
p(2*c + 2*d*x)*(2*a - b))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x)
+ exp(6*c + 6*d*x) - 1)
```

3.9 $\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

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3.9.1 Optimal result

Integrand size = 23, antiderivative size = 118

$$\begin{aligned}\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = {}& \frac{1}{8}\left(3a^2+30ab+35b^2\right)x \\ & -\frac{(a+b)(a+9b)\cosh(c+dx)\sinh(c+dx)}{8d} \\ & -\frac{\left(a^2+10ab+13b^2\right)\tanh(c+dx)}{4d} \\ & +\frac{(a+b)^2\sinh^4(c+dx)\tanh(c+dx)}{4d} \\ & -\frac{b^2\tanh^3(c+dx)}{3d}\end{aligned}$$

output

```
1/8*(3*a^2+30*a*b+35*b^2)*x-1/8*(a+b)*(a+9*b)*cosh(d*x+c)*sinh(d*x+c)/d-1/
4*(a^2+10*a*b+13*b^2)*tanh(d*x+c)/d+1/4*(a+b)^2*sinh(d*x+c)^4*tanh(d*x+c)/
d-1/3*b^2*tanh(d*x+c)^3/d
```

3.9.2 Mathematica [A] (verified)

Time = 2.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{12(3a^2+30ab+35b^2)(c+dx)-24(a^2+4ab+3b^2)\sinh(2(c+dx))+3(a+b)^2\sinh(4(c+dx))+32b(-}{96d}$$

input
```
Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(12*(3*a^2 + 30*a*b + 35*b^2)*(c + d*x) - 24*(a^2 + 4*a*b + 3*b^2)*Sinh[2*
(c + d*x)] + 3*(a + b)^2*Sinh[4*(c + d*x)] + 32*b*(-6*a - 10*b + b*Sech[c
+ d*x]^2)*Tanh[c + d*x])/(96*d)
```

3.9.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4146, 366, 360, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$\downarrow 3042$$
$$\int \sin(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)^2\,dx$$
$$\downarrow 4146$$
$$\frac{\int \frac{\tanh^4(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{\left(1-\tanh^2(c+dx)\right)^3}\,d\tanh(c+dx)}{d}$$
$$\downarrow 366$$
$$\frac{\frac{(a+b)^2\tanh^5(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{1}{4}\int \frac{\tanh^4(c+dx)\left(a^2+10ba+5b^2+4b^2\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)^2}\,d\tanh(c+dx)}{d}$$
$$\downarrow 360$$

$$\frac{\frac{1}{4}\left(-\frac{1}{2}\int -\frac{8b^2\tanh^4(c+dx)+2(a+b)(a+9b)\tanh^2(c+dx)+(a+b)(a+9b)}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{(a+b)(a+9b)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}\right)+\frac{(a+b)^2\tanh^5(c+}{4(1-\tanh^2(c+d}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{8b^2\tanh^4(c+dx)+2(a+b)(a+9b)\tanh^2(c+dx)+(a+b)(a+9b)}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{(a+b)(a+9b)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}\right)+\frac{(a+b)^2\tanh^5(c+dx)}{4(1-\tanh^2(c+dx))^2}}{d}$$

$$\downarrow 1467$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \left(-8b^2\tanh^2(c+dx)-2\left(a^2+10ba+13b^2\right)+\frac{3a^2+30ba+35b^2}{1-\tanh^2(c+dx)}\right)d\tanh(c+dx)-\frac{(a+b)(a+9b)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}\right)+\frac{(a+}{4(1}}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(\left(3a^2+30ab+35b^2\right)\operatorname{arctanh}(\tanh(c+dx))-2\left(a^2+10ab+13b^2\right)\tanh(c+dx)-\frac{8}{3}b^2\tanh^3(c+dx)\right)-\frac{(a+}{2}\right.}{d}$$

input
```
Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(((a + b)^2*Tanh[c + d*x]^5)/(4*(1 - Tanh[c + d*x]^2)^2) + (-1/2*((a + b)*
(a + 9*b)*Tanh[c + d*x])/(1 - Tanh[c + d*x]^2) + ((3*a^2 + 30*a*b + 35*b^2
)*ArcTanh[Tanh[c + d*x]] - 2*(a^2 + 10*a*b + 13*b^2)*Tanh[c + d*x] - (8*b^
2*Tanh[c + d*x]^3)/3)/2)/4)/d
```

3.9.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.9.4 Maple [A] (verified)

Time = 3.17 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{a^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\sinh(dx+c)^5}{4\cosh(dx+c)}-\frac{5\sinh(dx+c)^3}{8\cosh(dx+c)}+\frac{15dx}{8}+\frac{15c}{8}-\frac{15\tanh(dx+c)}{8}\right.}{d}$
default	$\frac{a^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\sinh(dx+c)^5}{4\cosh(dx+c)}-\frac{5\sinh(dx+c)^3}{8\cosh(dx+c)}+\frac{15dx}{8}+\frac{15c}{8}-\frac{15\tanh(dx+c)}{8}\right.}{d}$
risch	$\frac{3a^2x}{8}+\frac{15abx}{4}+\frac{35b^2x}{8}+\frac{e^{4dx+4c}a^2}{64d}+\frac{e^{4dx+4c}ab}{32d}+\frac{e^{4dx+4c}b^2}{64d}-\frac{e^{2dx+2c}a^2}{8d}-\frac{e^{2dx+2c}ab}{2d}-\frac{3e^{2dx+2c}b^2}{8d}-$

input
```
int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+2
*a*b*(1/4*sinh(d*x+c)^5/cosh(d*x+c)-5/8*sinh(d*x+c)^3/cosh(d*x+c)+15/8*d*x
+15/8*c-15/8*tanh(d*x+c))+b^2*(1/4*sinh(d*x+c)^7/cosh(d*x+c)^3-7/8*sinh(d*
x+c)^5/cosh(d*x+c)^3+35/8*d*x+35/8*c-35/8*tanh(d*x+c)-35/24*tanh(d*x+c)^3)
)
```

3.9.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(108) = 216$.

Time $= 0.27$ (sec) , antiderivative size $= 394$, normalized size of antiderivative $= 3.34$

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{3\left(a^2+2\,ab+b^2\right)\sinh\left(dx+c\right)^7+3\left(21\left(a^2+2\,ab+b^2\right)\cosh\left(dx+c\right)^2-5\,a^2-26\,ab-21\,b^2\right)\sinh\left(dx+\right.}{}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/192*(3*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^7 + 3*(21*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^2 - 5*a^2 - 26*a*b - 21*b^2)*sinh(d*x + c)^5 + 8*(3*(3*a^2 +
30*a*b + 35*b^2)*d*x + 48*a*b + 80*b^2)*cosh(d*x + c)^3 + 24*(3*(3*a^2 + 3
0*a*b + 35*b^2)*d*x + 48*a*b + 80*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (10
5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 - 30*(5*a^2 + 26*a*b + 21*b^2)*cosh(
d*x + c)^2 - 63*a^2 - 654*a*b - 847*b^2)*sinh(d*x + c)^3 + 24*(3*(3*a^2 +
30*a*b + 35*b^2)*d*x + 48*a*b + 80*b^2)*cosh(d*x + c) + 3*(7*(a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^6 - 5*(5*a^2 + 26*a*b + 21*b^2)*cosh(d*x + c)^4 - (63
*a^2 + 654*a*b + 847*b^2)*cosh(d*x + c)^2 - 15*a^2 - 190*a*b - 175*b^2)*si
nh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*
cosh(d*x + c))
```

3.9.6 Sympy [F]

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2(c+dx)\right)^2 \sinh^4(c+dx)\,dx$$

input
```
integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*sinh(c + d*x)**4, x)
```

3.9.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(108) = 216.

Time = 0.20 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.50

$$\begin{aligned}
&\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx \\
&= \frac{1}{64}a^2\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) \\
&\quad + \frac{1}{192}b^2\left(\frac{840(dx+c)}{d} + \frac{3\left(24e^{(-2dx-2c)} - e^{(-4dx-4c)}\right)}{d} - \frac{63e^{(-2dx-2c)} + 1487e^{(-4dx-4c)} + 2517e^{(-6dx-}}{d(e^{(-4dx-4c)} + 3e^{(-6dx-6c)} + 3e^{(-8dx-}}\right. \\
&\quad + \frac{1}{32}ab\left(\frac{120(dx+c)}{d} + \frac{16e^{(-2dx-2c)} - e^{(-4dx-4c)}}{d} - \frac{15e^{(-2dx-2c)} + 144e^{(-4dx-4c)} - 1}{d(e^{(-4dx-4c)} + e^{(-6dx-6c)})}\right)
\end{aligned}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/64*a^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) + 1/192*b^2*(840*(d*x + c)/d + 3*(24*e^(-2*d*x
 - 2*c) - e^(-4*d*x - 4*c))/d - (63*e^(-2*d*x - 2*c) + 1487*e^(-4*d*x - 4*
c) + 2517*e^(-6*d*x - 6*c) + 1608*e^(-8*d*x - 8*c) - 3)/(d*(e^(-4*d*x - 4*
c) + 3*e^(-6*d*x - 6*c) + 3*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c)))) + 1/3
2*a*b*(120*(d*x + c)/d + (16*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (15*
e^(-2*d*x - 2*c) + 144*e^(-4*d*x - 4*c) - 1)/(d*(e^(-4*d*x - 4*c) + e^(-6*
d*x - 6*c))))
```

3.9.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(108) = 216$.

Time = 0.40 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.48

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$= \frac{3\,a^2e^{(4\,dx+4\,c)} + 6\,abe^{(4\,dx+4\,c)} + 3\,b^2e^{(4\,dx+4\,c)} - 24\,a^2e^{(2\,dx+2\,c)} - 96\,abe^{(2\,dx+2\,c)} - 72\,b^2e^{(2\,dx+2\,c)} + 24\,(3\,a^2 +}{}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/192*(3*a^2*e^(4*d*x + 4*c) + 6*a*b*e^(4*d*x + 4*c) + 3*b^2*e^(4*d*x + 4*
c) - 24*a^2*e^(2*d*x + 2*c) - 96*a*b*e^(2*d*x + 2*c) - 72*b^2*e^(2*d*x + 2
*c) + 24*(3*a^2 + 30*a*b + 35*b^2)*(d*x + c) - 3*(18*a^2*e^(4*d*x + 4*c) +
 180*a*b*e^(4*d*x + 4*c) + 210*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c)
 - 32*a*b*e^(2*d*x + 2*c) - 24*b^2*e^(2*d*x + 2*c) + a^2 + 2*a*b + b^2)*e^
(-4*d*x - 4*c) + 256*(3*a*b*e^(4*d*x + 4*c) + 6*b^2*e^(4*d*x + 4*c) + 6*a*
b*e^(2*d*x + 2*c) + 9*b^2*e^(2*d*x + 2*c) + 3*a*b + 5*b^2)/(e^(2*d*x + 2*c
) + 1)^3)/d
```

3.9.9 Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.48

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$= \frac{\frac{4\,(b^2+a\,b)}{3\,d} + \frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(2\,b^2+a\,b)}{3\,d}}{2\,\mathrm{e}^{2\,c+2\,d\,x} + \mathrm{e}^{4\,c+4\,d\,x} + 1} + x\left(\frac{3\,a^2}{8} + \frac{15\,a\,b}{4} + \frac{35\,b^2}{8}\right)$$
$$+ \frac{\frac{4\,(2\,b^2+a\,b)}{3\,d} + \frac{8\,\mathrm{e}^{2\,c+2\,d\,x}\,(b^2+a\,b)}{3\,d} + \frac{4\,\mathrm{e}^{4\,c+4\,d\,x}\,(2\,b^2+a\,b)}{3\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x} + 3\,\mathrm{e}^{4\,c+4\,d\,x} + \mathrm{e}^{6\,c+6\,d\,x} + 1}$$
$$+ \frac{4\,(2\,b^2+a\,b)}{3\,d\,(\mathrm{e}^{2\,c+2\,d\,x}+1)} + \frac{\mathrm{e}^{-2\,c-2\,d\,x}\,(a^2+4\,a\,b+3\,b^2)}{8\,d}$$
$$- \frac{\mathrm{e}^{2\,c+2\,d\,x}\,(a^2+4\,a\,b+3\,b^2)}{8\,d} - \frac{\mathrm{e}^{-4\,c-4\,d\,x}\,(a+b)^2}{64\,d} + \frac{\mathrm{e}^{4\,c+4\,d\,x}\,(a+b)^2}{64\,d}$$

input
```
int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2,x)
```

output

```
((4*(a*b + b^2))/(3*d) + (4*exp(2*c + 2*d*x)*(a*b + 2*b^2))/(3*d))/(2*exp(
2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + x*((15*a*b)/4 + (3*a^2)/8 + (35*b^2
)/8) + ((4*(a*b + 2*b^2))/(3*d) + (8*exp(2*c + 2*d*x)*(a*b + b^2))/(3*d) +
 (4*exp(4*c + 4*d*x)*(a*b + 2*b^2))/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c
 + 4*d*x) + exp(6*c + 6*d*x) + 1) + (4*(a*b + 2*b^2))/(3*d*(exp(2*c + 2*d*
x) + 1)) + (exp(- 2*c - 2*d*x)*(4*a*b + a^2 + 3*b^2))/(8*d) - (exp(2*c + 2
*d*x)*(4*a*b + a^2 + 3*b^2))/(8*d) - (exp(- 4*c - 4*d*x)*(a + b)^2)/(64*d)
 + (exp(4*c + 4*d*x)*(a + b)^2)/(64*d)
```

3.10 $\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

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3.10.1 Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{(a+b)(a+3b)\cosh(c+dx)}{d} + \frac{(a+b)^2\cosh^3(c+dx)}{3d} - \frac{b(2a+3b)\operatorname{sech}(c+dx)}{d} + \frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

output
```
-(a+b)*(a+3*b)*cosh(d*x+c)/d+1/3*(a+b)^2*cosh(d*x+c)^3/d-b*(2*a+3*b)*sech(
d*x+c)/d+1/3*b^2*sech(d*x+c)^3/d
```

3.10.2 Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{-3(3a^2+14ab+11b^2)\cosh(c+dx)+(a+b)^2\cosh(3(c+dx))+4b\operatorname{sech}(c+dx)\left(-6a-9b+b\operatorname{sech}^2(c+\right.}{12d}$$

input
```
Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-3*(3*a^2 + 14*a*b + 11*b^2)*Cosh[c + d*x] + (a + b)^2*Cosh[3*(c + d*x)]
+ 4*b*Sech[c + d*x]*(-6*a - 9*b + b*Sech[c + d*x]^2))/(12*d)
```

3.10.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 26, 4147, 25, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int i\sin(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\downarrow 26$$

$$i\int \sin(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\downarrow 4147$$

$$\frac{\int -\cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)\left(-b\text{sech}^2(c+dx)+a+b\right)^2 d\text{sech}(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{\int \cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)\left(-b\text{sech}^2(c+dx)+a+b\right)^2 d\text{sech}(c+dx)}{d}$$

$$\downarrow 355$$

$$-\frac{\int \left((a+b)^2\cosh^4(c+dx)+(-a-3b)(a+b)\cosh^2(c+dx)-b^2\text{sech}^2(c+dx)+b(2a+3b)\right)d\text{sech}(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{3}(a+b)^2\cosh^3(c+dx)-(a+b)(a+3b)\cosh(c+dx)-b(2a+3b)\text{sech}(c+dx)+\frac{1}{3}b^2\text{sech}^3(c+dx)}{d}$$

input
```
Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-((a + b)*(a + 3*b)*Cosh[c + d*x]) + ((a + b)^2*Cosh[c + d*x]^3)/3 - b*(2
*a + 3*b)*Sech[c + d*x] + (b^2*Sech[c + d*x]^3)/3)/d
```

3.10.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.10.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(73) = 146$.

Time $= 1.97$ (sec) , antiderivative size $= 148$, normalized size of antiderivative $= 1.92$

method	result
derivativedivides	$\frac{a^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+2ab\left(\frac{\sinh(dx+c)^4}{3\cosh(dx+c)}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)}-\frac{8}{3\cosh(dx+c)}\right)+b^2\left(\frac{\sinh(dx+c)^6}{3\cosh(dx+c)^3}-\frac{2\sinh(dx+c)^4}{\cosh(dx+c)^3}\right.}{d}$
default	$\frac{a^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+2ab\left(\frac{\sinh(dx+c)^4}{3\cosh(dx+c)}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)}-\frac{8}{3\cosh(dx+c)}\right)+b^2\left(\frac{\sinh(dx+c)^6}{3\cosh(dx+c)^3}-\frac{2\sinh(dx+c)^4}{\cosh(dx+c)^3}\right.}{d}$
risch	$\frac{e^{3dx+3c}a^2}{24d}+\frac{e^{3dx+3c}ab}{12d}+\frac{e^{3dx+3c}b^2}{24d}-\frac{3e^{dx+c}a^2}{8d}-\frac{7e^{dx+c}ab}{4d}-\frac{11b^2e^{dx+c}}{8d}-\frac{3e^{-dx-c}a^2}{8d}-\frac{7e^{-dx-c}ab}{4d}-$

input
```
int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+2*a*b*(1/3*sinh(d*x+c)^4/cos
h(d*x+c)-4/3*sinh(d*x+c)^2/cosh(d*x+c)-8/3/cosh(d*x+c))+b^2*(1/3*sinh(d*x+
c)^6/cosh(d*x+c)^3-2*sinh(d*x+c)^4/cosh(d*x+c)^3-8*sinh(d*x+c)^2/cosh(d*x+
c)^3-16/3/cosh(d*x+c)^3))
```

3.10.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(73) = 146$.

Time $= 0.25$ (sec) , antiderivative size $= 259$, normalized size of antiderivative $= 3.36$

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{(a^2+2\,ab+b^2)\cosh(dx+c)^6+(a^2+2\,ab+b^2)\sinh(dx+c)^6-6\,(a^2+6\,ab+5\,b^2)\cosh(dx+c)^4+3\,(}{}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/24*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + (a^2 + 2*a*b + b^2)*sinh(d*x +
 c)^6 - 6*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)
*cosh(d*x + c)^2 - 2*a^2 - 12*a*b - 10*b^2)*sinh(d*x + c)^4 - 3*(11*a^2 +
86*a*b + 91*b^2)*cosh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^
4 - 12*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 - 11*a^2 - 86*a*b - 91*b^2)*s
inh(d*x + c)^2 - 26*a^2 - 220*a*b - 210*b^2)/(d*cosh(d*x + c)^3 + 3*d*cosh
(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

3.10.6 Sympy [F]

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2(c+dx)\right)^2\sinh^3(c+dx)\,dx$$

input
```
integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*sinh(c + d*x)**3, x)
```

3.10.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(73) = 146$.

Time = 0.21 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.44

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$
$$-\frac{1}{24}b^2\left(\frac{33\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{30\,e^{(-2\,dx-2\,c)}+240\,e^{(-4\,dx-4\,c)}+322\,e^{(-6\,dx-6\,c)}+177\,e^{(-8\,dx-8\,c)}-1}{d(e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}+3\,e^{(-7\,dx-7\,c)}+e^{(-9\,dx-9\,c)})}\right.$$
$$-\frac{1}{12}ab\left(\frac{21\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{20\,e^{(-2\,dx-2\,c)}+69\,e^{(-4\,dx-4\,c)}-1}{d(e^{(-3\,dx-3\,c)}+e^{(-5\,dx-5\,c)})}\right)$$
$$+\frac{1}{24}a^2\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/24*b^2*((33*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (30*e^(-2*d*x - 2*c) +
 240*e^(-4*d*x - 4*c) + 322*e^(-6*d*x - 6*c) + 177*e^(-8*d*x - 8*c) - 1)/(
d*(e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c) + e^(-9*d*x
 - 9*c)))) - 1/12*a*b*((21*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (20*e^(-2*d
*x - 2*c) + 69*e^(-4*d*x - 4*c) - 1)/(d*(e^(-3*d*x - 3*c) + e^(-5*d*x - 5*
c)))) + 1/24*a^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d +
 e^(-3*d*x - 3*c)/d)
```

3.10.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(73) = 146$.

Time = 0.37 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.66

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+2\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-12\,a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)-}{24\,d}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/24*(a^2*(e^(d*x + c) + e^(-d*x - c))^3 + 2*a*b*(e^(d*x + c) + e^(-d*x -
c))^3 + b^2*(e^(d*x + c) + e^(-d*x - c))^3 - 12*a^2*(e^(d*x + c) + e^(-d*x
 - c)) - 48*a*b*(e^(d*x + c) + e^(-d*x - c)) - 36*b^2*(e^(d*x + c) + e^(-d
*x - c)) - 16*(6*a*b*(e^(d*x + c) + e^(-d*x - c))^2 + 9*b^2*(e^(d*x + c) +
 e^(-d*x - c))^2 - 4*b^2)/(e^(d*x + c) + e^(-d*x - c))^3)/d
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.79

$$\begin{aligned}
&\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx \\
&= \frac{\mathrm{e}^{-3c-3dx}(a+b)^2}{24d} - \frac{\mathrm{e}^{c+dx}(3a^2+14ab+11b^2)}{8d} + \frac{\mathrm{e}^{3c+3dx}(a+b)^2}{24d} \\
&\quad - \frac{\mathrm{e}^{-c-dx}(3a^2+14ab+11b^2)}{8d} - \frac{8b^2\mathrm{e}^{c+dx}}{3d\left(3\mathrm{e}^{2c+2dx}+3\mathrm{e}^{4c+4dx}+\mathrm{e}^{6c+6dx}+1\right)} \\
&\quad - \frac{2\mathrm{e}^{c+dx}(3b^2+2ab)}{d\left(\mathrm{e}^{2c+2dx}+1\right)} + \frac{8b^2\mathrm{e}^{c+dx}}{3d\left(2\mathrm{e}^{2c+2dx}+\mathrm{e}^{4c+4dx}+1\right)}
\end{aligned}$$

input
```
int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
(exp(- 3*c - 3*d*x)*(a + b)^2)/(24*d) - (exp(c + d*x)*(14*a*b + 3*a^2 + 11
*b^2))/(8*d) + (exp(3*c + 3*d*x)*(a + b)^2)/(24*d) - (exp(- c - d*x)*(14*a
*b + 3*a^2 + 11*b^2))/(8*d) - (8*b^2*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x
) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (2*exp(c + d*x)*(2*a*b +
 3*b^2))/(d*(exp(2*c + 2*d*x) + 1)) + (8*b^2*exp(c + d*x))/(3*d*(2*exp(2*c
 + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.11 $\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

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3.11.1 Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{1}{2}(a+b)(a+5b)x + \frac{(a+b)(a+5b)\tanh(c+dx)}{2d} + \frac{(a+b)^2\sinh^2(c+dx)\tanh(c+dx)}{2d} + \frac{b^2\tanh^3(c+dx)}{3d}$$

output
```
-1/2*(a+b)*(a+5*b)*x+1/2*(a+b)*(a+5*b)*tanh(d*x+c)/d+1/2*(a+b)^2*sinh(d*x+
c)^2*tanh(d*x+c)/d+1/3*b^2*tanh(d*x+c)^3/d
```

3.11.2 Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{-6(a^2+6ab+5b^2)(c+dx)+3(a+b)^2\sinh(2(c+dx))+4b\left(6a+7b-b\operatorname{sech}^2(c+dx)\right)\tanh(c+dx)}{12d}$$

input
```
Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-6*(a^2 + 6*a*b + 5*b^2)*(c + d*x) + 3*(a + b)^2*Sinh[2*(c + d*x)] + 4*b*
(6*a + 7*b - b*Sech[c + d*x]^2)*Tanh[c + d*x])/(12*d)
```

3.11.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 25, 4146, 366, 363, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int -\sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\downarrow 25$$

$$-\int \sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\downarrow 4146$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{\left(1-\tanh^2(c+dx)\right)^2} d\tanh(c+dx)}{d}$$

$$\downarrow 366$$

$$\frac{\frac{(a+b)^2\tanh^3(c+dx)}{2\left(1-\tanh^2(c+dx)\right)} - \frac{1}{2}\int \frac{\tanh^2(c+dx)\left(a^2+6ba+3b^2+2b^2\tanh^2(c+dx)\right)}{1-\tanh^2(c+dx)} d\tanh(c+dx)}{d}$$

$$\downarrow 363$$

$$\frac{\frac{1}{2}\left(\frac{2}{3}b^2\tanh^3(c+dx) - (a+b)(a+5b)\int \frac{\tanh^2(c+dx)}{1-\tanh^2(c+dx)} d\tanh(c+dx)\right) + \frac{(a+b)^2\tanh^3(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 262$$

$$\frac{\frac{1}{2}\left(\frac{2}{3}b^2\tanh^3(c+dx) - (a+b)(a+5b)\left(\int \frac{1}{1-\tanh^2(c+dx)} d\tanh(c+dx) - \tanh(c+dx)\right)\right) + \frac{(a+b)^2\tanh^3(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 219$$

$$\frac{\frac{1}{2}\left(\frac{2}{3}b^2\tanh^3(c+dx) - (a+b)(a+5b)(\operatorname{arctanh}(\tanh(c+dx)) - \tanh(c+dx))\right) + \frac{(a+b)^2\tanh^3(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}}{d}$$

input
```
Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(((a + b)^2*Tanh[c + d*x]^3)/(2*(1 - Tanh[c + d*x]^2)) + (-((a + b)*(a + 5
*b)*(ArcTanh[Tanh[c + d*x]] - Tanh[c + d*x])) + (2*b^2*Tanh[c + d*x]^3)/3)
/2)/d
```

3.11.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.11.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.49

method	result
derivativedivides	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)+b^2\left(\frac{\sinh(dx+c)^5}{2\cosh(dx+c)^3}-\frac{5dx}{2}-\frac{5c}{2}+\frac{5\tanh(d}{2}\right.}{d}$
default	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)+b^2\left(\frac{\sinh(dx+c)^5}{2\cosh(dx+c)^3}-\frac{5dx}{2}-\frac{5c}{2}+\frac{5\tanh(d}{2}\right.}{d}$
risch	$-\frac{a^2x}{2}-3abx-\frac{5b^2x}{2}+\frac{e^{2dx+2c}a^2}{8d}+\frac{e^{2dx+2c}ab}{4d}+\frac{e^{2dx+2c}b^2}{8d}-\frac{e^{-2dx-2c}a^2}{8d}-\frac{e^{-2dx-2c}ab}{4d}-\frac{e^{-2dx-2c}}{8d}$

input

```
int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output

```
1/d*(a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+2*a*b*(1/2*sinh(d*x+c
)^3/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c))+b^2*(1/2*sinh(d*x+c)^5/cosh
(d*x+c)^3-5/2*d*x-5/2*c+5/2*tanh(d*x+c)+5/6*tanh(d*x+c)^3))
```

3.11.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(71) = 142.

Time = 0.26 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.68

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{3\left(a^2+2ab+b^2\right)\sinh(dx+c)^5-4\left(3\left(a^2+6ab+5b^2\right)dx+12ab+14b^2\right)\cosh(dx+c)^3-12\left(3\left(a^2+6\right.\right.}{}$$

input

```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/24*(3*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^5 - 4*(3*(a^2 + 6*a*b + 5*b^2)*d
*x + 12*a*b + 14*b^2)*cosh(d*x + c)^3 - 12*(3*(a^2 + 6*a*b + 5*b^2)*d*x +
12*a*b + 14*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (30*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^2 + 9*a^2 + 66*a*b + 65*b^2)*sinh(d*x + c)^3 - 12*(3*(a^2 + 6
*a*b + 5*b^2)*d*x + 12*a*b + 14*b^2)*cosh(d*x + c) + 3*(5*(a^2 + 2*a*b + b
^2)*cosh(d*x + c)^4 + (9*a^2 + 66*a*b + 65*b^2)*cosh(d*x + c)^2 + 2*a^2 +
20*a*b + 10*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sin
h(d*x + c)^2 + 3*d*cosh(d*x + c))
```

3.11.6 Sympy [F]

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2(c+dx)\right)^2 \sinh^2(c+dx)\,dx$$

input
```
integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*sinh(c + d*x)**2, x)
```

3.11.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(71) = 142.

Time = 0.22 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.75

$$\begin{aligned}\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx &= -\frac{1}{8}a^2\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right)\\ &-\frac{1}{24}b^2\left(\frac{60\,(dx+c)}{d} + \frac{3\,e^{(-2dx-2c)}}{d} - \frac{121\,e^{(-2dx-2c)} + 201\,e^{(-4dx-4c)} + 147\,e^{(-6dx-6c)} + 3}{d(e^{(-2dx-2c)} + 3\,e^{(-4dx-4c)} + 3\,e^{(-6dx-6c)} + e^{(-8dx-8c)})}\right)\\ &-\frac{1}{4}ab\left(\frac{12\,(dx+c)}{d} + \frac{e^{(-2dx-2c)}}{d} - \frac{17\,e^{(-2dx-2c)}+1}{d(e^{(-2dx-2c)} + e^{(-4dx-4c)})}\right)\end{aligned}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/24*b^2*(60*(d*
x + c)/d + 3*e^(-2*d*x - 2*c)/d - (121*e^(-2*d*x - 2*c) + 201*e^(-4*d*x -
4*c) + 147*e^(-6*d*x - 6*c) + 3)/(d*(e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c)
 + 3*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c)))) - 1/4*a*b*(12*(d*x + c)/d + e^
(-2*d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*
d*x - 4*c))))
```

3.11.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(71) = 142$.

Time $= 0.34$ (sec) , antiderivative size $= 213$, normalized size of antiderivative $= 2.70$

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$= \frac{3\,a^2e^{(2\,dx+2\,c)} + 6\,abe^{(2\,dx+2\,c)} + 3\,b^2e^{(2\,dx+2\,c)} - 12\,(a^2+6\,ab+5\,b^2)(dx+c) + 3\left(2\,a^2e^{(2\,dx+2\,c)} + 12\,abe^{(2\,dx}\right.}{}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/24*(3*a^2*e^(2*d*x + 2*c) + 6*a*b*e^(2*d*x + 2*c) + 3*b^2*e^(2*d*x + 2*c
) - 12*(a^2 + 6*a*b + 5*b^2)*(d*x + c) + 3*(2*a^2*e^(2*d*x + 2*c) + 12*a*b
*e^(2*d*x + 2*c) + 10*b^2*e^(2*d*x + 2*c) - a^2 - 2*a*b - b^2)*e^(-2*d*x -
 2*c) - 16*(6*a*b*e^(4*d*x + 4*c) + 9*b^2*e^(4*d*x + 4*c) + 12*a*b*e^(2*d*
x + 2*c) + 12*b^2*e^(2*d*x + 2*c) + 6*a*b + 7*b^2)/(e^(2*d*x + 2*c) + 1)^3
)/d
```

3.11.9 Mupad [B] (verification not implemented)

Time $= 1.87$ (sec) , antiderivative size $= 248$, normalized size of antiderivative $= 3.14$

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$= \frac{e^{2\,c+2\,d\,x}\,(a+b)^2}{8\,d} - x\left(\frac{a^2}{2}+3\,a\,b+\frac{5\,b^2}{2}\right)$$
$$- \frac{\frac{2\,(3\,b^2+2\,a\,b)}{3\,d} + \frac{4\,e^{2\,c+2\,d\,x}\,(b^2+2\,a\,b)}{3\,d} + \frac{2\,e^{4\,c+4\,d\,x}\,(3\,b^2+2\,a\,b)}{3\,d}}{3\,e^{2\,c+2\,d\,x} + 3\,e^{4\,c+4\,d\,x} + e^{6\,c+6\,d\,x} + 1} - \frac{2\,(3\,b^2+2\,a\,b)}{3\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$
$$- \frac{e^{-2\,c-2\,d\,x}\,(a+b)^2}{8\,d} - \frac{\frac{2\,(b^2+2\,a\,b)}{3\,d} + \frac{2\,e^{2\,c+2\,d\,x}\,(3\,b^2+2\,a\,b)}{3\,d}}{2\,e^{2\,c+2\,d\,x} + e^{4\,c+4\,d\,x} + 1}$$

input
```
int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
(exp(2*c + 2*d*x)*(a + b)^2)/(8*d) - x*(3*a*b + a^2/2 + (5*b^2)/2) - ((2*(
2*a*b + 3*b^2))/(3*d) + (4*exp(2*c + 2*d*x)*(2*a*b + b^2))/(3*d) + (2*exp(
4*c + 4*d*x)*(2*a*b + 3*b^2))/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d
*x) + exp(6*c + 6*d*x) + 1) - (2*(2*a*b + 3*b^2))/(3*d*(exp(2*c + 2*d*x) +
 1)) - (exp(- 2*c - 2*d*x)*(a + b)^2)/(8*d) - ((2*(2*a*b + b^2))/(3*d) + (
2*exp(2*c + 2*d*x)*(2*a*b + 3*b^2))/(3*d))/(2*exp(2*c + 2*d*x) + exp(4*c +
 4*d*x) + 1)
```

3.12 $\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

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3.12.1 Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{(a+b)^2\cosh(c+dx)}{d} + \frac{2b(a+b)\operatorname{sech}(c+dx)}{d} - \frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

output
```
(a+b)^2*cosh(d*x+c)/d+2*b*(a+b)*sech(d*x+c)/d-1/3*b^2*sech(d*x+c)^3/d
```

3.12.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{3(a+b)^2\cosh(c+dx)+b\operatorname{sech}(c+dx)\left(6(a+b)-b\operatorname{sech}^2(c+dx)\right)}{3d}$$

input
```
Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(3*(a + b)^2*Cosh[c + d*x] + b*Sech[c + d*x]*(6*(a + b) - b*Sech[c + d*x]^
2))/(3*d)
```

3.12.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 26, 4147, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int -i\sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\downarrow 26$$

$$-i\int \sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\downarrow 4147$$

$$-\frac{\int \cosh^2(c+dx)\left(-b\text{sech}^2(c+dx)+a+b\right)^2 d\text{sech}(c+dx)}{d}$$

$$\downarrow 244$$

$$-\frac{\int \left((a+b)^2\cosh^2(c+dx)+b^2\text{sech}^2(c+dx)-2b(a+b)\right) d\text{sech}(c+dx)}{d}$$

$$\downarrow 2009$$

$$-\frac{-(a+b)^2\cosh(c+dx)-2b(a+b)\text{sech}(c+dx)+\frac{1}{3}b^2\text{sech}^3(c+dx)}{d}$$

input
```
Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-((-((a + b)^2*Cosh[c + d*x]) - 2*b*(a + b)*Sech[c + d*x] + (b^2*Sech[c +
d*x]^3)/3)/d)
```

3.12.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.12.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(47) = 94$.

Time $= 0.53$ (sec) , antiderivative size $= 98$, normalized size of antiderivative $= 2.00$

method	result
derivativedivides	$\frac{\cosh(dx+c)a^2+2ab\left(\frac{\sinh(dx+c)^2}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)+b^2\left(\frac{\sinh(dx+c)^4}{\cosh(dx+c)^3}+\frac{4\sinh(dx+c)^2}{\cosh(dx+c)^3}+\frac{8}{3\cosh(dx+c)^3}\right)}{d}$
default	$\frac{\cosh(dx+c)a^2+2ab\left(\frac{\sinh(dx+c)^2}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)+b^2\left(\frac{\sinh(dx+c)^4}{\cosh(dx+c)^3}+\frac{4\sinh(dx+c)^2}{\cosh(dx+c)^3}+\frac{8}{3\cosh(dx+c)^3}\right)}{d}$
risch	$\frac{e^{dx+c}a^2}{2d}+\frac{e^{dx+c}ab}{d}+\frac{b^2e^{dx+c}}{2d}+\frac{e^{-dx-c}a^2}{2d}+\frac{e^{-dx-c}ab}{d}+\frac{e^{-dx-c}b^2}{2d}+\frac{4be^{dx+c}\left(3ae^{4dx+4c}+3be^{4dx+4c}+6e\right.}{3d\left(e^{2dx+2c}\right.}$

input
```
int(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(cosh(d*x+c)*a^2+2*a*b*(sinh(d*x+c)^2/cosh(d*x+c)+2/cosh(d*x+c))+b^2*(
sinh(d*x+c)^4/cosh(d*x+c)^3+4*sinh(d*x+c)^2/cosh(d*x+c)^3+8/3/cosh(d*x+c)^
3))
```

3.12.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(47) = 94$.

Time $= 0.27$ (sec) , antiderivative size $= 167$, normalized size of antiderivative $= 3.41$

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{3\left(a^2+2ab+b^2\right)\cosh(dx+c)^4+3\left(a^2+2ab+b^2\right)\sinh(dx+c)^4+12\left(a^2+4ab+3b^2\right)\cosh(dx+c)^2-}{6\left(d\cosh(dx+c)^3+3d\cosh(dx+c)\sinh\right.}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/6*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*sinh(d*
x + c)^4 + 12*(a^2 + 4*a*b + 3*b^2)*cosh(d*x + c)^2 + 6*(3*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^2 + 2*a^2 + 8*a*b + 6*b^2)*sinh(d*x + c)^2 + 9*a^2 + 42
*a*b + 25*b^2)/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*
d*cosh(d*x + c))
```

3.12.6 Sympy [F]

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2(c+dx)\right)^2\sinh(c+dx)\,dx$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*sinh(c + d*x), x)
```

3.12.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(47) = 94$.

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 3.49

$$\begin{aligned}&\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx\\&=\frac{1}{6}b^2\left(\frac{3\,e^{(-dx-c)}}{d}+\frac{33\,e^{(-2\,dx-2\,c)}+41\,e^{(-4\,dx-4\,c)}+27\,e^{(-6\,dx-6\,c)}+3}{d(e^{(-dx-c)}+3\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}+e^{(-7\,dx-7\,c)})}\right)\\&\quad+ab\left(\frac{e^{(-dx-c)}}{d}+\frac{5\,e^{(-2\,dx-2\,c)}+1}{d(e^{(-dx-c)}+e^{(-3\,dx-3\,c)})}\right)+\frac{a^2\cosh(dx+c)}{d}\end{aligned}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/6*b^2*(3*e^(-d*x - c)/d + (33*e^(-2*d*x - 2*c) + 41*e^(-4*d*x - 4*c) + 2
7*e^(-6*d*x - 6*c) + 3)/(d*(e^(-d*x - c) + 3*e^(-3*d*x - 3*c) + 3*e^(-5*d*
x - 5*c) + e^(-7*d*x - 7*c)))) + a*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c)
 + 1)/(d*(e^(-d*x - c) + e^(-3*d*x - 3*c)))) + a^2*cosh(d*x + c)/d
```

3.12.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(47) = 94$.

Time = 0.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.84

$$\begin{aligned}&\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx\\&=\frac{3\,a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)+6\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)+3\,b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)+\frac{8\left(3\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+3\,b^2(e^{(}\right.}{(e^{(dx+c)}+e^{(-dx-}}}{6\,d}\end{aligned}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/6*(3*a^2*(e^(d*x + c) + e^(-d*x - c)) + 6*a*b*(e^(d*x + c) + e^(-d*x - c
)) + 3*b^2*(e^(d*x + c) + e^(-d*x - c)) + 8*(3*a*b*(e^(d*x + c) + e^(-d*x
 - c))^2 + 3*b^2*(e^(d*x + c) + e^(-d*x - c))^2 - 2*b^2)/(e^(d*x + c) + e^(
-d*x - c))^3)/d
```

3.12.9 Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 154, normalized size of antiderivative = 3.14

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\mathrm{e}^{c+dx}(a+b)^2}{2d} + \frac{\mathrm{e}^{-c-dx}(a+b)^2}{2d} + \frac{8b^2\,\mathrm{e}^{c+dx}}{3d\left(3\,\mathrm{e}^{2c+2dx}+3\,\mathrm{e}^{4c+4dx}+\mathrm{e}^{6c+6dx}+1\right)} + \frac{4\,\mathrm{e}^{c+dx}(b^2+ab)}{d\left(\mathrm{e}^{2c+2dx}+1\right)} - \frac{8b^2\,\mathrm{e}^{c+dx}}{3d\left(2\,\mathrm{e}^{2c+2dx}+\mathrm{e}^{4c+4dx}+1\right)}$$

input
```
int(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
(exp(c + d*x)*(a + b)^2)/(2*d) + (exp(- c - d*x)*(a + b)^2)/(2*d) + (8*b^2
*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6
*d*x) + 1)) + (4*exp(c + d*x)*(a*b + b^2))/(d*(exp(2*c + 2*d*x) + 1)) - (8
*b^2*exp(c + d*x))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.13 $\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

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3.13.1 Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{a^2\operatorname{arctanh}(\cosh(c+dx))}{d} - \frac{b(2a+b)\operatorname{sech}(c+dx)}{d} + \frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

output
```
-a^2*arctanh(cosh(d*x+c))/d-b*(2*a+b)*sech(d*x+c)/d+1/3*b^2*sech(d*x+c)^3/
d
```

3.13.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.25

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{3a^2\left(-\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\right)-3b(2a+b)\operatorname{sech}(c+dx)+b^2\operatorname{sech}^3(c+dx)}{3d}$$

input
```
Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(3*a^2*(-Log[Cosh[(c + d*x)/2]] + Log[Sinh[(c + d*x)/2]]) - 3*b*(2*a + b)*
Sech[c + d*x] + b^2*Sech[c + d*x]^3)/(3*d)
```

3.13.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 26, 4147, 25, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{i\left(a-b\tan(ic+idx)^2\right)^2}{\sin(ic+idx)}dx$$

$$\downarrow 26$$

$$i\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\sin(ic+idx)}dx$$

$$\downarrow 4147$$

$$\frac{\int -\frac{\left(-b\operatorname{sech}^2(c+dx)+a+b\right)^2}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\left(-b\operatorname{sech}^2(c+dx)+a+b\right)^2}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)}{d}$$

$$\downarrow 300$$

$$-\frac{\int \left(\frac{a^2}{1-\operatorname{sech}^2(c+dx)}-b^2\operatorname{sech}^2(c+dx)+b(2a+b)\right)d\operatorname{sech}(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{-a^2\operatorname{arctanh}(\operatorname{sech}(c+dx))-b(2a+b)\operatorname{sech}(c+dx)+\frac{1}{3}b^2\operatorname{sech}^3(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-(a^2*ArcTanh[Sech[c + d*x]]) - b*(2*a + b)*Sech[c + d*x] + (b^2*Sech[c +
 d*x]^3)/3)/d
```

3.13.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.13.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

method	result	siz
derivativedivides	$\frac{-2a^2 \operatorname{arctanh}\left(e^{dx+c}\right)-\frac{2ab}{\cosh(dx+c)}+b^2\left(-\frac{\sinh(dx+c)^2}{\cosh(dx+c)^3}-\frac{2}{3\cosh(dx+c)^3}\right)}{d}$	63
default	$\frac{-2a^2 \operatorname{arctanh}\left(e^{dx+c}\right)-\frac{2ab}{\cosh(dx+c)}+b^2\left(-\frac{\sinh(dx+c)^2}{\cosh(dx+c)^3}-\frac{2}{3\cosh(dx+c)^3}\right)}{d}$	63
risch	$-\frac{2b e^{dx+c}\left(6a e^{4dx+4c}+3b e^{4dx+4c}+12 e^{2dx+2c}a+2b e^{2dx+2c}+6a+3b\right)}{3d\left(e^{2dx+2c}+1\right)^3}+\frac{a^2\ln\left(e^{dx+c}-1\right)}{d}-\frac{a^2\ln\left(e^{dx+c}+1\right)}{d}$	11

input
```
int(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a^2*arctanh(exp(d*x+c))-2*a*b/cosh(d*x+c)+b^2*(-sinh(d*x+c)^2/cosh
(d*x+c)^3-2/3/cosh(d*x+c)^3))
```

3.13.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 890 vs. $2(49) = 98$.

Time $= 0.26$ (sec) , antiderivative size $= 890$, normalized size of antiderivative $= 17.45$

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(6*(2*a*b + b^2)*cosh(d*x + c)^5 + 30*(2*a*b + b^2)*cosh(d*x + c)*sin
h(d*x + c)^4 + 6*(2*a*b + b^2)*sinh(d*x + c)^5 + 4*(6*a*b + b^2)*cosh(d*x
+ c)^3 + 4*(15*(2*a*b + b^2)*cosh(d*x + c)^2 + 6*a*b + b^2)*sinh(d*x + c)^
3 + 12*(5*(2*a*b + b^2)*cosh(d*x + c)^3 + (6*a*b + b^2)*cosh(d*x + c))*sin
h(d*x + c)^2 + 6*(2*a*b + b^2)*cosh(d*x + c) + 3*(a^2*cosh(d*x + c)^6 + 6*
a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + 3*a^2*cosh(d*x +
 c)^4 + 3*(5*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 3*a^2*cosh(d*x +
 c)^2 + 4*(5*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 +
3*(5*a^2*cosh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 +
a^2 + 6*(a^2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*
sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 3*(a^2*cosh(d*x +
c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + 3*a^2*c
osh(d*x + c)^4 + 3*(5*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 3*a^2*c
osh(d*x + c)^2 + 4*(5*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*(5*a^2*cosh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x
+ c)^2 + a^2 + 6*(a^2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + a^2*cosh(d
*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 6*(5*(2*a
*b + b^2)*cosh(d*x + c)^4 + 2*(6*a*b + b^2)*cosh(d*x + c)^2 + 2*a*b + b^2)
*sinh(d*x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d
*sinh(d*x + c)^6 + 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + d)*si...
```

3.13.6 Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^2 \operatorname{csch}\left(c+dx\right)dx$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x), x)
```

3.13.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(49) = 98$.

Time $= 0.20$ (sec) , antiderivative size $= 196$, normalized size of antiderivative $= 3.84$

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$
$$-\frac{2}{3}b^2\left(\frac{3\,e^{(-dx-c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{2\,e^{(-3\,dx-3\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right.$$
$$\left.+\frac{a^2\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}-\frac{4\,ab}{d(e^{(dx+c)}+e^{(-dx-c)})}\right.$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-2/3*b^2*(3*e^(-d*x - c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1)) + 2*e^(-3*d*x - 3*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*
d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 3*e^(-5*d*x - 5*c)/(d*(3*e^(-2*d*x -
 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + a^2*log(tanh(1/2*d*
x + 1/2*c))/d - 4*a*b/(d*(e^(d*x + c) + e^(-d*x - c)))
```

3.13.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(49) = 98$.

Time $= 0.33$ (sec) , antiderivative size $= 123$, normalized size of antiderivative $= 2.41$

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{3a^2\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-3a^2\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)+\frac{4\left(6ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+3b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4b^2\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3}}{6d}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/6*(3*a^2*log(e^(d*x + c) + e^(-d*x - c) + 2) - 3*a^2*log(e^(d*x + c) +
e^(-d*x - c) - 2) + 4*(6*a*b*(e^(d*x + c) + e^(-d*x - c))^2 + 3*b^2*(e^(d*
x + c) + e^(-d*x - c))^2 - 4*b^2)/(e^(d*x + c) + e^(-d*x - c))^3)/d
```

3.13.9 Mupad [B] (verification not implemented)

Time $= 0.17$ (sec) , antiderivative size $= 160$, normalized size of antiderivative $= 3.14$

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{8b^2\,\mathrm{e}^{c+dx}}{3d\left(2\,\mathrm{e}^{2c+2dx}+\mathrm{e}^{4c+4dx}+1\right)} - \frac{8b^2\,\mathrm{e}^{c+dx}}{3d\left(3\,\mathrm{e}^{2c+2dx}+3\,\mathrm{e}^{4c+4dx}+\mathrm{e}^{6c+6dx}+1\right)} - \frac{2\,\mathrm{e}^{c+dx}\left(b^2+2ab\right)}{d\left(\mathrm{e}^{2c+2dx}+1\right)} - \frac{2\operatorname{atan}\left(\frac{a^2\,\mathrm{e}^{dx}\,\mathrm{e}^{c}\sqrt{-d^2}}{d\sqrt{a^4}}\right)\sqrt{a^4}}{\sqrt{-d^2}}$$

input
```
int((a + b*tanh(c + d*x)^2)^2/sinh(c + d*x),x)
```

output
```
(8*b^2*exp(c + d*x))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (
8*b^2*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*
c + 6*d*x) + 1)) - (2*exp(c + d*x)*(2*a*b + b^2))/(d*(exp(2*c + 2*d*x) + 1
)) - (2*atan((a^2*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^4)^(1/2)))*(a^4)^(1/
2))/(-d^2)^(1/2)
```

3.14 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

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3.14.1 Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{a^2\coth(c+dx)}{d}+\frac{2ab\tanh(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}$$

output
```
-a^2*coth(d*x+c)/d+2*a*b*tanh(d*x+c)/d+1/3*b^2*tanh(d*x+c)^3/d
```

3.14.2 Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{-3a^2\coth(c+dx)+b\left(6a+b-b\operatorname{sech}^2(c+dx)\right)\tanh(c+dx)}{3d}$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-3*a^2*Coth[c + d*x] + b*(6*a + b - b*Sech[c + d*x]^2)*Tanh[c + d*x])/(3*
d)
```

3.14.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 25, 4146, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a-b\tan(ic+idx)^2\right)^2}{\sin(ic+idx)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\sin(ic+idx)^2}dx$$

$$\downarrow 4146$$

$$\frac{\int \coth^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^2 d\tanh(c+dx)}{d}$$

$$\downarrow 244$$

$$\frac{\int \left(a^2\coth^2(c+dx)+b^2\tanh^2(c+dx)+2ab\right) d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{-a^2\coth(c+dx)+2ab\tanh(c+dx)+\frac{1}{3}b^2\tanh^3(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-(a^2*Coth[c + d*x]) + 2*a*b*Tanh[c + d*x] + (b^2*Tanh[c + d*x]^3)/3)/d
```

3.14.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.14.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

method	result
derivativedivides	$\frac{-a^2\coth(dx+c)+2ab\tanh(dx+c)+b^2\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)}{d}$
default	$\frac{-a^2\coth(dx+c)+2ab\tanh(dx+c)+b^2\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)}{d}$
risch	$-\frac{2(3a^2e^{6dx+6c}+6ab\,e^{6dx+6c}+3b^2e^{6dx+6c}+9a^2e^{4dx+4c}+6ab\,e^{4dx+4c}-3\,e^{4dx+4c}b^2+9a^2e^{2dx+2c}-6ab\,e^{2dx+2c}+e^{2dx+2}}{3d\left(e^{2dx+2c}+1\right)^3\left(e^{2dx+2c}-1\right)}$

input
```
int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2*coth(d*x+c)+2*a*b*tanh(d*x+c)+b^2*(-1/2*sinh(d*x+c)/cosh(d*x+c)^
3+1/2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)))
```

3.14.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(44) = 88$.

Time $= 0.25$ (sec) , antiderivative size $= 264$, normalized size of antiderivative $= 5.74$

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$

$$-\frac{4\left(\left(3\,a^2+b^2\right)\cosh\left(dx+c\right)^3+3\left(3\,a^2+b^2\right)\cosh\left(dx+c\right)\sin}{3\left(d\cosh\left(dx+c\right)^5+5\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4+d\sinh\left(dx+c\right)^5+d\cosh\left(dx+c\right)^3+\left(10\,d\cosh\right.\right.}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-4/3*((3*a^2 + b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + b^2)*cosh(d*x + c)*sinh(d
*x + c)^2 + 2*(3*a*b + b^2)*sinh(d*x + c)^3 + (9*a^2 - b^2)*cosh(d*x + c)
+ 2*(3*(3*a*b + b^2)*cosh(d*x + c)^2 + 3*a*b - b^2)*sinh(d*x + c))/(d*cosh
(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 + d*co
sh(d*x + c)^3 + (10*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^3 + (10*d*cosh(
d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 - 2*d*cosh(d*x + c) + (5*d
*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))
```

3.14.6 Sympy [F]

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^2\operatorname{csch}^2\left(c+dx\right)dx$$

input
```
integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x)**2, x)
```

3.14.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(44) = 88$.

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.96

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{2}{3}b^2\left(\frac{3\,e^{(-4\,dx-4\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)$$
$$+\frac{4\,ab}{d(e^{(-2\,dx-2\,c)}+1)}+\frac{2\,a^2}{d(e^{(-2\,dx-2\,c)}-1)}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
2/3*b^2*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) +
e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c) + 1))) + 4*a*b/(d*(e^(-2*d*x - 2*c) + 1)) + 2*a^2/(d*(e^(-
2*d*x - 2*c) - 1))
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.87

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= -\frac{2\left(\frac{3\,a^2}{e^{(2\,dx+2\,c)}-1}+\frac{6\,abe^{(4\,dx+4\,c)}+3\,b^2e^{(4\,dx+4\,c)}+12\,abe^{(2\,dx+2\,c)}+6\,ab+b^2}{\left(e^{(2\,dx+2\,c)}+1\right)^3}\right)}{3\,d}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-2/3*(3*a^2/(e^(2*d*x + 2*c) - 1) + (6*a*b*e^(4*d*x + 4*c) + 3*b^2*e^(4*d*
x + 4*c) + 12*a*b*e^(2*d*x + 2*c) + 6*a*b + b^2)/(e^(2*d*x + 2*c) + 1)^3)/
d
```

3.14.9 Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 209, normalized size of antiderivative = 4.54

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= -\frac{\frac{2\,(2\,a\,b-b^2)}{3\,d}+\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,(b^2+2\,a\,b)}{3\,d}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1}-\frac{\frac{2\,(b^2+2\,a\,b)}{3\,d}+\frac{2\,\mathrm{e}^{4\,c+4\,d\,x}\,(b^2+2\,a\,b)}{3\,d}+\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(2\,a\,b-b^2)}{3\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1}$$
$$-\frac{2\,a^2}{d\,(\mathrm{e}^{2\,c+2\,d\,x}-1)}-\frac{2\,(b^2+2\,a\,b)}{3\,d\,(\mathrm{e}^{2\,c+2\,d\,x}+1)}$$

input
```
int((a + b*tanh(c + d*x)^2)^2/sinh(c + d*x)^2,x)
```

output
```
- ((2*(2*a*b - b^2))/(3*d) + (2*exp(2*c + 2*d*x)*(2*a*b + b^2))/(3*d))/(2*
exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*(2*a*b + b^2))/(3*d) + (2*e
xp(4*c + 4*d*x)*(2*a*b + b^2))/(3*d) + (4*exp(2*c + 2*d*x)*(2*a*b - b^2))/
(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) -
(2*a^2)/(d*(exp(2*c + 2*d*x) - 1)) - (2*(2*a*b + b^2))/(3*d*(exp(2*c + 2*d
*x) + 1))
```

3.15 $\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

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3.15.1 Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{a(a-4b)\operatorname{arctanh}(\cosh(c+dx))}{2d} - \frac{a(a-4b)\operatorname{sech}(c+dx)}{2d} - \frac{a^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{2d} - \frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

output
```
1/2*a*(a-4*b)*arctanh(cosh(d*x+c))/d-1/2*a*(a-4*b)*sech(d*x+c)/d-1/2*a^2*c
sch(d*x+c)^2*sech(d*x+c)/d-1/3*b^2*sech(d*x+c)^3/d
```

3.15.2 Mathematica [A] (verified)

Time = 5.64 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.55

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{3a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right) - 12a^2\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right) + 48ab\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right) + 12a^2\log\left(\sinh\left(\frac{1}{2}(c\right.\right.}{24}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output

```
-1/24*(3*a^2*Csch[(c + d*x)/2]^2 - 12*a^2*Log[Cosh[(c + d*x)/2]] + 48*a*b*
Log[Cosh[(c + d*x)/2]] + 12*a^2*Log[Sinh[(c + d*x)/2]] - 48*a*b*Log[Sinh[(
c + d*x)/2]] + 3*a^2*Sech[(c + d*x)/2]^2 - 48*a*b*Sech[c + d*x] + 8*b^2*Se
ch[c + d*x]^3)/d
```

3.15.3 Rubi [A] (verified)

Time $= 0.29$ (sec) , antiderivative size $= 79$, normalized size of antiderivative $= 0.96$, number of steps used $= 8$, number of rules used $= 7$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used $= \{3042, 26, 4147, 366, 363, 262, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int -\frac{i\left(a-b\tan(ic+idx)^2\right)^2}{\sin(ic+idx)^3}dx$$

$$\downarrow 26$$

$$-i\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\sin(ic+idx)^3}dx$$

$$\downarrow 4147$$

$$-\frac{\int \frac{\operatorname{sech}^2(c+dx)\left(-b\operatorname{sech}^2(c+dx)+a+b\right)^2}{\left(1-\operatorname{sech}^2(c+dx)\right)^2}d\operatorname{sech}(c+dx)}{d}$$

$$\downarrow 366$$

$$-\frac{\frac{a^2\operatorname{sech}^3(c+dx)}{2\left(1-\operatorname{sech}^2(c+dx)\right)}-\frac{1}{2}\int \frac{\operatorname{sech}^2(c+dx)\left(3a^2-2(a+b)^2+2b^2\operatorname{sech}^2(c+dx)\right)}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)}{d}$$

$$\downarrow 363$$

$$-\frac{\frac{1}{2}\left(\frac{2}{3}b^2\operatorname{sech}^3(c+dx)-a(a-4b)\int \frac{\operatorname{sech}^2(c+dx)}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)\right)+\frac{a^2\operatorname{sech}^3(c+dx)}{2\left(1-\operatorname{sech}^2(c+dx)\right)}}{d}$$

$$\downarrow 262$$

$$-\frac{\frac{1}{2}\left(\frac{2}{3}b^2\text{sech}^3(c+dx)-a(a-4b)\left(\int\frac{1}{1-\text{sech}^2(c+dx)}d\text{sech}(c+dx)-\text{sech}(c+dx)\right)\right)+\frac{a^2\text{sech}^3(c+dx)}{2\left(1-\text{sech}^2(c+dx)\right)}}{d}$$

$$\downarrow 219$$

$$-\frac{\frac{a^2\text{sech}^3(c+dx)}{2\left(1-\text{sech}^2(c+dx)\right)}+\frac{1}{2}\left(\frac{2}{3}b^2\text{sech}^3(c+dx)-a(a-4b)(\text{arctanh}(\text{sech}(c+dx))-\text{sech}(c+dx))\right)}{d}$$

input
```
Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-(((a^2*Sech[c + d*x]^3)/(2*(1 - Sech[c + d*x]^2)) + (-(a*(a - 4*b)*(ArcTa
nh[Sech[c + d*x]] - Sech[c + d*x])) + (2*b^2*Sech[c + d*x]^3)/3)/2)/d)
```

3.15.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.15.4 Maple [A] (verified)

Time = 3.71 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{a^2\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+2ab\left(\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)-\frac{b^2}{3\cosh(dx+c)^3}}{d}$
default	$\frac{a^2\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+2ab\left(\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)-\frac{b^2}{3\cosh(dx+c)^3}}{d}$
risch	$-\frac{e^{dx+c}\left(3a^2e^{8dx+8c}-12ab\,e^{8dx+8c}+12a^2e^{6dx+6c}+8b^2e^{6dx+6c}+18a^2e^{4dx+4c}+24ab\,e^{4dx+4c}-16\,e^{4dx+4c}b^2+12a^2e^{2dx-}\right.}{3d\left(e^{2dx+2c}+1\right)^3\left(e^{2dx+2c}-1\right)^2}$

input
```
int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+2*a*b*(1/cosh(
d*x+c)-2*arctanh(exp(d*x+c)))-1/3*b^2/cosh(d*x+c)^3)
```

3.15.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2462 vs. 2(74) = 148.

Time = 0.29 (sec) , antiderivative size = 2462, normalized size of antiderivative = 30.02

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/6*(6*(a^2 - 4*a*b)*cosh(d*x + c)^9 + 54*(a^2 - 4*a*b)*cosh(d*x + c)*sin
h(d*x + c)^8 + 6*(a^2 - 4*a*b)*sinh(d*x + c)^9 + 8*(3*a^2 + 2*b^2)*cosh(d*
x + c)^7 + 8*(27*(a^2 - 4*a*b)*cosh(d*x + c)^2 + 3*a^2 + 2*b^2)*sinh(d*x +
 c)^7 + 56*(9*(a^2 - 4*a*b)*cosh(d*x + c)^3 + (3*a^2 + 2*b^2)*cosh(d*x + c
))*sinh(d*x + c)^6 + 4*(9*a^2 + 12*a*b - 8*b^2)*cosh(d*x + c)^5 + 4*(189*(
a^2 - 4*a*b)*cosh(d*x + c)^4 + 42*(3*a^2 + 2*b^2)*cosh(d*x + c)^2 + 9*a^2
+ 12*a*b - 8*b^2)*sinh(d*x + c)^5 + 4*(189*(a^2 - 4*a*b)*cosh(d*x + c)^5 +
 70*(3*a^2 + 2*b^2)*cosh(d*x + c)^3 + 5*(9*a^2 + 12*a*b - 8*b^2)*cosh(d*x
+ c))*sinh(d*x + c)^4 + 8*(3*a^2 + 2*b^2)*cosh(d*x + c)^3 + 8*(63*(a^2 - 4
*a*b)*cosh(d*x + c)^6 + 35*(3*a^2 + 2*b^2)*cosh(d*x + c)^4 + 5*(9*a^2 + 12
*a*b - 8*b^2)*cosh(d*x + c)^2 + 3*a^2 + 2*b^2)*sinh(d*x + c)^3 + 8*(27*(a^
2 - 4*a*b)*cosh(d*x + c)^7 + 21*(3*a^2 + 2*b^2)*cosh(d*x + c)^5 + 5*(9*a^2
 + 12*a*b - 8*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + 2*b^2)*cosh(d*x + c))*sinh
(d*x + c)^2 + 6*(a^2 - 4*a*b)*cosh(d*x + c) - 3*((a^2 - 4*a*b)*cosh(d*x +
c)^10 + 10*(a^2 - 4*a*b)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^2 - 4*a*b)*sin
h(d*x + c)^10 + (a^2 - 4*a*b)*cosh(d*x + c)^8 + (45*(a^2 - 4*a*b)*cosh(d*x
 + c)^2 + a^2 - 4*a*b)*sinh(d*x + c)^8 + 8*(15*(a^2 - 4*a*b)*cosh(d*x + c)
^3 + (a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(a^2 - 4*a*b)*cosh(d
*x + c)^6 + 2*(105*(a^2 - 4*a*b)*cosh(d*x + c)^4 + 14*(a^2 - 4*a*b)*cosh(d
*x + c)^2 - a^2 + 4*a*b)*sinh(d*x + c)^6 + 4*(63*(a^2 - 4*a*b)*cosh(d*x...
```

3.15.6 Sympy [F]

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^2 \operatorname{csch}^3\left(c+dx\right)\,dx$$

input
```
integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x)**3, x)
```

3.15.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(74) = 148$.

Time = 0.20 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.21

$$\begin{aligned}
&\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx \\
&= \frac{1}{2}a^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right) \\
&\quad - 2\,ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} - \frac{2\,e^{(-dx-c)}}{d(e^{(-2\,dx-2\,c)}+1)}\right) \\
&\quad - \frac{8\,b^2}{3\,d\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3}
\end{aligned}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*a^2*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x -
c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) -
2*a*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/
(d*(e^(-2*d*x - 2*c) + 1))) - 8/3*b^2/(d*(e^(d*x + c) + e^(-d*x - c))^3)
```

3.15.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(74) = 148$.

Time = 0.36 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.87

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$

$$-\frac{\frac{12\,a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4} - 3\left(a^2-4\,ab\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right) + 3\left(a^2-4\,ab\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right.}{12\,d}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/12*(12*a^2*(e^(d*x + c) + e^(-d*x - c))/((e^(d*x + c) + e^(-d*x - c))^2
 - 4) - 3*(a^2 - 4*a*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) + 3*(a^2 - 4*a
*b)*log(e^(d*x + c) + e^(-d*x - c) - 2) - 16*(3*a*b*(e^(d*x + c) + e^(-d*x
 - c))^2 - 2*b^2)/(e^(d*x + c) + e^(-d*x - c))^3)/d
```

3.15.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 261, normalized size of antiderivative = 3.18

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{e^{dx}e^c\left(a^2\sqrt{-d^2}-4ab\sqrt{-d^2}\right)}{d\sqrt{a^4-8a^3b+16a^2b^2}}\right)\sqrt{a^4-8a^3b+16a^2b^2}}{\sqrt{-d^2}}$$
$$+\frac{8b^2e^{c+dx}}{3d\left(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1\right)}-\frac{a^2e^{c+dx}}{d\left(e^{2c+2dx}-1\right)}$$
$$-\frac{2a^2e^{c+dx}}{d\left(e^{4c+4dx}-2e^{2c+2dx}+1\right)}-\frac{8b^2e^{c+dx}}{3d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}+\frac{4abe^{c+dx}}{d\left(e^{2c+2dx}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)^2/sinh(c + d*x)^3,x)
```

output
```
(atan((exp(d*x)*exp(c)*(a^2*(-d^2)^(1/2) - 4*a*b*(-d^2)^(1/2)))/(d*(a^4 -
8*a^3*b + 16*a^2*b^2)^(1/2)))*(a^4 - 8*a^3*b + 16*a^2*b^2)^(1/2))/(-d^2)^(
1/2) + (8*b^2*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x)
+ exp(6*c + 6*d*x) + 1)) - (a^2*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) -
 (2*a^2*exp(c + d*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8
*b^2*exp(c + d*x))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (4*
a*b*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1))
```

3.16 $\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

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3.16.1 Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{a(a-2b)\coth(c+dx)}{d} - \frac{a^2\coth^3(c+dx)}{3d} - \frac{(2a-b)b\tanh(c+dx)}{d} - \frac{b^2\tanh^3(c+dx)}{3d}$$

output
```
a*(a-2*b)*coth(d*x+c)/d-1/3*a^2*coth(d*x+c)^3/d-(2*a-b)*b*tanh(d*x+c)/d-1/
3*b^2*tanh(d*x+c)^3/d
```

3.16.2 Mathematica [A] (verified)

Time = 2.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{-a\coth(c+dx)\left(-2a+6b+a\operatorname{csch}^2(c+dx)\right)+b\left(-6a+2b+b\operatorname{sech}^2(c+dx)\right)\tanh(c+dx)}{3d}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-(a*Coth[c + d*x]*(-2*a + 6*b + a*Csch[c + d*x]^2)) + b*(-6*a + 2*b + b*S
ech[c + d*x]^2)*Tanh[c + d*x])/(3*d)
```

3.16.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4146, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\sin(ic+idx)^4}dx$$

$$\downarrow 4146$$

$$\frac{\int \coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2 d\tanh(c+dx)}{d}$$

$$\downarrow 355$$

$$\frac{\int \left(a^2\coth^4(c+dx)-a(a-2b)\coth^2(c+dx)-b^2\tanh^2(c+dx)+b(b-2a)\right) d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{3}a^2\coth^3(c+dx)-b(2a-b)\tanh(c+dx)+a(a-2b)\coth(c+dx)-\frac{1}{3}b^2\tanh^3(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(a*(a - 2*b)*Coth[c + d*x] - (a^2*Coth[c + d*x]^3)/3 - (2*a - b)*b*Tanh[c
+ d*x] - (b^2*Tanh[c + d*x]^3)/3)/d
```

3.16.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.16.4 Maple [A] (verified)

Time = 8.71 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{a^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+2ab\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)+b^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{d}$
default	$\frac{a^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+2ab\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)+b^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{d}$
risch	$-\frac{4\left(3a^2e^{8dx+8c}+6ab\,e^{8dx+8c}+3b^2e^{8dx+8c}+8a^2e^{6dx+6c}-8b^2e^{6dx+6c}+6a^2e^{4dx+4c}-12ab\,e^{4dx+4c}+6\,e^{4dx+4c}b^2-a^2+6\right.}{3d\left(e^{2dx+2c}+1\right)^3\left(e^{2dx+2c}-1\right)^3}$

input
```
int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+2*a*b*(-1/sinh(d*x+c)/cosh(d*
x+c)-2*tanh(d*x+c))+b^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))
```

3.16.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(68) = 136$.

Time = 0.26 (sec) , antiderivative size = 393, normalized size of antiderivative = 5.46

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$

$$-\frac{8\left(\left(a^2+6ab+b^2\right)\cosh(dx+c)^4+8\left(a^2+b^2\right)\cosh(dx+c)^2\sinh(dx+c)^2+\left(a^2+6ab+b^2\right)\sinh(dx+c)^4\right)}{3\left(d\cosh(dx+c)^8+56\,d\cosh(dx+c)^3\sinh(dx+c)^5+28\,d\cosh(dx+c)^2\sinh(dx+c)^6+8\,d\cosh(dx+c)\sinh(dx+c)^7+d\sinh(dx+c)^8\right)}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-8/3*((a^2 + 6*a*b + b^2)*cosh(d*x + c)^4 + 8*(a^2 + b^2)*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^2 + 6*a*b + b^2)*sinh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d
*x + c)^2 + 2*(3*(a^2 + 6*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 - 2*b^2)*sinh
(d*x + c)^2 + 3*a^2 - 6*a*b + 3*b^2 + 8*((a^2 + b^2)*cosh(d*x + c)^3 + (a^
2 - b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 56*d*cosh(d*x
+ c)^3*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*d*cosh(d
*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^4 + 2*(35*
d*cosh(d*x + c)^4 - 2*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - d*cosh
(d*x + c))*sinh(d*x + c)^3 + 4*(7*d*cosh(d*x + c)^6 - 6*d*cosh(d*x + c)^2)
*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - d*cosh(d*x + c)^3)*sinh(d*x + c)
 + 3*d)
```

3.16.6 Sympy [F]

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2(c+dx)\right)^2 \operatorname{csch}^4(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x)**4, x)
```

3.16.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(68) = 136$.

Time $= 0.20$ (sec) , antiderivative size $= 210$, normalized size of antiderivative $= 2.92$

$$\begin{aligned}
&\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx \\
&= \frac{4}{3}b^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right) \\
&\quad+\frac{4}{3}a^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right. \\
&\quad\left.+\frac{8\,ab}{d(e^{(-4\,dx-4\,c)}-1)}\right.
\end{aligned}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
4/3*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) +
e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c) + 1))) + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c
) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c)
 - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 8*a*b/(d*(e^(-4*d*x - 4*
c) - 1))
```

3.16.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(68) = 136$.

Time $= 0.34$ (sec) , antiderivative size $= 143$, normalized size of antiderivative $= 1.99$

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{4\left(3a^2e^{(8dx+8c)}+6abe^{(8dx+8c)}+3b^2e^{(8dx+8c)}+8a^2e^{(6dx+6c)}-8b^2e^{(6dx+6c)}+6a^2e^{(4dx+4c)}-12abe^{(4}}{3d(e^{(4dx+4c)}-1)^3}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-4/3*(3*a^2*e^(8*d*x + 8*c) + 6*a*b*e^(8*d*x + 8*c) + 3*b^2*e^(8*d*x + 8*c
) + 8*a^2*e^(6*d*x + 6*c) - 8*b^2*e^(6*d*x + 6*c) + 6*a^2*e^(4*d*x + 4*c)
- 12*a*b*e^(4*d*x + 4*c) + 6*b^2*e^(4*d*x + 4*c) - a^2 + 6*a*b - b^2)/(d*(
e^(4*d*x + 4*c) - 1)^3)
```

3.16.9 Mupad [B] (verification not implemented)

Time $= 1.93$ (sec) , antiderivative size $= 143$, normalized size of antiderivative $= 1.99$

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{4\left(6ab-a^2-b^2+6a^2\,\mathrm{e}^{4c+4dx}+8a^2\,\mathrm{e}^{6c+6dx}+3a^2\,\mathrm{e}^{8c+8dx}+6b^2\,\mathrm{e}^{4c+4dx}-8b^2\,\mathrm{e}^{6c+6dx}+3b^2\,\mathrm{e}^{8c+8}\right.}{3d\left(\mathrm{e}^{4c+4dx}-1\right)^3}$$

input
```
int((a + b*tanh(c + d*x)^2)^2/sinh(c + d*x)^4,x)
```

output
```
-(4*(6*a*b - a^2 - b^2 + 6*a^2*exp(4*c + 4*d*x) + 8*a^2*exp(6*c + 6*d*x) +
 3*a^2*exp(8*c + 8*d*x) + 6*b^2*exp(4*c + 4*d*x) - 8*b^2*exp(6*c + 6*d*x)
+ 3*b^2*exp(8*c + 8*d*x) - 12*a*b*exp(4*c + 4*d*x) + 6*a*b*exp(8*c + 8*d*x
)))/(3*d*(exp(4*c + 4*d*x) - 1)^3)
```

3.17 $\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

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3.17.1 Optimal result

Integrand size = 23, antiderivative size = 182

$$\begin{aligned}
&\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx \\
&= \frac{3}{8}(a+b)\left(a^2+14ab+21b^2\right)x - \frac{3(a+b)\left(a^2+14ab+21b^2\right)\tanh(c+dx)}{8d} \\
&\quad - \frac{b(6a^2+35ab+21b^2)\tanh^3(c+dx)}{8d} - \frac{3b^2(5a+21b)\tanh^5(c+dx)}{40d} \\
&\quad - \frac{3(a+3b)\sinh^2(c+dx)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{8d} \\
&\quad + \frac{\cosh(c+dx)\sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{4d}
\end{aligned}$$

output

```
3/8*(a+b)*(a^2+14*a*b+21*b^2)*x-3/8*(a+b)*(a^2+14*a*b+21*b^2)*tanh(d*x+c)/
d-1/8*b*(6*a^2+35*a*b+21*b^2)*tanh(d*x+c)^3/d-3/40*b^2*(5*a+21*b)*tanh(d*x
+c)^5/d-3/8*(a+3*b)*sinh(d*x+c)^2*tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2/d+1/4*
cosh(d*x+c)*sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3/d
```

3.17.2 Mathematica [A] (verified)

Time = 5.62 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.69

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{60(a^3+15a^2b+35ab^2+21b^3)(c+dx)-40(a+b)^2(a+4b)\sinh(2(c+dx))+5(a+b)^3\sinh(4(c+dx))}{160d}$$

input
```
Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(60*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*(c + d*x) - 40*(a + b)^2*(a + 4*b
)*Sinh[2*(c + d*x)] + 5*(a + b)^3*Sinh[4*(c + d*x)] - 32*b*(15*a^2 + 50*a*
b + 36*b^2 - b*(5*a + 7*b)*Sech[c + d*x]^2 + b^2*Sech[c + d*x]^4)*Tanh[c +
 d*x])/(160*d)
```

3.17.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4146, 369, 27, 439, 25, 437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$\downarrow 3042$$
$$\int \sin(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)^3\,dx$$
$$\downarrow 4146$$
$$\frac{\int \frac{\tanh^4(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{\left(1-\tanh^2(c+dx)\right)^3}\,d\tanh(c+dx)}{d}$$
$$\downarrow 369$$
$$\frac{\frac{\tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{1}{4}\int \frac{3\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^2\left(3b\tanh^2(c+dx)+a\right)}{\left(1-\tanh^2(c+dx)\right)^2}\,d\tanh(c+dx)}{d}$$
$$\downarrow 27$$

$$\frac{\frac{\tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{3}{4}\int\frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^2\left(3b\tanh^2(c+dx)+a\right)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 439$$

$$\frac{\frac{\tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{3}{4}\left(\frac{1}{2}\int-\frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)\left(b(5a+21b)\tanh^2(c+dx)+a(a+9b)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{(a+}{d}$$

$$\downarrow 25$$

$$\frac{\frac{\tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{3}{4}\left(\frac{(a+3b)\tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{2\left(1-\tanh^2(c+dx)\right)}-\frac{1}{2}\int\frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)\left(b(5a+21b)\tanh^2(c\right.}{1-\tanh^2(c+dx)}}{d}$$

$$\downarrow 437$$

$$\frac{\frac{\tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{3}{4}\left(\frac{(a+3b)\tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{2\left(1-\tanh^2(c+dx)\right)}-\frac{1}{2}\int\left(-b^2(5a+21b)\tanh^4(c+dx)-b\left(6a^2+\right.\right.\right.}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{\tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{3}{4}\left(\frac{1}{2}\left(-(a+b)\left(a^2+14ab+21b^2\right)\operatorname{arctanh}(\tanh(c+dx))+\frac{1}{3}b\left(6a^2+35ab+21b^2\right)t\right.\right.}{}$$

input
```
Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3)/(4*(1 - Tanh[c + d*x]^2)^2) -
 (3*(((a + 3*b)*Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2)/(2*(1 - Tanh[c
+ d*x]^2)) + (-((a + b)*(a^2 + 14*a*b + 21*b^2)*ArcTanh[Tanh[c + d*x]]) +
(a + b)*(a^2 + 14*a*b + 21*b^2)*Tanh[c + d*x] + (b*(6*a^2 + 35*a*b + 21*b^
2)*Tanh[c + d*x]^3)/3 + (b^2*(5*a + 21*b)*Tanh[c + d*x]^5)/5)/2))/4)/d
```

3.17.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.17.4 Maple [A] (verified)

Time = 10.64 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{a^3\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\sinh(dx+c)^5}{4\cosh(dx+c)}-\frac{5\sinh(dx+c)^3}{8\cosh(dx+c)}+\frac{15dx}{8}+\frac{15c}{8}-\frac{15\tanh(dx+c}{8}$
default	$\frac{a^3\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\sinh(dx+c)^5}{4\cosh(dx+c)}-\frac{5\sinh(dx+c)^3}{8\cosh(dx+c)}+\frac{15dx}{8}+\frac{15c}{8}-\frac{15\tanh(dx+c}{8}$
risch	$\frac{3a^3x}{8}+\frac{45ba^2x}{8}+\frac{105ab^2x}{8}+\frac{63b^3x}{8}+\frac{e^{4dx+4c}a^3}{64d}+\frac{3e^{4dx+4c}a^2b}{64d}+\frac{3e^{4dx+4c}ab^2}{64d}+\frac{e^{4dx+4c}b^3}{64d}-\frac{e^{2dx+2c}}{8d}$

input

```
int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output

```
1/d*(a^3*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+3
*a^2*b*(1/4*sinh(d*x+c)^5/cosh(d*x+c)-5/8*sinh(d*x+c)^3/cosh(d*x+c)+15/8*d
*x+15/8*c-15/8*tanh(d*x+c))+3*a*b^2*(1/4*sinh(d*x+c)^7/cosh(d*x+c)^3-7/8*s
inh(d*x+c)^5/cosh(d*x+c)^3+35/8*d*x+35/8*c-35/8*tanh(d*x+c)-35/24*tanh(d*x
+c)^3)+b^3*(1/4*sinh(d*x+c)^9/cosh(d*x+c)^5-9/8*sinh(d*x+c)^7/cosh(d*x+c)^
5+63/8*d*x+63/8*c-63/8*tanh(d*x+c)-21/8*tanh(d*x+c)^3-63/40*tanh(d*x+c)^5)
)
```

3.17.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 879 vs. $2(170) = 340$.

Time = 0.27 (sec) , antiderivative size = 879, normalized size of antiderivative = 4.83

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input

```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/320*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^9 - 15*(a^3 + 11*a^
2*b + 19*a*b^2 + 9*b^3 - 12*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^7 + 8*(120*a^2*b + 400*a*b^2 + 288*b^3 + 15*(a^3 + 15*a^2
*b + 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c)^5 + 40*(120*a^2*b + 400*a*b^2 +
 288*b^3 + 15*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c)*sinh
(d*x + c)^4 + (630*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 - 150*a
^3 - 2010*a^2*b - 4850*a*b^2 - 3054*b^3 - 315*(a^3 + 11*a^2*b + 19*a*b^2 +
 9*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 40*(120*a^2*b + 400*a*b^2 + 288
*b^3 + 15*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c)^3 + 5*(8
4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 - 105*(a^3 + 11*a^2*b +
19*a*b^2 + 9*b^3)*cosh(d*x + c)^4 - 62*a^3 - 978*a^2*b - 2282*a*b^2 - 1302
*b^3 - 4*(75*a^3 + 1005*a^2*b + 2425*a*b^2 + 1527*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^3 + 40*(2*(120*a^2*b + 400*a*b^2 + 288*b^3 + 15*(a^3 + 15*a^2*
b + 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c)^3 + 3*(120*a^2*b + 400*a*b^2 + 2
88*b^3 + 15*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c))*sinh(
d*x + c)^2 + 80*(120*a^2*b + 400*a*b^2 + 288*b^3 + 15*(a^3 + 15*a^2*b + 35
*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c) + 5*(9*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*cosh(d*x + c)^8 - 21*(a^3 + 11*a^2*b + 19*a*b^2 + 9*b^3)*cosh(d*x + c)^6
- 2*(75*a^3 + 1005*a^2*b + 2425*a*b^2 + 1527*b^3)*cosh(d*x + c)^4 - 36*a^3
 - 612*a^2*b - 1372*a*b^2 - 924*b^3 - 6*(31*a^3 + 489*a^2*b + 1141*a*b^...
```

3.17.6 Sympy [F]

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^3\sinh^4\left(c+dx\right)dx$$

input
```
integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*sinh(c + d*x)**4, x)
```

3.17.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(170) = 340$.

Time = 0.20 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.64

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{1}{64}a^3\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right)$$
$$+ \frac{1}{320}b^3\left(\frac{2520(dx+c)}{d} + \frac{5\left(32e^{(-2dx-2c)} - e^{(-4dx-4c)}\right)}{d} - \frac{135e^{(-2dx-2c)} + 5358e^{(-4dx-4c)} + 18190e^{(-}}{d(e^{(-4dx-4c)} + 5e^{(-6dx-6c)} + 10}\right.$$
$$+ \frac{1}{64}ab^2\left(\frac{840(dx+c)}{d} + \frac{3\left(24e^{(-2dx-2c)} - e^{(-4dx-4c)}\right)}{d} - \frac{63e^{(-2dx-2c)} + 1487e^{(-4dx-4c)} + 2517e^{(-6dx-}}{d(e^{(-4dx-4c)} + 3e^{(-6dx-6c)} + 3e^{(-8dx-}}\right.$$
$$+ \frac{3}{64}a^2b\left(\frac{120(dx+c)}{d} + \frac{16e^{(-2dx-2c)} - e^{(-4dx-4c)}}{d} - \frac{15e^{(-2dx-2c)} + 144e^{(-4dx-4c)} - 1}{d(e^{(-4dx-4c)} + e^{(-6dx-6c)})}\right)$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/64*a^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) + 1/320*b^3*(2520*(d*x + c)/d + 5*(32*e^(-2*d*
x - 2*c) - e^(-4*d*x - 4*c))/d - (135*e^(-2*d*x - 2*c) + 5358*e^(-4*d*x -
4*c) + 18190*e^(-6*d*x - 6*c) + 28455*e^(-8*d*x - 8*c) + 19995*e^(-10*d*x
- 10*c) + 6560*e^(-12*d*x - 12*c) - 5)/(d*(e^(-4*d*x - 4*c) + 5*e^(-6*d*x
- 6*c) + 10*e^(-8*d*x - 8*c) + 10*e^(-10*d*x - 10*c) + 5*e^(-12*d*x - 12*c
) + e^(-14*d*x - 14*c)))) + 1/64*a*b^2*(840*(d*x + c)/d + 3*(24*e^(-2*d*x
- 2*c) - e^(-4*d*x - 4*c))/d - (63*e^(-2*d*x - 2*c) + 1487*e^(-4*d*x - 4*c
) + 2517*e^(-6*d*x - 6*c) + 1608*e^(-8*d*x - 8*c) - 3)/(d*(e^(-4*d*x - 4*c
) + 3*e^(-6*d*x - 6*c) + 3*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c)))) + 3/64
*a^2*b*(120*(d*x + c)/d + (16*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (15
*e^(-2*d*x - 2*c) + 144*e^(-4*d*x - 4*c) - 1)/(d*(e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c))))
```

3.17.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(170) = 340$.

Time $= 0.55$ (sec) , antiderivative size $= 505$, normalized size of antiderivative $= 2.77$

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{5\,a^3e^{(4\,dx+4\,c)} + 15\,a^2be^{(4\,dx+4\,c)} + 15\,ab^2e^{(4\,dx+4\,c)} + 5\,b^3e^{(4\,dx+4\,c)} - 40\,a^3e^{(2\,dx+2\,c)} - 240\,a^2be^{(2\,dx+2\,c)} - 360$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/320*(5*a^3*e^(4*d*x + 4*c) + 15*a^2*b*e^(4*d*x + 4*c) + 15*a*b^2*e^(4*d*
x + 4*c) + 5*b^3*e^(4*d*x + 4*c) - 40*a^3*e^(2*d*x + 2*c) - 240*a^2*b*e^(2
*d*x + 2*c) - 360*a*b^2*e^(2*d*x + 2*c) - 160*b^3*e^(2*d*x + 2*c) + 120*(a
^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*(d*x + c) - 5*(18*a^3*e^(4*d*x + 4*c) +
 270*a^2*b*e^(4*d*x + 4*c) + 630*a*b^2*e^(4*d*x + 4*c) + 378*b^3*e^(4*d*x
+ 4*c) - 8*a^3*e^(2*d*x + 2*c) - 48*a^2*b*e^(2*d*x + 2*c) - 72*a*b^2*e^(2*
d*x + 2*c) - 32*b^3*e^(2*d*x + 2*c) + a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-4
*d*x - 4*c) + 128*(15*a^2*b*e^(8*d*x + 8*c) + 60*a*b^2*e^(8*d*x + 8*c) + 5
0*b^3*e^(8*d*x + 8*c) + 60*a^2*b*e^(6*d*x + 6*c) + 210*a*b^2*e^(6*d*x + 6*
c) + 150*b^3*e^(6*d*x + 6*c) + 90*a^2*b*e^(4*d*x + 4*c) + 290*a*b^2*e^(4*d
*x + 4*c) + 210*b^3*e^(4*d*x + 4*c) + 60*a^2*b*e^(2*d*x + 2*c) + 190*a*b^2
*e^(2*d*x + 2*c) + 130*b^3*e^(2*d*x + 2*c) + 15*a^2*b + 50*a*b^2 + 36*b^3)
/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.17.9 Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 730, normalized size of antiderivative = 4.01

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$=\frac{\frac{2\left(3a^2b+12ab^2+10b^3\right)}{5d}+\frac{8e^{2c+2dx}\left(3a^2b+9ab^2+5b^3\right)}{5d}+\frac{12e^{4c+4dx}\left(3a^2b+8ab^2+6b^3\right)}{5d}+\frac{8e^{6c+6dx}\left(3a^2b+9ab^2+5b^3\right)}{5d}+\frac{2e^{8c+}}{}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1}$$
$$+\frac{\frac{2\left(3a^2b+9ab^2+5b^3\right)}{5d}+\frac{2e^{2c+2dx}\left(3a^2b+12ab^2+10b^3\right)}{5d}}{2e^{2c+2dx}+e^{4c+4dx}+1}+x\left(\frac{3a^3}{8}+\frac{45a^2b}{8}+\frac{105ab^2}{8}+\frac{63b^3}{8}\right)$$
$$+\frac{\frac{2\left(3a^2b+9ab^2+5b^3\right)}{5d}+\frac{6e^{2c+2dx}\left(3a^2b+8ab^2+6b^3\right)}{5d}+\frac{6e^{4c+4dx}\left(3a^2b+9ab^2+5b^3\right)}{5d}+\frac{2e^{6c+6dx}\left(3a^2b+12ab^2+10b^3\right)}{5d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1}$$
$$+\frac{\frac{2\left(3a^2b+8ab^2+6b^3\right)}{5d}+\frac{4e^{2c+2dx}\left(3a^2b+9ab^2+5b^3\right)}{5d}+\frac{2e^{4c+4dx}\left(3a^2b+12ab^2+10b^3\right)}{5d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}$$
$$+\frac{2\left(3a^2b+12ab^2+10b^3\right)}{5d\left(e^{2c+2dx}+1\right)}-\frac{e^{-4c-4dx}(a+b)^3}{64d}+\frac{e^{4c+4dx}(a+b)^3}{64d}$$
$$+\frac{e^{-2c-2dx}(a+b)^2(a+4b)}{8d}-\frac{e^{2c+2dx}(a+b)^2(a+4b)}{8d}$$

input
```
int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
((2*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d) + (8*exp(2*c + 2*d*x)*(9*a*b^2 +
3*a^2*b + 5*b^3))/(5*d) + (12*exp(4*c + 4*d*x)*(8*a*b^2 + 3*a^2*b + 6*b^3)
)/(5*d) + (8*exp(6*c + 6*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (2*exp(
8*c + 8*d*x)*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 1
0*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c +
 10*d*x) + 1) + ((2*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (2*exp(2*c + 2*d*
x)*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4
*d*x) + 1) + x*((105*a*b^2)/8 + (45*a^2*b)/8 + (3*a^3)/8 + (63*b^3)/8) + (
(2*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (6*exp(2*c + 2*d*x)*(8*a*b^2 + 3*a
^2*b + 6*b^3))/(5*d) + (6*exp(4*c + 4*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5
*d) + (2*exp(6*c + 6*d*x)*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d))/(4*exp(2*c
 + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1
) + ((2*(8*a*b^2 + 3*a^2*b + 6*b^3))/(5*d) + (4*exp(2*c + 2*d*x)*(9*a*b^2
+ 3*a^2*b + 5*b^3))/(5*d) + (2*exp(4*c + 4*d*x)*(12*a*b^2 + 3*a^2*b + 10*b
^3))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) +
1) + (2*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d*(exp(2*c + 2*d*x) + 1)) - (exp
(- 4*c - 4*d*x)*(a + b)^3)/(64*d) + (exp(4*c + 4*d*x)*(a + b)^3)/(64*d) +
(exp(- 2*c - 2*d*x)*(a + b)^2*(a + 4*b))/(8*d) - (exp(2*c + 2*d*x)*(a + b)
^2*(a + 4*b))/(8*d)
```

3.18 $\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

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3.18.1 Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{(a+b)^2(a+4b)\cosh(c+dx)}{d} + \frac{(a+b)^3\cosh^3(c+dx)}{3d} - \frac{3b(a+b)(a+2b)\operatorname{sech}(c+dx)}{d} + \frac{b^2(3a+4b)\operatorname{sech}^3(c+dx)}{3d} - \frac{b^3\operatorname{sech}^5(c+dx)}{5d}$$

output
```
-(a+b)^2*(a+4*b)*cosh(d*x+c)/d+1/3*(a+b)^3*cosh(d*x+c)^3/d-3*b*(a+b)*(a+2*
b)*sech(d*x+c)/d+1/3*b^2*(3*a+4*b)*sech(d*x+c)^3/d-1/5*b^3*sech(d*x+c)^5/d
```

3.18.2 Mathematica [A] (verified)

Time = 11.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.87

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{-45(a+b)^2(a+5b)\cosh(c+dx)+5(a+b)^3\cosh(3(c+dx))-180b(a+b)(a+2b)\operatorname{sech}(c+dx)+20b^2(}{60d}$$

input
```
Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-45*(a + b)^2*(a + 5*b)*Cosh[c + d*x] + 5*(a + b)^3*Cosh[3*(c + d*x)] - 1
80*b*(a + b)*(a + 2*b)*Sech[c + d*x] + 20*b^2*(3*a + 4*b)*Sech[c + d*x]^3
- 12*b^3*Sech[c + d*x]^5)/(60*d)
```

3.18.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 26, 4147, 25, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\downarrow 3042$$

$$\int i\sin(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^3\,dx$$

$$\downarrow 26$$

$$i\int \sin(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^3\,dx$$

$$\downarrow 4147$$

$$\frac{\int -\cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)\left(-b\text{sech}^2(c+dx)+a+b\right)^3\,d\text{sech}(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{\int \cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)\left(-b\text{sech}^2(c+dx)+a+b\right)^3\,d\text{sech}(c+dx)}{d}$$

$$\downarrow 355$$

$$-\frac{\int \left((a+b)^3\cosh^4(c+dx)-(a+b)^2(a+4b)\cosh^2(c+dx)+b^3\text{sech}^4(c+dx)-b^2(3a+4b)\text{sech}^2(c+dx)+3b(a$$

$$\downarrow 2009$$

$$\frac{\frac{1}{3}b^2(3a+4b)\text{sech}^3(c+dx)+\frac{1}{3}(a+b)^3\cosh^3(c+dx)-(a+b)^2(a+4b)\cosh(c+dx)-3b(a+b)(a+2b)\text{sech}(c+$$

$$d$$

input
```
Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-((a + b)^2*(a + 4*b)*Cosh[c + d*x]) + ((a + b)^3*Cosh[c + d*x]^3)/3 - 3*
b*(a + b)*(a + 2*b)*Sech[c + d*x] + (b^2*(3*a + 4*b)*Sech[c + d*x]^3)/3 -
(b^3*Sech[c + d*x]^5)/5)/d
```

3.18.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.18.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(99) = 198$.

Time = 5.00 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.28

method	result
derivativedivides	$\frac{a^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+3a^2b\left(\frac{\sinh(dx+c)^4}{3\cosh(dx+c)}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)}-\frac{8}{3\cosh(dx+c)}\right)+3ab^2\left(\frac{\sinh(dx+c)^6}{3\cosh(dx+c)^3}-\frac{2\sinh(dx+}{\cosh(dx+}\right.$
default	$\frac{a^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+3a^2b\left(\frac{\sinh(dx+c)^4}{3\cosh(dx+c)}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)}-\frac{8}{3\cosh(dx+c)}\right)+3ab^2\left(\frac{\sinh(dx+c)^6}{3\cosh(dx+c)^3}-\frac{2\sinh(dx-}{\cosh(dx+}\right.$
risch	$\frac{e^{3dx+3c}a^3}{24d}+\frac{e^{3dx+3c}a^2b}{8d}+\frac{e^{3dx+3c}ab^2}{8d}+\frac{e^{3dx+3c}b^3}{24d}-\frac{3e^{dx+c}a^3}{8d}-\frac{21e^{dx+c}a^2b}{8d}-\frac{33e^{dx+c}ab^2}{8d}-\frac{15b^3e^{dx}}{8d}$

input
```
int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+3*a^2*b*(1/3*sinh(d*x+c)^4/c
osh(d*x+c)-4/3*sinh(d*x+c)^2/cosh(d*x+c)-8/3/cosh(d*x+c))+3*a*b^2*(1/3*sin
h(d*x+c)^6/cosh(d*x+c)^3-2*sinh(d*x+c)^4/cosh(d*x+c)^3-8*sinh(d*x+c)^2/cos
h(d*x+c)^3-16/3/cosh(d*x+c)^3)+b^3*(1/3*sinh(d*x+c)^8/cosh(d*x+c)^5-8/3*si
nh(d*x+c)^6/cosh(d*x+c)^5-16*sinh(d*x+c)^4/cosh(d*x+c)^5-64/3*sinh(d*x+c)^
2/cosh(d*x+c)^5-128/15/cosh(d*x+c)^5))
```

3.18.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(99) = 198$.

Time = 0.25 (sec) , antiderivative size = 540, normalized size of antiderivative = 5.14

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{5\left(a^3+3a^2b+3ab^2+b^3\right)\cosh(dx+c)^8+5\left(a^3+3a^2b+3ab^2+b^3\right)\sinh(dx+c)^8-20\left(a^3+12a^2b+2\right.}{}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/120*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 5*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*sinh(d*x + c)^8 - 20*(a^3 + 12*a^2*b + 21*a*b^2 + 10*b^
3)*cosh(d*x + c)^6 - 20*(a^3 + 12*a^2*b + 21*a*b^2 + 10*b^3 - 7*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 - 20*(11*a^3 + 123*
a^2*b + 249*a*b^2 + 137*b^3)*cosh(d*x + c)^4 + 10*(35*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*cosh(d*x + c)^4 - 22*a^3 - 246*a^2*b - 498*a*b^2 - 274*b^3 - 3
0*(a^3 + 12*a^2*b + 21*a*b^2 + 10*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 -
425*a^3 - 5235*a^2*b - 10395*a*b^2 - 5649*b^3 - 20*(31*a^3 + 372*a^2*b + 7
47*a*b^2 + 390*b^3)*cosh(d*x + c)^2 + 20*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3
)*cosh(d*x + c)^6 - 15*(a^3 + 12*a^2*b + 21*a*b^2 + 10*b^3)*cosh(d*x + c)^
4 - 31*a^3 - 372*a^2*b - 747*a*b^2 - 390*b^3 - 6*(11*a^3 + 123*a^2*b + 249
*a*b^2 + 137*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2)/(d*cosh(d*x + c)^5 + 5
*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x +
 c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

3.18.6 Sympy [F]

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2(c+dx)\right)^3 \sinh^3(c+dx)\,dx$$

input
```
integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*sinh(c + d*x)**3, x)
```

3.18.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(99) = 198$.

Time $= 0.21$ (sec) , antiderivative size $= 439$, normalized size of antiderivative $= 4.18$

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$

$$-\frac{1}{120}b^3\left(\frac{5\left(45\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}\right)}{d}+\frac{200\,e^{(-2\,dx-2\,c)}+2515\,e^{(-4\,dx-4\,c)}+6680\,e^{(-6\,dx-6\,c)}+9073\,e^{(-8\,dx}}{d(e^{(-3\,dx-3\,c)}+5\,e^{(-5\,dx-5\,c)}+10\,e^{(-7\,dx-7\,c)}+10\,e^{(-9}}\right.$$

$$-\frac{1}{8}ab^2\left(\frac{33\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{30\,e^{(-2\,dx-2\,c)}+240\,e^{(-4\,dx-4\,c)}+322\,e^{(-6\,dx-6\,c)}+177\,e^{(-8\,dx-8\,c)}-1}{d(e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}+3\,e^{(-7\,dx-7\,c)}+e^{(-9\,dx-9\,c)})}\right.$$

$$-\frac{1}{8}a^2b\left(\frac{21\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{20\,e^{(-2\,dx-2\,c)}+69\,e^{(-4\,dx-4\,c)}-1}{d(e^{(-3\,dx-3\,c)}+e^{(-5\,dx-5\,c)})}\right)$$

$$+\frac{1}{24}a^3\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/120*b^3*(5*(45*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (200*e^(-2*d*x - 2*
c) + 2515*e^(-4*d*x - 4*c) + 6680*e^(-6*d*x - 6*c) + 9073*e^(-8*d*x - 8*c)
 + 5600*e^(-10*d*x - 10*c) + 1665*e^(-12*d*x - 12*c) - 5)/(d*(e^(-3*d*x -
3*c) + 5*e^(-5*d*x - 5*c) + 10*e^(-7*d*x - 7*c) + 10*e^(-9*d*x - 9*c) + 5*
e^(-11*d*x - 11*c) + e^(-13*d*x - 13*c)))) - 1/8*a*b^2*((33*e^(-d*x - c) -
 e^(-3*d*x - 3*c))/d + (30*e^(-2*d*x - 2*c) + 240*e^(-4*d*x - 4*c) + 322*e
^(-6*d*x - 6*c) + 177*e^(-8*d*x - 8*c) - 1)/(d*(e^(-3*d*x - 3*c) + 3*e^(-5
*d*x - 5*c) + 3*e^(-7*d*x - 7*c) + e^(-9*d*x - 9*c)))) - 1/8*a^2*b*((21*e^
(-d*x - c) - e^(-3*d*x - 3*c))/d + (20*e^(-2*d*x - 2*c) + 69*e^(-4*d*x - 4
*c) - 1)/(d*(e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c)))) + 1/24*a^3*(e^(3*d*x +
 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

3.18.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(99) = 198$.

Time $= 0.50$ (sec) , antiderivative size $= 330$, normalized size of antiderivative $= 3.14$

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{5\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+15\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+15\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+5\,b^3\left(e^{(dx+c)}+e^{(-dx-}\right.}{}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/120*(5*a^3*(e^(d*x + c) + e^(-d*x - c))^3 + 15*a^2*b*(e^(d*x + c) + e^(-
d*x - c))^3 + 15*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3 + 5*b^3*(e^(d*x + c)
 + e^(-d*x - c))^3 - 60*a^3*(e^(d*x + c) + e^(-d*x - c)) - 360*a^2*b*(e^(d
*x + c) + e^(-d*x - c)) - 540*a*b^2*(e^(d*x + c) + e^(-d*x - c)) - 240*b^3
*(e^(d*x + c) + e^(-d*x - c)) - 16*(45*a^2*b*(e^(d*x + c) + e^(-d*x - c))^
4 + 135*a*b^2*(e^(d*x + c) + e^(-d*x - c))^4 + 90*b^3*(e^(d*x + c) + e^(-d
*x - c))^4 - 60*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 - 80*b^3*(e^(d*x + c)
 + e^(-d*x - c))^2 + 48*b^3)/(e^(d*x + c) + e^(-d*x - c))^5)/d
```

3.18.9 Mupad [B] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 361, normalized size of antiderivative = 3.44

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{e^{-3c-3dx}(a+b)^3}{24d} + \frac{e^{3c+3dx}(a+b)^3}{24d} + \frac{8e^{c+dx}(4b^3+3ab^2)}{3d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}$$
$$+ \frac{64b^3e^{c+dx}}{5d\left(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1\right)}$$
$$- \frac{8e^{c+dx}(32b^3+15ab^2)}{15d\left(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1\right)}$$
$$- \frac{32b^3e^{c+dx}}{5d\left(5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1\right)}$$
$$- \frac{3e^{c+dx}(a+b)^2(a+5b)}{8d} - \frac{6e^{c+dx}(a^2b+3ab^2+2b^3)}{d\left(e^{2c+2dx}+1\right)} - \frac{3e^{-c-dx}(a+b)^2(a+5b)}{8d}$$

input
```
int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
(exp(- 3*c - 3*d*x)*(a + b)^3)/(24*d) + (exp(3*c + 3*d*x)*(a + b)^3)/(24*d
) + (8*exp(c + d*x)*(3*a*b^2 + 4*b^3))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c
+ 4*d*x) + 1)) + (64*b^3*exp(c + d*x))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*
c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (8*exp(c + d*x)
*(15*a*b^2 + 32*b^3))/(15*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp
(6*c + 6*d*x) + 1)) - (32*b^3*exp(c + d*x))/(5*d*(5*exp(2*c + 2*d*x) + 10*
exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 1
0*d*x) + 1)) - (3*exp(c + d*x)*(a + b)^2*(a + 5*b))/(8*d) - (6*exp(c + d*x
)*(3*a*b^2 + a^2*b + 2*b^3))/(d*(exp(2*c + 2*d*x) + 1)) - (3*exp(- c - d*x
)*(a + b)^2*(a + 5*b))/(8*d)
```

3.19 $\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

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3.19.1 Optimal result

Integrand size = 23, antiderivative size = 122

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{1}{2}(a+b)^2(a+7b)x + \frac{(a+b)^3}{4d(1-\tanh(c+dx))} + \frac{3b(a+b)^2\tanh(c+dx)}{d} + \frac{b^2(3a+2b)\tanh^3(c+dx)}{3d} + \frac{b^3\tanh^5(c+dx)}{5d} - \frac{(a+b)^3}{4d(1+\tanh(c+dx))}$$

output
```
-1/2*(a+b)^2*(a+7*b)*x+1/4*(a+b)^3/d/(1-tanh(d*x+c))+3*b*(a+b)^2*tanh(d*x+
c)/d+1/3*b^2*(3*a+2*b)*tanh(d*x+c)^3/d+1/5*b^3*tanh(d*x+c)^5/d-1/4*(a+b)^3
/d/(tanh(d*x+c)+1)
```

3.19.2 Mathematica [A] (verified)

Time = 3.99 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.78

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{-30(a+b)^2(a+7b)(c+dx)+15(a+b)^3\sinh(2(c+dx))+4b\left(45a^2+105ab+58b^2-b(15a+16b)\operatorname{sech}^2\right.}{60d}$$

input
```
Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-30*(a + b)^2*(a + 7*b)*(c + d*x) + 15*(a + b)^3*Sinh[2*(c + d*x)] + 4*b*
(45*a^2 + 105*a*b + 58*b^2 - b*(15*a + 16*b)*Sech[c + d*x]^2 + 3*b^2*Sech[
c + d*x]^4)*Tanh[c + d*x])/(60*d)
```

3.19.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.27, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 25, 4146, 369, 403, 25, 403, 25, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$$

$$\downarrow 3042$$

$$\int -\sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^3 dx$$

$$\downarrow 25$$

$$-\int \sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^3 dx$$

$$\downarrow 4146$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{\left(1-\tanh^2(c+dx)\right)^2} d\tanh(c+dx)}{d}$$

$$\downarrow 369$$

$$\frac{\frac{\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{2\left(1-\tanh^2(c+dx)\right)} - \frac{1}{2}\int \frac{\left(b\tanh^2(c+dx)+a\right)^2\left(7b\tanh^2(c+dx)+a\right)}{1-\tanh^2(c+dx)} d\tanh(c+dx)}{d}$$

$$\downarrow 403$$

$$\frac{\frac{1}{2}\left(\frac{1}{5}\int -\frac{\left(b\tanh^2(c+dx)+a\right)\left(b(33a+35b)\tanh^2(c+dx)+a(5a+7b)\right)}{1-\tanh^2(c+dx)} d\tanh(c+dx) + \frac{7}{5}b\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\right) -}{d}$$

$$\downarrow 25$$

$$\frac{\frac{1}{2}\left(\frac{7}{5}b\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2-\frac{1}{5}\int\frac{\left(b\tanh^2(c+dx)+a\right)\left(b(33a+35b)\tanh^2(c+dx)+a(5a+7b)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)\right)+}{d}$$

$$\downarrow 403$$

$$\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{1}{3}\int-\frac{b\left(81a^2+190ba+105b^2\right)\tanh^2(c+dx)+a\left(15a^2+54ba+35b^2\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{1}{3}b(33a+35b)\tanh(c+dx)\left(a+b\tan\right.\right.\right.}{d}$$

$$\downarrow 25$$

$$\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{1}{3}b(33a+35b)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)-\frac{1}{3}\int\frac{b\left(81a^2+190ba+105b^2\right)\tanh^2(c+dx)+a\left(15a^2+54ba+35b^2\right)}{1-\tanh^2(c+dx)}d\tan\right.\right.}{d}$$

$$\downarrow 299$$

$$\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{1}{3}\left(b\left(81a^2+190ab+105b^2\right)\tanh(c+dx)-15(a+b)^2(a+7b)\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)\right)+\frac{1}{3}b(33a+3\right.\right.\right.}{d}$$

$$\downarrow 219$$

$$\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{1}{3}\left(b\left(81a^2+190ab+105b^2\right)\tanh(c+dx)-15(a+b)^2(a+7b)\operatorname{arctanh}(\tanh(c+dx))\right)+\frac{1}{3}b(33a+35b)\tanh\right.\right.\right.}{d}$$

input
```
Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3)/(2*(1 - Tanh[c + d*x]^2)) + ((7
*b*Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2)/5 + ((-15*(a + b)^2*(a + 7*b)*
ArcTanh[Tanh[c + d*x]] + b*(81*a^2 + 190*a*b + 105*b^2)*Tanh[c + d*x])/3 +
 (b*(33*a + 35*b)*Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2))/3)/5)/2)/d
```

3.19.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.19.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.48

method	result
derivativedivides	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)+3ab^2\left(\frac{\sinh(dx+c)^5}{2\cosh(dx+c)^3}-\frac{5dx}{2}-\frac{5c}{2}+\frac{5\tan}{d}$
default	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)+3ab^2\left(\frac{\sinh(dx+c)^5}{2\cosh(dx+c)^3}-\frac{5dx}{2}-\frac{5c}{2}+\frac{5\tan}{d}$
risch	$-\frac{a^3x}{2}-\frac{9ba^2x}{2}-\frac{15ab^2x}{2}-\frac{7b^3x}{2}+\frac{e^{2dx+2c}a^3}{8d}+\frac{3e^{2dx+2c}a^2b}{8d}+\frac{3e^{2dx+2c}ab^2}{8d}+\frac{e^{2dx+2c}b^3}{8d}-\frac{e^{-2dx-2c}}{8d}$

input
```
int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+3*a^2*b*(1/2*sinh(d*x
+c)^3/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c))+3*a*b^2*(1/2*sinh(d*x+c)^
5/cosh(d*x+c)^3-5/2*d*x-5/2*c+5/2*tanh(d*x+c)+5/6*tanh(d*x+c)^3)+b^3*(1/2*
sinh(d*x+c)^7/cosh(d*x+c)^5-7/2*d*x-7/2*c+7/2*tanh(d*x+c)+7/6*tanh(d*x+c)^
3+7/10*tanh(d*x+c)^5))
```

3.19.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 725 vs. 2(110) = 220.

Time = 0.27 (sec) , antiderivative size = 725, normalized size of antiderivative = 5.94

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{15\left(a^3+3a^2b+3ab^2+b^3\right)\sinh(dx+c)^7-4\left(90a^2b+210ab^2+116b^3+15\left(a^3+9a^2b+15ab^2+7b^3\right)a\right.}{}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/120*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^7 - 4*(90*a^2*b +
210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*d*x)*cosh(d*x
+ c)^5 - 20*(90*a^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2
 + 7*b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^4 + (75*a^3 + 585*a^2*b + 1065*
a*b^2 + 539*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^5 - 20*(90*a^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15
*a*b^2 + 7*b^3)*d*x)*cosh(d*x + c)^3 + 5*(105*(a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*cosh(d*x + c)^4 + 27*a^3 + 297*a^2*b + 489*a*b^2 + 203*b^3 + 2*(75*a^3
 + 585*a^2*b + 1065*a*b^2 + 539*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 - 20
*(2*(90*a^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3
)*d*x)*cosh(d*x + c)^3 + 3*(90*a^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a
^2*b + 15*a*b^2 + 7*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 - 40*(90*a^2*
b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*d*x)*cosh(
d*x + c) + 5*(21*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + (75*a^3
 + 585*a^2*b + 1065*a*b^2 + 539*b^3)*cosh(d*x + c)^4 + 15*a^3 + 189*a^2*b
+ 285*a*b^2 + 175*b^3 + 3*(27*a^3 + 297*a^2*b + 489*a*b^2 + 203*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x
 + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)
)*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

3.19.6 Sympy [F]

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^3 \sinh^2\left(c+dx\right)\,dx$$

input
```
integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*sinh(c + d*x)**2, x)
```

3.19.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(110) = 220$.

Time = 0.21 (sec) , antiderivative size = 377, normalized size of antiderivative = 3.09

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{1}{8}a^3\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)$$
$$-\frac{1}{120}b^3\left(\frac{420\,(dx+c)}{d}+\frac{15\,e^{(-2dx-2c)}}{d}-\frac{1003\,e^{(-2dx-2c)}+3350\,e^{(-4dx-4c)}+5590\,e^{(-6dx-6c)}+3915\,e^{(-}}{d(e^{(-2dx-2c)}+5\,e^{(-4dx-4c)}+10\,e^{(-6dx-6c)}+10\,e^{(-8dx-8c)}+}\right.$$
$$-\frac{1}{8}ab^2\left(\frac{60\,(dx+c)}{d}+\frac{3\,e^{(-2dx-2c)}}{d}-\frac{121\,e^{(-2dx-2c)}+201\,e^{(-4dx-4c)}+147\,e^{(-6dx-6c)}+3}{d(e^{(-2dx-2c)}+3\,e^{(-4dx-4c)}+3\,e^{(-6dx-6c)}+e^{(-8dx-8c)})}\right)$$
$$-\frac{3}{8}a^2b\left(\frac{12\,(dx+c)}{d}+\frac{e^{(-2dx-2c)}}{d}-\frac{17\,e^{(-2dx-2c)}+1}{d(e^{(-2dx-2c)}+e^{(-4dx-4c)})}\right)$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/120*b^3*(420*(
d*x + c)/d + 15*e^(-2*d*x - 2*c)/d - (1003*e^(-2*d*x - 2*c) + 3350*e^(-4*d
*x - 4*c) + 5590*e^(-6*d*x - 6*c) + 3915*e^(-8*d*x - 8*c) + 1455*e^(-10*d*
x - 10*c) + 15)/(d*(e^(-2*d*x - 2*c) + 5*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
 6*c) + 10*e^(-8*d*x - 8*c) + 5*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c))))
 - 1/8*a*b^2*(60*(d*x + c)/d + 3*e^(-2*d*x - 2*c)/d - (121*e^(-2*d*x - 2*c
) + 201*e^(-4*d*x - 4*c) + 147*e^(-6*d*x - 6*c) + 3)/(d*(e^(-2*d*x - 2*c)
+ 3*e^(-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c)))) - 3/8*a^2*
b*(12*(d*x + c)/d + e^(-2*d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(
-2*d*x - 2*c) + e^(-4*d*x - 4*c))))
```

3.19.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(110) = 220.

Time = 0.48 (sec) , antiderivative size = 393, normalized size of antiderivative = 3.22

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{15\,a^3e^{(2dx+2c)}+45\,a^2be^{(2dx+2c)}+45\,ab^2e^{(2dx+2c)}+15\,b^3e^{(2dx+2c)}-60\,(a^3+9\,a^2b+15\,ab^2+7\,b^3)(dx+c}{}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/120*(15*a^3*e^(2*d*x + 2*c) + 45*a^2*b*e^(2*d*x + 2*c) + 45*a*b^2*e^(2*d
*x + 2*c) + 15*b^3*e^(2*d*x + 2*c) - 60*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)
*(d*x + c) + 15*(2*a^3*e^(2*d*x + 2*c) + 18*a^2*b*e^(2*d*x + 2*c) + 30*a*b
^2*e^(2*d*x + 2*c) + 14*b^3*e^(2*d*x + 2*c) - a^3 - 3*a^2*b - 3*a*b^2 - b^
3)*e^(-2*d*x - 2*c) - 16*(45*a^2*b*e^(8*d*x + 8*c) + 135*a*b^2*e^(8*d*x +
8*c) + 90*b^3*e^(8*d*x + 8*c) + 180*a^2*b*e^(6*d*x + 6*c) + 450*a*b^2*e^(6
*d*x + 6*c) + 240*b^3*e^(6*d*x + 6*c) + 270*a^2*b*e^(4*d*x + 4*c) + 600*a*
b^2*e^(4*d*x + 4*c) + 340*b^3*e^(4*d*x + 4*c) + 180*a^2*b*e^(2*d*x + 2*c)
+ 390*a*b^2*e^(2*d*x + 2*c) + 200*b^3*e^(2*d*x + 2*c) + 45*a^2*b + 105*a*b
^2 + 58*b^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.19.9 Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 668, normalized size of antiderivative = 5.48

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{e^{2c+2dx}(a+b)^3}{8d} - \frac{\frac{2(3a^2b+6ab^2+2b^3)}{5d} + \frac{6e^{2c+2dx}(a^2b+3ab^2+2b^3)}{5d}}{2e^{2c+2dx}+e^{4c+4dx}+1}$$
$$- \frac{\frac{2(3a^2b+6ab^2+2b^3)}{5d} + \frac{6e^{6c+6dx}(a^2b+3ab^2+2b^3)}{5d} + \frac{6e^{4c+4dx}(3a^2b+6ab^2+2b^3)}{5d} + \frac{2e^{2c+2dx}(9a^2b+15ab^2+10b^3)}{5d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1}$$
$$- \frac{\frac{2(9a^2b+15ab^2+10b^3)}{15d} + \frac{6e^{4c+4dx}(a^2b+3ab^2+2b^3)}{5d} + \frac{4e^{2c+2dx}(3a^2b+6ab^2+2b^3)}{5d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}$$
$$- \frac{6(a^2b+3ab^2+2b^3)}{5d\,(e^{2c+2dx}+1)} - \frac{e^{-2c-2dx}(a+b)^3}{8d}$$
$$- \frac{\frac{6(a^2b+3ab^2+2b^3)}{5d} + \frac{8e^{2c+2dx}(3a^2b+6ab^2+2b^3)}{5d} + \frac{6e^{8c+8dx}(a^2b+3ab^2+2b^3)}{5d} + \frac{8e^{6c+6dx}(3a^2b+6ab^2+2b^3)}{5d} + \frac{4e^{4c+4dx}}{}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1}$$
$$- \frac{x(a+b)^2(a+7b)}{2}$$

input
```
int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3,x)
```

output

```
(exp(2*c + 2*d*x)*(a + b)^3)/(8*d) - ((2*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d
) + (6*exp(2*c + 2*d*x)*(3*a*b^2 + a^2*b + 2*b^3))/(5*d))/(2*exp(2*c + 2*d
*x) + exp(4*c + 4*d*x) + 1) - ((2*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (6*
exp(6*c + 6*d*x)*(3*a*b^2 + a^2*b + 2*b^3))/(5*d) + (6*exp(4*c + 4*d*x)*(6
*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (2*exp(2*c + 2*d*x)*(15*a*b^2 + 9*a^2*b
 + 10*b^3))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c +
6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(15*a*b^2 + 9*a^2*b + 10*b^3))/(15*d)
 + (6*exp(4*c + 4*d*x)*(3*a*b^2 + a^2*b + 2*b^3))/(5*d) + (4*exp(2*c + 2*d
*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c +
4*d*x) + exp(6*c + 6*d*x) + 1) - (6*(3*a*b^2 + a^2*b + 2*b^3))/(5*d*(exp(2
*c + 2*d*x) + 1)) - (exp(- 2*c - 2*d*x)*(a + b)^3)/(8*d) - ((6*(3*a*b^2 +
a^2*b + 2*b^3))/(5*d) + (8*exp(2*c + 2*d*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(
5*d) + (6*exp(8*c + 8*d*x)*(3*a*b^2 + a^2*b + 2*b^3))/(5*d) + (8*exp(6*c +
 6*d*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (4*exp(4*c + 4*d*x)*(15*a*b^2
 + 9*a^2*b + 10*b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 1
0*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - (x*(a
+ b)^2*(a + 7*b))/2
```

3.20 $\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

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3.20.1 Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{(a+b)^3\cosh(c+dx)}{d} + \frac{3b(a+b)^2\operatorname{sech}(c+dx)}{d} - \frac{b^2(a+b)\operatorname{sech}^3(c+dx)}{d} + \frac{b^3\operatorname{sech}^5(c+dx)}{5d}$$

output
```
(a+b)^3*cosh(d*x+c)/d+3*b*(a+b)^2*sech(d*x+c)/d-b^2*(a+b)*sech(d*x+c)^3/d+
1/5*b^3*sech(d*x+c)^5/d
```

3.20.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{(a+b)^3\cosh(c+dx) + 3b(a+b)^2\operatorname{sech}(c+dx) - b^2(a+b)\operatorname{sech}^3(c+dx) + \frac{1}{5}b^3\operatorname{sech}^5(c+dx)}{d}$$

input
```
Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((a + b)^3*Cosh[c + d*x] + 3*b*(a + b)^2*Sech[c + d*x] - b^2*(a + b)*Sech[
c + d*x]^3 + (b^3*Sech[c + d*x]^5)/5)/d
```

3.20.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 26, 4147, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\downarrow 3042$$

$$\int -i\sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)^3\,dx$$

$$\downarrow 26$$

$$-i\int \sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)^3\,dx$$

$$\downarrow 4147$$

$$-\frac{\int \cosh^2(c+dx)\left(-b\text{sech}^2(c+dx)+a+b\right)^3\,d\text{sech}(c+dx)}{d}$$

$$\downarrow 244$$

$$-\frac{\int \left(-b^3\text{sech}^4(c+dx)+3b^2(a+b)\text{sech}^2(c+dx)-3b(a+b)^2+(a+b)^3\cosh^2(c+dx)\right)d\text{sech}(c+dx)}{d}$$

$$\downarrow 2009$$

$$-\frac{b^2(a+b)\text{sech}^3(c+dx)-(a+b)^3\cosh(c+dx)-3b(a+b)^2\text{sech}(c+dx)-\frac{1}{5}b^3\text{sech}^5(c+dx)}{d}$$

input
```
Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-((-((a + b)^3*Cosh[c + d*x]) - 3*b*(a + b)^2*Sech[c + d*x] + b^2*(a + b)*
Sech[c + d*x]^3 - (b^3*Sech[c + d*x]^5)/5)/d)
```

3.20.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.20.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(68) = 136$.

Time $= 1.81$ (sec) , antiderivative size $= 170$, normalized size of antiderivative $= 2.43$

method	result
derivativedivides	$\frac{a^3\cosh(dx+c)+3a^2b\left(\frac{\sinh(dx+c)^2}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)+3a\,b^2\left(\frac{\sinh(dx+c)^4}{\cosh(dx+c)^3}+\frac{4\sinh(dx+c)^2}{\cosh(dx+c)^3}+\frac{8}{3\cosh(dx+c)^3}\right)+b^3\left(\frac{\sinh(dx+c)^6}{\cosh(dx+c)^5}\right.}{d}$
default	$\frac{a^3\cosh(dx+c)+3a^2b\left(\frac{\sinh(dx+c)^2}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)+3a\,b^2\left(\frac{\sinh(dx+c)^4}{\cosh(dx+c)^3}+\frac{4\sinh(dx+c)^2}{\cosh(dx+c)^3}+\frac{8}{3\cosh(dx+c)^3}\right)+b^3\left(\frac{\sinh(dx+c)^6}{\cosh(dx+c)^5}\right.}{d}$
risch	$\frac{\mathrm{e}^{dx+c}a^3}{2d}+\frac{3\,\mathrm{e}^{dx+c}a^2b}{2d}+\frac{3\,\mathrm{e}^{dx+c}a\,b^2}{2d}+\frac{b^3\mathrm{e}^{dx+c}}{2d}+\frac{\mathrm{e}^{-dx-c}a^3}{2d}+\frac{3\,\mathrm{e}^{-dx-c}a^2b}{2d}+\frac{3\,\mathrm{e}^{-dx-c}a\,b^2}{2d}+\frac{\mathrm{e}^{-dx-c}b^3}{2d}+$

input
```
int(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*cosh(d*x+c)+3*a^2*b*(sinh(d*x+c)^2/cosh(d*x+c)+2/cosh(d*x+c))+3*a
*b^2*(sinh(d*x+c)^4/cosh(d*x+c)^3+4*sinh(d*x+c)^2/cosh(d*x+c)^3+8/3/cosh(d
*x+c)^3)+b^3*(sinh(d*x+c)^6/cosh(d*x+c)^5+6*sinh(d*x+c)^4/cosh(d*x+c)^5+8*
sinh(d*x+c)^2/cosh(d*x+c)^5+16/5/cosh(d*x+c)^5))
```

3.20.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(68) = 136$.

Time $= 0.28$ (sec) , antiderivative size $= 383$, normalized size of antiderivative $= 5.47$

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{5\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\cosh\left(dx+c\right)^6+5\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sinh\left(dx+c\right)^6+30\left(a^3+5\,a^2b+7\,a\right.}{}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/10*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 5*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*sinh(d*x + c)^6 + 30*(a^3 + 5*a^2*b + 7*a*b^2 + 3*b^3)*c
osh(d*x + c)^4 + 15*(2*a^3 + 10*a^2*b + 14*a*b^2 + 6*b^3 + 5*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 50*a^3 + 330*a^2*b +
 430*a*b^2 + 182*b^3 + 5*(15*a^3 + 93*a^2*b + 125*a*b^2 + 47*b^3)*cosh(d*x
 + c)^2 + 5*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 15*a^3 +
 93*a^2*b + 125*a*b^2 + 47*b^3 + 36*(a^3 + 5*a^2*b + 7*a*b^2 + 3*b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^2)/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(
d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x +
 c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

3.20.6 Sympy [F]

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^3\sinh\left(c+dx\right)dx$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*sinh(c + d*x), x)
```

3.20.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(68) = 136$.

Time = 0.20 (sec) , antiderivative size = 321, normalized size of antiderivative = 4.59

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{1}{10}b^3\left(\frac{5\,e^{(-dx-c)}}{d} + \frac{85\,e^{(-2\,dx-2\,c)} + 210\,e^{(-4\,dx-4\,c)} + 314\,e^{(-6\,dx-6\,c)} + 185\,e^{(-8\,dx-8\,c)} + 65\,e^{(-10\,dx-10\,c)} + 5}{d(e^{(-dx-c)} + 5\,e^{(-3\,dx-3\,c)} + 10\,e^{(-5\,dx-5\,c)} + 10\,e^{(-7\,dx-7\,c)} + 5\,e^{(-9\,dx-9\,c)} + e^{(-11\,dx-11\,c)}}\right)$$
$$+ \frac{1}{2}ab^2\left(\frac{3\,e^{(-dx-c)}}{d} + \frac{33\,e^{(-2\,dx-2\,c)} + 41\,e^{(-4\,dx-4\,c)} + 27\,e^{(-6\,dx-6\,c)} + 3}{d(e^{(-dx-c)} + 3\,e^{(-3\,dx-3\,c)} + 3\,e^{(-5\,dx-5\,c)} + e^{(-7\,dx-7\,c)})}\right)$$
$$+ \frac{3}{2}a^2b\left(\frac{e^{(-dx-c)}}{d} + \frac{5\,e^{(-2\,dx-2\,c)} + 1}{d(e^{(-dx-c)} + e^{(-3\,dx-3\,c)})}\right) + \frac{a^3\cosh(dx+c)}{d}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/10*b^3*(5*e^(-d*x - c)/d + (85*e^(-2*d*x - 2*c) + 210*e^(-4*d*x - 4*c) +
 314*e^(-6*d*x - 6*c) + 185*e^(-8*d*x - 8*c) + 65*e^(-10*d*x - 10*c) + 5)/
(d*(e^(-d*x - c) + 5*e^(-3*d*x - 3*c) + 10*e^(-5*d*x - 5*c) + 10*e^(-7*d*x
 - 7*c) + 5*e^(-9*d*x - 9*c) + e^(-11*d*x - 11*c)))) + 1/2*a*b^2*(3*e^(-d*
x - c)/d + (33*e^(-2*d*x - 2*c) + 41*e^(-4*d*x - 4*c) + 27*e^(-6*d*x - 6*c
) + 3)/(d*(e^(-d*x - c) + 3*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c) + e^(-7*
d*x - 7*c)))) + 3/2*a^2*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e
^(-d*x - c) + e^(-3*d*x - 3*c)))) + a^3*cosh(d*x + c)/d
```

3.20.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(68) = 136$.

Time = 0.41 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.37

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{5\,a^3\left(e^{(dx+c)} + e^{(-dx-c)}\right) + 15\,a^2b\left(e^{(dx+c)} + e^{(-dx-c)}\right) + 15\,ab^2\left(e^{(dx+c)} + e^{(-dx-c)}\right) + 5\,b^3\left(e^{(dx+c)} + e^{(-dx-c)}\right)}{}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/10*(5*a^3*(e^(d*x + c) + e^(-d*x - c)) + 15*a^2*b*(e^(d*x + c) + e^(-d*x
 - c)) + 15*a*b^2*(e^(d*x + c) + e^(-d*x - c)) + 5*b^3*(e^(d*x + c) + e^(-
d*x - c)) + 4*(15*a^2*b*(e^(d*x + c) + e^(-d*x - c))^4 + 30*a*b^2*(e^(d*x
+ c) + e^(-d*x - c))^4 + 15*b^3*(e^(d*x + c) + e^(-d*x - c))^4 - 20*a*b^2*
(e^(d*x + c) + e^(-d*x - c))^2 - 20*b^3*(e^(d*x + c) + e^(-d*x - c))^2 + 1
6*b^3)/(e^(d*x + c) + e^(-d*x - c))^5)/d
```

3.20.9 Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 308, normalized size of antiderivative = 4.40

$$
\begin{aligned}
&\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx \\
&= \frac{\mathrm{e}^{c+dx}\,(a+b)^3}{2\,d} + \frac{\mathrm{e}^{-c-dx}\,(a+b)^3}{2\,d} + \frac{6\,\mathrm{e}^{c+dx}\,(a^2\,b+2\,a\,b^2+b^3)}{d\,\left(\mathrm{e}^{2\,c+2\,dx}+1\right)} \\
&\quad - \frac{64\,b^3\,\mathrm{e}^{c+dx}}{5\,d\,\left(4\,\mathrm{e}^{2\,c+2\,dx}+6\,\mathrm{e}^{4\,c+4\,dx}+4\,\mathrm{e}^{6\,c+6\,dx}+\mathrm{e}^{8\,c+8\,dx}+1\right)} \\
&\quad + \frac{8\,\mathrm{e}^{c+dx}\,(9\,b^3+5\,a\,b^2)}{5\,d\,\left(3\,\mathrm{e}^{2\,c+2\,dx}+3\,\mathrm{e}^{4\,c+4\,dx}+\mathrm{e}^{6\,c+6\,dx}+1\right)} \\
&\quad + \frac{32\,b^3\,\mathrm{e}^{c+dx}}{5\,d\,\left(5\,\mathrm{e}^{2\,c+2\,dx}+10\,\mathrm{e}^{4\,c+4\,dx}+10\,\mathrm{e}^{6\,c+6\,dx}+5\,\mathrm{e}^{8\,c+8\,dx}+\mathrm{e}^{10\,c+10\,dx}+1\right)} \\
&\quad - \frac{8\,\mathrm{e}^{c+dx}\,(b^3+a\,b^2)}{d\,\left(2\,\mathrm{e}^{2\,c+2\,dx}+\mathrm{e}^{4\,c+4\,dx}+1\right)}
\end{aligned}
$$

input
```
int(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
(exp(c + d*x)*(a + b)^3)/(2*d) + (exp(- c - d*x)*(a + b)^3)/(2*d) + (6*exp
(c + d*x)*(2*a*b^2 + a^2*b + b^3))/(d*(exp(2*c + 2*d*x) + 1)) - (64*b^3*ex
p(c + d*x))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*
d*x) + exp(8*c + 8*d*x) + 1)) + (8*exp(c + d*x)*(5*a*b^2 + 9*b^3))/(5*d*(3
*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (32*b^3*
exp(c + d*x))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c
+ 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (8*exp(c + d*x)
*(a*b^2 + b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.21 $\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

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3.21.1 Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{a^3\operatorname{arctanh}(\cosh(c+dx))}{d} - \frac{b(3a^2+3ab+b^2)\operatorname{sech}(c+dx)}{d} + \frac{b^2(3a+2b)\operatorname{sech}^3(c+dx)}{3d} - \frac{b^3\operatorname{sech}^5(c+dx)}{5d}$$

output
```
-a^3*arctanh(cosh(d*x+c))/d-b*(3*a^2+3*a*b+b^2)*sech(d*x+c)/d+1/3*b^2*(3*a
+2*b)*sech(d*x+c)^3/d-1/5*b^3*sech(d*x+c)^5/d
```

3.21.2 Mathematica [A] (verified)

Time = 11.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{15a^3\left(-\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\right)-15b(3a^2+3ab+b^2)\operatorname{sech}(c+dx)+5b^2(3a+2}{15d}$$

input
```
Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(15*a^3*(-Log[Cosh[(c + d*x)/2]] + Log[Sinh[(c + d*x)/2]]) - 15*b*(3*a^2 +
 3*a*b + b^2)*Sech[c + d*x] + 5*b^2*(3*a + 2*b)*Sech[c + d*x]^3 - 3*b^3*Se
ch[c + d*x]^5)/(15*d)
```

3.21.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 26, 4147, 25, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$$

$$\downarrow 3042$$

$$\int \frac{i\left(a-b\tan(ic+idx)^2\right)^3}{\sin(ic+idx)}dx$$

$$\downarrow 26$$

$$i\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\sin(ic+idx)}dx$$

$$\downarrow 4147$$

$$\frac{\int -\frac{\left(-b\operatorname{sech}^2(c+dx)+a+b\right)^3}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\left(-b\operatorname{sech}^2(c+dx)+a+b\right)^3}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)}{d}$$

$$\downarrow 300$$

$$-\frac{\int \left(b^3\operatorname{sech}^4(c+dx)-b^2(3a+2b)\operatorname{sech}^2(c+dx)+b\left(3a^2+3ba+b^2\right)+\frac{a^3}{1-\operatorname{sech}^2(c+dx)}\right)d\operatorname{sech}(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{-a^3\text{arctanh}(\text{sech}(c+dx)) - b\left(3a^2+3ab+b^2\right)\text{sech}(c+dx) + \frac{1}{3}b^2(3a+2b)\text{sech}^3(c+dx) - \frac{1}{5}b^3\text{sech}^5(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-(a^3*ArcTanh[Sech[c + d*x]]) - b*(3*a^2 + 3*a*b + b^2)*Sech[c + d*x] + (
b^2*(3*a + 2*b)*Sech[c + d*x]^3)/3 - (b^3*Sech[c + d*x]^5)/5)/d
```

3.21.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.21.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{-2a^3 \operatorname{arctanh}\left(e^{dx+c}\right)-\frac{3a^2b}{\cosh(dx+c)}+3ab^2\left(-\frac{\sinh(dx+c)^2}{\cosh(dx+c)^3}-\frac{2}{3\cosh(dx+c)^3}\right)+b^3\left(-\frac{\sinh(dx+c)^4}{\cosh(dx+c)^5}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)^5}-\frac{8}{15\cosh(d}\right.}{d}$
default	$\frac{-2a^3 \operatorname{arctanh}\left(e^{dx+c}\right)-\frac{3a^2b}{\cosh(dx+c)}+3ab^2\left(-\frac{\sinh(dx+c)^2}{\cosh(dx+c)^3}-\frac{2}{3\cosh(dx+c)^3}\right)+b^3\left(-\frac{\sinh(dx+c)^4}{\cosh(dx+c)^5}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)^5}-\frac{8}{15\cosh(d}\right.}{d}$
risch	$-\frac{2b\,e^{dx+c}\left(45a^2e^{8dx+8c}+45ab\,e^{8dx+8c}+15b^2e^{8dx+8c}+180a^2e^{6dx+6c}+120ab\,e^{6dx+6c}+20b^2e^{6dx+6c}+270a^2e^{4dx+4c}+15\right.}{15d\left(e^{2dx+2c}+1\right)^5}$

input
```
int(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a^3*arctanh(exp(d*x+c))-3*a^2*b/cosh(d*x+c)+3*a*b^2*(-sinh(d*x+c)^
2/cosh(d*x+c)^3-2/3/cosh(d*x+c)^3)+b^3*(-sinh(d*x+c)^4/cosh(d*x+c)^5-4/3*s
inh(d*x+c)^2/cosh(d*x+c)^5-8/15/cosh(d*x+c)^5))
```

3.21.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2277 vs. 2(80) = 160.

Time = 0.28 (sec) , antiderivative size = 2277, normalized size of antiderivative = 27.11

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output

```
-1/15*(30*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 270*(3*a^2*b + 3*a*b
^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^8 + 30*(3*a^2*b + 3*a*b^2 + b^3)*sin
h(d*x + c)^9 + 40*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^7 + 40*(9*a^2*b
+ 6*a*b^2 + b^3 + 27*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x +
 c)^7 + 280*(9*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (9*a^2*b + 6*a*
b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 4*(135*a^2*b + 75*a*b^2 + 29*b
^3)*cosh(d*x + c)^5 + 4*(945*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 1
35*a^2*b + 75*a*b^2 + 29*b^3 + 210*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)
^2)*sinh(d*x + c)^5 + 20*(189*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 +
70*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^3 + (135*a^2*b + 75*a*b^2 + 29*
b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 40*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*
x + c)^3 + 40*(63*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 35*(9*a^2*b
+ 6*a*b^2 + b^3)*cosh(d*x + c)^4 + 9*a^2*b + 6*a*b^2 + b^3 + (135*a^2*b +
75*a*b^2 + 29*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 40*(27*(3*a^2*b + 3*
a*b^2 + b^3)*cosh(d*x + c)^7 + 21*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^
5 + (135*a^2*b + 75*a*b^2 + 29*b^3)*cosh(d*x + c)^3 + 3*(9*a^2*b + 6*a*b^2
 + b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 30*(3*a^2*b + 3*a*b^2 + b^3)*cosh
(d*x + c) + 15*(a^3*cosh(d*x + c)^10 + 10*a^3*cosh(d*x + c)*sinh(d*x + c)^
9 + a^3*sinh(d*x + c)^10 + 5*a^3*cosh(d*x + c)^8 + 10*a^3*cosh(d*x + c)^6
+ 5*(9*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^8 + 40*(3*a^3*cosh(d*x ...
```

3.21.6 Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^3 \operatorname{csch}\left(c+dx\right)dx$$

input

```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

output

```
Integral((a + b*tanh(c + d*x)**2)**3*csch(c + d*x), x)
```

3.21.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 560 vs. $2(80) = 160$.

Time $= 0.21$ (sec) , antiderivative size $= 560$, normalized size of antiderivative $= 6.67$

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$
$$-\frac{2}{15}b^3\left(\frac{15\,e^{(-dx-c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{}{d(5\,e^{(-2\,dx-2\,c}\right.$$
$$-2\,ab^2\left(\frac{3\,e^{(-dx-c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{2\,e^{(-3\,dx-3\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right.$$
$$+\frac{a^3\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}-\frac{6\,a^2b}{d(e^{(dx+c)}+e^{(-dx-c)})}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-2/15*b^3*(15*e^(-d*x - c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 20*e
^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*
x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 58*e^(-5*d*x -
5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) +
5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 20*e^(-7*d*x - 7*c)/(d*(5*
e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x
 - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-9*d*x - 9*c)/(d*(5*e^(-2*d*x -
 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e
^(-10*d*x - 10*c) + 1))) - 2*a*b^2*(3*e^(-d*x - c)/(d*(3*e^(-2*d*x - 2*c)
+ 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 2*e^(-3*d*x - 3*c)/(d*(3*e
^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 3*e^(-5*d*
x - 5*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) +
1))) + a^3*log(tanh(1/2*d*x + 1/2*c))/d - 6*a^2*b/(d*(e^(d*x + c) + e^(-d*
x - c)))
```

3.21.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(80) = 160$.

Time $= 0.42$ (sec) , antiderivative size $= 196$, normalized size of antiderivative $= 2.33$

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$

$$-\frac{15\,a^3\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-15\,a^3\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)+\frac{4\left(45\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4+45\,ab^2\left(e^{(dx}\right.}{30\,d}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/30*(15*a^3*log(e^(d*x + c) + e^(-d*x - c) + 2) - 15*a^3*log(e^(d*x + c)
 + e^(-d*x - c) - 2) + 4*(45*a^2*b*(e^(d*x + c) + e^(-d*x - c))^4 + 45*a*b
^2*(e^(d*x + c) + e^(-d*x - c))^4 + 15*b^3*(e^(d*x + c) + e^(-d*x - c))^4
- 60*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 - 40*b^3*(e^(d*x + c) + e^(-d*x
- c))^2 + 48*b^3)/(e^(d*x + c) + e^(-d*x - c))^5)/d
```

3.21.9 Mupad [B] (verification not implemented)

Time $= 0.27$ (sec) , antiderivative size $= 317$, normalized size of antiderivative $= 3.77$

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$=\frac{8\,\mathrm{e}^{c+dx}\left(2\,b^3+3\,a\,b^2\right)}{3\,d\,\left(2\,\mathrm{e}^{2\,c+2\,dx}+\mathrm{e}^{4\,c+4\,dx}+1\right)}-\frac{2\,\operatorname{atan}\left(\frac{a^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^6}}\right)\,\sqrt{a^6}}{\sqrt{-d^2}}$$
$$-\frac{2\,\mathrm{e}^{c+dx}\left(3\,a^2\,b+3\,a\,b^2+b^3\right)}{d\,\left(\mathrm{e}^{2\,c+2\,dx}+1\right)}+\frac{64\,b^3\,\mathrm{e}^{c+dx}}{5\,d\,\left(4\,\mathrm{e}^{2\,c+2\,dx}+6\,\mathrm{e}^{4\,c+4\,dx}+4\,\mathrm{e}^{6\,c+6\,dx}+\mathrm{e}^{8\,c+8\,dx}+1\right)}$$
$$-\frac{8\,\mathrm{e}^{c+dx}\left(22\,b^3+15\,a\,b^2\right)}{15\,d\,\left(3\,\mathrm{e}^{2\,c+2\,dx}+3\,\mathrm{e}^{4\,c+4\,dx}+\mathrm{e}^{6\,c+6\,dx}+1\right)}$$
$$-\frac{32\,b^3\,\mathrm{e}^{c+dx}}{5\,d\,\left(5\,\mathrm{e}^{2\,c+2\,dx}+10\,\mathrm{e}^{4\,c+4\,dx}+10\,\mathrm{e}^{6\,c+6\,dx}+5\,\mathrm{e}^{8\,c+8\,dx}+\mathrm{e}^{10\,c+10\,dx}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)^3/sinh(c + d*x),x)
```

output

```
(8*exp(c + d*x)*(3*a*b^2 + 2*b^3))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*
d*x) + 1)) - (2*atan((a^3*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^6)^(1/2)))*(
a^6)^(1/2))/(-d^2)^(1/2) - (2*exp(c + d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(d*(
exp(2*c + 2*d*x) + 1)) + (64*b^3*exp(c + d*x))/(5*d*(4*exp(2*c + 2*d*x) +
6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (8*exp(
c + d*x)*(15*a*b^2 + 22*b^3))/(15*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*
x) + exp(6*c + 6*d*x) + 1)) - (32*b^3*exp(c + d*x))/(5*d*(5*exp(2*c + 2*d*
x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(
10*c + 10*d*x) + 1))
```

3.22 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

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3.22.1 Optimal result

Integrand size = 23, antiderivative size = 64

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{a^3\coth(c+dx)}{d}+\frac{3a^2b\tanh(c+dx)}{d}+\frac{ab^2\tanh^3(c+dx)}{d}+\frac{b^3\tanh^5(c+dx)}{5d}$$

output
```
-a^3*coth(d*x+c)/d+3*a^2*b*tanh(d*x+c)/d+a*b^2*tanh(d*x+c)^3/d+1/5*b^3*tanh(d*x+c)^5/d
```

3.22.2 Mathematica [A] (verified)

Time = 2.80 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{-5a^3\coth(c+dx)+b\left(15a^2+5ab+b^2-b(5a+2b)\operatorname{sech}^2(c+dx)+b^2\operatorname{sech}^4(c+dx)\right)\tanh(c+dx)}{5d}$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-5*a^3*Coth[c + d*x] + b*(15*a^2 + 5*a*b + b^2 - b*(5*a + 2*b)*Sech[c + d*x]^2 + b^2*Sech[c + d*x]^4)*Tanh[c + d*x])/(5*d)
```

3.22.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 25, 4146, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a-b\tan(ic+idx)^2\right)^3}{\sin(ic+idx)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\sin(ic+idx)^2}dx$$

$$\downarrow 4146$$

$$\frac{\int \coth^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^3 d\tanh(c+dx)}{d}$$

$$\downarrow 244$$

$$\frac{\int \left(b^3\tanh^4(c+dx)+3ab^2\tanh^2(c+dx)+a^3\coth^2(c+dx)+3a^2b\right) d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{-a^3\coth(c+dx)+3a^2b\tanh(c+dx)+ab^2\tanh^3(c+dx)+\frac{1}{5}b^3\tanh^5(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-(a^3*Coth[c + d*x]) + 3*a^2*b*Tanh[c + d*x] + a*b^2*Tanh[c + d*x]^3 + (b
^3*Tanh[c + d*x]^5)/5)/d
```

3.22.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.22.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(62) = 124$.

Time = 5.62 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.20

method	result
derivativedivides	$\frac{-a^3\coth(dx+c)+3a^2b\tanh(dx+c)+3ab^2\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+b^3\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\,\mathrm{s}}{8\,\mathrm{co}}\right.}{d}$
default	$\frac{-a^3\coth(dx+c)+3a^2b\tanh(dx+c)+3ab^2\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+b^3\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\,\mathrm{s}}{8\,\mathrm{co}}\right.}{d}$
risch	$-\frac{2\left(5a^3\mathrm{e}^{10dx+10c}+15a^2b\,\mathrm{e}^{10dx+10c}+15a\,b^2\mathrm{e}^{10dx+10c}+5b^3\mathrm{e}^{10dx+10c}+25a^3\mathrm{e}^{8dx+8c}+45a^2b\,\mathrm{e}^{8dx+8c}+15a\,b^2\mathrm{e}^{8dx+8c}-5\right.}{}$

input
```
int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^3*coth(d*x+c)+3*a^2*b*tanh(d*x+c)+3*a*b^2*(-1/2*sinh(d*x+c)/cosh(d
*x+c)^3+1/2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(-1/2*sinh(d*x+c)^3/c
osh(d*x+c)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/1
5*sech(d*x+c)^2)*tanh(d*x+c)))
```

3.22.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(62) = 124$.

Time $= 0.26$ (sec) , antiderivative size $= 572$, normalized size of antiderivative $= 8.94$

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$
$$-\frac{4\left(\left(5\,a^3+5\,ab^2+2\,b^3\right)\cosh\left(dx+c\right)^5+5\left(5\,a^3+5\,ab^2+2\,b^3\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4+\left(15\,a^2b+1\right.\right.}{5\left(d\cosh\left(dx+c\right)^7+7\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^6+d\sinh\left(dx+c\right)^7+3\,d\cosh\left(d\right.\right.}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-4/5*((5*a^3 + 5*a*b^2 + 2*b^3)*cosh(d*x + c)^5 + 5*(5*a^3 + 5*a*b^2 + 2*b
^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (15*a^2*b + 10*a*b^2 + 3*b^3)*sinh(d*x
 + c)^5 + (25*a^3 + 5*a*b^2 - 2*b^3)*cosh(d*x + c)^3 + (45*a^2*b + 10*a*b^
2 - 3*b^3 + 10*(15*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^3 + (10*(5*a^3 + 5*a*b^2 + 2*b^3)*cosh(d*x + c)^3 + 3*(25*a^3 + 5*a*b^2
 - 2*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^3 - a*b^2)*cosh(d*x + c)
 + (5*(15*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^4 + 30*a^2*b + 10*b^3 +
3*(45*a^2*b + 10*a*b^2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*
x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 3*d*cos
h(d*x + c)^5 + (21*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(
d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (35*
d*cosh(d*x + c)^4 + 50*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3 + 3*(7*d*c
osh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^2 -
 5*d*cosh(d*x + c) + (7*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 27*d*co
sh(d*x + c)^2 + 5*d)*sinh(d*x + c))
```

3.22.6 Sympy [F]

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2(c+dx)\right)^3 \operatorname{csch}^2(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*csch(c + d*x)**2, x)
```

3.22.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(62) = 124$.

Time = 0.20 (sec) , antiderivative size = 348, normalized size of antiderivative = 5.44

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{2}{5}b^3\left(\frac{10\,e^{(-4\,dx-4\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{}{d(5\,e^{(-2\,dx-2\,c)}+}\right.$$
$$+2\,ab^2\left(\frac{3\,e^{(-4\,dx-4\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right.$$
$$\left.+\frac{6\,a^2b}{d(e^{(-2\,dx-2\,c)}+1)}+\frac{2\,a^3}{d(e^{(-2\,dx-2\,c)}-1)}\right.$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
2/5*b^3*(10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
+ 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*
e^(-8*d*x - 8*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d
*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*
d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c
) + e^(-10*d*x - 10*c) + 1))) + 2*a*b^2*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*
x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x
 - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 6*a^2*b/(d*(e^(-2
*d*x - 2*c) + 1)) + 2*a^3/(d*(e^(-2*d*x - 2*c) - 1))
```

3.22.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(62) = 124$.

Time $= 0.42$ (sec) , antiderivative size $= 202$, normalized size of antiderivative $= 3.16$

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{2\left(\frac{5a^3}{e^{(2dx+2c)}-1}+\frac{15a^2be^{(8dx+8c)}+15ab^2e^{(8dx+8c)}+5b^3e^{(8dx+8c)}+60a^2be^{(6dx+6c)}+30ab^2e^{(6dx+6c)}+90a^2be^{(4dx+4c)}+20ab^2e^{(4dx}}{\left(e^{(2dx+2c)}+1\right)^5}\right.}{5d}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-2/5*(5*a^3/(e^(2*d*x + 2*c) - 1) + (15*a^2*b*e^(8*d*x + 8*c) + 15*a*b^2*e
^(8*d*x + 8*c) + 5*b^3*e^(8*d*x + 8*c) + 60*a^2*b*e^(6*d*x + 6*c) + 30*a*b
^2*e^(6*d*x + 6*c) + 90*a^2*b*e^(4*d*x + 4*c) + 20*a*b^2*e^(4*d*x + 4*c) +
 10*b^3*e^(4*d*x + 4*c) + 60*a^2*b*e^(2*d*x + 2*c) + 10*a*b^2*e^(2*d*x + 2
*c) + 15*a^2*b + 5*a*b^2 + b^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.22.9 Mupad [B] (verification not implemented)

Time $= 1.88$ (sec) , antiderivative size $= 590$, normalized size of antiderivative $= 9.22$

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{\frac{2\left(3a^2b-b^3\right)}{5d}+\frac{2e^{2c+2dx}\left(3a^2b+3ab^2+b^3\right)}{5d}}{2e^{2c+2dx}+e^{4c+4dx}+1}$$
$$-\frac{\frac{2\left(3a^2b-ab^2+b^3\right)}{5d}+\frac{2e^{4c+4dx}\left(3a^2b+3ab^2+b^3\right)}{5d}+\frac{4e^{2c+2dx}\left(3a^2b-b^3\right)}{5d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}$$
$$-\frac{\frac{2\left(3a^2b-b^3\right)}{5d}+\frac{6e^{2c+2dx}\left(3a^2b-ab^2+b^3\right)}{5d}+\frac{2e^{6c+6dx}\left(3a^2b+3ab^2+b^3\right)}{5d}+\frac{6e^{4c+4dx}\left(3a^2b-b^3\right)}{5d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1}$$
$$-\frac{\frac{2\left(3a^2b+3ab^2+b^3\right)}{5d}+\frac{12e^{4c+4dx}\left(3a^2b-ab^2+b^3\right)}{5d}+\frac{2e^{8c+8dx}\left(3a^2b+3ab^2+b^3\right)}{5d}+\frac{8e^{2c+2dx}\left(3a^2b-b^3\right)}{5d}+\frac{8e^{6c+6dx}\left(3a^2b-b^3\right.}{5d}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1}$$
$$-\frac{2a^3}{d\left(e^{2c+2dx}-1\right)}-\frac{2\left(3a^2b+3ab^2+b^3\right)}{5d\left(e^{2c+2dx}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)^3/sinh(c + d*x)^2,x)
```

output

```
- ((2*(3*a^2*b - b^3))/(5*d) + (2*exp(2*c + 2*d*x)*(3*a*b^2 + 3*a^2*b + b^
3))/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*(3*a^2*b - a*
b^2 + b^3))/(5*d) + (2*exp(4*c + 4*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(5*d) +
 (4*exp(2*c + 2*d*x)*(3*a^2*b - b^3))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4
*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((2*(3*a^2*b - b^3))/(5*d) + (6*exp(
2*c + 2*d*x)*(3*a^2*b - a*b^2 + b^3))/(5*d) + (2*exp(6*c + 6*d*x)*(3*a*b^2
 + 3*a^2*b + b^3))/(5*d) + (6*exp(4*c + 4*d*x)*(3*a^2*b - b^3))/(5*d))/(4*
exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d
*x) + 1) - ((2*(3*a*b^2 + 3*a^2*b + b^3))/(5*d) + (12*exp(4*c + 4*d*x)*(3*
a^2*b - a*b^2 + b^3))/(5*d) + (2*exp(8*c + 8*d*x)*(3*a*b^2 + 3*a^2*b + b^3
))/(5*d) + (8*exp(2*c + 2*d*x)*(3*a^2*b - b^3))/(5*d) + (8*exp(6*c + 6*d*x
)*(3*a^2*b - b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*e
xp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - (2*a^3)/(
d*(exp(2*c + 2*d*x) - 1)) - (2*(3*a*b^2 + 3*a^2*b + b^3))/(5*d*(exp(2*c +
2*d*x) + 1))
```

3.23 $\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

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3.23.1 Optimal result

Integrand size = 23, antiderivative size = 152

$$\begin{aligned}
&\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx \\
&= \frac{a^2(a-6b)\operatorname{arctanh}(\cosh(c+dx))}{2d} + \frac{b(81a^2-28ab-4b^2)\operatorname{sech}(c+dx)}{30d} \\
&\quad + \frac{(33a-2b)b\operatorname{sech}(c+dx)\left(a+b-b\operatorname{sech}^2(c+dx)\right)}{30d} \\
&\quad + \frac{7b\operatorname{sech}(c+dx)\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2}{10d} \\
&\quad - \frac{\coth(c+dx)\operatorname{csch}(c+dx)\left(a+b-b\operatorname{sech}^2(c+dx)\right)^3}{2d}
\end{aligned}$$

output

```
1/2*a^2*(a-6*b)*arctanh(cosh(d*x+c))/d+1/30*b*(81*a^2-28*a*b-4*b^2)*sech(d
*x+c)/d+1/30*(33*a-2*b)*b*sech(d*x+c)*(a+b-b*sech(d*x+c)^2)/d+7/10*b*sech(
d*x+c)*(a+b-b*sech(d*x+c)^2)^2/d-1/2*coth(d*x+c)*csch(d*x+c)*(a+b-b*sech(d
*x+c)^2)^3/d
```

3.23.2 Mathematica [A] (verified)

Time = 17.15 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{(a^3-6a^2b)\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{(-a^3+6a^2b)\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a^3\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{3a^2b\operatorname{sech}(c+dx)}{d} - \frac{b^2(3a+b)\operatorname{sech}^3(c+dx)}{3d} + \frac{b^3\operatorname{sech}^5(c+dx)}{5d}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-1/8*(a^3*Csch[(c + d*x)/2]^2)/d + ((a^3 - 6*a^2*b)*Log[Cosh[(c + d*x)/2]]
)/(2*d) + ((-a^3 + 6*a^2*b)*Log[Sinh[(c + d*x)/2]])/(2*d) - (a^3*Sech[(c +
 d*x)/2]^2)/(8*d) + (3*a^2*b*Sech[c + d*x])/d - (b^2*(3*a + b)*Sech[c + d*
x]^3)/(3*d) + (b^3*Sech[c + d*x]^5)/(5*d)
```

3.23.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 26, 4147, 369, 403, 25, 403, 25, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\left(a-b\tan(ic+idx)^2\right)^3}{\sin(ic+idx)^3}\,dx$$

$$\downarrow 26$$

$$-i\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\sin(ic+idx)^3}\,dx$$

$$\downarrow 4147$$

$$-\frac{\int \frac{\text{sech}^2(c+dx)\left(-b\text{sech}^2(c+dx)+a+b\right)^3}{\left(1-\text{sech}^2(c+dx)\right)^2}d\text{sech}(c+dx)}{d}$$

$$\downarrow 369$$

$$-\frac{\frac{\text{sech}(c+dx)\left(a-b\text{sech}^2(c+dx)+b\right)^3}{2\left(1-\text{sech}^2(c+dx)\right)}-\frac{1}{2}\int\frac{\left(-7b\text{sech}^2(c+dx)+a+b\right)\left(-b\text{sech}^2(c+dx)+a+b\right)^2}{1-\text{sech}^2(c+dx)}d\text{sech}(c+dx)}{d}$$

$$\downarrow 403$$

$$-\frac{\frac{1}{2}\left(\frac{1}{5}\int-\frac{\left(-b\text{sech}^2(c+dx)+a+b\right)\left((5a-2b)(a+b)-(33a-2b)b\text{sech}^2(c+dx)\right)}{1-\text{sech}^2(c+dx)}d\text{sech}(c+dx)-\frac{7}{5}b\text{sech}(c+dx)\left(a-b\text{sech}^2(c+d\right.\right.}{d}$$

$$\downarrow 25$$

$$-\frac{\frac{1}{2}\left(-\frac{1}{5}\int\frac{\left(-b\text{sech}^2(c+dx)+a+b\right)\left((5a-2b)(a+b)-(33a-2b)b\text{sech}^2(c+dx)\right)}{1-\text{sech}^2(c+dx)}d\text{sech}(c+dx)-\frac{7}{5}b\text{sech}(c+dx)\left(a-b\text{sech}^2(c+d\right.\right.}{d}$$

$$\downarrow 403$$

$$-\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{1}{3}\int-\frac{(a+b)(15a^2-24ba-4b^2)-b(81a^2-28ba-4b^2)\text{sech}^2(c+dx)}{1-\text{sech}^2(c+dx)}d\text{sech}(c+dx)-\frac{1}{3}b(33a-2b)\text{sech}(c+dx)\left(a-b\text{sech}\right.\right.\right.}{d}$$

$$\downarrow 25$$

$$-\frac{\frac{1}{2}\left(\frac{1}{5}\left(-\frac{1}{3}\int\frac{(a+b)(15a^2-24ba-4b^2)-b(81a^2-28ba-4b^2)\text{sech}^2(c+dx)}{1-\text{sech}^2(c+dx)}d\text{sech}(c+dx)-\frac{1}{3}b(33a-2b)\text{sech}(c+dx)\left(a-b\text{sech}\right.\right.\right.}{d}$$

$$\downarrow 299$$

$$-\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{1}{3}\left(-15a^2(a-6b)\int\frac{1}{1-\text{sech}^2(c+dx)}d\text{sech}(c+dx)-b\left(81a^2-28ab-4b^2\right)\text{sech}(c+dx)\right)-\frac{1}{3}b(33a-2b)\text{sec}\right.\right.\right.}{d}$$

$$\downarrow 219$$

$$-\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{1}{3}\left(-15a^2(a-6b)\operatorname{arctanh}(\operatorname{sech}(c+dx))-b\left(81a^2-28ab-4b^2\right)\operatorname{sech}(c+dx)\right)-\frac{1}{3}b(33a-2b)\operatorname{sech}(c+dx)\right.\right.}{d}$$

input
```
Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-(((Sech[c + d*x]*(a + b - b*Sech[c + d*x]^2)^3)/(2*(1 - Sech[c + d*x]^2))
 + ((-7*b*Sech[c + d*x]*(a + b - b*Sech[c + d*x]^2)^2)/5 + ((-15*a^2*(a -
6*b)*ArcTanh[Sech[c + d*x]] - b*(81*a^2 - 28*a*b - 4*b^2)*Sech[c + d*x])/3
 - ((33*a - 2*b)*b*Sech[c + d*x]*(a + b - b*Sech[c + d*x]^2))/3)/5)/2)/d)
```

3.23.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.23.4 Maple [A] (verified)

Time = 11.55 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.68

method	result
derivativedivides	$\frac{a^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3a^2b\left(\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)-\frac{a\,b^2}{\cosh(dx+c)^3}+b^3\left(-\frac{\sinh(dx+c)^2}{3\cosh(dx+c)}\right.}{d}$
default	$\frac{a^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3a^2b\left(\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)-\frac{a\,b^2}{\cosh(dx+c)^3}+b^3\left(-\frac{\sinh(dx+c)^2}{3\cosh(dx+c)}\right.}{d}$
risch	$-\frac{e^{dx+c}\left(15a^3e^{12dx+12c}-90a^2b\,e^{12dx+12c}+90a^3e^{10dx+10c}-180a^2b\,e^{10dx+10c}+120a\,b^2e^{10dx+10c}+40b^3e^{10dx+10c}+225\right.}{}$

input
```
int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+3*a^2*b*(1/cos
h(d*x+c)-2*arctanh(exp(d*x+c)))-a*b^2/cosh(d*x+c)^3+b^3*(-1/3*sinh(d*x+c)^
2/cosh(d*x+c)^5-2/15/cosh(d*x+c)^5))
```

3.23.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5037 vs. 2(151) = 302.

Time = 0.30 (sec) , antiderivative size = 5037, normalized size of antiderivative = 33.14

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.23.6 Sympy [F]

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2(c+dx)\right)^3 \operatorname{csch}^3(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*csch(c + d*x)**3, x)
```

3.23.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(151) = 302.

Time = 0.21 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.65

$$\begin{aligned}
&\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx \\
&= \frac{1}{2}a^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right) \\
&\quad - 3\,a^2b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} - \frac{2\,e^{(-dx-c)}}{d(e^{(-2\,dx-2\,c)}+1)}\right) \\
&\quad - \frac{8}{15}b^3\left(\frac{5\,e^{(-3\,dx-3\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)} - \frac{}{d(5\,e^{(-2\,dx-2\,c}\right. \\
&\quad \left. - \frac{8\,ab^2}{d(e^{(dx+c)}+e^{(-dx-c)})^3}\right.
\end{aligned}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/2*a^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x -
c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) -
3*a^2*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c
)/(d*(e^(-2*d*x - 2*c) + 1))) - 8/15*b^3*(5*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d
*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c)
 + e^(-10*d*x - 10*c) + 1)) - 2*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) +
10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*
x - 10*c) + 1)) + 5*e^(-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x
 - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) +
1))) - 8*a*b^2/(d*(e^(d*x + c) + e^(-d*x - c))^3)
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.36

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$

$$-\frac{\frac{60\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4}-15\left(a^3-6\,a^2b\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)+15\left(a^3-6\,a^2b\right)\log\left(e^{(dx+c)}+e^{(-dx-c}\right.}{60\,d}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/60*(60*a^3*(e^(d*x + c) + e^(-d*x - c))/((e^(d*x + c) + e^(-d*x - c))^2
 - 4) - 15*(a^3 - 6*a^2*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) + 15*(a^3 -
 6*a^2*b)*log(e^(d*x + c) + e^(-d*x - c) - 2) - 8*(45*a^2*b*(e^(d*x + c) +
 e^(-d*x - c))^4 - 60*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 - 20*b^3*(e^(d*
x + c) + e^(-d*x - c))^2 + 48*b^3)/(e^(d*x + c) + e^(-d*x - c))^5)/d
```

3.23.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.71

$$
\begin{aligned}
&\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx \\
&= \frac{\operatorname{atan}\left(\frac{e^{dx}e^{c}\left(a^3\sqrt{-d^2}-6a^2b\sqrt{-d^2}\right)}{d\sqrt{a^6-12a^5b+36a^4b^2}}\right)\sqrt{a^6-12a^5b+36a^4b^2}}{\sqrt{-d^2}} \\
&\quad - \frac{64b^3e^{c+dx}}{5d\left(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1\right)} \\
&\quad + \frac{8e^{c+dx}\left(17b^3+15ab^2\right)}{15d\left(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1\right)} \\
&\quad + \frac{32b^3e^{c+dx}}{5d\left(5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1\right)} \\
&\quad - \frac{a^3e^{c+dx}}{d\left(e^{2c+2dx}-1\right)} - \frac{8e^{c+dx}\left(b^3+3ab^2\right)}{3d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)} \\
&\quad - \frac{2a^3e^{c+dx}}{d\left(e^{4c+4dx}-2e^{2c+2dx}+1\right)} + \frac{6a^2be^{c+dx}}{d\left(e^{2c+2dx}+1\right)}
\end{aligned}
$$

input
```
int((a + b*tanh(c + d*x)^2)^3/sinh(c + d*x)^3,x)
```

output
```
(atan((exp(d*x)*exp(c)*(a^3*(-d^2)^(1/2) - 6*a^2*b*(-d^2)^(1/2)))/(d*(a^6
- 12*a^5*b + 36*a^4*b^2)^(1/2)))*(a^6 - 12*a^5*b + 36*a^4*b^2)^(1/2))/(-d^
2)^(1/2) - (64*b^3*exp(c + d*x))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*
d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (8*exp(c + d*x)*(15*a
*b^2 + 17*b^3))/(15*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c +
 6*d*x) + 1)) + (32*b^3*exp(c + d*x))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*
c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x)
 + 1)) - (a^3*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (8*exp(c + d*x)*(
3*a*b^2 + b^3))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (2*a^3
*exp(c + d*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) + (6*a^2*b*
exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1))
```

3.24 $\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

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3.24.1 Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{a^2(a-3b)\coth(c+dx)}{d} - \frac{a^3\coth^3(c+dx)}{3d} - \frac{3a(a-b)b\tanh(c+dx)}{d} - \frac{(3a-b)b^2\tanh^3(c+dx)}{3d} - \frac{b^3\tanh^5(c+dx)}{5d}$$

output
```
a^2*(a-3*b)*coth(d*x+c)/d-1/3*a^3*coth(d*x+c)^3/d-3*a*(a-b)*b*tanh(d*x+c)/
d-1/3*(3*a-b)*b^2*tanh(d*x+c)^3/d-1/5*b^3*tanh(d*x+c)^5/d
```

3.24.2 Mathematica [A] (verified)

Time = 3.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.89

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{-5a^2\coth(c+dx)\left(-2a+9b+a\operatorname{csch}^2(c+dx)\right)+b\left(-45a^2+30ab+2b^2+b(15a+b)\operatorname{sech}^2(c+dx)-3b\right.}{15d}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-5*a^2*Coth[c + d*x]*(-2*a + 9*b + a*Csch[c + d*x]^2) + b*(-45*a^2 + 30*a
*b + 2*b^2 + b*(15*a + b)*Sech[c + d*x]^2 - 3*b^2*Sech[c + d*x]^4)*Tanh[c
+ d*x])/(15*d)
```

3.24.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4146, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\sin(ic+idx)^4}dx$$

$$\downarrow 4146$$

$$\frac{\int \coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3 d\tanh(c+dx)}{d}$$

$$\downarrow 355$$

$$\frac{\int \left(a^3\coth^4(c+dx)-a^2(a-3b)\coth^2(c+dx)-b^3\tanh^4(c+dx)-(3a-b)b^2\tanh^2(c+dx)-3a(a-b)b\right)d\tan}{d}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{3}a^3\coth^3(c+dx)+a^2(a-3b)\coth(c+dx)-\frac{1}{3}b^2(3a-b)\tanh^3(c+dx)-3ab(a-b)\tanh(c+dx)-\frac{1}{5}b^3\tanh^5}{d}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(a^2*(a - 3*b)*Coth[c + d*x] - (a^3*Coth[c + d*x]^3)/3 - 3*a*(a - b)*b*Tan
h[c + d*x] - ((3*a - b)*b^2*Tanh[c + d*x]^3)/3 - (b^3*Tanh[c + d*x]^5)/5)/
d
```

3.24.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.24.4 Maple [A] (verified)

Time = 27.66 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{a^3\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+3a^2b\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)+3ab^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{d}$
default	$\frac{a^3\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+3a^2b\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)+3ab^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{d}$
risch	$-\frac{4\left(-15ab^2+15a^3e^{12dx+12c}+15b^3e^{12dx+12c}+70a^3e^{10dx+10c}-5a^3-30ab^2e^{10dx+10c}+17e^{4dx+4c}b^3+45a^2b-30e^{2dx+2}\right.}{}$

input
```
int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(-1/sinh(d*x+c)/cosh(
d*x+c)-2*tanh(d*x+c))+3*a*b^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+b^3*(-1/
4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2
)*tanh(d*x+c)))
```

3.24.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(92) = 184$.

Time $= 0.25$ (sec) , antiderivative size $= 925$, normalized size of antiderivative $= 9.44$

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-8/15*((5*a^3 + 45*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 + 12*(5*a^3 +
 15*a*b^2 + 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^3 + 45*a^2*b + 15*
a*b^2 + 7*b^3)*sinh(d*x + c)^6 + 2*(15*a^3 + 45*a^2*b - 15*a*b^2 - 13*b^3)
*cosh(d*x + c)^4 + (30*a^3 + 90*a^2*b - 30*a*b^2 - 26*b^3 + 15*(5*a^3 + 45
*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(5*a^3
+ 15*a*b^2 + 4*b^3)*cosh(d*x + c)^3 + 4*(5*a^3 - 3*b^3)*cosh(d*x + c))*sin
h(d*x + c)^3 + 50*a^3 - 90*a^2*b + 30*a*b^2 - 22*b^3 + (75*a^3 - 45*a^2*b
- 15*a*b^2 + 41*b^3)*cosh(d*x + c)^2 + (15*(5*a^3 + 45*a^2*b + 15*a*b^2 +
7*b^3)*cosh(d*x + c)^4 + 75*a^3 - 45*a^2*b - 15*a*b^2 + 41*b^3 + 12*(15*a^
3 + 45*a^2*b - 15*a*b^2 - 13*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(3*
(5*a^3 + 15*a*b^2 + 4*b^3)*cosh(d*x + c)^5 + 8*(5*a^3 - 3*b^3)*cosh(d*x +
c)^3 + (25*a^3 - 45*a*b^2 + 12*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(
d*x + c)^10 + 10*d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + 2*
d*cosh(d*x + c)^8 + (45*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^8 + 8*(15*d
*cosh(d*x + c)^3 + 2*d*cosh(d*x + c))*sinh(d*x + c)^7 - 3*d*cosh(d*x + c)^
6 + (210*d*cosh(d*x + c)^4 + 56*d*cosh(d*x + c)^2 - 3*d)*sinh(d*x + c)^6 +
 2*(126*d*cosh(d*x + c)^5 + 56*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh
(d*x + c)^5 - 8*d*cosh(d*x + c)^4 + (210*d*cosh(d*x + c)^6 + 140*d*cosh(d*
x + c)^4 - 45*d*cosh(d*x + c)^2 - 8*d)*sinh(d*x + c)^4 + 4*(30*d*cosh(d*x
+ c)^7 + 28*d*cosh(d*x + c)^5 - 5*d*cosh(d*x + c)^3 - 4*d*cosh(d*x + c)...
```

3.24.6 Sympy [F]

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2(c+dx)\right)^3 \operatorname{csch}^4(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*csch(c + d*x)**4, x)
```

3.24.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(92) = 184$.

Time $= 0.20$ (sec) , antiderivative size $= 493$, normalized size of antiderivative $= 5.03$

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{4}{15}b^3\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)} - \frac{}{d(5\,e^{(-2\,dx-2\,c)}+}\right.$$
$$+4\,ab^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)} + \frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right.$$
$$+\frac{4}{3}a^3\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)} - \frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right.$$
$$+\frac{12\,a^2b}{d(e^{(-4\,dx-4\,c)}-1)}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
4/15*b^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
+ 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*
e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d
*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x -
 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) +
 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c)
+ 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*
d*x - 10*c) + 1))) + 4*a*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) +
3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3
*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 4/3*a^3*(3*e^(-2*d*x - 2*c)/
(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(
d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 12*
a^2*b/(d*(e^(-4*d*x - 4*c) - 1))
```

3.24.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(92) = 184$.

Time $= 0.46$ (sec) , antiderivative size $= 257$, normalized size of antiderivative $= 2.62$

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$

$$-\frac{2\left(\frac{5\left(9a^2be^{(4dx+4c)}+6a^3e^{(2dx+2c)}-18a^2be^{(2dx+2c)}-2a^3+9a^2b\right)}{\left(e^{(2dx+2c)}-1\right)^3}-\frac{45a^2be^{(8dx+8c)}+180a^2be^{(6dx+6c)}-90ab^2e^{(6dx+6c)}-30b^3e^{(6dx}}{15}\right.$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-2/15*(5*(9*a^2*b*e^(4*d*x + 4*c) + 6*a^3*e^(2*d*x + 2*c) - 18*a^2*b*e^(2*
d*x + 2*c) - 2*a^3 + 9*a^2*b)/(e^(2*d*x + 2*c) - 1)^3 - (45*a^2*b*e^(8*d*x
 + 8*c) + 180*a^2*b*e^(6*d*x + 6*c) - 90*a*b^2*e^(6*d*x + 6*c) - 30*b^3*e^
(6*d*x + 6*c) + 270*a^2*b*e^(4*d*x + 4*c) - 210*a*b^2*e^(4*d*x + 4*c) + 10
*b^3*e^(4*d*x + 4*c) + 180*a^2*b*e^(2*d*x + 2*c) - 150*a*b^2*e^(2*d*x + 2*
c) - 10*b^3*e^(2*d*x + 2*c) + 45*a^2*b - 30*a*b^2 - 2*b^3)/(e^(2*d*x + 2*c
) + 1)^5)/d
```

3.24.9 Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 622, normalized size of antiderivative = 6.35

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$=\frac{\frac{2\left(9a^2b-12ab^2+4b^3\right)}{15d}-\frac{4e^{2c+2dx}\left(-3a^2b+3ab^2+b^3\right)}{5d}+\frac{6a^2be^{4c+4dx}}{5d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}$$
$$-\frac{\frac{2\left(-3a^2b+3ab^2+b^3\right)}{5d}+\frac{6e^{4c+4dx}\left(-3a^2b+3ab^2+b^3\right)}{5d}-\frac{2e^{2c+2dx}\left(9a^2b-12ab^2+4b^3\right)}{5d}-\frac{6a^2be^{6c+6dx}}{5d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1}$$
$$+\frac{\frac{6a^2b}{5d}-\frac{8e^{6c+6dx}\left(-3a^2b+3ab^2+b^3\right)}{5d}-\frac{8e^{2c+2dx}\left(-3a^2b+3ab^2+b^3\right)}{5d}+\frac{4e^{4c+4dx}\left(9a^2b-12ab^2+4b^3\right)}{5d}+\frac{6a^2be^{8c+8dx}}{5d}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1}$$
$$-\frac{\frac{2\left(-3a^2b+3ab^2+b^3\right)}{5d}-\frac{6a^2be^{2c+2dx}}{5d}}{2e^{2c+2dx}+e^{4c+4dx}+1}-\frac{4a^3}{d\left(e^{4c+4dx}-2e^{2c+2dx}+1\right)}$$
$$-\frac{8a^3}{3d\left(3e^{2c+2dx}-3e^{4c+4dx}+e^{6c+6dx}-1\right)}-\frac{6a^2b}{d\left(e^{2c+2dx}-1\right)}+\frac{6a^2b}{5d\left(e^{2c+2dx}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)^3/sinh(c + d*x)^4,x)
```

output
```
((2*(9*a^2*b - 12*a*b^2 + 4*b^3))/(15*d) - (4*exp(2*c + 2*d*x)*(3*a*b^2 -
3*a^2*b + b^3))/(5*d) + (6*a^2*b*exp(4*c + 4*d*x))/(5*d))/(3*exp(2*c + 2*d
*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((2*(3*a*b^2 - 3*a^2*b
+ b^3))/(5*d) + (6*exp(4*c + 4*d*x)*(3*a*b^2 - 3*a^2*b + b^3))/(5*d) - (2*
exp(2*c + 2*d*x)*(9*a^2*b - 12*a*b^2 + 4*b^3))/(5*d) - (6*a^2*b*exp(6*c +
6*d*x))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*
x) + exp(8*c + 8*d*x) + 1) + ((6*a^2*b)/(5*d) - (8*exp(6*c + 6*d*x)*(3*a*b
^2 - 3*a^2*b + b^3))/(5*d) - (8*exp(2*c + 2*d*x)*(3*a*b^2 - 3*a^2*b + b^3)
)/(5*d) + (4*exp(4*c + 4*d*x)*(9*a^2*b - 12*a*b^2 + 4*b^3))/(5*d) + (6*a^2
*b*exp(8*c + 8*d*x))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10
*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*(3*
a*b^2 - 3*a^2*b + b^3))/(5*d) - (6*a^2*b*exp(2*c + 2*d*x))/(5*d))/(2*exp(2
*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (4*a^3)/(d*(exp(4*c + 4*d*x) - 2*exp
(2*c + 2*d*x) + 1)) - (8*a^3)/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x
) + exp(6*c + 6*d*x) - 1)) - (6*a^2*b)/(d*(exp(2*c + 2*d*x) - 1)) + (6*a^2
*b)/(5*d*(exp(2*c + 2*d*x) + 1))
```

3.25 $\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.25.1 Optimal result

Integrand size = 23, antiderivative size = 118

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(3a^2-6ab-b^2)\,x}{8(a+b)^3} + \frac{a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a+b)^3 d} - \frac{(5a+b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2 d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d}$$

output
```
1/8*(3*a^2-6*a*b-b^2)*x/(a+b)^3-1/8*(5*a+b)*cosh(d*x+c)*sinh(d*x+c)/(a+b)^
2/d+1/4*cosh(d*x+c)^3*sinh(d*x+c)/(a+b)/d+a^(3/2)*arctan(b^(1/2)*tanh(d*x+
c)/a^(1/2))*b^(1/2)/(a+b)^3/d
```

3.25.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.79

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{4(3a^2-6ab-b^2)(c+dx)+32a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)-8a(a+b)\sinh(2(c+dx))+(a+b)^2\sinh(}{32(a+b)^3 d}$$

input
```
Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(4*(3*a^2 - 6*a*b - b^2)*(c + d*x) + 32*a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*Ta
nh[c + d*x])/Sqrt[a]] - 8*a*(a + b)*Sinh[2*(c + d*x)] + (a + b)^2*Sinh[4*(
c + d*x)])/(32*(a + b)^3*d)
```

3.25.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.28, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4146, 372, 402, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(ic+idx)^4}{a-b\tan(ic+idx)^2}dx$$

$$\downarrow 4146$$

$$\frac{\int \frac{\tanh^4(c+dx)}{(1-\tanh^2(c+dx))^3(b\tanh^2(c+dx)+a)}d\tanh(c+dx)}{d}$$

$$\downarrow 372$$

$$\frac{\frac{\tanh(c+dx)}{4(a+b)(1-\tanh^2(c+dx))^2}-\frac{\int \frac{(4a+b)\tanh^2(c+dx)+a}{(1-\tanh^2(c+dx))^2(b\tanh^2(c+dx)+a)}d\tanh(c+dx)}{4(a+b)}}{d}$$

$$\downarrow 402$$

$$\frac{\frac{\tanh(c+dx)}{4(a+b)(1-\tanh^2(c+dx))^2}-\frac{\frac{\int -\frac{a(3a-b)-b(5a+b)\tanh^2(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh(c+dx)}{2(a+b)}+\frac{(5a+b)\tanh(c+dx)}{2(a+b)(1-\tanh^2(c+dx))}}{4(a+b)}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a+b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}-\frac{\int\frac{a(3a-b)-b(5a+b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2(a+b)}}{4(a+b)}}{d}$$

$$\downarrow 397$$

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a+b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}-\frac{\frac{\left(3a^2-6ab-b^2\right)\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{8a^2b\int\frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{2(a+b)}}{4(a+b)}}{d}$$

$$\downarrow 218$$

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a+b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}-\frac{\frac{\left(3a^2-6ab-b^2\right)\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{8a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a+b}}{2(a+b)}}{4(a+b)}}{d}$$

$$\downarrow 219$$

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a+b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}-\frac{\frac{8a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a+b}+\frac{\left(3a^2-6ab-b^2\right)\operatorname{arctanh}(\tanh(c+dx))}{a+b}}{2(a+b)}}{4(a+b)}}{d}$$

input
```
Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(Tanh[c + d*x]/(4*(a + b)*(1 - Tanh[c + d*x]^2)^2) - (-1/2*((8*a^(3/2)*Sqr
t[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a + b) + ((3*a^2 - 6*a*b -
b^2)*ArcTanh[Tanh[c + d*x]])/(a + b))/(a + b) + ((5*a + b)*Tanh[c + d*x])/
(2*(a + b)*(1 - Tanh[c + d*x]^2)))/(4*(a + b)))/d
```

3.25.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.25.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(104) = 208.

Time = 11.14 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.82

method	result
risch	$\frac{3a^2x}{8(a+b)^3}-\frac{3axb}{4(a+b)^3}-\frac{xb^2}{8(a+b)^3}+\frac{e^{4dx+4c}}{64d(a+b)}-\frac{ae^{2dx+2c}}{8(a+b)^2d}+\frac{ae^{-2dx-2c}}{8(a^2+2ab+b^2)d}-\frac{e^{-4dx-4c}}{64d(a+b)}+\frac{\sqrt{-ab}\,a\ln\left(e^{2dx+}\right.}{2(a-}$
derivativedivides	$\frac{8}{(32a+32b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{32}{(64a+64b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a-3b}{8(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3a-b}{8(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(-}$
default	$\frac{8}{(32a+32b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{32}{(64a+64b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a-3b}{8(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3a-b}{8(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(-}$

input
```
int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
3/8*a^2*x/(a+b)^3-3/4*a*x/(a+b)^3*b-1/8*x/(a+b)^3*b^2+1/64/d/(a+b)*exp(4*d
*x+4*c)-1/8*a/(a+b)^2/d*exp(2*d*x+2*c)+1/8*a/(a^2+2*a*b+b^2)/d*exp(-2*d*x-
2*c)-1/64/d/(a+b)*exp(-4*d*x-4*c)+1/2*(-a*b)^(1/2)*a/(a+b)^3/d*ln(exp(2*d*
x+2*c)+(2*(-a*b)^(1/2)+a-b)/(a+b))-1/2*(-a*b)^(1/2)*a/(a+b)^3/d*ln(exp(2*d
*x+2*c)-(2*(-a*b)^(1/2)-a+b)/(a+b))
```

3.25.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 860 vs. 2(104) = 208.

Time = 0.30 (sec) , antiderivative size = 2024, normalized size of antiderivative = 17.15

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/64*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 8*(3*a^2 -
6*a*b - b^2)*d*x*cosh(d*x + c)^4 - 8*(a^2 + a*b)*cosh(d*x + c)^6 + 4*(7*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - 2*a^2 - 2*a*b)*sinh(d*x + c)^6 + 8*(7*
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c))*sinh(d*
x + c)^5 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(3*a^2 - 6*a*b -
b^2)*d*x - 60*(a^2 + a*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)^5 + 4*(3*a^2 - 6*a*b - b^2)*d*x*cosh(d*x + c) -
20*(a^2 + a*b)*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 8*(a^2 + a*b)*cosh(d*x +
 c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 12*(3*a^2 - 6*a*b - b^2
)*d*x*cosh(d*x + c)^2 - 30*(a^2 + a*b)*cosh(d*x + c)^4 + 2*a^2 + 2*a*b)*si
nh(d*x + c)^2 + 32*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)^3*sinh(d*x + c)
+ 6*a*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3
+ a*sinh(d*x + c)^4)*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 +
 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)
*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2
)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a
^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x +
c) + 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) +
(a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 +...
```

3.25.6 Sympy [F]

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(sinh(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)
```

3.25.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(104) = 208$.

Time $= 0.34$ (sec) , antiderivative size $= 514$, normalized size of antiderivative $= 4.36$

$$\begin{aligned}\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = &-\frac{(ab-b^2)(dx+c)}{2\left(a^3+3a^2b+3ab^2+b^3\right)d}+\frac{\left(8be^{(-2dx-2c)}+a+b\right)e^{(4dx+4c)}}{64\left(a^2+2ab+b^2\right)d}\\&-\frac{b\log\left((a+b)e^{(4dx+4c)}+2(a-b)e^{(2dx+2c)}+a+b\right)}{4\left(a^2+2ab+b^2\right)d}\\&+\frac{b\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{4\left(a^2+2ab+b^2\right)d}\\&+\frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{4\left(a^2+2ab+b^2\right)\sqrt{ab}d}\\&-\frac{\left(a^2b-6ab^2+b^3\right)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{8\left(a^3+3a^2b+3ab^2+b^3\right)\sqrt{ab}d}\\&-\frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{4\left(a^2+2ab+b^2\right)\sqrt{ab}d}\\&-\frac{3b\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{8\sqrt{ab}(a+b)d}\\&-\frac{8be^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}}{64\left(a^2+2ab+b^2\right)d}\\&+\frac{3(dx+c)}{8(a+b)d}-\frac{e^{(2dx+2c)}}{8(a+b)d}+\frac{e^{(-2dx-2c)}}{8(a+b)d}\end{aligned}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/2*(a*b - b^2)*(d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 1/64*(8*b
*e^(-2*d*x - 2*c) + a + b)*e^(4*d*x + 4*c)/((a^2 + 2*a*b + b^2)*d) - 1/4*b
*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^2 +
2*a*b + b^2)*d) + 1/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x
 - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d) + 1/4*(a*b - b^2)*arctan(1/2*((a
+ b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d)
 - 1/8*(a^2*b - 6*a*b^2 + b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a -
b)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)*d) - 1/4*(a*b - b
^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b
 + b^2)*sqrt(a*b)*d) - 3/8*b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)
/sqrt(a*b))/(sqrt(a*b)*(a + b)*d) - 1/64*(8*b*e^(-2*d*x - 2*c) + (a + b)*e
^(-4*d*x - 4*c))/((a^2 + 2*a*b + b^2)*d) + 3/8*(d*x + c)/((a + b)*d) - 1/8
*e^(2*d*x + 2*c)/((a + b)*d) + 1/8*e^(-2*d*x - 2*c)/((a + b)*d)
```

3.25.8 Giac [F]

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\sinh\left(dx+c\right)^4}{b\tanh\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.25.9 Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.12

$$\begin{aligned}
&\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx \\
&= \frac{\mathrm{e}^{4c+4dx}}{64\,d\,(a+b)} - \frac{\mathrm{e}^{-4c-4dx}}{64\,d\,(a+b)} - \frac{x\left(-3a^2+6ab+b^2\right)}{8\,(a+b)^3} + \frac{a\,\mathrm{e}^{-2c-2dx}}{8\,d\,(a+b)^2} - \frac{a\,\mathrm{e}^{2c+2dx}}{8\,d\,(a+b)^2} \\
&\quad + \frac{(-a)^{3/2}\sqrt{b}\,\ln\left((-a)^{3/2}b^{3/2}\left(\mathrm{e}^{2c+2dx}-1\right) - 2a^2b\,\mathrm{e}^{2c+2dx} + (-a)^{5/2}\sqrt{b}\left(\mathrm{e}^{2c+2dx}+1\right)\right)}{2\,d\,(a+b)^3} \\
&\quad - \frac{(-a)^{3/2}\sqrt{b}\,\ln\left(2a^2b\,\mathrm{e}^{2c+2dx} + (-a)^{3/2}b^{3/2}\left(\mathrm{e}^{2c+2dx}-1\right) + (-a)^{5/2}\sqrt{b}\left(\mathrm{e}^{2c+2dx}+1\right)\right)}{2\,d\,(a+b)^3}
\end{aligned}$$

input
```
int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2),x)
```

output
```
exp(4*c + 4*d*x)/(64*d*(a + b)) - exp(- 4*c - 4*d*x)/(64*d*(a + b)) - (x*(
6*a*b - 3*a^2 + b^2))/(8*(a + b)^3) + (a*exp(- 2*c - 2*d*x))/(8*d*(a + b)^
2) - (a*exp(2*c + 2*d*x))/(8*d*(a + b)^2) + ((-a)^(3/2)*b^(1/2)*log((-a)^(
3/2)*b^(3/2)*(exp(2*c + 2*d*x) - 1) - 2*a^2*b*exp(2*c + 2*d*x) + (-a)^(5/2
)*b^(1/2)*(exp(2*c + 2*d*x) + 1)))/(2*d*(a + b)^3) - ((-a)^(3/2)*b^(1/2)*l
og(2*a^2*b*exp(2*c + 2*d*x) + (-a)^(3/2)*b^(3/2)*(exp(2*c + 2*d*x) - 1) +
(-a)^(5/2)*b^(1/2)*(exp(2*c + 2*d*x) + 1)))/(2*d*(a + b)^3)
```

3.26 $\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.26.1 Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{a\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}d} - \frac{a\cosh(c+dx)}{(a+b)^2 d} + \frac{\cosh^3(c+dx)}{3(a+b)d}$$

output

```
-a*cosh(d*x+c)/(a+b)^2/d+1/3*cosh(d*x+c)^3/(a+b)/d+a*arctanh(sech(d*x+c)*b
^(1/2)/(a+b)^(1/2))*b^(1/2)/(a+b)^(5/2)/d
```

3.26.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.80

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{12ia\sqrt{b}\left(\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)-3(3a-b)\sqrt{a+b}\cosh(c+}{12(a+b)^{5/2}d}$$

input

```
Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
((12*I)*a*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/S
qrt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])
- 3*(3*a - b)*Sqrt[a + b]*Cosh[c + d*x] + (a + b)^(3/2)*Cosh[3*(c + d*x)])
/(12*(a + b)^(5/2)*d)
```

3.26.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4147, 25, 359, 264, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{i\sin(ic+idx)^3}{a-b\tan(ic+idx)^2}dx$$

$$\downarrow 26$$

$$i\int \frac{\sin(ic+idx)^3}{a-b\tan(ic+idx)^2}dx$$

$$\downarrow 4147$$

$$\frac{\int -\frac{\cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)}{-b\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)}{-b\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{d}$$

$$\downarrow 359$$

$$\frac{a\int \frac{\cosh^2(c+dx)}{-b\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{a+b}+\frac{\cosh^3(c+dx)}{3(a+b)}}{d}$$

$$\downarrow 264$$

$$\frac{a\left(\frac{b\int \frac{1}{-b\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{a+b}-\frac{\cosh(c+dx)}{a+b}\right)}{a+b}+\frac{\cosh^3(c+dx)}{3(a+b)}}{d}$$

$$\downarrow 221$$

$$\frac{a\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{\cosh(c+dx)}{a+b}\right)}{a+b}+\frac{\cosh^3(c+dx)}{3(a+b)}}{d}$$

input
```
Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(Cosh[c + d*x]^3/(3*(a + b)) + (a*((Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x]
)/Sqrt[a + b]])/(a + b)^(3/2) - Cosh[c + d*x]/(a + b)))/(a + b))/d
```

3.26.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.26.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(65) = 130$.

Time = 3.90 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.69

method	result
derivativedivides	$\frac{-\frac{8}{(16a+16b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{16}{3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3(16a+16b)}-\frac{a-b}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{ab\arctanh\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-}{4\sqrt{ab+b^2}}\right.}{(a+b)^2\sqrt{ab+b^2}}}{d}$
default	$\frac{-\frac{8}{(16a+16b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{16}{3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3(16a+16b)}-\frac{a-b}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{ab\arctanh\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-}{4\sqrt{ab+b^2}}\right.}{(a+b)^2\sqrt{ab+b^2}}}{d}$
risch	$\frac{e^{3dx+3c}}{24d(a+b)}-\frac{3e^{dx+c}a}{8(a+b)^2d}+\frac{e^{dx+c}b}{8(a+b)^2d}-\frac{3e^{-dx-c}a}{8(a+b)^2d}+\frac{e^{-dx-c}b}{8(a+b)^2d}+\frac{e^{-3dx-3c}}{24d(a+b)}+\frac{\sqrt{(a+b)b}a\ln\left(e^{2dx+2c}+\frac{2\sqrt{(a+b)}}{a+}\right.}{2(a+b)^3d}$

input
```
int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-8/(16*a+16*b)/(1+tanh(1/2*d*x+1/2*c))^2+16/3/(1+tanh(1/2*d*x+1/2*c))
^3/(16*a+16*b)-1/2*(a-b)/(a+b)^2/(1+tanh(1/2*d*x+1/2*c))+a*b/(a+b)^2/(a*b+
b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2)
)-16/3/(tanh(1/2*d*x+1/2*c)-1)^3/(16*a+16*b)-8/(16*a+16*b)/(tanh(1/2*d*x+1
/2*c)-1)^2-1/2/(a+b)^2*(-a+b)/(tanh(1/2*d*x+1/2*c)-1))
```

3.26.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 616 vs. $2(65) = 130$.

Time $= 0.28$ (sec) , antiderivative size $= 1367$, normalized size of antiderivative $= 18.23$

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/24*((a + b)*cosh(d*x + c)^6 + 6*(a + b)*cosh(d*x + c)*sinh(d*x + c)^5 +
 (a + b)*sinh(d*x + c)^6 - 3*(3*a - b)*cosh(d*x + c)^4 + 3*(5*(a + b)*cosh
(d*x + c)^2 - 3*a + b)*sinh(d*x + c)^4 + 4*(5*(a + b)*cosh(d*x + c)^3 - 3*
(3*a - b)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(3*a - b)*cosh(d*x + c)^2 + 3
*(5*(a + b)*cosh(d*x + c)^4 - 6*(3*a - b)*cosh(d*x + c)^2 - 3*a + b)*sinh(
d*x + c)^2 + 12*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)^2*sinh(d*x + c) + 3
*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3)*sqrt(b/(a + b))*log(
((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a +
b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x +
 c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*
cosh(d*x + c))*sinh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh
(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c
) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a
 + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d
*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*
cosh(d*x + c))*sinh(d*x + c) + a + b)) + 6*((a + b)*cosh(d*x + c)^5 - 2*(3
*a - b)*cosh(d*x + c)^3 - (3*a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)/
((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x
+ c)^2*sinh(d*x + c) + 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x +...
```

3.26.6 Sympy [F]

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(sinh(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(sinh(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

3.26.7 Maxima [F]

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\sinh(dx+c)^3}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/24*((a*e^(6*c) + b*e^(6*c))*e^(6*d*x) - 3*(3*a*e^(4*c) - b*e^(4*c))*e^(4
*d*x) - 3*(3*a*e^(2*c) - b*e^(2*c))*e^(2*d*x) + a + b)*e^(-3*d*x)/(a^2*d*e
^(3*c) + 2*a*b*d*e^(3*c) + b^2*d*e^(3*c)) - 1/8*integrate(16*(a*b*e^(3*d*x
 + 3*c) - a*b*e^(d*x + c))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + (a^3*e^(4*c) +
 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*
c) + a^2*b*e^(2*c) - a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

3.26.8 Giac [F]

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\sinh(dx+c)^3}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.26.9 Mupad [B] (verification not implemented)

Time = 3.34 (sec) , antiderivative size = 955, normalized size of antiderivative = 12.73

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{e^{-3c-3dx}}{24d\,(a+b)} + \frac{e^{3c+3dx}}{24d\,(a+b)} - \frac{\sqrt{a^2b}\left(2\,\mathrm{atan}\left(\frac{\left(e^{dx}e^{c}\left(\frac{4\left(2a^2b^3d\sqrt{a^2b}+4a^3b^2d\sqrt{a^2b}+2a^4bd\sqrt{a^2b}\right)}{a(a+b)\sqrt{-d^2(a+b)^5}\left(a^2+2ab+b^2\right)\left(a^3+3a^2b+3ab^2+b^3\right)\sqrt{-a^5d^2-5a^4bd^2-10a^3b^2d^2-10a^2b^3d^2-5ab^4d^2-b^5}}\right.\right.}{}\right.\right.}{} - \frac{e^{-c-dx}(3a-b)}{8d(a+b)^2} - \frac{e^{c+dx}(3a-b)}{8d(a+b)^2}$$

input
```
int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^2),x)
```

output
```
exp(- 3*c - 3*d*x)/(24*d*(a + b)) + exp(3*c + 3*d*x)/(24*d*(a + b)) - ((a^
2*b)^(1/2)*(2*atan(((exp(d*x)*exp(c)*((4*(2*a^2*b^3*d*(a^2*b)^(1/2) + 4*a^
3*b^2*d*(a^2*b)^(1/2) + 2*a^4*b*d*(a^2*b)^(1/2)))/(a*(a + b)*(-d^2*(a + b)
^5)^(1/2)*(2*a*b + a^2 + b^2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(- a^5*d^2 -
 b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^(1
/2)) + (2*a^3*b)/(d*(a + b)^3*(a^2*b)^(1/2)*(2*a*b + a^2 + b^2)*(3*a*b^2 +
 3*a^2*b + a^3 + b^3))) + (2*a^3*b*exp(3*c)*exp(3*d*x))/(d*(a + b)^3*(a^2*
b)^(1/2)*(2*a*b + a^2 + b^2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)))*(a^6*(- a^5
*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d
^2)^(1/2) + b^6*(- a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*
b^3*d^2 - 10*a^3*b^2*d^2)^(1/2) + 15*a^2*b^4*(- a^5*d^2 - b^5*d^2 - 5*a*b^
4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^(1/2) + 20*a^3*b^3*
(- a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3
*b^2*d^2)^(1/2) + 15*a^4*b^2*(- a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*
d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^(1/2) + 6*a*b^5*(- a^5*d^2 - b^5*d^
2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^(1/2) + 6
*a^5*b*(- a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 -
 10*a^3*b^2*d^2)^(1/2)))/(4*a^2*b)) - 2*atan((a*exp(d*x)*exp(c)*(-d^2*(a +
 b)^5)^(1/2))/(2*d*(a + b)^2*(a^2*b)^(1/2)))))/(2*(- a^5*d^2 - b^5*d^2 - 5
*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^(1/2)) - (e...
```

3.27 $\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.27.1 Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{(a-b)x}{2(a+b)^2} - \frac{\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a+b)^2 d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d}$$

output
```
-1/2*(a-b)*x/(a+b)^2+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d-arctan(b^(1/2)*ta
nh(d*x+c)/a^(1/2))*a^(1/2)*b^(1/2)/(a+b)^2/d
```

3.27.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{-2(a-b)(c+dx) - 4\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) + (a+b)\sinh(2(c+dx))}{4(a+b)^2 d}$$

input
```
Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-2*(a - b)*(c + d*x) - 4*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/S
qrt[a]] + (a + b)*Sinh[2*(c + d*x)])/(4*(a + b)^2*d)
```

3.27.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 25, 4146, 373, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin(ic+idx)^2}{a-b\tan(ic+idx)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin(ic+idx)^2}{a-b\tan(ic+idx)^2}dx$$

$$\downarrow 4146$$

$$\frac{\int \frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^2\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{d}$$

$$\downarrow 373$$

$$\frac{\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}-\frac{\int \frac{a-b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2(a+b)}}{d}$$

$$\downarrow 397$$

$$\frac{\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}-\frac{\frac{(a-b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{2ab\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{2(a+b)}}{d}$$

$$\downarrow 218$$

$$\frac{\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}-\frac{\frac{(a-b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{2\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a+b}}{2(a+b)}}{d}$$

$$\downarrow 219$$

$$\frac{\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}-\frac{\frac{2\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a+b}+\frac{(a-b)\operatorname{arctanh}(\tanh(c+dx))}{a+b}}{2(a+b)}}{d}$$

input
```
Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-1/2*((2*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a + b)
 + ((a - b)*ArcTanh[Tanh[c + d*x]])/(a + b))/(a + b) + Tanh[c + d*x]/(2*(a
 + b)*(1 - Tanh[c + d*x]^2)))/d
```

3.27.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.27.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(66) = 132$.

Time = 1.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.94

method	result
risch	$-\frac{ax}{2(a+b)^2}+\frac{xb}{2(a+b)^2}+\frac{e^{2dx+2c}}{8d(a+b)}-\frac{e^{-2dx-2c}}{8d(a+b)}+\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}-a+b}{a+b}\right)}{2(a+b)^2d}-\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{-}}{}\right.}{2(a+b)^2d}$
derivativedivides	$-\frac{4}{(8a+8b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{8}{(16a+16b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(-a+b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)^2}+2a^2b\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\right.\right.}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(}\right.}$
default	$-\frac{4}{(8a+8b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{8}{(16a+16b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(-a+b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)^2}+2a^2b\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\right.\right.}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(}\right.}$

input

```
int(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output

```
-1/2*a*x/(a+b)^2+1/2*x/(a+b)^2*b+1/8/d/(a+b)*exp(2*d*x+2*c)-1/8/d/(a+b)*ex
p(-2*d*x-2*c)+1/2*(-a*b)^(1/2)/(a+b)^2/d*ln(exp(2*d*x+2*c)-(2*(-a*b)^(1/2)
-a+b)/(a+b))-1/2*(-a*b)^(1/2)/(a+b)^2/d*ln(exp(2*d*x+2*c)+(2*(-a*b)^(1/2)+
a-b)/(a+b))
```

3.27.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(66) = 132$.

Time $= 0.29$ (sec) , antiderivative size $= 916$, normalized size of antiderivative $= 11.74$

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/8*(4*(a - b)*d*x*cosh(d*x + c)^2 - (a + b)*cosh(d*x + c)^4 - 4*(a + b)
*cosh(d*x + c)*sinh(d*x + c)^3 - (a + b)*sinh(d*x + c)^4 + 2*(2*(a - b)*d*
x - 3*(a + b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 4*sqrt(-a*b)*(cosh(d*x +
c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*log(((a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)
^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 +
 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a
^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*co
sh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d
*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a +
 b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*si
nh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 +
 a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x +
c))*sinh(d*x + c) + a + b)) + 4*(2*(a - b)*d*x*cosh(d*x + c) - (a + b)*cos
h(d*x + c)^3)*sinh(d*x + c) + a + b)/((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^
2 + 2*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b
^2)*d*sinh(d*x + c)^2), -1/8*(4*(a - b)*d*x*cosh(d*x + c)^2 - (a + b)*cosh
(d*x + c)^4 - 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 - (a + b)*sinh(d*x +
 c)^4 + 2*(2*(a - b)*d*x - 3*(a + b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*
sqrt(a*b)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +...
```

3.27.6 Sympy [F]

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(sinh(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(sinh(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

3.27.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(66) = 132$.

Time = 0.30 (sec) , antiderivative size = 316, normalized size of antiderivative = 4.05

$$\begin{aligned}\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = {} & \frac{b\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d} \\ & - \frac{b\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d} \\ & - \frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}d} \\ & + \frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}d} \\ & + \frac{b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}(a+b)d} \\ & - \frac{dx+c}{2\,(a+b)d} + \frac{e^{(2\,dx+2\,c)}}{8\,(a+b)d} - \frac{e^{(-2\,dx-2\,c)}}{8\,(a+b)d}\end{aligned}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a
^2 + 2*a*b + b^2)*d) - 1/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-
4*d*x - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/4*(a*b - b^2)*arctan(1/2
*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*
b)*d) + 1/4*(a*b - b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt
(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d) + 1/2*b*arctan(1/2*((a + b)*e^(-2
*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a + b)*d) - 1/2*(d*x + c)/((a
+ b)*d) + 1/8*e^(2*d*x + 2*c)/((a + b)*d) - 1/8*e^(-2*d*x - 2*c)/((a + b)*
d)
```

3.27.8 Giac [F]

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\sinh(dx+c)^2}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.27.9 Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.54

$$\begin{aligned}
&\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx \\
&= \frac{e^{2c+2dx}}{8d(a+b)} - \frac{e^{-2c-2dx}}{8d(a+b)} - \frac{x(a-b)}{2(a+b)^2} \\
&\quad - \frac{\sqrt{-a}\sqrt{b}\ln\left(\sqrt{-a}b^{3/2}\left(e^{2c+2dx}-1\right)+(-a)^{3/2}\sqrt{b}\left(e^{2c+2dx}+1\right)-2ab\,e^{2c+2dx}\right)}{2d(a+b)^2} \\
&\quad + \frac{\sqrt{-a}\sqrt{b}\ln\left(\sqrt{-a}b^{3/2}\left(e^{2c+2dx}-1\right)+(-a)^{3/2}\sqrt{b}\left(e^{2c+2dx}+1\right)+2ab\,e^{2c+2dx}\right)}{2d(a+b)^2}
\end{aligned}$$

input
```
int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2),x)
```

output
```
exp(2*c + 2*d*x)/(8*d*(a + b)) - exp(- 2*c - 2*d*x)/(8*d*(a + b)) - (x*(a
- b))/(2*(a + b)^2) - ((-a)^(1/2)*b^(1/2)*log((-a)^(1/2)*b^(3/2)*(exp(2*c
+ 2*d*x) - 1) + (-a)^(3/2)*b^(1/2)*(exp(2*c + 2*d*x) + 1) - 2*a*b*exp(2*c
+ 2*d*x)))/(2*d*(a + b)^2) + ((-a)^(1/2)*b^(1/2)*log((-a)^(1/2)*b^(3/2)*(e
xp(2*c + 2*d*x) - 1) + (-a)^(3/2)*b^(1/2)*(exp(2*c + 2*d*x) + 1) + 2*a*b*e
xp(2*c + 2*d*x)))/(2*d*(a + b)^2)
```

3.28 $\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.28.1 Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d} + \frac{\cosh(c+dx)}{(a+b)d}$$

output
```
cosh(d*x+c)/(a+b)/d-arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/(a+b)
^(3/2)/d
```

3.28.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.02

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{-i\sqrt{b}\left(\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)+\sqrt{a+b}\cosh(c+dx)}{(a+b)^{3/2}d}$$

input
```
Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
((-I)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[
b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]]) + Sq
rt[a + b]*Cosh[c + d*x])/((a + b)^(3/2)*d)
```

3.28.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 26, 4147, 264, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\sin(ic+idx)}{a-b\tan(ic+idx)^2}dx$$

$$\downarrow 26$$

$$-i\int \frac{\sin(ic+idx)}{a-b\tan(ic+idx)^2}dx$$

$$\downarrow 4147$$

$$-\frac{\int \frac{\cosh^2(c+dx)}{-b\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{d}$$

$$\downarrow 264$$

$$-\frac{\frac{b\int \frac{1}{-b\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{a+b}-\frac{\cosh(c+dx)}{a+b}}{d}$$

$$\downarrow 221$$

$$-\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{\cosh(c+dx)}{a+b}}{d}$$

input
```
Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
-(((Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(a + b)^(3/2) -
Cosh[c + d*x]/(a + b))/d)
```

3.28.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.28.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(45) = 90$.

Time $= 0.48$ (sec) , antiderivative size $= 104$, normalized size of antiderivative $= 1.96$

method	result
derivativedivides	$\dfrac{-\dfrac{b\,\operatorname{arctanh}\left(\dfrac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+2a+4b}{4\sqrt{ab+b^2}}\right)}{(a+b)\sqrt{ab+b^2}}+\dfrac{4}{(4a+4b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\dfrac{4}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{d}$
default	$\dfrac{-\dfrac{b\,\operatorname{arctanh}\left(\dfrac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+2a+4b}{4\sqrt{ab+b^2}}\right)}{(a+b)\sqrt{ab+b^2}}+\dfrac{4}{(4a+4b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\dfrac{4}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{d}$
risch	$\dfrac{e^{dx+c}}{2d(a+b)}+\dfrac{e^{-dx-c}}{2d(a+b)}+\dfrac{\sqrt{(a+b)b}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{(a+b)b}\,e^{dx+c}}{a+b}+1\right)}{2(a+b)^2 d}-\dfrac{\sqrt{(a+b)b}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{(a+b)b}\,e^{dx+c}}{a+b}+1\right)}{2(a+b)^2 d}$

input
```
int(sinh(d*x+c)/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-b/(a+b)/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4
*b)/(a*b+b^2)^(1/2))+4/(4*a+4*b)/(1+tanh(1/2*d*x+1/2*c))-4/(4*a+4*b)/(tanh
(1/2*d*x+1/2*c)-1))
```

3.28.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(45) = 90$.

Time $= 0.31$ (sec) , antiderivative size $= 666$, normalized size of antiderivative $= 12.57$

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$= \left[\frac{\sqrt{\frac{b}{a+b}}(\cosh(dx+c)+\sinh(dx+c))\log\left(\frac{(a+b)\cosh(dx+c)^4+4(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)\sinh(dx+c)^4+2(a+3b}{(a+b)}\right.\right.$$

$$\left.\left. -\frac{2\sqrt{-\frac{b}{a+b}}(\cosh(dx+c)+\sinh(dx+c))\arctan\left(\frac{\left((a+b)\cosh(dx+c)^3+3(a+b)\cosh(dx+c)\sinh(dx+c)^2+(a+b)\sinh(dx+\right.\right.\right.\right.$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b/(a + b))*(cosh(d*x + c) + sinh(d*x + c))*log(((a + b)*cosh(d*
x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)
^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a + 3*b)
*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*cosh(d*x + c))*s
inh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d
*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c) + (3*(a + b)*c
osh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a + b)/((a + b)*c
osh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*
x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a -
b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*s
inh(d*x + c) + a + b)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2 + 1)/((a + b)*d*cosh(d*x + c) + (a + b)*d*sinh(d*x + c)),
 -1/2*(2*sqrt(-b/(a + b))*(cosh(d*x + c) + sinh(d*x + c))*arctan(1/2*((a +
 b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*si
nh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a -
 3*b)*sinh(d*x + c))*sqrt(-b/(a + b))/b) - 2*sqrt(-b/(a + b))*(cosh(d*x +
c) + sinh(d*x + c))*arctan(1/2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x +
 c))*sqrt(-b/(a + b))/b) - cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c)
 - sinh(d*x + c)^2 - 1)/((a + b)*d*cosh(d*x + c) + (a + b)*d*sinh(d*x + c)
)]
```

3.28.6 Sympy [F]

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\sinh\left(c+dx\right)}{a+b\tanh^2\left(c+dx\right)}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(sinh(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

3.28.7 Maxima [F]

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\sinh(dx+c)}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x)/(a*d*e^c + b*d*e^c) + 1/2*integrate(4*(
b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^2 + 2*a*b + b^2 + (a^2*e^(4*c) + 2*a
*b*e^(4*c) + b^2*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) - b^2*e^(2*c))*e^(2*d
*x)), x)
```

3.28.8 Giac [F]

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\sinh(dx+c)}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.28.9 Mupad [B] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 520, normalized size of antiderivative = 9.81

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{e^{c+d\,x}}{2\,d\,(a+b)} + \frac{e^{-c-d\,x}}{2\,d\,(a+b)} - \frac{\sqrt{b}\left(2\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\,\sqrt{-d^2\,(a+b)^3}}{2\,\sqrt{b}\,d\,(a+b)}\right) - 2\,\mathrm{atan}\left(\frac{\left(e^{d\,x}\,e^{c}\left(\frac{2\,a\,\sqrt{b}}{d\,(a+b)^2\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)} + \frac{4\,\left(2\,a^2\,b^{3/2}\,d+2\,a\right.}{(a+b)\,\sqrt{-d^2\,(a+b)^3}\,\sqrt{-a^3\,d^2-3\,a^2\,b\,d^2-3\,a\,b}}\right.\right.}{}\right.\right.}{}$$

input
```
int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2),x)
```

output

```
exp(c + d*x)/(2*d*(a + b)) + exp(- c - d*x)/(2*d*(a + b)) - (b^(1/2)*(2*at
an((exp(d*x)*exp(c)*(-d^2*(a + b)^3)^(1/2))/(2*b^(1/2)*d*(a + b))) - 2*ata
n(((exp(d*x)*exp(c)*((2*a*b^(1/2))/(d*(a + b)^2*(3*a*b^2 + 3*a^2*b + a^3 +
 b^3)) + (4*(2*a^2*b^(3/2)*d + 2*a*b^(5/2)*d))/((a + b)*(-d^2*(a + b)^3)^(
1/2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)*(3*a*b^2 + 3*
a^2*b + a^3 + b^3))) + (2*a*b^(1/2)*exp(3*c)*exp(3*d*x))/(d*(a + b)^2*(3*a
*b^2 + 3*a^2*b + a^3 + b^3)))*(a^4*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*
a^2*b*d^2)^(1/2) + b^4*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(
1/2) + 4*a*b^3*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + 4
*a^3*b*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + 6*a^2*b^2
*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)))/(4*a*b))))/(2*(
- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2))
```

3.29 $\int \frac{\text{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.29.1 Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{\text{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\text{arctanh}(\cosh(c+dx))}{ad} + \frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}d}$$

output
```
-arctanh(cosh(d*x+c))/a/d+arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)
/a/d/(a+b)^(1/2)
```

3.29.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.45

$$\int \frac{\text{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{\frac{i\sqrt{b}\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{\sqrt{a+b}} + \frac{i\sqrt{b}\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{\sqrt{a+b}} - \log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\sinh\left(\frac{1}{2}(\right.\right.}{ad}$$

input
```
Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
((I*Sqrt[b]*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]]
)/Sqrt[a + b] + (I*Sqrt[b]*ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*
x)/2])/Sqrt[b]])/Sqrt[a + b] - Log[Cosh[(c + d*x)/2]] + Log[Sinh[(c + d*x)
/2]])/(a*d)
```

3.29.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 4147, 25, 303, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{i}{\sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\downarrow 26$$

$$i\int \frac{1}{\sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\downarrow 4147$$

$$\frac{\int -\frac{1}{\left(1-\operatorname{sech}^2(c+dx)\right)\left(-b\operatorname{sech}^2(c+dx)+a+b\right)}d\operatorname{sech}(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{\int \frac{1}{\left(1-\operatorname{sech}^2(c+dx)\right)\left(-b\operatorname{sech}^2(c+dx)+a+b\right)}d\operatorname{sech}(c+dx)}{d}$$

$$\downarrow 303$$

$$\frac{b\int \frac{1}{-b\operatorname{sech}^2(c+dx)+a+b}d\operatorname{sech}(c+dx)}{a} - \frac{\int \frac{1}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)}{a}}{d}$$

$$\downarrow 219$$

$$\frac{b\int \frac{1}{-b\operatorname{sech}^2(c+dx)+a+b}d\operatorname{sech}(c+dx)}{a} - \frac{\operatorname{arctanh}(\operatorname{sech}(c+dx))}{a}}{d}$$

$$\downarrow 221$$

$$\frac{\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}} - \frac{\operatorname{arctanh}(\operatorname{sech}(c+dx))}{a}}{d}$$

input
```
Int[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-(ArcTanh[Sech[c + d*x]]/a) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sq
rt[a + b]])/(a*Sqrt[a + b]))/d
```

3.29.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.29.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{b\,\operatorname{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+2a+4b}{4\sqrt{ab+b^2}}\right)}{a\sqrt{ab+b^2}}}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{b\,\operatorname{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+2a+4b}{4\sqrt{ab+b^2}}\right)}{a\sqrt{ab+b^2}}}{d}$
risch	$-\frac{\ln(e^{dx+c}+1)}{da}+\frac{\ln(e^{dx+c}-1)}{da}+\frac{\sqrt{(a+b)b}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{(a+b)b}\,e^{dx+c}}{a+b}+1\right)}{2(a+b)da}-\frac{\sqrt{(a+b)b}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{(a+b)b}}{a+}\right.}{2(a+b)da}$

input
```
int(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a*ln(tanh(1/2*d*x+1/2*c))+b/a/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1
/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2)))
```

3.29.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(47) = 94$.

Time = 0.29 (sec) , antiderivative size = 587, normalized size of antiderivative = 10.67

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \left[\frac{\sqrt{\frac{b}{a+b}}\log\left(\frac{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)\sinh(dx+c)^4+2\,(a+3\,b)\cosh(dx+c)^2+2\left(3\,(a+b)\cosh(dx+c)^2+a\right.}{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx+c)\sinh(dx+\right.}\right.$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b/(a + b))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x +
c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2
 + 2*(3*(a + b)*cosh(d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*co
sh(d*x + c)^3 + (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a + b)*cosh(d
*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c
)^3 + (a + b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x
 + c))*sqrt(b/(a + b)) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(
d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x +
c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*
cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 2*log(c
osh(d*x + c) + sinh(d*x + c) + 1) + 2*log(cosh(d*x + c) + sinh(d*x + c) -
1))/(a*d), (sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a +
b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a - 3*b)*cos
h(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x + c))*sqrt(-b/
(a + b))/b) - sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c) + (a + b)
*sinh(d*x + c))*sqrt(-b/(a + b))/b) - log(cosh(d*x + c) + sinh(d*x + c) +
1) + log(cosh(d*x + c) + sinh(d*x + c) - 1))/(a*d)]
```

3.29.6 Sympy [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

3.29.7 Maxima [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-log((e^(d*x + c) + 1)*e^(-c))/(a*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d)
 - 2*integrate((b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^2 + a*b + (a^2*e^(4*
c) + a*b*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) - a*b*e^(2*c))*e^(2*d*x)), x)
```

3.29.8 Giac [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.29.9 Mupad [B] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 284, normalized size of antiderivative = 5.16

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{2\operatorname{atan}\left(\frac{e^{dx}e^{c}\left(9b^4\sqrt{-a^2d^2}+16a^2b^2\sqrt{-a^2d^2}+24ab^3\sqrt{-a^2d^2}\right)}{16da^3b^2+24da^2b^3+9dab^4}\right)}{\sqrt{-a^2d^2}} - \frac{\sqrt{b}\left(2\operatorname{atan}\left(\frac{e^{dx}e^{c}\sqrt{-a^3d^2-ba^2d^2}\sqrt{-a^2d^2(a+b)}+e^{3c}e^{3dx}\sqrt{-a^3d^2-ba^2d^2}\sqrt{-a^2d^2(a+b)}+4a^2bd^2e^{dx}e^{c}}{2a\sqrt{b}d\sqrt{-a^2d^2(a+b)}}\right)-2\operatorname{atan}\left(\frac{e^{dx}e^{}}{}\right.\right.}{2\sqrt{-a^3d^2-ba^2d^2}}$$

input
```
int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)),x)
```

output
```
- (2*atan((exp(d*x)*exp(c)*(9*b^4*(-a^2*d^2)^(1/2) + 16*a^2*b^2*(-a^2*d^2)
^(1/2) + 24*a*b^3*(-a^2*d^2)^(1/2)))/(24*a^2*b^3*d + 16*a^3*b^2*d + 9*a*b^
4*d)))/(-a^2*d^2)^(1/2) - (b^(1/2)*(2*atan((exp(d*x)*exp(c)*(- a^3*d^2 - a
^2*b*d^2)^(1/2)*(-a^2*d^2*(a + b))^(1/2) + exp(3*c)*exp(3*d*x)*(- a^3*d^2
- a^2*b*d^2)^(1/2)*(-a^2*d^2*(a + b))^(1/2) + 4*a^2*b*d^2*exp(d*x)*exp(c))
/(2*a*b^(1/2)*d*(-a^2*d^2*(a + b))^(1/2))) - 2*atan((exp(d*x)*exp(c)*(-a^2
*d^2*(a + b))^(1/2))/(2*a*b^(1/2)*d))))/(2*(- a^3*d^2 - a^2*b*d^2)^(1/2))
```

3.30 $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.30.1 Optimal result

Integrand size = 23, antiderivative size = 48

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\coth(c+dx)}{ad}$$

output
```
-coth(d*x+c)/a/d-arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/a^(3/2)/d
```

3.30.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\coth(c+dx)}{ad}$$

input
```
Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
-((Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*d)) - Coth[c
+ d*x]/(a*d)
```

3.30.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 25, 4146, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\downarrow 4146$$

$$\frac{\int \frac{\coth^2(c+dx)}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{d}$$

$$\downarrow 264$$

$$\frac{-\frac{b\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a}-\frac{\coth(c+dx)}{a}}{d}$$

$$\downarrow 218$$

$$\frac{-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\coth(c+dx)}{a}}{d}$$

input
```
Int[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-((Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2)) - Coth[c + d
*x]/a)/d
```

3.30.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.30.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(40) = 80$.

Time $= 0.49$ (sec) , antiderivative size $= 112$, normalized size of antiderivative $= 2.33$

method	result
risch	$-\frac{2}{da\left(e^{2dx+2c}-1\right)}+\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}-a+b}{a+b}\right)}{2a^2d}-\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{-ab}+a-b}{a+b}\right)}{2a^2d}$
derivativedivides	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+2b\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(-a+\sqrt{(a+b)b}-b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{d}$
default	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+2b\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(-a+\sqrt{(a+b)b}-b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{d}$

input
```
int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-2/d/a/(exp(2*d*x+2*c)-1)+1/2/a^2*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)-(2*(-a*
b)^(1/2)-a+b)/(a+b))-1/2/a^2*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)+(2*(-a*b)^(1
/2)+a-b)/(a+b))
```

3.30.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(40)=80$.

Time $=0.28$ (sec) , antiderivative size $=618$, normalized size of antiderivative $=12.88$

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$=\left[\frac{\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2-1\right)\sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+2ab+b^2)\cosh(dx+c)^4+4(\right.}{}\right.$$
$$\left.-\frac{\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2-1\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{\left((a+b)\cosh(dx+c)^2+2(a+\right.}{ad\cosh(dx+c)^2+2\,ad\cosh(dx+c)\sinh(dx+c)+ad\sinh(dx\right.}\right]$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 -
 1)*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b +
 b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4
+ 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2
 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)
*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 + a*
b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*
b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(
a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)
*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 +
 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b
)) - 4)/(a*d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sin
h(d*x + c)^2 - a*d), -((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
sinh(d*x + c)^2 - 1)*sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a
+ b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b
/a)/b) + 2)/(a*d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d
*sinh(d*x + c)^2 - a*d)]
```

3.30.6 Sympy [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}ad} + \frac{2}{(ae^{(-2\,dx-2\,c)}-a)d}$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a*d)
 + 2/((a*e^(-2*d*x - 2*c) - a)*d)
```

3.30.8 Giac [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^2}{b\tanh\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.30.9 Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.83

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{2}{a\,d-a\,d\,\mathrm{e}^{2\,c+2\,d\,x}} - \frac{\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{a^3\,d^2}}{2\,a\,\sqrt{b}\,d} - \frac{\sqrt{b}\,\sqrt{a^3\,d^2}}{2\,a^2\,d} + \frac{\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{a^3\,d^2}}{2\,a\,\sqrt{b}\,d} + \frac{\sqrt{b}\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{a^3\,d^2}}{2\,a^2\,d}\right)}{\sqrt{a^3\,d^2}}$$

input
```
int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)),x)
```

output
```
2/(a*d - a*d*exp(2*c + 2*d*x)) - (b^(1/2)*atan((a^3*d^2)^(1/2)/(2*a*b^(1/2
)*d) - (b^(1/2)*(a^3*d^2)^(1/2))/(2*a^2*d) + (exp(2*c)*exp(2*d*x)*(a^3*d^2
)^(1/2))/(2*a*b^(1/2)*d) + (b^(1/2)*exp(2*c)*exp(2*d*x)*(a^3*d^2)^(1/2))/(
2*a^2*d)))/(a^3*d^2)^(1/2)
```

3.31 $\int \frac{\text{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.31.1 Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\text{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(a+2b)\text{arctanh}(\cosh(c+dx))}{2a^2d} - \frac{\sqrt{b}\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{a^2d} - \frac{\coth(c+dx)\text{csch}(c+dx)}{2ad}$$

output
```
1/2*(a+2*b)*arctanh(cosh(d*x+c))/a^2/d-1/2*coth(d*x+c)*csch(d*x+c)/a/d-arc
tanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)*(a+b)^(1/2)/a^2/d
```

3.31.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.33

$$\int \frac{\text{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{8i\sqrt{b}\sqrt{a+b}\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right) + 8i\sqrt{b}\sqrt{a+b}\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right) + a\text{csch}^2($$

input
```
Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
-1/8*((8*I)*Sqrt[b]*Sqrt[a + b]*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c
 + d*x)/2])/Sqrt[b]] + (8*I)*Sqrt[b]*Sqrt[a + b]*ArcTan[((-I)*Sqrt[a + b]
+ Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]] + a*Csch[(c + d*x)/2]^2 - 4*a*Log[Co
sh[(c + d*x)/2]] - 8*b*Log[Cosh[(c + d*x)/2]] + 4*a*Log[Sinh[(c + d*x)/2]]
 + 8*b*Log[Sinh[(c + d*x)/2]] + a*Sech[(c + d*x)/2]^2)/(a^2*d)
```

3.31.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4147, 373, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i}{\sin(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\downarrow 26$$

$$-i\int \frac{1}{\sin(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\downarrow 4147$$

$$-\frac{\int \frac{\operatorname{sech}^2(c+dx)}{\left(1-\operatorname{sech}^2(c+dx)\right)^2\left(-b\operatorname{sech}^2(c+dx)+a+b\right)}d\operatorname{sech}(c+dx)}{d}$$

$$\downarrow 373$$

$$-\frac{\frac{\operatorname{sech}(c+dx)}{2a\left(1-\operatorname{sech}^2(c+dx)\right)}-\frac{\int \frac{b\operatorname{sech}^2(c+dx)+a+b}{\left(1-\operatorname{sech}^2(c+dx)\right)\left(-b\operatorname{sech}^2(c+dx)+a+b\right)}d\operatorname{sech}(c+dx)}{2a}}{d}$$

$$\downarrow 397$$

$$-\frac{\frac{\operatorname{sech}(c+dx)}{2a\left(1-\operatorname{sech}^2(c+dx)\right)}-\frac{(a+2b)\int \frac{1}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)}{a}-\frac{2b(a+b)\int \frac{1}{-b\operatorname{sech}^2(c+dx)+a+b}d\operatorname{sech}(c+dx)}{a}}{2a}}{d}$$

$$\downarrow 219$$

$$-\frac{\frac{\operatorname{sech}(c+dx)}{2a\left(1-\operatorname{sech}^2(c+dx)\right)}-\frac{\frac{(a+2b)\operatorname{arctanh}(\operatorname{sech}(c+dx))}{a}-\frac{2b(a+b)\int\frac{1}{-b\operatorname{sech}^2(c+dx)+a+b}d\operatorname{sech}(c+dx)}{a}}{2a}}{d}$$

$$\downarrow 221$$

$$-\frac{\frac{\operatorname{sech}(c+dx)}{2a\left(1-\operatorname{sech}^2(c+dx)\right)}-\frac{\frac{(a+2b)\operatorname{arctanh}(\operatorname{sech}(c+dx))}{a}-\frac{2\sqrt{b}\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{a}}{2a}}{d}$$

input
```
Int[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
-((-1/2*(((a + 2*b)*ArcTanh[Sech[c + d*x]])/a - (2*Sqrt[b]*Sqrt[a + b]*Arc
Tanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/a)/a + Sech[c + d*x]/(2*a*(1 -
Sech[c + d*x]^2)))/d)
```

3.31.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.31.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\dfrac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a}-\frac{(a+b)b\,\operatorname{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+2a+4b}{4\sqrt{ab+b^2}}\right)}{a^2\sqrt{ab+b^2}}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(-2a-4b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2}}{d}$
default	$\dfrac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a}-\frac{(a+b)b\,\operatorname{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+2a+4b}{4\sqrt{ab+b^2}}\right)}{a^2\sqrt{ab+b^2}}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(-2a-4b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2}}{d}$
risch	$-\frac{e^{dx+c}(e^{2dx+2c}+1)}{da\left(e^{2dx+2c}-1\right)^2}+\frac{\ln\left(e^{dx+c}+1\right)}{2da}+\frac{\ln\left(e^{dx+c}+1\right)b}{da^2}-\frac{\ln\left(e^{dx+c}-1\right)}{2da}-\frac{\ln\left(e^{dx+c}-1\right)b}{da^2}+\frac{\sqrt{ab+b^2}\ln\left(e^{2dx+2}\right.}{2}$

input
```
int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a-(a+b)*b/a^2/(a*b+b^2)^(1/2)*arctanh(1/4*(
2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))-1/8/a/tanh(1/2*d*x+1/2
*c)^2+1/4/a^2*(-2*a-4*b)*ln(tanh(1/2*d*x+1/2*c)))
```

3.31.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 858 vs. 2(73) = 146.

Time = 0.30 (sec) , antiderivative size = 1790, normalized size of antiderivative = 21.06

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/2*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*sinh(
d*x + c)^3 - (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x
 + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 +
4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(a*b + b^2)*log
(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a +
 b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x
+ c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)
*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(
d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cos
h(d*x + c))*sqrt(a*b + b^2) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*
cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d
*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a
+ b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 2*
a*cosh(d*x + c) - ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*s
inh(d*x + c)^3 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 +
 2*(3*(a + 2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*
cosh(d*x + c)^3 - (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(co
sh(d*x + c) + sinh(d*x + c) + 1) + ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b
)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*
cosh(d*x + c)^2 + 2*(3*(a + 2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + ...
```

3.31.6 Sympy [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

3.31.7 Maxima [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-(e^(3*d*x + 3*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4*c) - 2*a*d*e^(2*d*x + 2
*c) + a*d) + 1/2*(a + 2*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) - 1/2*(a
+ 2*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^2*d) + 8*integrate(1/4*((a*b*e^(3*
c) + b^2*e^(3*c))*e^(3*d*x) - (a*b*e^c + b^2*e^c)*e^(d*x))/(a^3 + a^2*b +
(a^3*e^(4*c) + a^2*b*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*c) - a^2*b*e^(2*c))*
e^(2*d*x)), x)
```

3.31.8 Giac [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.31.9 Mupad [B] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 787, normalized size of antiderivative = 9.26

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$= \frac{\operatorname{atan}\left(\frac{e^{dx}e^{c}\left(18b^7\sqrt{-a^4d^2}+48a^2b^5\sqrt{-a^4d^2}+27a^3b^4\sqrt{-a^4d^2}+8a^4b^3\sqrt{-a^4d^2}+a^5b^2\sqrt{-a^4d^2}+45ab^6\sqrt{-a^4d^2}\right)}{9a^2b^6d\sqrt{a^2+4ab+4b^2}+18a^3b^5d\sqrt{a^2+4ab+4b^2}+15a^4b^4d\sqrt{a^2+4ab+4b^2}+6a^5b^3d\sqrt{a^2+4ab+4b^2}+a^6b^2d\sqrt{a^2+4ab+4b^2}}\right)}{\sqrt{-a^4d^2}}$$

$$- \left(2\operatorname{atan}\left(\frac{e^{dx}e^{c}(a+b)\sqrt{-a^4d^2}}{2a^2d\sqrt{b(a+b)}}\right) + 2\operatorname{atan}\left(\frac{\left(e^{dx}e^{c}\left(\frac{64\left(2a^4bd\sqrt{b^2+ab}+6a^2b^3d\sqrt{b^2+ab}+6a^3b^2d\sqrt{b^2+ab}\right)}{a^9d^2(a+b)^2\left(a^2+2ab+b^2\right)} - \frac{32\left(3b^4\sqrt{-a^4d^2}+4a^2\right.\right.\right.\right.}{a^7d(a+b)}\right.\right.\right.\right.$$

$$- \frac{e^{c+dx}}{ad\left(e^{2c+2dx}-1\right)} - \frac{2e^{c+dx}}{ad\left(e^{4c+4dx}-2e^{2c+2dx}+1\right)}$$

input
```
int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)),x)
```

output
```
(atan((exp(d*x)*exp(c)*(18*b^7*(-a^4*d^2)^(1/2) + 48*a^2*b^5*(-a^4*d^2)^(1
/2) + 27*a^3*b^4*(-a^4*d^2)^(1/2) + 8*a^4*b^3*(-a^4*d^2)^(1/2) + a^5*b^2*(
-a^4*d^2)^(1/2) + 45*a*b^6*(-a^4*d^2)^(1/2)))/(9*a^2*b^6*d*(4*a*b + a^2 +
4*b^2)^(1/2) + 18*a^3*b^5*d*(4*a*b + a^2 + 4*b^2)^(1/2) + 15*a^4*b^4*d*(4*
a*b + a^2 + 4*b^2)^(1/2) + 6*a^5*b^3*d*(4*a*b + a^2 + 4*b^2)^(1/2) + a^6*b
^2*d*(4*a*b + a^2 + 4*b^2)^(1/2)))*(4*a*b + a^2 + 4*b^2)^(1/2))/(-a^4*d^2)
^(1/2) - ((2*atan((exp(d*x)*exp(c)*(a + b)*(-a^4*d^2)^(1/2))/(2*a^2*d*(b*(
a + b))^(1/2))) + 2*atan(((exp(d*x)*exp(c)*((64*(2*a^4*b*d*(a*b + b^2)^(1/
2) + 6*a^2*b^3*d*(a*b + b^2)^(1/2) + 6*a^3*b^2*d*(a*b + b^2)^(1/2)))/(a^9*
d^2*(a + b)^2*(2*a*b + a^2 + b^2)) - (32*(3*b^4*(-a^4*d^2)^(1/2) + 4*a^2*b
^2*(-a^4*d^2)^(1/2) + 6*a*b^3*(-a^4*d^2)^(1/2) + a^3*b*(-a^4*d^2)^(1/2)))/
(a^7*d*(a + b)*(-a^4*d^2)^(1/2)*(b*(a + b))^(1/2)*(2*a*b + a^2 + b^2))) -
(32*exp(3*c)*exp(3*d*x)*(3*b^4*(-a^4*d^2)^(1/2) + 4*a^2*b^2*(-a^4*d^2)^(1/
2) + 6*a*b^3*(-a^4*d^2)^(1/2) + a^3*b*(-a^4*d^2)^(1/2)))/(a^7*d*(a + b)*(-
a^4*d^2)^(1/2)*(b*(a + b))^(1/2)*(2*a*b + a^2 + b^2)))*(a^8*(-a^4*d^2)^(1/
2) + a^5*b^3*(-a^4*d^2)^(1/2) + 3*a^6*b^2*(-a^4*d^2)^(1/2) + 3*a^7*b*(-a^4
*d^2)^(1/2)))/(192*a*b^2 + 64*a^2*b + 192*b^3)))*(a*b + b^2)^(1/2))/(2*(-a
^4*d^2)^(1/2)) - exp(c + d*x)/(a*d*(exp(2*c + 2*d*x) - 1)) - (2*exp(c + d*
x))/(a*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.32 $\int \frac{\text{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.32.1 Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{\text{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\sqrt{b}(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(a+b)\coth(c+dx)}{a^2 d} - \frac{\coth^3(c+dx)}{3ad}$$

output
```
(a+b)*coth(d*x+c)/a^2/d-1/3*coth(d*x+c)^3/a/d+(a+b)*arctan(b^(1/2)*tanh(d*
x+c)/a^(1/2))*b^(1/2)/a^(5/2)/d
```

3.32.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{\text{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{3\sqrt{b}(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) + \sqrt{a}\coth(c+dx)\left(2a+3b-a\text{csch}^2(c+dx)\right)}{3a^{5/2}d}$$

input
```
Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(3*Sqrt[b]*(a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*Coth[
c + d*x]*(2*a + 3*b - a*Csch[c + d*x]^2))/(3*a^(5/2)*d)
```

3.32.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4146, 359, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\downarrow 4146$$

$$\frac{\int \frac{\coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{d}$$

$$\downarrow 359$$

$$\frac{-\frac{(a+b)\int \frac{\coth^2(c+dx)}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a} - \frac{\coth^3(c+dx)}{3a}}{d}$$

$$\downarrow 264$$

$$\frac{-\frac{(a+b)\left(-\frac{b\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a} - \frac{\coth(c+dx)}{a}\right)}{a} - \frac{\coth^3(c+dx)}{3a}}{d}$$

$$\downarrow 218$$

$$\frac{-\frac{(a+b)\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\coth(c+dx)}{a}\right)}{a} - \frac{\coth^3(c+dx)}{3a}}{d}$$

input
```
Int[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-1/3*Coth[c + d*x]^3/a - ((a + b)*(-((Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*
x])/Sqrt[a]])/a^(3/2)) - Coth[c + d*x]/a))/a)/d
```

3.32.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.32.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(60) = 120$.

Time = 1.57 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.51

method	result
risch	$-\frac{2\left(-3b\,\mathrm{e}^{4dx+4c}+6\,\mathrm{e}^{2dx+2c}a+6b\,\mathrm{e}^{2dx+2c}-2a-3b\right)}{3d\,a^2\left(\mathrm{e}^{2dx+2c}-1\right)^3}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2dx+2c}+\frac{2\sqrt{-ab}+a-b}{a+b}\right)}{2a^2d}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2dx+2c}+\frac{2\sqrt{-}}{}\right.}{2a^3d}$
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2}-\frac{2b(a+b)\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(-a+\sqrt{(}\right.}{a}\right.}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2}-\frac{2b(a+b)\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(-a+\sqrt{(}\right.}{a}\right.}{d}$

input
```
int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-3*b*exp(4*d*x+4*c)+6*exp(2*d*x+2*c)*a+6*b*exp(2*d*x+2*c)-2*a-3*b)/d
/a^2/(exp(2*d*x+2*c)-1)^3+1/2/a^2*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)+(2*(-a*
b)^(1/2)+a-b)/(a+b))+1/2/a^3*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)+(2*(-a*b)^(1
/2)+a-b)/(a+b))*b-1/2/a^2*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)-(2*(-a*b)^(1/2)
-a+b)/(a+b))-1/2/a^3*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)-(2*(-a*b)^(1/2)-a+b)
/(a+b))*b
```

3.32.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(60) = 120$.

Time $= 0.28$ (sec) , antiderivative size $= 1628$, normalized size of antiderivative $= 23.26$

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/6*(12*b*cosh(d*x + c)^4 + 48*b*cosh(d*x + c)*sinh(d*x + c)^3 + 12*b*sin
h(d*x + c)^4 - 24*(a + b)*cosh(d*x + c)^2 + 24*(3*b*cosh(d*x + c)^2 - a -
b)*sinh(d*x + c)^2 + 3*((a + b)*cosh(d*x + c)^6 + 6*(a + b)*cosh(d*x + c)*
sinh(d*x + c)^5 + (a + b)*sinh(d*x + c)^6 - 3*(a + b)*cosh(d*x + c)^4 + 3*
(5*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c)^4 + 4*(5*(a + b)*cosh(d*
x + c)^3 - 3*(a + b)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a + b)*cosh(d*x +
 c)^2 + 3*(5*(a + b)*cosh(d*x + c)^4 - 6*(a + b)*cosh(d*x + c)^2 + a + b)*
sinh(d*x + c)^2 + 6*((a + b)*cosh(d*x + c)^5 - 2*(a + b)*cosh(d*x + c)^3 +
 (a + b)*cosh(d*x + c))*sinh(d*x + c) - a - b)*sqrt(-b/a)*log(((a^2 + 2*a*
b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x +
c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2
 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 +
 a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*
cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a
*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)
*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x +
 c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)
*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (
a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 48*(b*cosh(d*x + c)^3 - (a
 + b)*cosh(d*x + c))*sinh(d*x + c) + 8*a + 12*b)/(a^2*d*cosh(d*x + c)^6...
```

3.32.6 Sympy [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)
```

3.32.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(60) = 120$.

Time $= 0.30$ (sec) , antiderivative size $= 134$, normalized size of antiderivative $= 1.91$

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{2\left(6\,(a+b)e^{(-2\,dx-2\,c)} - 3\,be^{(-4\,dx-4\,c)} - 2\,a - 3\,b\right)}{3\left(3\,a^2e^{(-2\,dx-2\,c)} - 3\,a^2e^{(-4\,dx-4\,c)} + a^2e^{(-6\,dx-6\,c)} - a^2\right)d} - \frac{(ab+b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}a^2d}$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
2/3*(6*(a + b)*e^(-2*d*x - 2*c) - 3*b*e^(-4*d*x - 4*c) - 2*a - 3*b)/((3*a^
2*e^(-2*d*x - 2*c) - 3*a^2*e^(-4*d*x - 4*c) + a^2*e^(-6*d*x - 6*c) - a^2)*
d) - (a*b + b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/
(sqrt(a*b)*a^2*d)
```

3.32.8 Giac [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^4}{b\tanh\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.32.9 Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.63

$$\begin{aligned}
&\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx \\
&= \frac{2b}{a^2 d\left(\mathrm{e}^{2c+2dx}-1\right)} - \frac{8}{3ad\left(3\,\mathrm{e}^{2c+2dx}-3\,\mathrm{e}^{4c+4dx}+\mathrm{e}^{6c+6dx}-1\right)} \\
&\quad - \frac{4}{ad\left(\mathrm{e}^{4c+4dx}-2\,\mathrm{e}^{2c+2dx}+1\right)} \\
&\quad + \frac{\sqrt{-b}\,\ln\left(-\frac{4b\,\mathrm{e}^{2c+2dx}}{a^2} - \frac{2\sqrt{-b}\,(ad+bd+ad\,\mathrm{e}^{2c+2dx}-bd\,\mathrm{e}^{2c+2dx})}{a^{5/2}d}\right)(a+b)}{2a^{5/2}d} \\
&\quad - \frac{\sqrt{-b}\,\ln\left(\frac{2\sqrt{-b}\,(ad+bd+ad\,\mathrm{e}^{2c+2dx}-bd\,\mathrm{e}^{2c+2dx})}{a^{5/2}d} - \frac{4b\,\mathrm{e}^{2c+2dx}}{a^2}\right)(a+b)}{2a^{5/2}d}
\end{aligned}$$

input
```
int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)),x)
```

output
```
(2*b)/(a^2*d*(exp(2*c + 2*d*x) - 1)) - 8/(3*a*d*(3*exp(2*c + 2*d*x) - 3*ex
p(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - 4/(a*d*(exp(4*c + 4*d*x) - 2*exp
(2*c + 2*d*x) + 1)) + ((-b)^(1/2)*log(- (4*b*exp(2*c + 2*d*x))/a^2 - (2*(-
b)^(1/2)*(a*d + b*d + a*d*exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/(a^(5/
2)*d))*(a + b))/(2*a^(5/2)*d) - ((-b)^(1/2)*log((2*(-b)^(1/2)*(a*d + b*d +
 a*d*exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/(a^(5/2)*d) - (4*b*exp(2*c
+ 2*d*x))/a^2)*(a + b))/(2*a^(5/2)*d)
```

3.33 $\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

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3.33.1 Optimal result

Integrand size = 23, antiderivative size = 192

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{3(a^2-6ab+b^2)\,x}{8(a+b)^4} + \frac{3\sqrt{a}(a-b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2(a+b)^4 d} - \frac{(5a-b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2 d\left(a+b\tanh^2(c+dx)\right)} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{3(3a-b)b\tanh(c+dx)}{8(a+b)^3 d\left(a+b\tanh^2(c+dx)\right)}$$

output

```
3/8*(a^2-6*a*b+b^2)*x/(a+b)^4+3/2*(a-b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2)
)*a^(1/2)*b^(1/2)/(a+b)^4/d-1/8*(5*a-b)*cosh(d*x+c)*sinh(d*x+c)/(a+b)^2/d/
(a+b*tanh(d*x+c)^2)+1/4*cosh(d*x+c)^3*sinh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c)
^2)+3/8*(3*a-b)*b*tanh(d*x+c)/(a+b)^3/d/(a+b*tanh(d*x+c)^2)
```

3.33.2 Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.69

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{12(a^2-6ab+b^2)(c+dx)+48\sqrt{a}(a-b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)-8(a-b)(a+b)\sinh(2(c+dx))+\frac{1}{a}}{32(a+b)^4 d}$$

input
```
Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(12*(a^2 - 6*a*b + b^2)*(c + d*x) + 48*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[(Sqr
t[b]*Tanh[c + d*x])/Sqrt[a]] - 8*(a - b)*(a + b)*Sinh[2*(c + d*x)] + (16*a
*b*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]) + (a + b
)^2*Sinh[4*(c + d*x)])/(32*(a + b)^4*d)
```

3.33.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4146, 372, 402, 27, 402, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sin(ic+idx)^4}{(a-b\tan(ic+idx)^2)^2}\,dx$$
$$\downarrow 4146$$
$$\frac{\int \frac{\tanh^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)^3\left(b\tanh^2(c+dx)+a\right)^2}\,d\tanh(c+dx)}{d}$$
$$\downarrow 372$$

$$\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{\int\frac{(4a-b)\tanh^2(c+dx)+a}{\left(1-\tanh^2(c+dx)\right)^2\left(b\tanh^2(c+dx)+a\right)^2}\,d\tanh(c+dx)}{4(a+b)}$$

$$d$$

$$\downarrow 402$$

$$\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{\int-\frac{3\left(a(a-b)-(5a-b)b\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}\,d\tanh(c+dx)}{2(a+b)}+\frac{(5a-b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}}{4(a+b)}$$

$$d$$

$$\downarrow 27$$

$$\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{\frac{(5a-b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}-\frac{3\int\frac{a(a-b)-(5a-b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}\,d\tanh(c+dx)}{2(a+b)}}{4(a+b)}$$

$$d$$

$$\downarrow 402$$

$$\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{\frac{(5a-b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}-3\left(\frac{b(3a-b)\tanh(c+dx)}{(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\int-\frac{2a\left(a(a-3b)-(3a-b)b\right.}{\left(1-\tanh^2(c+dx)\right)\left(b\ta\right.}}{2a}\right.}{2(a+b)}}{4(a+b)}$$

$$d$$

$$\downarrow 27$$

$$\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{\frac{(5a-b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}-3\left(\frac{\int\frac{a(a-3b)-(3a-b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}\,d\tanh(c+dx)}{a+b}\right.}{2(a+b)}}{4(a+b)}$$

$$d$$

$$\downarrow 397$$

$$\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{\frac{(5a-b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}-3\left(\frac{\left(a^2-6ab+b^2\right)\int\frac{1}{1-\tanh^2(c+dx)}\,d\tanh(c+dx)}{a+b}+\frac{4ab(a\right.}{a+b}\right.}{4(a+b)}$$

$$d$$

$$\downarrow 218$$

$$\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{\frac{(5a-b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}-\frac{3\left(\frac{\left(a^2-6ab+b^2\right)\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{4\sqrt{a}\sqrt{}}{a+b}\right.}{4(a+b)}}{d}$$

$$\downarrow 219$$

$$\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{\frac{(5a-b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}-\frac{3\left(\frac{\left(a^2-6ab+b^2\right)\operatorname{arctanh}(\tanh(c+dx))}{a+b}+\frac{4\sqrt{a}\sqrt{b}(a-b)}{a+b}\right.}{4(a+b)}}{d}$$

input
```
Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(Tanh[c + d*x]/(4*(a + b)*(1 - Tanh[c + d*x]^2)^2*(a + b*Tanh[c + d*x]^2))
 - (((5*a - b)*Tanh[c + d*x])/(2*(a + b)*(1 - Tanh[c + d*x]^2)*(a + b*Tanh
[c + d*x]^2)) - (3*(((4*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d
*x])/Sqrt[a]])/(a + b) + ((a^2 - 6*a*b + b^2)*ArcTanh[Tanh[c + d*x]])/(a +
 b))/(a + b) + ((3*a - b)*b*Tanh[c + d*x])/((a + b)*(a + b*Tanh[c + d*x]^2
))))/(2*(a + b)))/(4*(a + b)))/d
```

3.33.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.33.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(174) = 348$.

Time = 69.26 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.67

method	result
derivativedivides	$2ab\left(\frac{\left(-\frac{b}{2}-\frac{a}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-\frac{b}{2}-\frac{a}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a}+\frac{(3a-3b)a\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(a-\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(-2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(-2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2}\right)}{(a+b)^4}$
default	$2ab\left(\frac{\left(-\frac{b}{2}-\frac{a}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-\frac{b}{2}-\frac{a}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a}+\frac{(3a-3b)a\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(a-\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(-2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(-2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2}\right)}{(a+b)^4}$
risch	$\frac{3x\,a^2}{8(a+b)^2(a^2+2ab+b^2)}-\frac{9xab}{4(a+b)^2(a^2+2ab+b^2)}+\frac{3x\,b^2}{8(a+b)^2(a^2+2ab+b^2)}+\frac{e^{4dx+4c}}{64d(a^2+2ab+b^2)}-\frac{e^{2dx+2c}a}{8(a+b)(a^2+2ab+$

input
```
int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a*b/(a+b)^4*(((-1/2*b-1/2*a)*tanh(1/2*d*x+1/2*c)^3+(-1/2*b-1/2*a)*
tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*
tanh(1/2*d*x+1/2*c)^2*b+a)+1/2*(3*a-3*b)*a*(1/2*(a+((a+b)*b)^(1/2)+b)/a/((
a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/
2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(-a+((a+b)*b)^(1/2)-b)/a/((a
+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/
2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))+1/4/(a+b)^2/(tanh(1/2*d*x+1/2*
c)-1)^4+1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^3-1/8*(a-7*b)/(a+b)^3/(tanh(1/
2*d*x+1/2*c)-1)^2-1/8*(3*a-5*b)/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)+1/8/(a+b)^
4*(-3*a^2+18*a*b-3*b^2)*ln(tanh(1/2*d*x+1/2*c)-1)-1/4/(a+b)^2/(1+tanh(1/2*
d*x+1/2*c))^4+1/2/(a+b)^2/(1+tanh(1/2*d*x+1/2*c))^3-1/8*(-a+7*b)/(a+b)^3/(
1+tanh(1/2*d*x+1/2*c))^2-1/8*(3*a-5*b)/(a+b)^3/(1+tanh(1/2*d*x+1/2*c))+1/8
/(a+b)^4*(3*a^2-18*a*b+3*b^2)*ln(1+tanh(1/2*d*x+1/2*c)))
```

3.33. $\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

3.33.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3531 vs. $2(174) = 348$.

Time $= 0.38$ (sec) , antiderivative size $= 7366$, normalized size of antiderivative $= 38.36$

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.33.6 Sympy [F]

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(sinh(c + d*x)**4/(a + b*tanh(c + d*x)**2)**2, x)
```

3.33.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1690 vs. $2(174) = 348$.

Time $= 0.48$ (sec) , antiderivative size $= 1690$, normalized size of antiderivative $= 8.80$

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output

```
-1/4*(a*b - 2*b^2)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c)
 + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 1/2*b*log((a +
 b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d) + 1/4*(a*b - 2*b^2)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a
 + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^
4)*d) + 1/2*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) +
a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 1/32*(3*a^3*b - 33*a^2*b^2 +
13*a*b^3 + b^4)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((
a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*sqrt(a*b)*d) + 1/8*(3*a^2*b
 - 6*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/
((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) - 1/32*(3*a^3*b - 33*a^2
*b^2 + 13*a*b^3 + b^4)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(
a*b))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*sqrt(a*b)*d) - 1/8*
(3*a^2*b - 6*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sq
rt(a*b))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) - 3/16*(3*a*b +
 b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^3 + 2*a
^2*b + a*b^2)*sqrt(a*b)*d) - 1/16*(a^3*b - 5*a^2*b^2 - 5*a*b^3 + b^4 + (a^
3*b - 15*a^2*b^2 + 15*a*b^3 - b^4)*e^(2*d*x + 2*c))/((a^6 + 5*a^5*b + 10*a
^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5 + (a^6 + 5*a^5*b + 10*a^4*b^2 + 10
*a^3*b^3 + 5*a^2*b^4 + a*b^5)*e^(4*d*x + 4*c) + 2*(a^6 + 3*a^5*b + 2*a^...
```

3.33.8 Giac [F]

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\sinh\left(dx+c\right)^4}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input

```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output

```
sage0*x
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\sinh(c+d\,x)^4}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2, x)
```

3.34 $\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

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3.34.1 Optimal result

Integrand size = 23, antiderivative size = 124

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{(3a-2b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}d} - \frac{(a-b)\cosh(c+dx)}{(a+b)^3 d} + \frac{\cosh^3(c+dx)}{3(a+b)^2 d} + \frac{ab\operatorname{sech}(c+dx)}{2(a+b)^3 d\left(a+b-b\operatorname{sech}^2(c+dx)\right)}$$

output
```
-(a-b)*cosh(d*x+c)/(a+b)^3/d+1/3*cosh(d*x+c)^3/(a+b)^2/d+1/2*a*b*sech(d*x+
c)/(a+b)^3/d/(a+b-b*sech(d*x+c)^2)+1/2*(3*a-2*b)*arctanh(sech(d*x+c)*b^(1/
2)/(a+b)^(1/2))*b^(1/2)/(a+b)^(7/2)/d
```

3.34.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.29

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{6i(3a-2b)\sqrt{b}\left(\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)}{(a+b)^{7/2}} + \frac{3\cosh(c+dx)\left(5b+a\left(-3+\frac{4b}{a-b+(a+b)\cosh(2(c+}\right.}{(a+b)^3}}{12d}$$

input
```
Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(((6*I)*(3*a - 2*b)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c +
d*x)/2])/Sqrt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/
Sqrt[b]]))/(a + b)^(7/2) + (3*Cosh[c + d*x]*(5*b + a*(-3 + (4*b)/(a - b +
(a + b)*Cosh[2*(c + d*x)])))/(a + b)^3 + Cosh[3*(c + d*x)]/(a + b)^2)/(12
*d)
```

3.34.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4147, 25, 361, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{i\sin(ic+idx)^3}{\left(a-b\tan(ic+idx)^2\right)^2}\,dx$$

$$\downarrow 26$$

$$i\int \frac{\sin(ic+idx)^3}{\left(a-b\tan(ic+idx)^2\right)^2}\,dx$$

$$\downarrow 4147$$

$$\frac{\int -\frac{\cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)}{\left(-b\text{sech}^2(c+dx)+a+b\right)^2}\,d\text{sech}(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{\int \frac{\cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)}{\left(-b\text{sech}^2(c+dx)+a+b\right)^2}\,d\text{sech}(c+dx)}{d}$$

$$\downarrow 361$$

$$\frac{\frac{ab\operatorname{sech}(c+dx)}{2(a+b)^3\left(a-b\operatorname{sech}^2(c+dx)+b\right)}-\frac{1}{2}b\int\frac{\cosh^4(c+dx)\left(-\frac{a\operatorname{sech}^4(c+dx)}{(a+b)^3}-\frac{2a\operatorname{sech}^2(c+dx)}{b(a+b)^2}+\frac{2}{b(a+b)}\right)}{-b\operatorname{sech}^2(c+dx)+a+b}d\operatorname{sech}(c+dx)}{d}$$

$$\downarrow 1584$$

$$\frac{\frac{ab\operatorname{sech}(c+dx)}{2(a+b)^3\left(a-b\operatorname{sech}^2(c+dx)+b\right)}-\frac{1}{2}b\int\left(\frac{2\cosh^4(c+dx)}{b(a+b)^2}-\frac{2(a-b)\cosh^2(c+dx)}{b(a+b)^3}+\frac{2b-3a}{(a+b)^3\left(-b\operatorname{sech}^2(c+dx)+a+b\right)}\right)d\operatorname{sech}(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{ab\operatorname{sech}(c+dx)}{2(a+b)^3\left(a-b\operatorname{sech}^2(c+dx)+b\right)}-\frac{1}{2}b\left(-\frac{(3a-2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}(a+b)^{7/2}}-\frac{2\cosh^3(c+dx)}{3b(a+b)^2}+\frac{2(a-b)\cosh(c+dx)}{b(a+b)^3}\right)}{d}$$

input
```
Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-1/2*(b*(-(((3*a - 2*b)*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(Sq
rt[b]*(a + b)^(7/2))) + (2*(a - b)*Cosh[c + d*x])/(b*(a + b)^3) - (2*Cosh[
c + d*x]^3)/(3*b*(a + b)^2))) + (a*b*Sech[c + d*x])/(2*(a + b)^3*(a + b -
b*Sech[c + d*x]^2)))/d
```

3.34.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.34.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(110) = 220$.

Time = 14.74 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.15

method	result
derivativedivides	$\frac{1}{3(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{1}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{a-3b}{2(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{4b\left(\frac{\left(-\frac{a}{4}-\frac{b}{2}\right)\tanh\left(\frac{dx}{2}\right.}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\right.}{}$
default	$\frac{1}{3(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{1}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{a-3b}{2(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{4b\left(\frac{\left(-\frac{a}{4}-\frac{b}{2}\right)\tanh\left(\frac{dx}{2}\right.}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\right.}{}$
risch	$\frac{e^{3dx+3c}}{24d(a^2+2ab+b^2)}-\frac{3e^{dx+c}a}{8(a+b)(a^2+2ab+b^2)d}+\frac{5e^{dx+c}b}{8(a+b)(a^2+2ab+b^2)d}-\frac{3e^{-dx-c}a}{8(a^3+3a^2b+3ab^2+b^3)d}+\frac{5e^{-dx-c}}{8(a^3+3a^2b+3a}$

input
```
int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3/(a+b)^2/(1+tanh(1/2*d*x+1/2*c))^3-1/2/(a+b)^2/(1+tanh(1/2*d*x+1/2
*c))^2-1/2*(a-3*b)/(a+b)^3/(1+tanh(1/2*d*x+1/2*c))-4*b/(a+b)^3*(((-1/4*a-1
/2*b)*tanh(1/2*d*x+1/2*c)^2-1/4*a)/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x
+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)-1/8*(3*a-2*b)/(a*b+b^2)^(1/2)*arc
tanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2)))-1/3/(a+b)^2
/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/(a+b)
^3*(-a+3*b)/(tanh(1/2*d*x+1/2*c)-1))
```

3.34.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2650 vs. $2(113) = 226$.

Time $= 0.34$ (sec) , antiderivative size $= 5025$, normalized size of antiderivative $= 40.52$

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.34.6 Sympy [F]

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(sinh(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)
```

3.34.7 Maxima [F]

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\sinh\left(dx+c\right)^3}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/24*(a^2 + 2*a*b + b^2 + (a^2*e^(10*c) + 2*a*b*e^(10*c) + b^2*e^(10*c))*e
^(10*d*x) - (7*a^2*e^(8*c) - 6*a*b*e^(8*c) - 13*b^2*e^(8*c))*e^(8*d*x) - 2
*(13*a^2*e^(6*c) - 40*a*b*e^(6*c) + 7*b^2*e^(6*c))*e^(6*d*x) - 2*(13*a^2*e
^(4*c) - 40*a*b*e^(4*c) + 7*b^2*e^(4*c))*e^(4*d*x) - (7*a^2*e^(2*c) - 6*a*
b*e^(2*c) - 13*b^2*e^(2*c))*e^(2*d*x))/((a^4*d*e^(7*c) + 4*a^3*b*d*e^(7*c)
 + 6*a^2*b^2*d*e^(7*c) + 4*a*b^3*d*e^(7*c) + b^4*d*e^(7*c))*e^(7*d*x) + 2*
(a^4*d*e^(5*c) + 2*a^3*b*d*e^(5*c) - 2*a*b^3*d*e^(5*c) - b^4*d*e^(5*c))*e^
(5*d*x) + (a^4*d*e^(3*c) + 4*a^3*b*d*e^(3*c) + 6*a^2*b^2*d*e^(3*c) + 4*a*b
^3*d*e^(3*c) + b^4*d*e^(3*c))*e^(3*d*x)) - 1/8*integrate(8*((3*a*b*e^(3*c)
 - 2*b^2*e^(3*c))*e^(3*d*x) - (3*a*b*e^c - 2*b^2*e^c)*e^(d*x))/(a^4 + 4*a^
3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 + (a^4*e^(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*b
^2*e^(4*c) + 4*a*b^3*e^(4*c) + b^4*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 2
*a^3*b*e^(2*c) - 2*a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

3.34.8 Giac [F]

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\sinh\left(dx+c\right)^3}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\sinh(c+d\,x)^3}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2, x)
```

3.35 $\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

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3.35.1 Optimal result

Integrand size = 23, antiderivative size = 132

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{(a-3b)x}{2(a+b)^3} - \frac{(3a-b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}(a+b)^3 d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{b\tanh(c+dx)}{(a+b)^2 d\left(a+b\tanh^2(c+dx)\right)}$$

output

```
-1/2*(a-3*b)*x/(a+b)^3-1/2*(3*a-b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(
1/2)/(a+b)^3/d/a^(1/2)+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c
)^2)-b*tanh(d*x+c)/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

3.35.2 Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.80

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{-2(a-3b)(c+dx) + \frac{2\sqrt{b}(-3a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}} + (a+b)\sinh(2(c+dx)) - \frac{2b(a+b)\sinh(2(c+dx))}{a-b+(a+b)\cosh(2(c+dx))}}{4(a+b)^3 d}$$

input
```
Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-2*(a - 3*b)*(c + d*x) + (2*Sqrt[b]*(-3*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d
*x])/Sqrt[a]])/Sqrt[a] + (a + b)*Sinh[2*(c + d*x)] - (2*b*(a + b)*Sinh[2*(
c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(4*(a + b)^3*d)
```

3.35.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 25, 4146, 373, 402, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin(ic+idx)^2}{\left(a-b\tan(ic+idx)^2\right)^2}\,dx$$

$$\downarrow 25$$

$$-\int \frac{\sin(ic+idx)^2}{\left(a-b\tan(ic+idx)^2\right)^2}\,dx$$

$$\downarrow 4146$$

$$\frac{\int \frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^2\left(b\tanh^2(c+dx)+a\right)^2}\,d\tanh(c+dx)}{d}$$

$$\downarrow 373$$

$$\frac{\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)} - \frac{\int \frac{a-3b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}\,d\tanh(c+dx)}{2(a+b)}}{d}$$

$$\downarrow 402$$

$$\frac{\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)} - \frac{\frac{2b\tanh(c+dx)}{(a+b)\left(a+b\tanh^2(c+dx)\right)} - \frac{\int -\frac{2a\left(-2b\tanh^2(c+dx)+a-b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}\,d\tanh(c+dx)}{2a(a+b)}}{2(a+b)}}{d}$$

$$\downarrow 27$$

$$\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)} - \frac{\frac{\int \frac{-2b\tanh^2(c+dx)+a-b}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}\,d\tanh(c+dx)}{a+b} + \frac{2b\tanh(c+dx)}{(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)}$$

$$d$$

$$\downarrow 397$$

$$\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)} - \frac{\frac{(a-3b)\int \frac{1}{1-\tanh^2(c+dx)}\,d\tanh(c+dx)}{a+b} + \frac{b(3a-b)\int \frac{1}{b\tanh^2(c+dx)+a}\,d\tanh(c+dx)}{a+b}}{a+b} + \frac{2b\tanh(c+dx)}{(a+b)\left(a+b\tanh^2(c+}$$

$$d$$

$$\downarrow 218$$

$$\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)} - \frac{\frac{(a-3b)\int \frac{1}{1-\tanh^2(c+dx)}\,d\tanh(c+dx)}{a+b} + \frac{\sqrt{b}(3a-b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a+b} + \frac{2b\tanh(c+dx)}{(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)}$$

$$d$$

$$\downarrow 219$$

$$\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)} - \frac{\frac{\frac{\sqrt{b}(3a-b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)} + \frac{(a-3b)\operatorname{arctanh}(\tanh(c+dx))}{a+b}}{a+b} + \frac{2b\tanh(c+dx)}{(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)}$$

$$d$$

input
```
Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(Tanh[c + d*x]/(2*(a + b)*(1 - Tanh[c + d*x]^2)*(a + b*Tanh[c + d*x]^2)) -
 ((((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a
 + b)) + ((a - 3*b)*ArcTanh[Tanh[c + d*x]])/(a + b))/(a + b) + (2*b*Tanh[c
 + d*x])/((a + b)*(a + b*Tanh[c + d*x]^2)))/(2*(a + b)))/d
```

3.35. $\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

3.35.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.35.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(118) = 236$.

Time = 4.50 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.89

method	result
risch	$-\frac{xa}{2(a+b)(a^2+2ab+b^2)}+\frac{3xb}{2(a+b)(a^2+2ab+b^2)}+\frac{e^{2dx+2c}}{8d(a^2+2ab+b^2)}-\frac{e^{-2dx-2c}}{8d(a^2+2ab+b^2)}+\frac{b\left(e^{2dx+2}\right.}{d(a+b)^3(a\,e^{4dx+4c}+b\,e^{4}}$
derivativedivides	$-\frac{1}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{1}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(-a+3b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)^3}+4b\left(\frac{\left(-\frac{a}{4}-\frac{b}{4}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\right.}\right.$
default	$-\frac{1}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{1}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(-a+3b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)^3}+4b\left(\frac{\left(-\frac{a}{4}-\frac{b}{4}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\right.}\right.$

input

```
int(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/(a+b)/(a^2+2*a*b+b^2)*a+3/2*x/(a+b)/(a^2+2*a*b+b^2)*b+1/8/d/(a^2+2*
a*b+b^2)*exp(2*d*x+2*c)-1/8/d/(a^2+2*a*b+b^2)*exp(-2*d*x-2*c)+b*(exp(2*d*x
+2*c)*a-b*exp(2*d*x+2*c)+a+b)/d/(a+b)^3/(a*exp(4*d*x+4*c)+b*exp(4*d*x+4*c)
+2*exp(2*d*x+2*c)*a-2*b*exp(2*d*x+2*c)+a+b)+3/4*(-a*b)^(1/2)/(a+b)^3/d*ln(
exp(2*d*x+2*c)-(2*(-a*b)^(1/2)-a+b)/(a+b))-1/4/a*(-a*b)^(1/2)/(a+b)^3/d*ln
(exp(2*d*x+2*c)-(2*(-a*b)^(1/2)-a+b)/(a+b))*b-3/4*(-a*b)^(1/2)/(a+b)^3/d*l
n(exp(2*d*x+2*c)+(2*(-a*b)^(1/2)+a-b)/(a+b))+1/4/a*(-a*b)^(1/2)/(a+b)^3/d*
ln(exp(2*d*x+2*c)+(2*(-a*b)^(1/2)+a-b)/(a+b))*b
```

3.35.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1798 vs. $2(118) = 236$.

Time $= 0.34$ (sec) , antiderivative size $= 3918$, normalized size of antiderivative $= 29.68$

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 - 2*(2*(a^2 -
2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^6 - 2*(2*(a^2 - 2*a*b - 3*b^
2)*d*x - 14*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - a^2 + b^2)*sinh(d*x + c)
^6 + 4*(14*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - 3*(2*(a^2 - 2*a*b - 3*b^2
)*d*x - a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*((a^2 - 4*a*b + 3*b^
2)*d*x - a*b + b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x +
 c)^4 - 4*(a^2 - 4*a*b + 3*b^2)*d*x - 15*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^
2 + b^2)*cosh(d*x + c)^2 + 4*a*b - 4*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*
a*b + b^2)*cosh(d*x + c)^5 - 5*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*c
osh(d*x + c)^3 - 4*((a^2 - 4*a*b + 3*b^2)*d*x - a*b + b^2)*cosh(d*x + c))*
sinh(d*x + c)^3 - 2*(2*(a^2 - 2*a*b - 3*b^2)*d*x + a^2 - 4*a*b - 5*b^2)*co
sh(d*x + c)^2 + 2*(14*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 - 15*(2*(a^2 - 2
*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^4 - 2*(a^2 - 2*a*b - 3*b^2)*d
*x - 24*((a^2 - 4*a*b + 3*b^2)*d*x - a*b + b^2)*cosh(d*x + c)^2 - a^2 + 4*
a*b + 5*b^2)*sinh(d*x + c)^2 - 2*((3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^6 +
6*(3*a^2 + 2*a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^2 + 2*a*b - b
^2)*sinh(d*x + c)^6 + 2*(3*a^2 - 4*a*b + b^2)*cosh(d*x + c)^4 + (15*(3*a^2
 + 2*a*b - b^2)*cosh(d*x + c)^2 + 6*a^2 - 8*a*b + 2*b^2)*sinh(d*x + c)^4 +
 4*(5*(3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^3 + 2*(3*a^2 - 4*a*b + b^2)*c...
```

3.35.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

3.35.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 840 vs. 2(118) = 236.

Time = 0.40 (sec) , antiderivative size = 840, normalized size of antiderivative = 6.36

$$\begin{aligned}
\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx &= \frac{b\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)d} \\
&- \frac{b\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)d} \\
&- \frac{(3\,a^2b-6\,ab^2-b^3)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{8\,(a^4+3\,a^3b+3\,a^2b^2+ab^3)\sqrt{ab}d} \\
&+ \frac{(3\,a^2b-6\,ab^2-b^3)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{8\,(a^4+3\,a^3b+3\,a^2b^2+ab^3)\sqrt{ab}d} \\
&+ \frac{(3\,ab+b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{4\,(a^3+2\,a^2b+ab^2)\sqrt{ab}d} \\
&+ \frac{a^2b-b^3+(a^2b-6\,ab^2+b^3)e^{(2\,dx+2\,c)}}{4\,(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4+(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)e^{(4\,dx+4\,c)}+2\,(a^5+2\,a^4b-2\,a^2b^3-ab^4)e^{(2\,dx+2\,c)}+(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)e^{(-4\,dx-4\,c)}+2\,(a^5+2\,a^4b-2\,a^2b^3-ab^4)e^{(-2\,dx-2\,c)}+a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)} \\
&- \frac{a^2b-b^3+(a^2b-6\,ab^2+b^3)e^{(-2\,dx-2\,c)}}{4\,(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4+2\,(a^5+2\,a^4b-2\,a^2b^3-ab^4)e^{(-2\,dx-2\,c)}+(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)e^{(-4\,dx-4\,c)}+a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)} \\
&- \frac{ab+b^2+(ab-b^2)e^{(-2\,dx-2\,c)}}{2\,(a^4+3\,a^3b+3\,a^2b^2+ab^3+2\,(a^4+a^3b-a^2b^2-ab^3)e^{(-2\,dx-2\,c)}+(a^4+3\,a^3b+3\,a^2b^2+ab^3)e^{(-4\,dx-4\,c)}} \\
&- \frac{dx+c}{2\,(a^2+2\,ab+b^2)d} + \frac{e^{(2\,dx+2\,c)}}{8\,(a^2+2\,ab+b^2)d} - \frac{e^{(-2\,dx-2\,c)}}{8\,(a^2+2\,ab+b^2)d}
\end{aligned}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/2*b*log(2*(a - b)*e^(-2*d*x - 2*c) +
(a + b)*e^(-4*d*x - 4*c) + a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/
8*(3*a^2*b - 6*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/s
qrt(a*b))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) + 1/8*(3*a^2*b
 - 6*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))
/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) + 1/4*(3*a*b + b^2)*arc
tan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^3 + 2*a^2*b + a*
b^2)*sqrt(a*b)*d) + 1/4*(a^2*b - b^3 + (a^2*b - 6*a*b^2 + b^3)*e^(2*d*x +
2*c))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + (a^5 + 4*a^4*b + 6
*a^3*b^2 + 4*a^2*b^3 + a*b^4)*e^(4*d*x + 4*c) + 2*(a^5 + 2*a^4*b - 2*a^2*b
^3 - a*b^4)*e^(2*d*x + 2*c))*d) - 1/4*(a^2*b - b^3 + (a^2*b - 6*a*b^2 + b^
3)*e^(-2*d*x - 2*c))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + 2*(
a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*e^(-2*d*x - 2*c) + (a^5 + 4*a^4*b + 6*a
^3*b^2 + 4*a^2*b^3 + a*b^4)*e^(-4*d*x - 4*c))*d) - 1/2*(a*b + b^2 + (a*b -
 b^2)*e^(-2*d*x - 2*c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3
*b - a^2*b^2 - a*b^3)*e^(-2*d*x - 2*c) + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^
3)*e^(-4*d*x - 4*c))*d) - 1/2*(d*x + c)/((a^2 + 2*a*b + b^2)*d) + 1/8*e^(2
*d*x + 2*c)/((a^2 + 2*a*b + b^2)*d) - 1/8*e^(-2*d*x - 2*c)/((a^2 + 2*a*b +
 b^2)*d)
```

3.35.8 Giac [F]

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\sinh\left(dx+c\right)^2}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\sinh(c+d\,x)^2}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2, x)
```

3.36 $\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

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3.36.1 Optimal result

Integrand size = 21, antiderivative size = 92

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{3\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}d} + \frac{3\cosh(c+dx)}{2(a+b)^2 d} - \frac{\cosh(c+dx)}{2(a+b)d\left(a+b-b\operatorname{sech}^2(c+dx)\right)}$$

output
```
3/2*cosh(d*x+c)/(a+b)^2/d-1/2*cosh(d*x+c)/(a+b)/d/(a+b-b*sech(d*x+c)^2)-3/
2*arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/(a+b)^(5/2)/d
```

3.36.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.45

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{-\frac{3i\sqrt{b}\left(\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)}{(a+b)^{5/2}} + \frac{2\cosh(c+dx)\left(1-\frac{b}{a-b+(a+b)\cosh(2(c+dx))}\right)}{(a+b)^2}}{2d}$$

input
```
Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(((-3*I)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sq
rt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]]))/
(a + b)^(5/2) + (2*Cosh[c + d*x]*(1 - b/(a - b + (a + b)*Cosh[2*(c + d*x)]
)))/(a + b)^2)/(2*d)
```

3.36.3 Rubi [A] (verified)

Time $= 0.27$ (sec) , antiderivative size $= 93$, normalized size of antiderivative $= 1.01$, number of steps used $= 7$, number of rules used $= 6$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used $= \{3042, 26, 4147, 253, 264, 221\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\sin(ic+idx)}{\left(a-b\tan(ic+idx)^2\right)^2}\,dx$$

$$\downarrow 26$$

$$-i\int \frac{\sin(ic+idx)}{\left(a-b\tan(ic+idx)^2\right)^2}\,dx$$

$$\downarrow 4147$$

$$-\frac{\int \frac{\cosh^2(c+dx)}{\left(-b\text{sech}^2(c+dx)+a+b\right)^2}\,d\text{sech}(c+dx)}{d}$$

$$\downarrow 253$$

$$-\frac{\frac{3\int \frac{\cosh^2(c+dx)}{-b\text{sech}^2(c+dx)+a+b}\,d\text{sech}(c+dx)}{2(a+b)}+\frac{\cosh(c+dx)}{2(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)}}{d}$$

$$\downarrow 264$$

$$-\frac{3\left(\frac{b\int\frac{1}{-b\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{a+b}-\frac{\cosh(c+dx)}{a+b}\right)}{2(a+b)}+\frac{\cosh(c+dx)}{2(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)}}{d}$$

$$\downarrow 221$$

$$-\frac{3\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{\cosh(c+dx)}{a+b}\right)}{2(a+b)}+\frac{\cosh(c+dx)}{2(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)}}{d}$$

input
```
Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-(((3*((Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(a + b)^(3/2
) - Cosh[c + d*x]/(a + b)))/(2*(a + b)) + Cosh[c + d*x]/(2*(a + b)*(a + b
- b*Sech[c + d*x]^2)))/d)
```

3.36.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.36.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(78) = 156.

Time = 1.72 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.82

method	result
derivativedivides	$\frac{2b\left(\frac{-\frac{(a+2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2a}-\frac{1}{2}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a}-\frac{3\ \mathrm{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+2a+4b}{4\sqrt{ab+b^2}}\right)}{4\sqrt{ab+b^2}}\right)}{(a+b)^2}-\frac{1}{(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d}$
default	$\frac{2b\left(\frac{-\frac{(a+2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2a}-\frac{1}{2}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a}-\frac{3\ \mathrm{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+2a+4b}{4\sqrt{ab+b^2}}\right)}{4\sqrt{ab+b^2}}\right)}{(a+b)^2}-\frac{1}{(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d}$
risch	$\frac{e^{dx+c}}{2d(a^2+2ab+b^2)}+\frac{e^{-dx-c}}{2d(a^2+2ab+b^2)}-\frac{b\,e^{dx+c}\left(e^{2dx+2c}+1\right)}{d(a+b)^2\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)}+\frac{3\sqrt{(a+b)b}}{}$

input
```
int(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*b/(a+b)^2*((-1/2*(a+2*b)/a*tanh(1/2*d*x+1/2*c)^2-1/2)/(tanh(1/2*d*x
+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)-3/4/(a*
b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/
2)))-1/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)+1/(a+b)^2/(1+tanh(1/2*d*x+1/2*c)))
```

3.36.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1114 vs. 2(81) = 162.

Time = 0.31 (sec) , antiderivative size = 2252, normalized size of antiderivative = 24.48

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*(a + b)*cosh(d*x + c)^6 + 12*(a + b)*cosh(d*x + c)*sinh(d*x + c)^5
 + 2*(a + b)*sinh(d*x + c)^6 + 6*(a - b)*cosh(d*x + c)^4 + 6*(5*(a + b)*co
sh(d*x + c)^2 + a - b)*sinh(d*x + c)^4 + 8*(5*(a + b)*cosh(d*x + c)^3 + 3*
(a - b)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(a - b)*cosh(d*x + c)^2 + 6*(5*
(a + b)*cosh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)
^2 + 3*((a + b)*cosh(d*x + c)^5 + 5*(a + b)*cosh(d*x + c)*sinh(d*x + c)^4
+ (a + b)*sinh(d*x + c)^5 + 2*(a - b)*cosh(d*x + c)^3 + 2*(5*(a + b)*cosh(
d*x + c)^2 + a - b)*sinh(d*x + c)^3 + 2*(5*(a + b)*cosh(d*x + c)^3 + 3*(a
- b)*cosh(d*x + c))*sinh(d*x + c)^2 + (a + b)*cosh(d*x + c) + (5*(a + b)*c
osh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/
(a + b))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x +
 c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a +
b)*cosh(d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3
 + (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3
*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)
*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b
/(a + b)) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh
(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(
a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)
^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 12*((a + b)*cosh(...
```

3.36.6 Sympy [F]

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\sinh\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(sinh(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

3.36.7 Maxima [F]

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\sinh\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*((a*e^(6*c) + b*e^(6*c))*e^(6*d*x) + 3*(a*e^(4*c) - b*e^(4*c))*e^(4*d*
x) + 3*(a*e^(2*c) - b*e^(2*c))*e^(2*d*x) + a + b)/((a^3*d*e^(5*c) + 3*a^2*
b*d*e^(5*c) + 3*a*b^2*d*e^(5*c) + b^3*d*e^(5*c))*e^(5*d*x) + 2*(a^3*d*e^(3
*c) + a^2*b*d*e^(3*c) - a*b^2*d*e^(3*c) - b^3*d*e^(3*c))*e^(3*d*x) + (a^3*
d*e^c + 3*a^2*b*d*e^c + 3*a*b^2*d*e^c + b^3*d*e^c)*e^(d*x)) + 1/2*integrat
e(6*(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + (
a^3*e^(4*c) + 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) +
 2*(a^3*e^(2*c) + a^2*b*e^(2*c) - a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)),
 x)
```

3.36.8 Giac [F]

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\sinh\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\sinh(c+d\,x)}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2)^2, x)
```

3.37 $\int \frac{\text{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

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3.37.1 Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \frac{\text{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{\text{arctanh}(\cosh(c+dx))}{a^2 d} + \frac{\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}d} + \frac{b\,\text{sech}(c+dx)}{2a(a+b)d\left(a+b-b\,\text{sech}^2(c+dx)\right)}$$

output
```
-arctanh(cosh(d*x+c))/a^2/d+1/2*b*sech(d*x+c)/a/(a+b)/d/(a+b-b*sech(d*x+c)
^2)+1/2*(3*a+2*b)*arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a^2/(a+
b)^(3/2)/d
```

3.37.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.50 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.83

$$\int \frac{\text{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{i\sqrt{b}(3a+2b)\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{(a+b)^{3/2}} + \frac{i\sqrt{b}(3a+2b)\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{(a+b)^{3/2}} + \frac{2ab\cosh(c+dx)}{(a+b)(a-b+(a+b)\cosh(2(c+dx)))}$$

input
```
Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((I*Sqrt[b]*(3*a + 2*b)*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/
2])/Sqrt[b]])/(a + b)^(3/2) + (I*Sqrt[b]*(3*a + 2*b)*ArcTan[((-I)*Sqrt[a +
 b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/(a + b)^(3/2) + (2*a*b*Cosh[c +
 d*x])/((a + b)*(a - b + (a + b)*Cosh[2*(c + d*x)])) - 2*Log[Cosh[(c + d*x
)/2]] + 2*Log[Sinh[(c + d*x)/2]])/(2*a^2*d)
```

3.37.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 26, 4147, 25, 316, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{i}{\sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)^2}\,dx$$

$$\downarrow 26$$

$$i\int \frac{1}{\sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)^2}\,dx$$

$$\downarrow 4147$$

$$\frac{\int -\frac{1}{\left(1-\operatorname{sech}^2(c+dx)\right)\left(-b\operatorname{sech}^2(c+dx)+a+b\right)^2}\,d\operatorname{sech}(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{\int \frac{1}{\left(1-\operatorname{sech}^2(c+dx)\right)\left(-b\operatorname{sech}^2(c+dx)+a+b\right)^2}\,d\operatorname{sech}(c+dx)}{d}$$

$$\downarrow 316$$

$$\frac{\int -\frac{b\operatorname{sech}^2(c+dx)+2a+b}{\left(1-\operatorname{sech}^2(c+dx)\right)\left(-b\operatorname{sech}^2(c+dx)+a+b\right)}d\operatorname{sech}(c+dx)}{2a(a+b)}+\frac{b\operatorname{sech}(c+dx)}{2a(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{b\operatorname{sech}(c+dx)}{2a(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}-\frac{\int \frac{b\operatorname{sech}^2(c+dx)+2a+b}{\left(1-\operatorname{sech}^2(c+dx)\right)\left(-b\operatorname{sech}^2(c+dx)+a+b\right)}d\operatorname{sech}(c+dx)}{2a(a+b)}}{d}$$

$$\downarrow 397$$

$$\frac{\frac{b\operatorname{sech}(c+dx)}{2a(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}-\frac{\frac{2(a+b)\int \frac{1}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)}{a}-\frac{b(3a+2b)\int \frac{1}{-b\operatorname{sech}^2(c+dx)+a+b}d\operatorname{sech}(c+dx)}{a}}{2a(a+b)}}{d}$$

$$\downarrow 219$$

$$\frac{\frac{b\operatorname{sech}(c+dx)}{2a(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}-\frac{\frac{2(a+b)\operatorname{arctanh}(\operatorname{sech}(c+dx))}{a}-\frac{b(3a+2b)\int \frac{1}{-b\operatorname{sech}^2(c+dx)+a+b}d\operatorname{sech}(c+dx)}{a}}{2a(a+b)}}{d}$$

$$\downarrow 221$$

$$\frac{\frac{b\operatorname{sech}(c+dx)}{2a(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}-\frac{\frac{2(a+b)\operatorname{arctanh}(\operatorname{sech}(c+dx))}{a}-\frac{\sqrt{b}(3a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}}{d}$$

input
```
Int[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-1/2*((2*(a + b)*ArcTanh[Sech[c + d*x]])/a - (Sqrt[b]*(3*a + 2*b)*ArcTanh
[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a*(a + b)) + (b*S
ech[c + d*x])/(2*a*(a + b)*(a + b - b*Sech[c + d*x]^2)))/d
```

3.37.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.37.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.56

method	result
derivativedivides	$\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{4b\left(\frac{-\frac{(a+2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4(a+b)}-\frac{a}{4(a+b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a}-\frac{(3a+2b)\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+2a+4b}{4\sqrt{ab+b^2}}\right.}{8(a+b)\sqrt{ab+b^2}}\right.}{a^2}}{d}$
default	$\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{4b\left(\frac{-\frac{(a+2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4(a+b)}-\frac{a}{4(a+b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a}-\frac{(3a+2b)\arctan\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+2a+4b}{4\sqrt{ab+b^2}}\right.}{8(a+b)\sqrt{ab+b^2}}\right.}{a^2}}{d}$
risch	$\frac{b\,e^{dx+c}\left(e^{2dx+2c}+1\right)}{a(a+b)d\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)}+\frac{\ln\left(e^{dx+c}-1\right)}{a^2d}-\frac{\ln\left(e^{dx+c}+1\right)}{a^2d}+\frac{3\sqrt{(a+b)b}\ln\left(e^{}\right.}{}$

input
```
int(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a^2*ln(tanh(1/2*d*x+1/2*c))-4/a^2*b*((-1/4*(a+2*b)/(a+b)*tanh(1/2*d
*x+1/2*c)^2-1/4*a/(a+b))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*
a+4*tanh(1/2*d*x+1/2*c)^2*b+a)-1/8*(3*a+2*b)/(a+b)/(a*b+b^2)^(1/2)*arctanh
(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))))
```

3.37.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1324 vs. 2(94) = 188.

Time = 0.35 (sec) , antiderivative size = 2614, normalized size of antiderivative = 25.38

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*a*b*cosh(d*x + c)^3 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 4*a*b
*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + ((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x
 + c)^4 + 4*(3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2
 + 5*a*b + 2*b^2)*sinh(d*x + c)^4 + 2*(3*a^2 - a*b - 2*b^2)*cosh(d*x + c)^
2 + 2*(3*(3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 3*a^2 - a*b - 2*b^2)*si
nh(d*x + c)^2 + 3*a^2 + 5*a*b + 2*b^2 + 4*((3*a^2 + 5*a*b + 2*b^2)*cosh(d*
x + c)^3 + (3*a^2 - a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a +
 b))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^
3 + (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*c
osh(d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (
a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 + 3*(a
+ b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cos
h(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a
+ b)) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x
 + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a +
b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 +
 (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 4*((a^2 + 2*a*b + b^2)*c
osh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^
2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 + ...
```

3.37.6 Sympy [F]

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

3.37.7 Maxima [F]

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
(b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^3*d + 2*a^2*b*d + a*b^2*d + (a^3*d*
e^(4*c) + 2*a^2*b*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(a^3*d*e^(2*c
) - a*b^2*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) +
log((e^(d*x + c) - 1)*e^(-c))/(a^2*d) - 2*integrate(1/2*((3*a*b*e^(3*c) +
2*b^2*e^(3*c))*e^(3*d*x) - (3*a*b*e^c + 2*b^2*e^c)*e^(d*x))/(a^4 + 2*a^3*b
 + a^2*b^2 + (a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c))*e^(4*d*x) +
 2*(a^4*e^(2*c) - a^2*b^2*e^(2*c))*e^(2*d*x)), x)
```

3.37.8 Giac [F]

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^2),x)
```

output
```
int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^2), x)
```

3.38 $\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

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3.38.1 Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d} - \frac{3\coth(c+dx)}{2a^2d} + \frac{\coth(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)}$$

output

```
-3/2*coth(d*x+c)/a^2/d-3/2*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/a^(
5/2)/d+1/2*coth(d*x+c)/a/d/(a+b*tanh(d*x+c)^2)
```

3.38.2 Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{-3\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) - 2\sqrt{a}\coth(c+dx) - \frac{\sqrt{a}b\sinh(2(c+dx))}{a-b+(a+b)\cosh(2(c+dx))}}{2a^{5/2}d}$$

input

```
Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-3*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] - 2*Sqrt[a]*Coth[c + d
*x] - (Sqrt[a]*b*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(
2*a^(5/2)*d)
```

3.38.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 25, 4146, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\downarrow 4146$$

$$\frac{\int \frac{\coth^2(c+dx)}{\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 253$$

$$\frac{\frac{3\int \frac{\coth^2(c+dx)}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{2a}+\frac{\coth(c+dx)}{2a\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 264$$

$$\frac{\frac{3\left(-\frac{b\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a}-\frac{\coth(c+dx)}{a}\right)}{2a}+\frac{\coth(c+dx)}{2a\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 218$$

$$\frac{3\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\coth(c+dx)}{a}\right)}{2a}+\frac{\coth(c+dx)}{2a\left(a+b\tanh^2(c+dx)\right)}$$
$$d$$

input
```
Int[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((3*(-((Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2)) - Coth[c
 + d*x]/a))/(2*a) + Coth[c + d*x]/(2*a*(a + b*Tanh[c + d*x]^2)))/d
```

3.38.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.38.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(68) = 136.

Time = 1.55 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.07

method	result
risch	$-\frac{2a^2e^{4dx+4c}+3abe^{4dx+4c}+3e^{4dx+4c}b^2+4a^2e^{2dx+2c}-6e^{2dx+2c}b^2+2a^2+5ab+3b^2}{(a+b)da^2\left(ae^{4dx+4c}+be^{4dx+4c}+2e^{2dx+2c}a-2be^{2dx+2c}+a+b\right)\left(e^{2dx+2c}-1\right)}+\frac{3\sqrt{-ab}\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}-a}{a+b}\right.}{4a^3d}$
derivativedivides	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}-\frac{1}{2a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{4b\left(\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a}+\frac{3a\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan}{2a\sqrt{(a+b)b}\sqrt{}}\right.\right.}{a^2}}{d}$
default	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}-\frac{1}{2a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{4b\left(\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a}+\frac{3a\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan}{2a\sqrt{(a+b)b}\sqrt{}}\right.\right.}{a^2}}{d}$

input
```
int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-(2*a^2*exp(4*d*x+4*c)+3*a*b*exp(4*d*x+4*c)+3*exp(4*d*x+4*c)*b^2+4*a^2*exp
(2*d*x+2*c)-6*exp(2*d*x+2*c)*b^2+2*a^2+5*a*b+3*b^2)/(a+b)/d/a^2/(a*exp(4*d
*x+4*c)+b*exp(4*d*x+4*c)+2*exp(2*d*x+2*c)*a-2*b*exp(2*d*x+2*c)+a+b)/(exp(2
*d*x+2*c)-1)+3/4/a^3*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)-(2*(-a*b)^(1/2)-a+b)
/(a+b))-3/4/a^3*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)+(2*(-a*b)^(1/2)+a-b)/(a+b
))
```

3.38.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1120 vs. 2(68) = 136.

Time = 0.31 (sec) , antiderivative size = 2562, normalized size of antiderivative = 31.24

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*(2*a^2 + 3*a*b + 3*b^2)*cosh(d*x + c)^4 + 16*(2*a^2 + 3*a*b + 3*b
^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(2*a^2 + 3*a*b + 3*b^2)*sinh(d*x + c
)^4 + 8*(2*a^2 - 3*b^2)*cosh(d*x + c)^2 + 8*(3*(2*a^2 + 3*a*b + 3*b^2)*cos
h(d*x + c)^2 + 2*a^2 - 3*b^2)*sinh(d*x + c)^2 - 3*((a^2 + 2*a*b + b^2)*cos
h(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2
 + 2*a*b + b^2)*sinh(d*x + c)^6 + (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^4 + (
15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - 2*a*b - 3*b^2)*sinh(d*x + c
)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - 2*a*b - 3*b^2)*cos
h(d*x + c))*sinh(d*x + c)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^2 + (15*
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 - 2*a*b - 3*b^2)*cosh(d*x + c
)^2 - a^2 + 2*a*b + 3*b^2)*sinh(d*x + c)^2 - a^2 - 2*a*b - b^2 + 2*(3*(a^2
 + 2*a*b + b^2)*cosh(d*x + c)^5 + 2*(a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^3
 - (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2
 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh
(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x
 + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x +
 c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2
 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(
a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2
 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*...
```

3.38.6 Sympy [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**2)**2, x)
```

3.38.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(68) = 136$.

Time $= 0.34$ (sec) , antiderivative size $= 212$, normalized size of antiderivative $= 2.59$

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{2a^2+5ab+3b^2+2(2a^2-3b^2)e^{(-2dx-2c)}+(2a^2+3ab+3b^2)e^{(-4dx-4c)}}{(a^4+2a^3b+a^2b^2+(a^4-2a^3b-3a^2b^2)e^{(-2dx-2c)}-(a^4-2a^3b-3a^2b^2)e^{(-4dx-4c)}-(a^4+2a^3b+a} + \frac{3b\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}a^2d}$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-(2*a^2 + 5*a*b + 3*b^2 + 2*(2*a^2 - 3*b^2)*e^(-2*d*x - 2*c) + (2*a^2 + 3*
a*b + 3*b^2)*e^(-4*d*x - 4*c))/((a^4 + 2*a^3*b + a^2*b^2 + (a^4 - 2*a^3*b
- 3*a^2*b^2)*e^(-2*d*x - 2*c) - (a^4 - 2*a^3*b - 3*a^2*b^2)*e^(-4*d*x - 4*
c) - (a^4 + 2*a^3*b + a^2*b^2)*e^(-6*d*x - 6*c))*d) + 3/2*b*arctan(1/2*((a
 + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*d)
```

3.38.8 Giac [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^2}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)^2\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2),x)
```

output
```
int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2), x)
```

3.39 $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

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3.39.1 Optimal result

Integrand size = 23, antiderivative size = 141

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{(a+4b)\operatorname{arctanh}(\cosh(c+dx))}{2a^3d} - \frac{\sqrt{b}(3a+4b)\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^3\sqrt{a+b}d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)} - \frac{b\operatorname{sech}(c+dx)}{a^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)}$$

output

```
1/2*(a+4*b)*arctanh(cosh(d*x+c))/a^3/d-1/2*coth(d*x+c)*csch(d*x+c)/a/d/(a+
b-b*sech(d*x+c)^2)-b*sech(d*x+c)/a^2/d/(a+b-b*sech(d*x+c)^2)-1/2*(3*a+4*b)
*arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a^3/d/(a+b)^(1/2)
```

3.39.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.85 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.57

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx =$$

$$-\frac{4i\sqrt{b}(3a+4b)\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{\sqrt{a+b}}+\frac{4i\sqrt{b}(3a+4b)\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{\sqrt{a+b}}+\frac{8ab\cosh(c+dx)}{a-b+(a+b)\cosh(2(c+dx))}+$$

input
```
Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-1/8*(((4*I)*Sqrt[b]*(3*a + 4*b)*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(
c + d*x)/2])/Sqrt[b]])/Sqrt[a + b] + ((4*I)*Sqrt[b]*(3*a + 4*b)*ArcTan[((-
I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/Sqrt[a + b] + (8*a*b
*Cosh[c + d*x])/(a - b + (a + b)*Cosh[2*(c + d*x)]) + a*Csch[(c + d*x)/2]^
2 - 4*(a + 4*b)*Log[Cosh[(c + d*x)/2]] + 4*(a + 4*b)*Log[Sinh[(c + d*x)/2]
] + a*Sech[(c + d*x)/2]^2)/(a^3*d)
```

3.39.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 26, 4147, 373, 402, 27, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i}{\sin(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^2}\,dx$$

$$\downarrow 26$$

$$-i\int \frac{1}{\sin(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^2}\,dx$$

$$\downarrow 4147$$

$$-\frac{\int \frac{\operatorname{sech}^2(c+dx)}{\left(1-\operatorname{sech}^2(c+dx)\right)^2\left(-b\operatorname{sech}^2(c+dx)+a+b\right)^2}\,d\operatorname{sech}(c+dx)}{d}$$

$$\downarrow 373$$

$$-\frac{\frac{\operatorname{sech}(c+dx)}{2a\left(1-\operatorname{sech}^2(c+dx)\right)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}-\frac{\int \frac{3b\operatorname{sech}^2(c+dx)+a+b}{\left(1-\operatorname{sech}^2(c+dx)\right)\left(-b\operatorname{sech}^2(c+dx)+a+b\right)^2}\,d\operatorname{sech}(c+dx)}{2a}}{d}$$

$$\downarrow 402$$

$$-\frac{\frac{\operatorname{sech}(c+dx)}{2a\left(1-\operatorname{sech}^2(c+dx)\right)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}-\frac{-\frac{\int -\frac{2(a+b)\left(2b\operatorname{sech}^2(c+dx)+a+2b\right)}{\left(1-\operatorname{sech}^2(c+dx)\right)\left(-b\operatorname{sech}^2(c+dx)+a+b\right)}\,d\operatorname{sech}(c+dx)}{2a(a+b)}-\frac{2b\operatorname{sech}(c+dx)}{a\left(a-b\operatorname{sech}^2(c+dx)+b\right)}}{2a}}{d}$$

$$\downarrow 27$$

$$-\frac{\frac{\operatorname{sech}(c+dx)}{2a\left(1-\operatorname{sech}^2(c+dx)\right)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}-\frac{\frac{\int \frac{2b\operatorname{sech}^2(c+dx)+a+2b}{\left(1-\operatorname{sech}^2(c+dx)\right)\left(-b\operatorname{sech}^2(c+dx)+a+b\right)}\,d\operatorname{sech}(c+dx)}{a}-\frac{2b\operatorname{sech}(c+dx)}{a\left(a-b\operatorname{sech}^2(c+dx)+b\right)}}{2a}}{d}$$

$$\downarrow 397$$

$$-\frac{\frac{\operatorname{sech}(c+dx)}{2a\left(1-\operatorname{sech}^2(c+dx)\right)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}-\frac{\frac{(a+4b)\int \frac{1}{1-\operatorname{sech}^2(c+dx)}\,d\operatorname{sech}(c+dx)}{a}-\frac{b(3a+4b)\int \frac{1}{-b\operatorname{sech}^2(c+dx)+a+b}\,d\operatorname{sech}(c+dx)}{a}}{2a}}{d}$$

$$\downarrow 219$$

$$-\frac{\frac{\operatorname{sech}(c+dx)}{2a\left(1-\operatorname{sech}^2(c+dx)\right)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}-\frac{\frac{(a+4b)\operatorname{arctanh}(\operatorname{sech}(c+dx))}{a}-\frac{b(3a+4b)\int \frac{1}{-b\operatorname{sech}^2(c+dx)+a+b}\,d\operatorname{sech}(c+dx)}{a}}{2a}}{d}$$

$$\downarrow 221$$

3.39. $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

$$-\frac{\frac{\operatorname{sech}(c+dx)}{2a\left(1-\operatorname{sech}^2(c+dx)\right)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}-\frac{\frac{(a+4b)\operatorname{arctanh}(\operatorname{sech}(c+dx))}{a}-\frac{\sqrt{b}(3a+4b)\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a}-\frac{2b\operatorname{sech}(c+dx)}{a\left(a-b\operatorname{sech}^2(c+dx)+\right.}}{d}$$

input
```
Int[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-((Sech[c + d*x]/(2*a*(1 - Sech[c + d*x]^2)*(a + b - b*Sech[c + d*x]^2)) -
 ((((a + 4*b)*ArcTanh[Sech[c + d*x]])/a - (Sqrt[b]*(3*a + 4*b)*ArcTanh[(Sq
rt[b]*Sech[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/a - (2*b*Sech[c + d*x]
)/(a*(a + b - b*Sech[c + d*x]^2)))/(2*a))/d)
```

3.39.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.39.4 Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a^2}-\frac{1}{8a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(-2a-8b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{2b\left(\frac{\left(-\frac{a}{2}-b\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{a}{2}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-}\right.}{a^3}}{d}$
default	$\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a^2}-\frac{1}{8a^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(-2a-8b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{2b\left(\frac{\left(-\frac{a}{2}-b\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{a}{2}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-}\right.}{a^3}}{d}$
risch	$-\frac{e^{dx+c}\left(a\,e^{6dx+6c}+2b\,e^{6dx+6c}+3a\,e^{4dx+4c}-2b\,e^{4dx+4c}+3\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+2b\right)}{d\,a^2\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)\left(e^{2dx+2c}-1\right)^2}+\frac{\ln\left(e^{dx+c}+1\right)}{2a^2d}+\frac{2\ln(e^{d}}{d}$

input
```
int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a^2-1/8/a^2/tanh(1/2*d*x+1/2*c)^2+1/4/a^3*(
-2*a-8*b)*ln(tanh(1/2*d*x+1/2*c))+2*b/a^3*(((-1/2*a-b)*tanh(1/2*d*x+1/2*c)
^2-1/2*a)/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*
x+1/2*c)^2*b+a)-1/4*(3*a+4*b)/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+
1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))))
```

3.39.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3368 vs. $2(132) = 264$.

Time $= 0.35$ (sec) , antiderivative size $= 6335$, normalized size of antiderivative $= 44.93$

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.39.6 Sympy [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)
```

3.39.7 Maxima [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{\left(b\tanh(dx+c)^2+a\right)^2}\,dx$$

input
```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
((a*e^(7*c) + 2*b*e^(7*c))*e^(7*d*x) + (3*a*e^(5*c) - 2*b*e^(5*c))*e^(5*d*
x) + (3*a*e^(3*c) - 2*b*e^(3*c))*e^(3*d*x) + (a*e^c + 2*b*e^c)*e^(d*x))/(4
*a^2*b*d*e^(6*d*x + 6*c) + 4*a^2*b*d*e^(2*d*x + 2*c) - a^3*d - a^2*b*d - (
a^3*d*e^(8*c) + a^2*b*d*e^(8*c))*e^(8*d*x) + 2*(a^3*d*e^(4*c) - 3*a^2*b*d*
e^(4*c))*e^(4*d*x)) + 1/2*(a + 4*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^3*d)
- 1/2*(a + 4*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^3*d) + 8*integrate(1/8*((
3*a*b*e^(3*c) + 4*b^2*e^(3*c))*e^(3*d*x) - (3*a*b*e^c + 4*b^2*e^c)*e^(d*x)
)/(a^4 + a^3*b + (a^4*e^(4*c) + a^3*b*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c)
- a^3*b*e^(2*c))*e^(2*d*x)), x)
```

3.39.8 Giac [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{\left(b\tanh(dx+c)^2+a\right)^2}\,dx$$

input
```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)^3\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2),x)
```

output
```
int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2), x)
```

3.40 $\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

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3.40.1 Optimal result

Integrand size = 23, antiderivative size = 113

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\sqrt{b}(3a+5b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d} + \frac{(a+2b)\coth(c+dx)}{a^3 d} - \frac{\coth^3(c+dx)}{3a^2 d} + \frac{b(a+b)\tanh(c+dx)}{2a^3 d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
(a+2*b)*coth(d*x+c)/a^3/d-1/3*coth(d*x+c)^3/a^2/d+1/2*(3*a+5*b)*arctan(b^(
1/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/a^(7/2)/d+1/2*b*(a+b)*tanh(d*x+c)/a^3/d/
(a+b*tanh(d*x+c)^2)
```

3.40.2 Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{3\sqrt{b}(3a+5b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) + 2\sqrt{a}\coth(c+dx)\left(2a+6b-a\operatorname{csch}^2(c+dx)\right) + \frac{3\sqrt{a}b(a+b)\sinh(2(c+dx))}{a-b+(a+b)\cosh(2(c+dx))}}{6a^{7/2}d}$$

input
```
Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(3*Sqrt[b]*(3*a + 5*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] + 2*Sqrt[a]
*Coth[c + d*x]*(2*a + 6*b - a*Csch[c + d*x]^2) + (3*Sqrt[a]*b*(a + b)*Sinh
[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(6*a^(7/2)*d)
```

3.40.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4146, 361, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\downarrow 4146$$

$$\frac{\int \frac{\coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)}{\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 361$$

$$\frac{\frac{b(a+b)\tanh(c+dx)}{2a^3\left(a+b\tanh^2(c+dx)\right)} - \frac{1}{2}b\int -\frac{\coth^4(c+dx)\left(\frac{(a+b)\tanh^4(c+dx)}{a^3} - \frac{2(a+b)\tanh^2(c+dx)}{a^2b} + \frac{2}{ab}\right)}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{d}$$

$$\downarrow 25$$

$$\frac{\frac{1}{2}b\int \frac{\coth^4(c+dx)\left(\frac{(a+b)\tanh^4(c+dx)}{a^3} - \frac{2(a+b)\tanh^2(c+dx)}{a^2b} + \frac{2}{ab}\right)}{b\tanh^2(c+dx)+a}d\tanh(c+dx) + \frac{b(a+b)\tanh(c+dx)}{2a^3\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 1584$$

$$\frac{\frac{1}{2}b\int \left(\frac{2\coth^4(c+dx)}{a^2b} - \frac{2(a+2b)\coth^2(c+dx)}{a^3b} + \frac{3a+5b}{a^3\left(b\tanh^2(c+dx)+a\right)}\right)d\tanh(c+dx) + \frac{b(a+b)\tanh(c+dx)}{2a^3\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{b(a+b)\tanh(c+dx)}{2a^3(a+b\tanh^2(c+dx))}+\frac{1}{2}b\left(\frac{(3a+5b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{b}}+\frac{2(a+2b)\coth(c+dx)}{a^3b}-\frac{2\coth^3(c+dx)}{3a^2b}\right)}{d}$$

input
```
Int[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((b*(((3*a + 5*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(7/2)*Sqrt[b
]) + (2*(a + 2*b)*Coth[c + d*x])/(a^3*b) - (2*Coth[c + d*x]^3)/(3*a^2*b)))
/2 + (b*(a + b)*Tanh[c + d*x])/(2*a^3*(a + b*Tanh[c + d*x]^2)))/d
```

3.40.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.40.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(99) = 198$.

Time = 3.72 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.99

method	result
derivativedivides	$-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-8b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3}-2b\left(\frac{\left(-\frac{b}{2}-\frac{a}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-\frac{b}{2}-\frac{a}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a}+\frac{(3a+5b)}{}\right.$
default	$-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-8b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3}-2b\left(\frac{\left(-\frac{b}{2}-\frac{a}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-\frac{b}{2}-\frac{a}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a}+\frac{(3a+5b)}{}\right.$
risch	$-\frac{-9ab\,e^{8dx+8c}-15b^2e^{8dx+8c}+12a^2e^{6dx+6c}+6ab\,e^{6dx+6c}+60b^2e^{6dx+6c}+20a^2e^{4dx+4c}-4ab\,e^{4dx+4c}-90\,e^{4dx+4c}b^2+4}{3d\,a^3\left(e^{2dx+2c}-1\right)^3\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}\right.}$

input

```
int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/8/a^3*(1/3*tanh(1/2*d*x+1/2*c)^3*a-3*tanh(1/2*d*x+1/2*c)*a-8*b*tan
h(1/2*d*x+1/2*c))-2*b/a^3*(((-1/2*b-1/2*a)*tanh(1/2*d*x+1/2*c)^3+(-1/2*b-1
/2*a)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^
2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)+1/2*(3*a+5*b)*a*(1/2*(a+((a+b)*b)^(1/2)+b
)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*
d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(-a+((a+b)*b)^(1/2)-b)
/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*
d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))-1/24/a^2/tanh(1/2*d*x+1/
2*c)^3-1/8/a^3*(-3*a-8*b)/tanh(1/2*d*x+1/2*c))
```

3.40.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2370 vs. $2(99) = 198$.

Time $= 0.33$ (sec) , antiderivative size $= 5062$, normalized size of antiderivative $= 44.80$

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.40.6 Sympy [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**2)**2, x)
```

3.40.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(99) = 198$.

Time = 0.38 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.50

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{4\,a^2+19\,ab+15\,b^2-2\,(2\,a^2+13\,ab+30\,b^2)e^{(-2\,dx-2\,c)}-2\,(10\,a^2-2\,ab-45\,b^2)e^{(-4\,dx-4\,c)}-6\,(2\,a^2+ab}{3\,(a^4+a^3b-(a^4+5\,a^3b)e^{(-2\,dx-2\,c)}-2\,(a^4-5\,a^3b)e^{(-4\,dx-4\,c)}+2\,(a^4-5\,a^3b)e^{(-6\,dx-6\,c)}+(a^4+5\,a}$$
$$-\frac{(3\,ab+5\,b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}a^3d}$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/3*(4*a^2 + 19*a*b + 15*b^2 - 2*(2*a^2 + 13*a*b + 30*b^2)*e^(-2*d*x - 2*c
) - 2*(10*a^2 - 2*a*b - 45*b^2)*e^(-4*d*x - 4*c) - 6*(2*a^2 + a*b + 10*b^2
)*e^(-6*d*x - 6*c) + 3*(3*a*b + 5*b^2)*e^(-8*d*x - 8*c))/((a^4 + a^3*b - (
a^4 + 5*a^3*b)*e^(-2*d*x - 2*c) - 2*(a^4 - 5*a^3*b)*e^(-4*d*x - 4*c) + 2*(
a^4 - 5*a^3*b)*e^(-6*d*x - 6*c) + (a^4 + 5*a^3*b)*e^(-8*d*x - 8*c) - (a^4
+ a^3*b)*e^(-10*d*x - 10*c))*d) - 1/2*(3*a*b + 5*b^2)*arctan(1/2*((a + b)*
e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^3*d)
```

3.40.8 Giac [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^4}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)^4\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2),x)
```

output
```
int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2), x)
```

3.41 $\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

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3.41.1 Optimal result

Integrand size = 23, antiderivative size = 240

$$\begin{aligned}\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx &= \frac{3(a^2-10ab+5b^2)\,x}{8(a+b)^5} \\ &+ \frac{3\sqrt{b}(5a^2-10ab+b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}(a+b)^5 d} \\ &- \frac{(5a-3b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2 d\left(a+b\tanh^2(c+dx)\right)^2} \\ &+ \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} \\ &+ \frac{(7a-5b)b\tanh(c+dx)}{8(a+b)^3 d\left(a+b\tanh^2(c+dx)\right)^2} \\ &+ \frac{3(a-b)b\tanh(c+dx)}{2(a+b)^4 d\left(a+b\tanh^2(c+dx)\right)}\end{aligned}$$

output

```
3/8*(a^2-10*a*b+5*b^2)*x/(a+b)^5+3/8*(5*a^2-10*a*b+b^2)*arctan(b^(1/2)*tan
h(d*x+c)/a^(1/2))*b^(1/2)/(a+b)^5/d/a^(1/2)-1/8*(5*a-3*b)*cosh(d*x+c)*sinh
(d*x+c)/(a+b)^2/d/(a+b*tanh(d*x+c)^2)^2+1/4*cosh(d*x+c)^3*sinh(d*x+c)/(a+b
)/d/(a+b*tanh(d*x+c)^2)^2+1/8*(7*a-5*b)*b*tanh(d*x+c)/(a+b)^3/d/(a+b*tanh(
d*x+c)^2)^2+3/2*(a-b)*b*tanh(d*x+c)/(a+b)^4/d/(a+b*tanh(d*x+c)^2)
```

3.41.2 Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.77

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$= \frac{12(a^2-10ab+5b^2)(c+dx)+\frac{12\sqrt{b}(5a^2-10ab+b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}}-8(a-2b)(a+b)\sinh(2(c+dx))+\frac{1}{(a}}{32(a+b)^5 d}$$

input
```
Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(12*(a^2 - 10*a*b + 5*b^2)*(c + d*x) + (12*Sqrt[b]*(5*a^2 - 10*a*b + b^2)*
ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a] - 8*(a - 2*b)*(a + b)*Sin
h[2*(c + d*x)] + (16*a*b^2*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cos
h[2*(c + d*x)])^2 + (4*(9*a - 5*b)*b*(a + b)*Sinh[2*(c + d*x)])/(a - b + (
a + b)*Cosh[2*(c + d*x)]) + (a + b)^2*Sinh[4*(c + d*x)])/(32*(a + b)^5*d)
```

3.41.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 4146, 372, 402, 25, 402, 27, 402, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(ic+idx)^4}{\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$

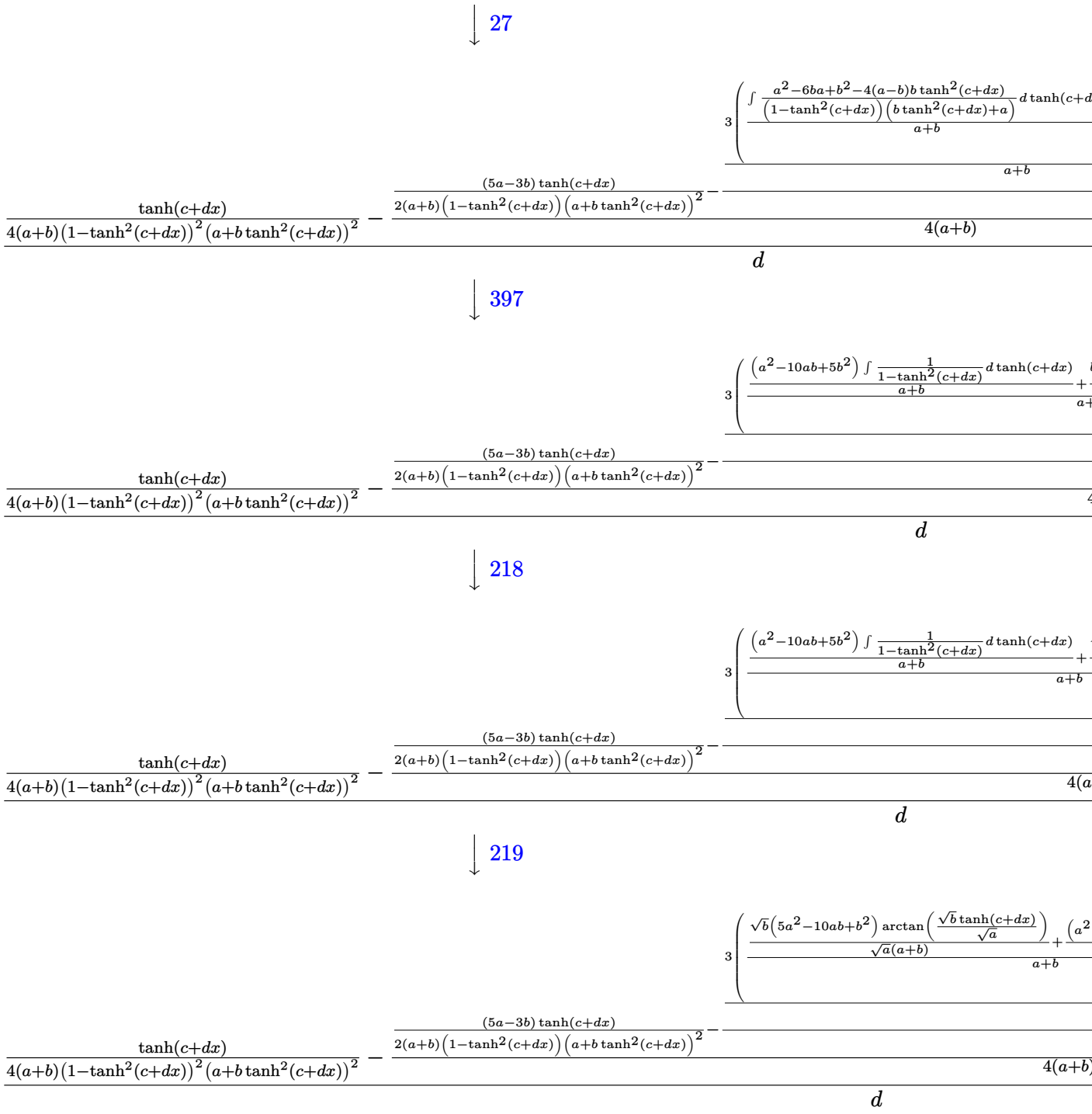
$$\downarrow 4146$$

$$\frac{\int \frac{\tanh^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)^3\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh(c+dx)}{d}$$

$$\downarrow 372$$

$$\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\int\frac{(4a-3b)\tanh^2(c+dx)+a}{\left(1-\tanh^2(c+dx)\right)^2\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh(c+dx)}{4(a+b)}$$
$$d$$
$$\downarrow 402$$
$$\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\int-\frac{a(3a-5b)-5(5a-3b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh(c+dx)}{2(a+b)}+\frac{(5a-3b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}$$
$$d$$
$$\downarrow 25$$
$$\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a-3b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\int\frac{a(3a-5b)-5(5a-3b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh(c+dx)}{2(a+b)}}{4(a+b)}$$
$$d$$
$$\downarrow 402$$
$$\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a-3b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{b(7a-5b)\tanh(c+dx)}{(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\int-\frac{12a\left(a(a-3b)-(7a-5\right.}{\left(1-\tanh^2(c+dx)\right)\left(b\right.}}{2(a+b)}}{4(a+b)}$$
$$d$$
$$\downarrow 27$$
$$\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a-3b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{3\int\frac{a(a-3b)-(7a-5b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}\,d\tanh(c+dx)}{a+b}}{2(a+b)}}{4(a+b)}$$
$$d$$
$$\downarrow 402$$
$$\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a-3b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{3\left(\frac{4b(a-b)\tanh(c+dx)}{(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\int-\frac{2a\left(a^2-6ba+b^2-4(\right.}{\left(1-\tanh^2(c+dx)\right)}\right.}{a+b}}{4(a+b)}$$
$$d$$

3.41. $\int\frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$



input `Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]`

output
```
(Tanh[c + d*x]/(4*(a + b)*(1 - Tanh[c + d*x]^2)^2*(a + b*Tanh[c + d*x]^2)^
2) - (((5*a - 3*b)*Tanh[c + d*x])/(2*(a + b)*(1 - Tanh[c + d*x]^2)*(a + b*
Tanh[c + d*x]^2)^2) - (((7*a - 5*b)*b*Tanh[c + d*x])/((a + b)*(a + b*Tanh[
c + d*x]^2)^2) + (3*(((Sqrt[b]*(5*a^2 - 10*a*b + b^2)*ArcTan[(Sqrt[b]*Tanh
[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)) + ((a^2 - 10*a*b + 5*b^2)*ArcTanh[T
anh[c + d*x]])/(a + b))/(a + b) + (4*(a - b)*b*Tanh[c + d*x])/((a + b)*(a
+ b*Tanh[c + d*x]^2))))/(a + b))/(2*(a + b)))/(4*(a + b)))/d
```

3.41.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.41.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(220) = 440$.

Time = 193.92 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.54

method	result
derivativedivides	$-\frac{1}{4(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{1}{2(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{3a-9b}{8(a+b)^4\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{11b-a}{8(a+b)^4\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{\left(3a^2$
default	$-\frac{1}{4(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{1}{2(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{3a-9b}{8(a+b)^4\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{11b-a}{8(a+b)^4\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{\left(3a^2$
risch	$\frac{3x\,a^2}{8(a^3+3a^2b+3a\,b^2+b^3)(a+b)^2}-\frac{15xab}{4(a^3+3a^2b+3a\,b^2+b^3)(a+b)^2}+\frac{15x\,b^2}{8(a^3+3a^2b+3a\,b^2+b^3)(a+b)^2}+\frac{\mathrm{e}^{4dx+4c}}{64(a+b)^3d}-$

input
```
int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4/(a+b)^3/(1+tanh(1/2*d*x+1/2*c))^4+1/2/(a+b)^3/(1+tanh(1/2*d*x+1/
2*c))^3-1/8*(3*a-9*b)/(a+b)^4/(1+tanh(1/2*d*x+1/2*c))-1/8*(11*b-a)/(a+b)^4
/(1+tanh(1/2*d*x+1/2*c))^2+1/8/(a+b)^5*(3*a^2-30*a*b+15*b^2)*ln(1+tanh(1/2
*d*x+1/2*c))+1/4/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^4+1/2/(a+b)^3/(tanh(1/2*d
*x+1/2*c)-1)^3-1/8*(3*a-9*b)/(a+b)^4/(tanh(1/2*d*x+1/2*c)-1)-1/8*(-11*b+a)
/(a+b)^4/(tanh(1/2*d*x+1/2*c)-1)^2+1/8/(a+b)^5*(-3*a^2+30*a*b-15*b^2)*ln(t
anh(1/2*d*x+1/2*c)-1)-2*b/(a+b)^5*((-3/8*a*(3*a^2+2*a*b-b^2)*tanh(1/2*d*x+
1/2*c)^7+(-27/8*a^3-23/4*a^2*b+1/8*a*b^2+5/2*b^3)*tanh(1/2*d*x+1/2*c)^5+(-
27/8*a^3-23/4*a^2*b+1/8*a*b^2+5/2*b^3)*tanh(1/2*d*x+1/2*c)^3+(-9/8*a^3-3/4
*a^2*b+3/8*a*b^2)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2
*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2+1/8*(15*a^2-30*a*b+3*b^2)*a
*(1/2*(a+((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a
)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-
1/2*(-a+((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)
^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)))
))
```

3.41.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9248 vs. $2(220) = 440$.

Time $= 0.53$ (sec) , antiderivative size $= 18818$, normalized size of antiderivative $= 78.41$

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.41.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.41.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3392 vs. 2(220) = 440.

Time = 0.73 (sec) , antiderivative size = 3392, normalized size of antiderivative = 14.13

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-3/8*(a*b - 3*b^2)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c)
 + a + b)/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d) -
3/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a
^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) + 3/8*(a*b - 3*b^2)*log(2*(a
 - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^5 + 5*a^4*b
 + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d) + 3/4*b*log(2*(a - b)*e^(-2*
d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2
 + 4*a*b^3 + b^4)*d) + 3/128*(5*a^4*b - 80*a^3*b^2 + 50*a^2*b^3 + 8*a*b^4
 + b^5)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^7 + 5*a
^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*sqrt(a*b)*d) + 3/32*
(5*a^3*b - 15*a^2*b^2 - 5*a*b^3 - b^4)*arctan(1/2*((a + b)*e^(2*d*x + 2*c)
 + a - b)/sqrt(a*b))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sq
rt(a*b)*d) - 3/128*(5*a^4*b - 80*a^3*b^2 + 50*a^2*b^3 + 8*a*b^4 + b^5)*arc
tan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^7 + 5*a^6*b + 10
*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*sqrt(a*b)*d) - 3/32*(5*a^3*b
 - 15*a^2*b^2 - 5*a*b^3 - b^4)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b
)/sqrt(a*b))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)*
d) - 3/64*(15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*
c) + a - b)/sqrt(a*b))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)*d)
 - 1/64*(9*a^5*b - 65*a^4*b^2 - 134*a^3*b^3 - 34*a^2*b^4 + 29*a*b^5 + 3...
```

3.41.8 Giac [F]

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\sinh\left(dx+c\right)^4}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\sinh(c+d\,x)^4}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3, x)
```

3.42 $\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

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3.42.1 Optimal result

Integrand size = 23, antiderivative size = 166

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{5(3a-4b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}d} - \frac{(a-2b)\cosh(c+dx)}{(a+b)^4 d} + \frac{\cosh^3(c+dx)}{3(a+b)^3 d} + \frac{ab\operatorname{sech}(c+dx)}{4(a+b)^3 d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} + \frac{(7a-4b)b\operatorname{sech}(c+dx)}{8(a+b)^4 d\left(a+b-b\operatorname{sech}^2(c+dx)\right)}$$

output

```
-(a-2*b)*cosh(d*x+c)/(a+b)^4/d+1/3*cosh(d*x+c)^3/(a+b)^3/d+1/4*a*b*sech(d*
x+c)/(a+b)^3/d/(a+b-b*sech(d*x+c)^2)^2+1/8*(7*a-4*b)*b*sech(d*x+c)/(a+b)^4
/d/(a+b-b*sech(d*x+c)^2)+5/8*(3*a-4*b)*arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(
1/2))*b^(1/2)/(a+b)^(9/2)/d
```

3.42.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.09 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.37

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$= \frac{15i(3a-4b)\sqrt{b}\left(\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)}{(a+b)^{9/2}} - \frac{6\cosh(c+dx)(3a^3-24a^2b+30ab^2-13b^3+(6a}{(a+}$$

$$\frac{}{24d}$$

input
```
Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(((15*I)*(3*a - 4*b)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c +
 d*x)/2])/Sqrt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])
/Sqrt[b]]))/(a + b)^(9/2) - (6*Cosh[c + d*x]*(3*a^3 - 24*a^2*b + 30*a*b^2
- 13*b^3 + (6*a^3 - 27*a^2*b - 11*a*b^2 + 22*b^3)*Cosh[2*(c + d*x)] + 3*(a
 - 3*b)*(a + b)^2*Cosh[2*(c + d*x)]^2))/((a + b)^4*(a - b + (a + b)*Cosh[2
*(c + d*x)])^2) + (2*Cosh[3*(c + d*x)])/(a + b)^3)/(24*d)
```

3.42.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 26, 4147, 25, 361, 1582, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{i\sin(ic+idx)^3}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\downarrow 26$$

$$i\int \frac{\sin(ic+idx)^3}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\downarrow 4147$$

$$\frac{\int -\frac{\cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)}{\left(-b\text{sech}^2(c+dx)+a+b\right)^3}d\text{sech}(c+dx)}{d}$$

$$\downarrow 25$$

$$\frac{-\int \frac{\cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)}{\left(-b\text{sech}^2(c+dx)+a+b\right)^3}d\text{sech}(c+dx)}{d}$$

$$\downarrow 361$$

$$\frac{\frac{ab\text{sech}(c+dx)}{4(a+b)^3\left(a-b\text{sech}^2(c+dx)+b\right)^2}-\frac{1}{4}b\int \frac{\cosh^4(c+dx)\left(-\frac{3a\text{sech}^4(c+dx)}{(a+b)^3}-\frac{4a\text{sech}^2(c+dx)}{b(a+b)^2}+\frac{4}{b(a+b)}\right)}{\left(-b\text{sech}^2(c+dx)+a+b\right)^2}d\text{sech}(c+dx)}{d}$$

$$\downarrow 1582$$

$$\frac{\frac{ab\text{sech}(c+dx)}{4(a+b)^3\left(a-b\text{sech}^2(c+dx)+b\right)^2}-\frac{1}{4}b\left(\frac{\int \frac{\cosh^4(c+dx)\left(-\frac{(7a-4b)b^2\text{sech}^4(c+dx)}{a+b}-8(a-b)b\text{sech}^2(c+dx)+8b(a+b)\right)}{-b\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{2b^2(a+b)^3}-\frac{(7a-4b}{2(a+b)^4\left(a-\right.}\right.}{d}$$

$$\downarrow 1584$$

$$\frac{\frac{ab\text{sech}(c+dx)}{4(a+b)^3\left(a-b\text{sech}^2(c+dx)+b\right)^2}-\frac{1}{4}b\left(\frac{\int \left(8b\cosh^4(c+dx)+\frac{8b(2b-a)\cosh^2(c+dx)}{a+b}-\frac{5b^2(4b-3a)}{(a+b)\left(b\text{sech}^2(c+dx)-a-b\right)}\right)d\text{sech}(c+dx)}{2b^2(a+b)^3}-\frac{(7a-4}{2(a+b)^4\left(a-\right.}\right.}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{ab\text{sech}(c+dx)}{4(a+b)^3\left(a-b\text{sech}^2(c+dx)+b\right)^2}-\frac{1}{4}b\left(\frac{-\frac{5b^{3/2}(3a-4b)\text{arctanh}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\frac{8b(a-2b)\cosh(c+dx)}{a+b}-\frac{8}{3}b\cosh^3(c+dx)}{2b^2(a+b)^3}-\frac{(7a-4b)\text{se}}{2(a+b)^4\left(a-b\text{se}\right.}\right.}{d}$$

input
```
Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

3.42. $\int \frac{\sinh^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

output
```
((a*b*Sech[c + d*x])/(4*(a + b)^3*(a + b - b*Sech[c + d*x]^2)^2) - (b*(((-
5*(3*a - 4*b)*b^(3/2)*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(a + b
)^(3/2) + (8*(a - 2*b)*b*Cosh[c + d*x])/(a + b) - (8*b*Cosh[c + d*x]^3)/3)
/(2*b^2*(a + b)^3) - ((7*a - 4*b)*Sech[c + d*x])/(2*(a + b)^4*(a + b - b*S
ech[c + d*x]^2))))/4)/d
```

3.42.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.42.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(150) = 300.

Time = 77.41 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.05

method	result
derivativedivides	$\frac{1}{3(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{1}{2(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{a-5b}{2(a+b)^4\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2b\left(-\frac{(9a+20b)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8}-\frac{(27$
default	$\frac{1}{3(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{1}{2(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{a-5b}{2(a+b)^4\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2b\left(-\frac{(9a+20b)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8}-\frac{(27$
risch	$\frac{e^{3dx+3c}}{24(a^3+3a^2b+3ab^2+b^3)d}-\frac{3e^{dx+c}a}{8(a^3+3a^2b+3ab^2+b^3)(a+b)d}+\frac{9e^{dx+c}b}{8(a^3+3a^2b+3ab^2+b^3)(a+b)d}-\frac{3e^{-dx-c}}{8(a^4+4a^3b+6a^2b^2+$

input
```
int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3/(a+b)^3/(1+tanh(1/2*d*x+1/2*c))^3-1/2/(a+b)^3/(1+tanh(1/2*d*x+1/2
*c))^2-1/2*(a-5*b)/(a+b)^4/(1+tanh(1/2*d*x+1/2*c))-2*b/(a+b)^4*((-1/8*(9*a
+20*b)*a*tanh(1/2*d*x+1/2*c)^6-1/8*(27*a^3+66*a^2*b+56*a*b^2-16*b^3)/a*tan
h(1/2*d*x+1/2*c)^4+(-27/8*a^2-11/2*a*b+2*b^2)*tanh(1/2*d*x+1/2*c)^2-9/8*a^
2+1/4*a*b)/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d
*x+1/2*c)^2*b+a)^2-5/16*(3*a-4*b)/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*
d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2)))-1/3/(a+b)^3/(tanh(1/2*d*x+1/2*c)
-1)^3-1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/(a+b)^4*(-a+5*b)/(tanh(1/2
*d*x+1/2*c)-1))
```

3.42.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7129 vs. $2(156) = 312$.

Time $= 0.45$ (sec) , antiderivative size $= 13095$, normalized size of antiderivative $= 78.89$

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.42.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.42.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.42.8 Giac [F]

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\sinh\left(dx+c\right)^3}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\sinh(c+d\,x)^3}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3, x)
```

3.43 $\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

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3.43.1 Optimal result

Integrand size = 23, antiderivative size = 185

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{(a-5b)x}{2(a+b)^4} - \frac{\sqrt{b}(15a^2-10ab-b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}(a+b)^4 d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{3b\tanh(c+dx)}{4(a+b)^2 d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(11a-b)b\tanh(c+dx)}{8a(a+b)^3 d\left(a+b\tanh^2(c+dx)\right)}$$

output

```
-1/2*(a-5*b)*x/(a+b)^4-1/8*(15*a^2-10*a*b-b^2)*arctan(b^(1/2)*tanh(d*x+c)/
a^(1/2))*b^(1/2)/a^(3/2)/(a+b)^4/d+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d/(a+
b*tanh(d*x+c)^2)^2-3/4*b*tanh(d*x+c)/(a+b)^2/d/(a+b*tanh(d*x+c)^2)^2-1/8*(
11*a-b)*b*tanh(d*x+c)/a/(a+b)^3/d/(a+b*tanh(d*x+c)^2)
```

3.43.2 Mathematica [A] (verified)

Time = 2.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.85

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{-4(a-5b)(c+dx) + \frac{\sqrt{b}(-15a^2+10ab+b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + 2(a+b)\sinh(2(c+dx)) - \frac{4b^2(a+b)\sinh(2(c+dx)}{(a-b+(a+b)\cosh(2(c+dx}}{8(a+b)^4 d}$$

input
```
Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-4*(a - 5*b)*(c + d*x) + (Sqrt[b]*(-15*a^2 + 10*a*b + b^2)*ArcTan[(Sqrt[b
]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2) + 2*(a + b)*Sinh[2*(c + d*x)] - (4*b^2*
(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)])^2 - ((9*a -
 b)*b*(a + b)*Sinh[2*(c + d*x)])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])))/
(8*(a + b)^4*d)
```

3.43.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.19, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 25, 4146, 373, 402, 27, 402, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{\sin(ic+idx)^2}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$
$$\downarrow 25$$
$$-\int \frac{\sin(ic+idx)^2}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$
$$\downarrow 4146$$

$$\frac{\int \frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^2\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh(c+dx)}{d}$$

$$\downarrow 373$$

$$\frac{\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\int \frac{a-5b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh(c+dx)}{2(a+b)}}{d}$$

$$\downarrow 402$$

$$\frac{\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{3b\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\int -\frac{2a\left(-9b\tanh^2(c+dx)+2a-b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}\,d\tanh(c+dx)}{4a(a+b)}}{2(a+b)}}{d}$$

$$\downarrow 27$$

$$\frac{\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\int \frac{-9b\tanh^2(c+dx)+2a-b}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}\,d\tanh(c+dx)}{2(a+b)}+\frac{3b\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{2(a+b)}}{d}$$

$$\downarrow 402$$

$$\frac{\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{b(11a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\int -\frac{4a^2-9ba-b^2-(11a-b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}\,d\tanh(c+dx)}{2a(a+b)}}{2(a+b)}+\frac{3b\tanh(c}{2(a+b)\left(a+b\tanh\right.}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\int \frac{4a^2-9ba-b^2-(11a-b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}\,d\tanh(c+dx)}{2a(a+b)}+\frac{b(11a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)}+\frac{3b\tanh(c+}{2(a+b)\left(a+b\tanh^2\right.}}{d}$$

$$\downarrow 397$$

3.43. $\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

$$\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\frac{b\left(15a^2-10ab-b^2\right)\int \frac{1}{b\tanh^2(c+dx)+a}\,d\tanh(c+dx)}{a+b} + \frac{4a(a-5b)\int \frac{1}{1-\tanh^2(c+dx)}\,d\tanh(c+dx)}{a+b}}{2a(a+b)} + \frac{b(1}{2a(a+}}{2(a+b)}}{d}$$

$$\downarrow 218$$

$$\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\frac{4a(a-5b)\int \frac{1}{1-\tanh^2(c+dx)}\,d\tanh(c+dx)}{a+b} + \frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2a(a+b)} + \frac{b(11a-b)}{2a(a+b)\left(a+}}{2(a+b)}}{d}$$

$$\downarrow 219$$

$$\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)} + \frac{4a(a-5b)\operatorname{arctanh}(\tanh(c+dx))}{a+b}}{2a(a+b)} + \frac{b(11a-b)\tanh(c}{2a(a+b)\left(a+b\tanh^2}}{2(a+b)}}{d}$$

input
```
Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(Tanh[c + d*x]/(2*(a + b)*(1 - Tanh[c + d*x]^2)*(a + b*Tanh[c + d*x]^2)^2)
 - ((3*b*Tanh[c + d*x])/(2*(a + b)*(a + b*Tanh[c + d*x]^2)^2) + (((Sqrt[b]
*(15*a^2 - 10*a*b - b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]
*(a + b)) + (4*a*(a - 5*b)*ArcTanh[Tanh[c + d*x]])/(a + b))/(2*a*(a + b))
+ ((11*a - b)*b*Tanh[c + d*x])/(2*a*(a + b)*(a + b*Tanh[c + d*x]^2)))/(2*(
a + b)))/(2*(a + b)))/d
```

3.43.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.43.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(167) = 334$.

Time = 15.26 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.64

method	result
derivativedivides	$\frac{1}{2(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{2(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(a-5b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2(a+b)^4}+2b\left(\frac{\left(-\frac{9}{8}a^2-\frac{5}{4}ab-\frac{1}{8}b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{}\right.$
default	$\frac{1}{2(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{2(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(a-5b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2(a+b)^4}+2b\left(\frac{\left(-\frac{9}{8}a^2-\frac{5}{4}ab-\frac{1}{8}b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{}\right.$
risch	$-\frac{xa}{2(a+b)(a^3+3a^2b+3ab^2+b^3)}+\frac{5xb}{2(a+b)(a^3+3a^2b+3ab^2+b^3)}+\frac{e^{2dx+2c}}{8(a^3+3a^2b+3ab^2+b^3)d}-\frac{e^{-2dx-2c}}{8(a^3+3a^2b+3ab^2+b^3$

input
```
int(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c
)-1)+1/2*(a-5*b)/(a+b)^4*ln(tanh(1/2*d*x+1/2*c)-1)+2*b/(a+b)^4*(((-9/8*a^2
-5/4*a*b-1/8*b^2)*tanh(1/2*d*x+1/2*c)^7-1/8*(27*a^3+58*a^2*b+27*a*b^2-4*b^
3)/a*tanh(1/2*d*x+1/2*c)^5-1/8*(27*a^3+58*a^2*b+27*a*b^2-4*b^3)/a*tanh(1/2
*d*x+1/2*c)^3+(-9/8*a^2-5/4*a*b-1/8*b^2)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*
x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2+1/8*
(15*a^2-10*a*b-b^2)*(1/2*(a+((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b
)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2
)+a+2*b)*a)^(1/2))-1/2*(-a+((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)
*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2
)-a-2*b)*a)^(1/2))))-1/2/(a+b)^3/(1+tanh(1/2*d*x+1/2*c))^2+1/2/(a+b)^3/(1+
tanh(1/2*d*x+1/2*c))+1/2/(a+b)^4*(-a+5*b)*ln(1+tanh(1/2*d*x+1/2*c)))
```

3.43.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6319 vs. $2(167) = 334$.

Time $= 0.45$ (sec) , antiderivative size $= 12965$, normalized size of antiderivative $= 70.08$

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.43.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.43.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1806 vs. 2(167) = 334.

Time = 0.59 (sec) , antiderivative size = 1806, normalized size of antiderivative = 9.76

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
3/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a
^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 3/4*b*log(2*(a - b)*e^(-2*d
*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2
 + 4*a*b^3 + b^4)*d) - 3/32*(5*a^3*b - 15*a^2*b^2 - 5*a*b^3 - b^4)*arctan(1
/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^6 + 4*a^5*b + 6*a^4*b^
2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) + 3/32*(5*a^3*b - 15*a^2*b^2 - 5*a*b
^3 - b^4)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^6 +
 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) + 1/16*(15*a^2*b
 + 10*a*b^2 + 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b
))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)*d) + 1/16*(9*a^4*b + 4
*a^3*b^2 - 22*a^2*b^3 - 20*a*b^4 - 3*b^5 + 3*(3*a^4*b - 22*a^3*b^2 - 20*a^
2*b^3 + 6*a*b^4 + b^5)*e^(6*d*x + 6*c) + (27*a^4*b - 156*a^3*b^2 + 110*a^2
*b^3 - 36*a*b^4 - 9*b^5)*e^(4*d*x + 4*c) + (27*a^4*b - 86*a^3*b^2 - 84*a^2
*b^3 + 38*a*b^4 + 9*b^5)*e^(2*d*x + 2*c))/((a^8 + 6*a^7*b + 15*a^6*b^2 + 2
0*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6 + (a^8 + 6*a^7*b + 15*a^6*b^2
 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*e^(8*d*x + 8*c) + 4*(a^8
 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*e^(6*d*x + 6*c)
+ 2*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5
+ 3*a^2*b^6)*e^(4*d*x + 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 -
4*a^3*b^5 - a^2*b^6)*e^(2*d*x + 2*c))*d) - 1/16*(9*a^4*b + 4*a^3*b^2 - ...
```

3.43.8 Giac [F]

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\sinh\left(dx+c\right)^2}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\sinh(c+d\,x)^2}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3, x)
```

3.44 $\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

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3.44.1 Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{15\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}d} + \frac{15\cosh(c+dx)}{8(a+b)^3 d} - \frac{\cosh(c+dx)}{4(a+b)d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{5\cosh(c+dx)}{8(a+b)^2 d\left(a+b-b\operatorname{sech}^2(c+dx)\right)}$$

output
```
15/8*cosh(d*x+c)/(a+b)^3/d-1/4*cosh(d*x+c)/(a+b)/d/(a+b-b*sech(d*x+c)^2)^2
-5/8*cosh(d*x+c)/(a+b)^2/d/(a+b-b*sech(d*x+c)^2)-15/8*arctanh(sech(d*x+c)*
b^(1/2)/(a+b)^(1/2))*b^(1/2)/(a+b)^(7/2)/d
```

3.44.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.25

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{-\frac{15i\sqrt{b}\left(\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)}{(a+b)^{7/2}} + \frac{2\cosh(c+dx)\left(4-\frac{4b^2}{(a-b+(a+b)\cosh(2(c+dx)))^2}-\frac{a-b}{a-b}\right.}{(a+b)^3}}{8d}$$

input
```
Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(((-15*I)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/S
qrt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]]))
/(a + b)^(7/2) + (2*Cosh[c + d*x]*(4 - (4*b^2)/(a - b + (a + b)*Cosh[2*(c
+ d*x)])^2 - (9*b)/(a - b + (a + b)*Cosh[2*(c + d*x)])))/(a + b)^3)/(8*d)
```

3.44.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 4147, 253, 253, 264, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\sin(ic+idx)}{\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$

$$\downarrow 26$$

$$-i\int \frac{\sin(ic+idx)}{\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$

$$\downarrow 4147$$

$$-\frac{\int \frac{\cosh^2(c+dx)}{\left(-b\text{sech}^2(c+dx)+a+b\right)^3}\,d\text{sech}(c+dx)}{d}$$

$$\downarrow 253$$

$$-\frac{\frac{5\int \frac{\cosh^2(c+dx)}{\left(-b\text{sech}^2(c+dx)+a+b\right)^2}\,d\text{sech}(c+dx)}{4(a+b)}+\frac{\cosh(c+dx)}{4(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)^2}}{d}$$

$$\downarrow 253$$

$$-\frac{5\left(\frac{3\int \frac{\cosh^2(c+dx)}{-b\operatorname{sech}^2(c+dx)+a+b}d\operatorname{sech}(c+dx)}{2(a+b)}+\frac{\cosh(c+dx)}{2(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}\right)}{4(a+b)}+\frac{\cosh(c+dx)}{4(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2}}{d}$$

$$\downarrow 264$$

$$-\frac{5\left(\frac{3\left(\frac{b\int \frac{1}{-b\operatorname{sech}^2(c+dx)+a+b}d\operatorname{sech}(c+dx)}{a+b}-\frac{\cosh(c+dx)}{a+b}\right)}{2(a+b)}+\frac{\cosh(c+dx)}{2(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}\right)}{4(a+b)}+\frac{\cosh(c+dx)}{4(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2}}{d}$$

$$\downarrow 221$$

$$-\frac{5\left(\frac{3\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{\cosh(c+dx)}{a+b}\right)}{2(a+b)}+\frac{\cosh(c+dx)}{2(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}\right)}{4(a+b)}+\frac{\cosh(c+dx)}{4(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2}}{d}$$

input
```
Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-((Cosh[c + d*x]/(4*(a + b)*(a + b - b*Sech[c + d*x]^2)^2) + (5*((3*((Sqrt
[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(a + b)^(3/2) - Cosh[c +
 d*x]/(a + b)))/(2*(a + b)) + Cosh[c + d*x]/(2*(a + b)*(a + b - b*Sech[c +
 d*x]^2))))/(4*(a + b)))/d)
```

3.44.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.44.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(110) = 220$.

Time = 5.91 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.00

method	result
derivativedivides	$-\frac{1}{(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2b\left(\frac{-\frac{\left(9a^2+24ab+8b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8a}-\frac{\left(27a^3+78a^2b+88ab^2+16b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8a^2}-\frac{(27a^2+56a}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}\right.}{(a+b)^3}}{d}$
default	$-\frac{1}{(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2b\left(\frac{-\frac{\left(9a^2+24ab+8b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8a}-\frac{\left(27a^3+78a^2b+88ab^2+16b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8a^2}-\frac{(27a^2+56a}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}\right.}{(a+b)^3}}{d}$
risch	$\frac{e^{dx+c}}{2(a^3+3a^2b+3ab^2+b^3)d}+\frac{e^{-dx-c}}{2(a^3+3a^2b+3ab^2+b^3)d}-\frac{e^{dx+c}b(9ae^{6dx+6c}+9be^{6dx+6c}+27ae^{4dx+4c}-be^{4dx+4c}+27e$

input
```
int(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)+2*b/(a+b)^3*((-1/8*(9*a^2+24*a*b+8
*b^2)/a*tanh(1/2*d*x+1/2*c)^6-1/8/a^2*(27*a^3+78*a^2*b+88*a*b^2+16*b^3)*ta
nh(1/2*d*x+1/2*c)^4-1/8*(27*a^2+56*a*b+8*b^2)/a*tanh(1/2*d*x+1/2*c)^2-9/8*
a-1/4*b)/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)^2-15/16/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^
2*a+2*a+4*b)/(a*b+b^2)^(1/2)))+1/(a+b)^3/(1+tanh(1/2*d*x+1/2*c)))
```

3.44.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3792 vs. $2(116) = 232$.

Time $= 0.37$ (sec) , antiderivative size $= 7119$, normalized size of antiderivative $= 56.50$

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.44.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.44.7 Maxima [F]

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\sinh\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*(2*a^2 + 4*a*b + 2*b^2 + 2*(a^2*e^(10*c) + 2*a*b*e^(10*c) + b^2*e^(10*
c))*e^(10*d*x) + 5*(2*a^2*e^(8*c) - a*b*e^(8*c) - 3*b^2*e^(8*c))*e^(8*d*x)
 + 5*(4*a^2*e^(6*c) - 7*a*b*e^(6*c) + b^2*e^(6*c))*e^(6*d*x) + 5*(4*a^2*e^
(4*c) - 7*a*b*e^(4*c) + b^2*e^(4*c))*e^(4*d*x) + 5*(2*a^2*e^(2*c) - a*b*e^
(2*c) - 3*b^2*e^(2*c))*e^(2*d*x))/((a^5*d*e^(9*c) + 5*a^4*b*d*e^(9*c) + 10
*a^3*b^2*d*e^(9*c) + 10*a^2*b^3*d*e^(9*c) + 5*a*b^4*d*e^(9*c) + b^5*d*e^(9
*c))*e^(9*d*x) + 4*(a^5*d*e^(7*c) + 3*a^4*b*d*e^(7*c) + 2*a^3*b^2*d*e^(7*c
) - 2*a^2*b^3*d*e^(7*c) - 3*a*b^4*d*e^(7*c) - b^5*d*e^(7*c))*e^(7*d*x) + 2
*(3*a^5*d*e^(5*c) + 7*a^4*b*d*e^(5*c) + 6*a^3*b^2*d*e^(5*c) + 6*a^2*b^3*d*
e^(5*c) + 7*a*b^4*d*e^(5*c) + 3*b^5*d*e^(5*c))*e^(5*d*x) + 4*(a^5*d*e^(3*c
) + 3*a^4*b*d*e^(3*c) + 2*a^3*b^2*d*e^(3*c) - 2*a^2*b^3*d*e^(3*c) - 3*a*b^
4*d*e^(3*c) - b^5*d*e^(3*c))*e^(3*d*x) + (a^5*d*e^c + 5*a^4*b*d*e^c + 10*a
^3*b^2*d*e^c + 10*a^2*b^3*d*e^c + 5*a*b^4*d*e^c + b^5*d*e^c)*e^(d*x)) + 1/
2*integrate(15/2*(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^4 + 4*a^3*b + 6*a^
2*b^2 + 4*a*b^3 + b^4 + (a^4*e^(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2*e^(4*c)
 + 4*a*b^3*e^(4*c) + b^4*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 2*a^3*b*e^(
2*c) - 2*a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

3.44.8 Giac [F]

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\sinh\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\sinh(c+d\,x)}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2)^3, x)
```

3.45 $\int \frac{\text{csch}(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

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3.45.1 Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{\text{csch}(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = -\frac{\text{arctanh}(\cosh(c+dx))}{a^3 d} + \frac{\sqrt{b}(15a^2+20ab+8b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}d} + \frac{b\,\text{sech}(c+dx)}{4a(a+b)d\left(a+b-b\,\text{sech}^2(c+dx)\right)^2} + \frac{b(7a+4b)\,\text{sech}(c+dx)}{8a^2(a+b)^2 d\left(a+b-b\,\text{sech}^2(c+dx)\right)}$$

output

```
-arctanh(cosh(d*x+c))/a^3/d+1/4*b*sech(d*x+c)/a/(a+b)/d/(a+b-b*sech(d*x+c)
^2)^2+1/8*b*(7*a+4*b)*sech(d*x+c)/a^2/(a+b)^2/d/(a+b-b*sech(d*x+c)^2)+1/8*
(15*a^2+20*a*b+8*b^2)*arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a^3
/(a+b)^(5/2)/d
```

3.45.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.23 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.60

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{i\sqrt{b}(15a^2+20ab+8b^2)\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{(a+b)^{5/2}} + \frac{i\sqrt{b}(15a^2+20ab+8b^2)\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{(a+b)^{5/2}} + \frac{8a^2b^2}{(a+b)^2(a-b+(}$$
$$8a^3d$$

input
```
Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((I*Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*T
anh[(c + d*x)/2])/Sqrt[b]])/(a + b)^(5/2) + (I*Sqrt[b]*(15*a^2 + 20*a*b +
8*b^2)*ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/(a
+ b)^(5/2) + (8*a^2*b^2*Cosh[c + d*x])/((a + b)^2*(a - b + (a + b)*Cosh[2*
(c + d*x)])^2) + (2*a*b*(9*a + 4*b)*Cosh[c + d*x])/((a + b)^2*(a - b + (a
+ b)*Cosh[2*(c + d*x)])) - 8*Log[Cosh[(c + d*x)/2]] + 8*Log[Sinh[(c + d*x)
/2]])/(8*a^3*d)
```

3.45.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 26, 4147, 25, 316, 25, 402, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{i}{\sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$
$$\downarrow 26$$
$$i\int \frac{1}{\sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$

$$\downarrow 4147$$

$$\frac{\int -\frac{1}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\text{sech}^2(c+dx)+a+b\right)^3}d\text{sech}(c+dx)}{d}$$

$$\downarrow 25$$

$$-\frac{\int \frac{1}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\text{sech}^2(c+dx)+a+b\right)^3}d\text{sech}(c+dx)}{d}$$

$$\downarrow 316$$

$$\frac{\int -\frac{3b\text{sech}^2(c+dx)+4a+b}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\text{sech}^2(c+dx)+a+b\right)^2}d\text{sech}(c+dx)}{4a(a+b)}+\frac{b\text{sech}(c+dx)}{4a(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)^2}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{b\text{sech}(c+dx)}{4a(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)^2}-\frac{\int \frac{3b\text{sech}^2(c+dx)+4a+b}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\text{sech}^2(c+dx)+a+b\right)^2}d\text{sech}(c+dx)}{4a(a+b)}}{d}$$

$$\downarrow 402$$

$$\frac{\frac{b\text{sech}(c+dx)}{4a(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)^2}-\frac{-\frac{\int -\frac{8a^2+9ba+4b^2+b(7a+4b)\text{sech}^2(c+dx)}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\text{sech}^2(c+dx)+a+b\right)}d\text{sech}(c+dx)}{2a(a+b)}-\frac{b(7a+4b)\text{sech}(c+dx)}{2a(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)}}{4a(a+b)}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{b\text{sech}(c+dx)}{4a(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)^2}-\frac{\frac{\int \frac{8a^2+9ba+4b^2+b(7a+4b)\text{sech}^2(c+dx)}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\text{sech}^2(c+dx)+a+b\right)}d\text{sech}(c+dx)}{2a(a+b)}-\frac{b(7a+4b)\text{sech}(c+dx)}{2a(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)}}{4a(a+b)}}{d}$$

$$\downarrow 397$$

$$\frac{\frac{b\text{sech}(c+dx)}{4a(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)^2}-\frac{\frac{8(a+b)^2\int \frac{1}{1-\text{sech}^2(c+dx)}d\text{sech}(c+dx)}{a}-\frac{b\left(15a^2+20ab+8b^2\right)\int \frac{1}{-b\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{a}}{2a(a+b)}-\frac{b(7a+}{2a(a+b)\left(a-\right.}}{4a(a+b)}}{d}$$

3.45. $\int \frac{\text{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}dx$

$$\downarrow 219$$

$$\frac{\frac{b\operatorname{sech}(c+dx)}{4a(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2}-\frac{\frac{8(a+b)^2\operatorname{arctanh}(\operatorname{sech}(c+dx))}{a}-\frac{b\left(15a^2+20ab+8b^2\right)\int\frac{1}{-b\operatorname{sech}^2(c+dx)+a+b}d\operatorname{sech}(c+dx)}{a}}{2a(a+b)}-\frac{b(7a+4b)\operatorname{sech}}{2a(a+b)\left(a-b\operatorname{sech}}}{4a(a+b)}}{d}$$

$$\downarrow 221$$

$$\frac{\frac{b\operatorname{sech}(c+dx)}{4a(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2}-\frac{\frac{8(a+b)^2\operatorname{arctanh}(\operatorname{sech}(c+dx))}{a}-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{b(7a+4b)\operatorname{sech}(c+dx)}{2a(a+b)\left(a-b\operatorname{sech}^2(c+dx)\right.}}{4a(a+b)}}{d}$$

input
```
Int[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((b*Sech[c + d*x])/(4*a*(a + b)*(a + b - b*Sech[c + d*x]^2)^2) - (((8*(a +
 b)^2*ArcTanh[Sech[c + d*x]])/a - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTa
nh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a*(a + b)) -
(b*(7*a + 4*b)*Sech[c + d*x])/(2*a*(a + b)*(a + b - b*Sech[c + d*x]^2)))/(
4*a*(a + b)))/d
```

3.45.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.45.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(142) = 284$.

Time = 2.88 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.95

method	result
derivativedivides	$\frac{2b\left(\frac{-\frac{\left(9a^2+28ab+16b^2\right)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8\left(a^2+2ab+b^2\right)}-\frac{3\left(9a^3+30a^2b+40ab^2+16b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8\left(a^2+2ab+b^2\right)}-\frac{a\left(27a^2+68ab+32b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8\left(a^2+2ab+b^2\right)}-}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}-\right)}{a^3}}{d}$
default	$\frac{2b\left(\frac{-\frac{\left(9a^2+28ab+16b^2\right)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8\left(a^2+2ab+b^2\right)}-\frac{3\left(9a^3+30a^2b+40ab^2+16b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8\left(a^2+2ab+b^2\right)}-\frac{a\left(27a^2+68ab+32b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8\left(a^2+2ab+b^2\right)}-}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}-\right)}{a^3}}{d}$
risch	$\frac{(9a^2e^{6dx+6c}+13abe^{6dx+6c}+4b^2e^{6dx+6c}+27a^2e^{4dx+4c}+11abe^{4dx+4c}-4e^{4dx+4c}b^2+27a^2e^{2dx+2c}+11abe^{2dx+2c}-4e^{2d}}{4(a^2+2ab+b^2)(ae^{4dx+4c}+be^{4dx+4c}+2e^{2dx+2c}a-2be^{2dx+2c}+a+b)^2da^2}$

input
```
int(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*b/a^3*((-1/8*(9*a^2+28*a*b+16*b^2)*a/(a^2+2*a*b+b^2)*tanh(1/2*d*x+
1/2*c)^6-3/8*(9*a^3+30*a^2*b+40*a*b^2+16*b^3)/(a^2+2*a*b+b^2)*tanh(1/2*d*x
+1/2*c)^4-1/8*a*(27*a^2+68*a*b+32*b^2)/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)
^2-3/8*a^2*(3*a+2*b)/(a^2+2*a*b+b^2))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*
d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2-1/16*(15*a^2+20*a*b+8*b^2)/(
a^2+2*a*b+b^2)/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+
4*b)/(a*b+b^2)^(1/2)))+1/a^3*ln(tanh(1/2*d*x+1/2*c)))
```

3.45.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5783 vs. $2(148) = 296$.

Time = 0.47 (sec) , antiderivative size = 10716, normalized size of antiderivative = 68.69

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.45.6 Sympy [F]

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{csch}\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**2)**3, x)
```

3.45.7 Maxima [F]

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*((9*a^2*b*e^(7*c) + 13*a*b^2*e^(7*c) + 4*b^3*e^(7*c))*e^(7*d*x) + (27*
a^2*b*e^(5*c) + 11*a*b^2*e^(5*c) - 4*b^3*e^(5*c))*e^(5*d*x) + (27*a^2*b*e^
(3*c) + 11*a*b^2*e^(3*c) - 4*b^3*e^(3*c))*e^(3*d*x) + (9*a^2*b*e^c + 13*a*
b^2*e^c + 4*b^3*e^c)*e^(d*x))/(a^6*d + 4*a^5*b*d + 6*a^4*b^2*d + 4*a^3*b^3
*d + a^2*b^4*d + (a^6*d*e^(8*c) + 4*a^5*b*d*e^(8*c) + 6*a^4*b^2*d*e^(8*c)
+ 4*a^3*b^3*d*e^(8*c) + a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(a^6*d*e^(6*c) +
2*a^5*b*d*e^(6*c) - 2*a^3*b^3*d*e^(6*c) - a^2*b^4*d*e^(6*c))*e^(6*d*x) + 2
*(3*a^6*d*e^(4*c) + 4*a^5*b*d*e^(4*c) + 2*a^4*b^2*d*e^(4*c) + 4*a^3*b^3*d*
e^(4*c) + 3*a^2*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^6*d*e^(2*c) + 2*a^5*b*d*e^
(2*c) - 2*a^3*b^3*d*e^(2*c) - a^2*b^4*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x
+ c) + 1)*e^(-c))/(a^3*d) + log((e^(d*x + c) - 1)*e^(-c))/(a^3*d) - 2*inte
grate(1/8*((15*a^2*b*e^(3*c) + 20*a*b^2*e^(3*c) + 8*b^3*e^(3*c))*e^(3*d*x)
 - (15*a^2*b*e^c + 20*a*b^2*e^c + 8*b^3*e^c)*e^(d*x))/(a^6 + 3*a^5*b + 3*a
^4*b^2 + a^3*b^3 + (a^6*e^(4*c) + 3*a^5*b*e^(4*c) + 3*a^4*b^2*e^(4*c) + a^
3*b^3*e^(4*c))*e^(4*d*x) + 2*(a^6*e^(2*c) + a^5*b*e^(2*c) - a^4*b^2*e^(2*c
) - a^3*b^3*e^(2*c))*e^(2*d*x)), x)
```

3.45.8 Giac [F]

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^3),x)
```

output
```
int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^3), x)
```

3.46 $\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

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3.46.1 Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{15\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d} - \frac{15\coth(c+dx)}{8a^3d} + \frac{\coth(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2} + \frac{5\coth(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right)}$$

output

```
-15/8*coth(d*x+c)/a^3/d-15/8*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/a
^(7/2)/d+1/4*coth(d*x+c)/a/d/(a+b*tanh(d*x+c)^2)^2+5/8*coth(d*x+c)/a^2/d/(
a+b*tanh(d*x+c)^2)
```

3.46.2 Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{-15\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) - 8\sqrt{a}\coth(c+dx) - \frac{\sqrt{a}b(9a-7b+(9a+7b)\cosh(2(c+dx)))\sinh(2(c+dx))}{(a-b+(a+b)\cosh(2(c+dx)))^2}}{8a^{7/2}d}$$

input

```
Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-15*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] - 8*Sqrt[a]*Coth[c +
d*x] - (Sqrt[a]*b*(9*a - 7*b + (9*a + 7*b)*Cosh[2*(c + d*x)])*Sinh[2*(c +
d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)])^2)/(8*a^(7/2)*d)
```

3.46.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 25, 4146, 253, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\downarrow 4146$$

$$\frac{\int \frac{\coth^2(c+dx)}{\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\downarrow 253$$

$$\frac{\frac{5\int \frac{\coth^2(c+dx)}{\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4a}+\frac{\coth(c+dx)}{4a\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 253$$

$$\frac{5\left(\frac{3\int \frac{\coth^2(c+dx)}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{2a}+\frac{\coth(c+dx)}{2a\left(a+b\tanh^2(c+dx)\right)}\right)}{4a}+\frac{\coth(c+dx)}{4a\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 264$$

$$\frac{5\left(\frac{3\left(-\frac{b\int\frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a}-\frac{\coth(c+dx)}{a}\right)}{2a}+\frac{\coth(c+dx)}{2a\left(a+b\tanh^2(c+dx)\right)}\right)}{4a}+\frac{\coth(c+dx)}{4a\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 218$$

$$\frac{5\left(\frac{3\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\coth(c+dx)}{a}\right)}{2a}+\frac{\coth(c+dx)}{2a\left(a+b\tanh^2(c+dx)\right)}\right)}{4a}+\frac{\coth(c+dx)}{4a\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

input
```
Int[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(Coth[c + d*x]/(4*a*(a + b*Tanh[c + d*x]^2)^2) + (5*((3*(-((Sqrt[b]*ArcTan
[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2)) - Coth[c + d*x]/a))/(2*a) + Co
th[c + d*x]/(2*a*(a + b*Tanh[c + d*x]^2))))/(4*a))/d
```

3.46.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.46.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(96) = 192$.

Time = 5.22 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.73

method	result
derivativedivides	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3}-\frac{1}{2a^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+2b\left(\frac{-\frac{9a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8}+\left(-\frac{27a}{8}-\frac{7b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-\frac{27a}{8}-\frac{7b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3-\frac{9\tan}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}\right.$
default	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3}-\frac{1}{2a^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+2b\left(\frac{-\frac{9a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8}+\left(-\frac{27a}{8}-\frac{7b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-\frac{27a}{8}-\frac{7b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3-\frac{9\tan}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}\right.$
risch	$-\frac{8e^{8dx+8c}a^4+23e^{8dx+8c}a^3b+45e^{8dx+8c}a^2b^2+45ab^3e^{8dx+8c}+15b^4e^{8dx+8c}+32e^{6dx+6c}a^4+46ba^3e^{6dx+6c}-90ab^3e^{6}}{4(a}$

input
```
int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/a^3*tanh(1/2*d*x+1/2*c)-1/2/a^3/tanh(1/2*d*x+1/2*c)+2*b/a^3*((-9
/8*a*tanh(1/2*d*x+1/2*c)^7+(-27/8*a-7/2*b)*tanh(1/2*d*x+1/2*c)^5+(-27/8*a-
7/2*b)*tanh(1/2*d*x+1/2*c)^3-9/8*tanh(1/2*d*x+1/2*c)*a)/(tanh(1/2*d*x+1/2*
c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2+15/8*a*(1/
2*(a+((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1
/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*
(-a+((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/
2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)))))
```

3.46.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3995 vs. $2(96) = 192$.

Time $= 0.38$ (sec) , antiderivative size $= 8312$, normalized size of antiderivative $= 74.21$

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.46.6 Sympy [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

input
```
integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**2)**3, x)
```

3.46.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(96) = 192$.

Time = 0.43 (sec) , antiderivative size = 478, normalized size of antiderivative = 4.27

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{8a^4+41a^3b+73a^2b^2+55ab^3+15b^4+2\left(16a^4+41a^3b-55ab^3-30b^4\right)e^{(-2a}}{4\left(a^7+4a^6b+6a^5b^2+4a^4b^3+a^3b^4+\left(3a^7+4a^6b-6a^5b^2-12a^4b^3-5a^3b^4\right)e^{(-2dx-2c)}+2\left(a^7+2a\right.\right.} + \frac{15b\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{8\sqrt{ab}a^3d}$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*(8*a^4 + 41*a^3*b + 73*a^2*b^2 + 55*a*b^3 + 15*b^4 + 2*(16*a^4 + 41*a
^3*b - 55*a*b^3 - 30*b^4)*e^(-2*d*x - 2*c) + 2*(24*a^4 + 32*a^3*b + 5*a^2*
b^2 + 50*a*b^3 + 45*b^4)*e^(-4*d*x - 4*c) + 2*(16*a^4 + 23*a^3*b - 45*a*b^
3 - 30*b^4)*e^(-6*d*x - 6*c) + (8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 +
 15*b^4)*e^(-8*d*x - 8*c))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b
^4 + (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*e^(-2*d*x - 2*
c) + 2*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*e^(-4*d*x - 4*c) - 2*(a^7
 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*e^(-6*d*x - 6*c) - (3*a^7 + 4*a^6*b
 - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*e^(-8*d*x - 8*c) - (a^7 + 4*a^6*b +
6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*e^(-10*d*x - 10*c))*d) + 15/8*b*arctan(1/
2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^3*d)
```

3.46.8 Giac [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^2}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)^2\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3),x)
```

output
```
int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3), x)
```

3.47 $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

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3.47.1 Optimal result

Integrand size = 23, antiderivative size = 196

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{(a+6b)\operatorname{arctanh}(\cosh(c+dx))}{2a^4d} - \frac{\sqrt{b}(15a^2+40ab+24b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{3/2}d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{3b\operatorname{sech}(c+dx)}{4a^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} - \frac{b(11a+12b)\operatorname{sech}(c+dx)}{8a^3(a+b)d\left(a+b-b\operatorname{sech}^2(c+dx)\right)}$$

output

```
1/2*(a+6*b)*arctanh(cosh(d*x+c))/a^4/d-1/2*coth(d*x+c)*csch(d*x+c)/a/d/(a+
b-b*sech(d*x+c)^2)^2-3/4*b*sech(d*x+c)/a^2/d/(a+b-b*sech(d*x+c)^2)^2-1/8*b
*(11*a+12*b)*sech(d*x+c)/a^3/(a+b)/d/(a+b-b*sech(d*x+c)^2)-1/8*(15*a^2+40*
a*b+24*b^2)*arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a^4/(a+b)^(3/
2)/d
```

3.47.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.61 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.05

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$= -\frac{i\sqrt{b}(15a^2+40ab+24b^2)\arctan\left(\frac{\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(-i\sqrt{a+b}\cosh\left(\frac{1}{2}(c+dx)\right)-\sqrt{a}\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{b}}\right)}{8a^4(a+b)^{3/2}d}$$

$$-\frac{i\sqrt{b}(15a^2+40ab+24b^2)\arctan\left(\frac{\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(-i\sqrt{a+b}\cosh\left(\frac{1}{2}(c+dx)\right)+\sqrt{a}\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{b}}\right)}{8a^4(a+b)^{3/2}d}$$

$$-\frac{b^2\cosh(c+dx)}{a^2(a+b)d(a-b+a\cosh(2(c+dx))+b\cosh(2(c+dx)))^2}$$

$$+\frac{-9ab\cosh(c+dx)-8b^2\cosh(c+dx)}{4a^3(a+b)d(a-b+a\cosh(2(c+dx))+b\cosh(2(c+dx)))}-\frac{\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8a^3d}$$

$$+\frac{(a+6b)\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d}+\frac{(-a-6b)\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d}-\frac{\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8a^3d}$$

input
```
Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((-1/8*I)*Sqrt[b]*(15*a^2 + 40*a*b + 24*b^2)*ArcTan[(Sech[(c + d*x)/2]*((-
I)*Sqrt[a + b]*Cosh[(c + d*x)/2] - Sqrt[a]*Sinh[(c + d*x)/2]))/Sqrt[b]])/(
a^4*(a + b)^(3/2)*d) - ((I/8)*Sqrt[b]*(15*a^2 + 40*a*b + 24*b^2)*ArcTan[(S
ech[(c + d*x)/2]*((-I)*Sqrt[a + b]*Cosh[(c + d*x)/2] + Sqrt[a]*Sinh[(c + d
*x)/2]))/Sqrt[b]])/(a^4*(a + b)^(3/2)*d) - (b^2*Cosh[c + d*x])/(a^2*(a + b
)*d*(a - b + a*Cosh[2*(c + d*x)] + b*Cosh[2*(c + d*x)])^2) + (-9*a*b*Cosh[
c + d*x] - 8*b^2*Cosh[c + d*x])/(4*a^3*(a + b)*d*(a - b + a*Cosh[2*(c + d*
x)] + b*Cosh[2*(c + d*x)])) - Csch[(c + d*x)/2]^2/(8*a^3*d) + ((a + 6*b)*L
og[Cosh[(c + d*x)/2]])/(2*a^4*d) + ((-a - 6*b)*Log[Sinh[(c + d*x)/2]])/(2*
a^4*d) - Sech[(c + d*x)/2]^2/(8*a^3*d)
```

3.47.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 26, 4147, 373, 402, 27, 402, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i}{\sin(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\downarrow 26$$

$$-i\int \frac{1}{\sin(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\downarrow 4147$$

$$-\frac{\int \frac{\operatorname{sech}^2(c+dx)}{\left(1-\operatorname{sech}^2(c+dx)\right)^2\left(-b\operatorname{sech}^2(c+dx)+a+b\right)^3}d\operatorname{sech}(c+dx)}{d}$$

$$\downarrow 373$$

$$-\frac{\frac{\operatorname{sech}(c+dx)}{2a\left(1-\operatorname{sech}^2(c+dx)\right)\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2}-\frac{\int \frac{5b\operatorname{sech}^2(c+dx)+a+b}{\left(1-\operatorname{sech}^2(c+dx)\right)\left(-b\operatorname{sech}^2(c+dx)+a+b\right)^3}d\operatorname{sech}(c+dx)}{2a}}{d}$$

$$\downarrow 402$$

$$-\frac{\frac{\operatorname{sech}(c+dx)}{2a\left(1-\operatorname{sech}^2(c+dx)\right)\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2}-\frac{-\frac{\int -\frac{2(a+b)\left(9b\operatorname{sech}^2(c+dx)+2a+3b\right)}{\left(1-\operatorname{sech}^2(c+dx)\right)\left(-b\operatorname{sech}^2(c+dx)+a+b\right)^2}d\operatorname{sech}(c+dx)}{4a(a+b)}-\frac{3b\operatorname{sech}(c+dx)}{2a\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2}}{2a}}{d}$$

$$\downarrow 27$$

$$-\frac{\operatorname{sech}(c+dx)}{2a\left(1-\operatorname{sech}^2(c+dx)\right)\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2}-\frac{\dfrac{\int \frac{9b\operatorname{sech}^2(c+dx)+2a+3b}{\left(1-\operatorname{sech}^2(c+dx)\right)\left(-b\operatorname{sech}^2(c+dx)+a+b\right)^2}\,d\operatorname{sech}(c+dx)}{2a}-\frac{3b\operatorname{sech}(c+dx)}{2a\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2}}{2a}}{d}$$

$$\downarrow 402$$

$$-\frac{\operatorname{sech}(c+dx)}{2a\left(1-\operatorname{sech}^2(c+dx)\right)\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2}-\frac{-\dfrac{\int -\frac{4a^2+17ba+12b^2+b(11a+12b)\operatorname{sech}^2(c+dx)}{\left(1-\operatorname{sech}^2(c+dx)\right)\left(-b\operatorname{sech}^2(c+dx)+a+b\right)}\,d\operatorname{sech}(c+dx)}{2a(a+b)}-\frac{b(11a+12b)\operatorname{sech}(c+dx)}{2a(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}}{2a}}{d}$$

$$\downarrow 25$$

$$-\frac{\operatorname{sech}(c+dx)}{2a\left(1-\operatorname{sech}^2(c+dx)\right)\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2}-\frac{\dfrac{\int \frac{4a^2+17ba+12b^2+b(11a+12b)\operatorname{sech}^2(c+dx)}{\left(1-\operatorname{sech}^2(c+dx)\right)\left(-b\operatorname{sech}^2(c+dx)+a+b\right)}\,d\operatorname{sech}(c+dx)}{2a(a+b)}-\frac{b(11a+12b)\operatorname{sech}(c+dx)}{2a(a+b)\left(a-b\operatorname{sech}^2(c+dx)+b\right)}}{2a}}{d}$$

$$\downarrow 397$$

$$-\frac{\operatorname{sech}(c+dx)}{2a\left(1-\operatorname{sech}^2(c+dx)\right)\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2}-\frac{\dfrac{4(a+b)(a+6b)\int \frac{1}{1-\operatorname{sech}^2(c+dx)}\,d\operatorname{sech}(c+dx)}{a}-\frac{b\left(15a^2+40ab+24b^2\right)\int \frac{1}{-b\operatorname{sech}^2(c+dx)+a+b}\,d\operatorname{se}}{a}}{2a}}{d}$$

$$\downarrow 219$$

$$-\frac{\operatorname{sech}(c+dx)}{2a\left(1-\operatorname{sech}^2(c+dx)\right)\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2}-\frac{\dfrac{4(a+b)(a+6b)\operatorname{arctanh}(\operatorname{sech}(c+dx))}{a}-\frac{b\left(15a^2+40ab+24b^2\right)\int \frac{1}{-b\operatorname{sech}^2(c+dx)+a+b}\,d\operatorname{sech}(c+d}{a}}{2a}}{d}$$

$$\downarrow 221$$

$$-\frac{\frac{\operatorname{sech}(c+dx)}{2a\left(1-\operatorname{sech}^2(c+dx)\right)\left(a-b\operatorname{sech}^2(c+dx)+b\right)^2}-\frac{\frac{\frac{4(a+b)(a+6b)\operatorname{arctanh}(\operatorname{sech}(c+dx))}{a}-\frac{\sqrt{b}\left(15a^2+40ab+24b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{}{2a}}{2a}}{d}$$

input
```
Int[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-((Sech[c + d*x]/(2*a*(1 - Sech[c + d*x]^2)*(a + b - b*Sech[c + d*x]^2)^2)
 - ((-3*b*Sech[c + d*x])/(2*a*(a + b - b*Sech[c + d*x]^2)^2) + (((4*(a + b
)*(a + 6*b)*ArcTanh[Sech[c + d*x]])/a - (Sqrt[b]*(15*a^2 + 40*a*b + 24*b^2
)*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a*(a +
 b)) - (b*(11*a + 12*b)*Sech[c + d*x])/(2*a*(a + b)*(a + b - b*Sech[c + d*
x]^2)))/(2*a))/(2*a))/d)
```

3.47.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

3.47.4 Maple [A] (verified)

Time = 7.89 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.55

method	result
derivativedivides	$\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a^3}+\frac{2b\left(\frac{-\frac{\left(9a^2+32ab+24b^2\right)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8(a+b)}-\frac{\left(27a^3+102a^2b+152ab^2+80b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8(a+b)}-\frac{a\left(27a^2+80ab+56b^2\right.}{8(a+}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}\right.}{a^4}$
default	$\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a^3}+\frac{2b\left(\frac{-\frac{\left(9a^2+32ab+24b^2\right)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8(a+b)}-\frac{\left(27a^3+102a^2b+152ab^2+80b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8(a+b)}-\frac{a\left(27a^2+80ab+56b^2\right.}{8(a+}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}\right.}{a^4}$
risch	$-\frac{(4a^3e^{10dx+10c}+21a^2be^{10dx+10c}+29ab^2e^{10dx+10c}+12b^3e^{10dx+10c}+20a^3e^{8dx+8c}+37a^2be^{8dx+8c}-15ab^2e^{8dx+8c}-3$

input
```
int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a^3+2*b/a^4*((-1/8*(9*a^2+32*a*b+24*b^2)*a/
(a+b)*tanh(1/2*d*x+1/2*c)^6-1/8*(27*a^3+102*a^2*b+152*a*b^2+80*b^3)/(a+b)*
tanh(1/2*d*x+1/2*c)^4-1/8*a*(27*a^2+80*a*b+56*b^2)/(a+b)*tanh(1/2*d*x+1/2*
c)^2-1/8*a^2*(9*a+10*b)/(a+b))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2
*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2-1/16*(15*a^2+40*a*b+24*b^2)/(a+b)/(
a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(
1/2)))-1/8/a^3/tanh(1/2*d*x+1/2*c)^2+1/4/a^4*(-2*a-12*b)*ln(tanh(1/2*d*x+1
/2*c)))
```

3.47.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11576 vs. 2(187) = 374.

Time = 0.56 (sec) , antiderivative size = 21301, normalized size of antiderivative = 108.68

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.47.6 Sympy [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

input
```
integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**2)**3, x)
```

3.47.7 Maxima [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{\left(b\tanh(dx+c)^2+a\right)^3}\,dx$$

input
```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output

```
-1/4*((4*a^3*e^(11*c) + 21*a^2*b*e^(11*c) + 29*a*b^2*e^(11*c) + 12*b^3*e^(
11*c))*e^(11*d*x) + (20*a^3*e^(9*c) + 37*a^2*b*e^(9*c) - 15*a*b^2*e^(9*c)
- 36*b^3*e^(9*c))*e^(9*d*x) + 2*(20*a^3*e^(7*c) + 3*a^2*b*e^(7*c) - 7*a*b^
2*e^(7*c) + 12*b^3*e^(7*c))*e^(7*d*x) + 2*(20*a^3*e^(5*c) + 3*a^2*b*e^(5*c
) - 7*a*b^2*e^(5*c) + 12*b^3*e^(5*c))*e^(5*d*x) + (20*a^3*e^(3*c) + 37*a^2
*b*e^(3*c) - 15*a*b^2*e^(3*c) - 36*b^3*e^(3*c))*e^(3*d*x) + (4*a^3*e^c + 2
1*a^2*b*e^c + 29*a*b^2*e^c + 12*b^3*e^c)*e^(d*x))/(a^6*d + 3*a^5*b*d + 3*a
^4*b^2*d + a^3*b^3*d + (a^6*d*e^(12*c) + 3*a^5*b*d*e^(12*c) + 3*a^4*b^2*d*
e^(12*c) + a^3*b^3*d*e^(12*c))*e^(12*d*x) + 2*(a^6*d*e^(10*c) - a^5*b*d*e^
(10*c) - 5*a^4*b^2*d*e^(10*c) - 3*a^3*b^3*d*e^(10*c))*e^(10*d*x) - (a^6*d*
e^(8*c) + 3*a^5*b*d*e^(8*c) - 13*a^4*b^2*d*e^(8*c) - 15*a^3*b^3*d*e^(8*c))
*e^(8*d*x) - 4*(a^6*d*e^(6*c) - a^5*b*d*e^(6*c) + 3*a^4*b^2*d*e^(6*c) + 5*
a^3*b^3*d*e^(6*c))*e^(6*d*x) - (a^6*d*e^(4*c) + 3*a^5*b*d*e^(4*c) - 13*a^4
*b^2*d*e^(4*c) - 15*a^3*b^3*d*e^(4*c))*e^(4*d*x) + 2*(a^6*d*e^(2*c) - a^5*
b*d*e^(2*c) - 5*a^4*b^2*d*e^(2*c) - 3*a^3*b^3*d*e^(2*c))*e^(2*d*x)) + 1/2*
(a + 6*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^4*d) - 1/2*(a + 6*b)*log((e^(d*
x + c) - 1)*e^(-c))/(a^4*d) + 8*integrate(1/32*((15*a^2*b*e^(3*c) + 40*a*b
^2*e^(3*c) + 24*b^3*e^(3*c))*e^(3*d*x) - (15*a^2*b*e^c + 40*a*b^2*e^c + 24
*b^3*e^c)*e^(d*x))/(a^6 + 2*a^5*b + a^4*b^2 + (a^6*e^(4*c) + 2*a^5*b*e^(4*
c) + a^4*b^2*e^(4*c))*e^(4*d*x) + 2*(a^6*e^(2*c) - a^4*b^2*e^(2*c))*e^(...
```

3.47.8 Giac [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^3}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input

```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output

```
sage0*x
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)^3\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3),x)
```

output
```
int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3), x)
```

3.48 $\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

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3.48.1 Optimal result

Integrand size = 23, antiderivative size = 151

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{5\sqrt{b}(3a+7b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d} + \frac{(a+3b)\coth(c+dx)}{a^4d} - \frac{\coth^3(c+dx)}{3a^3d} + \frac{b(a+b)\tanh(c+dx)}{4a^3d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(7a+11b)\tanh(c+dx)}{8a^4d\left(a+b\tanh^2(c+dx)\right)}$$

output

```
(a+3*b)*coth(d*x+c)/a^4/d-1/3*coth(d*x+c)^3/a^3/d+5/8*(3*a+7*b)*arctan(b^(
1/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/a^(9/2)/d+1/4*b*(a+b)*tanh(d*x+c)/a^3/d/
(a+b*tanh(d*x+c)^2)^2+1/8*b*(7*a+11*b)*tanh(d*x+c)/a^4/d/(a+b*tanh(d*x+c)^
2)
```

3.48.2 Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{15\sqrt{b}(3a+7b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) - 8\sqrt{a}\coth(c+dx)\left(-2a-9b+a\operatorname{csch}^2(c+dx)\right) + \frac{3\sqrt{a}b(9a^2+6ab-11b}{24a^{9/2}d}$$

input
```
Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(15*Sqrt[b]*(3*a + 7*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] - 8*Sqrt[a
]*Coth[c + d*x]*(-2*a - 9*b + a*Csch[c + d*x]^2) + (3*Sqrt[a]*b*(9*a^2 + 6
*a*b - 11*b^2 + (9*a^2 + 20*a*b + 11*b^2)*Cosh[2*(c + d*x)])*Sinh[2*(c + d
*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)])^2)/(24*a^(9/2)*d)
```

3.48.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4146, 361, 25, 1582, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\downarrow 4146$$

$$\frac{\int \frac{\coth^4(c+dx)(1-\tanh^2(c+dx))}{\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\downarrow 361$$

$$\frac{\frac{b(a+b)\tanh(c+dx)}{4a^3\left(a+b\tanh^2(c+dx)\right)^2} - \frac{1}{4}b\int -\frac{\coth^4(c+dx)\left(\frac{3(a+b)\tanh^4(c+dx)}{a^3}-\frac{4(a+b)\tanh^2(c+dx)}{a^2b}+\frac{4}{ab}\right)}{\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 25$$

$$\frac{\frac{1}{4}b\int \frac{\coth^4(c+dx)\left(\frac{3(a+b)\tanh^4(c+dx)}{a^3}-\frac{4(a+b)\tanh^2(c+dx)}{a^2b}+\frac{4}{ab}\right)}{\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx) + \frac{b(a+b)\tanh(c+dx)}{4a^3\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 1582$$

$$\frac{\frac{1}{4}b\left(\frac{\int \frac{\coth^4(c+dx)\left(\frac{b^2(7a+11b)\tanh^4(c+dx)}{a}-8b(a+2b)\tanh^2(c+dx)+8ab\right)}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{2a^3b^2}+\frac{(7a+11b)\tanh(c+dx)}{2a^4\left(a+b\tanh^2(c+dx)\right)}\right)+\frac{b(a+b)\tanh(c+dx)}{4a^3\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 1584$$

$$\frac{\frac{1}{4}b\left(\frac{\int\left(8b\coth^4(c+dx)-\frac{8b(a+3b)\coth^2(c+dx)}{a}+\frac{5b^2(3a+7b)}{a\left(b\tanh^2(c+dx)+a\right)}\right)d\tanh(c+dx)}{2a^3b^2}+\frac{(7a+11b)\tanh(c+dx)}{2a^4\left(a+b\tanh^2(c+dx)\right)}\right)+\frac{b(a+b)\tanh(c+dx)}{4a^3\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{b(a+b)\tanh(c+dx)}{4a^3\left(a+b\tanh^2(c+dx)\right)^2}+\frac{1}{4}b\left(\frac{(7a+11b)\tanh(c+dx)}{2a^4\left(a+b\tanh^2(c+dx)\right)}+\frac{\frac{5b^{3/2}(3a+7b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}+\frac{8b(a+3b)\coth(c+dx)}{a}-\frac{8}{3}b\coth^3(c+dx)}{2a^3b^2}\right)}{d}$$

input
```
Int[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((b*(a + b)*Tanh[c + d*x])/(4*a^3*(a + b*Tanh[c + d*x]^2)^2) + (b*(((5*b^(
3/2)*(3*a + 7*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2) + (8*b*(
a + 3*b)*Coth[c + d*x])/a - (8*b*Coth[c + d*x]^3)/3)/(2*a^3*b^2) + ((7*a +
 11*b)*Tanh[c + d*x])/(2*a^4*(a + b*Tanh[c + d*x]^2))))/4)/d
```

3.48.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
 + 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
 - a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.48.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(135) = 270$.

Time = 11.28 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.63

method	result
derivativedivides	$-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-12b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^4}-2b\left(\frac{-\frac{a(9a+13b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8}+\left(-\frac{27}{8}a^2-\frac{67}{8}ab-\frac{11}{2}b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)$
default	$-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-12b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^4}-2b\left(\frac{-\frac{a(9a+13b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8}+\left(-\frac{27}{8}a^2-\frac{67}{8}ab-\frac{11}{2}b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)$
risch	$-\frac{900ab^3e^{10dx+10c}-64a^3be^{2dx+2c}+502a^2b^2e^{2dx+2c}+1180ab^3e^{2dx+2c}+313a^3be^{4dx+4c}-1575b^4e^{8dx+8c}+630b^4e^{10dx}$

input
```
int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/8/a^4*(1/3*tanh(1/2*d*x+1/2*c)^3*a-3*tanh(1/2*d*x+1/2*c)*a-12*b*ta
nh(1/2*d*x+1/2*c))-2*b/a^4*((-1/8*a*(9*a+13*b)*tanh(1/2*d*x+1/2*c)^7+(-27/
8*a^2-67/8*a*b-11/2*b^2)*tanh(1/2*d*x+1/2*c)^5+(-27/8*a^2-67/8*a*b-11/2*b^
2)*tanh(1/2*d*x+1/2*c)^3+(-9/8*a^2-13/8*a*b)*tanh(1/2*d*x+1/2*c))/(tanh(1/
2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2+
1/8*(15*a+35*b)*a*(1/2*(a+((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*
b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+
a+2*b)*a)^(1/2))-1/2*(-a+((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b
)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-
a-2*b)*a)^(1/2))))-1/24/a^3/tanh(1/2*d*x+1/2*c)^3-1/8/a^4*(-3*a-12*b)/tanh
(1/2*d*x+1/2*c))
```

3.48. $\int \frac{\operatorname{csch}^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

3.48.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7006 vs. 2(135) = 270.

Time = 0.43 (sec) , antiderivative size = 14334, normalized size of antiderivative = 94.93

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.48.6 Sympy [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

input
```
integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**2)**3, x)
```

3.48.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. 2(135) = 270.

Time = 0.50 (sec) , antiderivative size = 615, normalized size of antiderivative = 4.07

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{16\,a^4+147\,a^3b+351\,a^2b^2+325\,ab^3+105\,b^4+2\,(8\,a^4+32\,a^3b-251\,a^2b^2-590\,ab^3-315\,b^4)e^{(-2\,dx-2\,c)}-}{12\,(a^7+3\,a^6b+3\,a^5b^2+a^4b^3+(a^7-5\,a^6b-13\,a^5b^2-7\,a^4b^3)e^{(-2\,dx-2\,c)}-(3\,a^7+a}$$
$$-\frac{5\,(3\,ab+7\,b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{8\sqrt{ab}a^4d}$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/12*(16*a^4 + 147*a^3*b + 351*a^2*b^2 + 325*a*b^3 + 105*b^4 + 2*(8*a^4 +
32*a^3*b - 251*a^2*b^2 - 590*a*b^3 - 315*b^4)*e^(-2*d*x - 2*c) - (96*a^4 +
 313*a^3*b + 19*a^2*b^2 - 1725*a*b^3 - 1575*b^4)*e^(-4*d*x - 4*c) - 4*(56*
a^4 + 80*a^3*b - 65*a^2*b^2 + 400*a*b^3 + 525*b^4)*e^(-6*d*x - 6*c) - (176
*a^4 + 135*a^3*b + 15*a^2*b^2 - 1375*a*b^3 - 1575*b^4)*e^(-8*d*x - 8*c) -
6*(8*a^4 + 45*a^2*b^2 + 150*a*b^3 + 105*b^4)*e^(-10*d*x - 10*c) + 15*(3*a^
3*b + 13*a^2*b^2 + 17*a*b^3 + 7*b^4)*e^(-12*d*x - 12*c))/((a^7 + 3*a^6*b +
 3*a^5*b^2 + a^4*b^3 + (a^7 - 5*a^6*b - 13*a^5*b^2 - 7*a^4*b^3)*e^(-2*d*x
- 2*c) - (3*a^7 + a^6*b - 23*a^5*b^2 - 21*a^4*b^3)*e^(-4*d*x - 4*c) - (3*a
^7 - 7*a^6*b + 25*a^5*b^2 + 35*a^4*b^3)*e^(-6*d*x - 6*c) + (3*a^7 - 7*a^6*
b + 25*a^5*b^2 + 35*a^4*b^3)*e^(-8*d*x - 8*c) + (3*a^7 + a^6*b - 23*a^5*b^
2 - 21*a^4*b^3)*e^(-10*d*x - 10*c) - (a^7 - 5*a^6*b - 13*a^5*b^2 - 7*a^4*b
^3)*e^(-12*d*x - 12*c) - (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*e^(-14*d*x
- 14*c))*d) - 5/8*(3*a*b + 7*b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a
 - b)/sqrt(a*b))/(sqrt(a*b)*a^4*d)
```

3.48.8 Giac [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^4}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)^4\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3),x)
```

output
```
int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3), x)
```

3.49 $\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

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3.49.1 Optimal result

Integrand size = 21, antiderivative size = 132

$$\begin{aligned}
&\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx \\
&= -\frac{3(a+8b)\log(1-\tanh(c+dx))}{16d} + \frac{3(a-8b)\log(1+\tanh(c+dx))}{16d} \\
&\quad - \frac{3a\tanh(c+dx)}{8d} - \frac{3b\tanh^2(c+dx)}{2d} + \frac{\sinh^4(c+dx)(b+a\tanh(c+dx))}{4d} \\
&\quad - \frac{\sinh^2(c+dx)\tanh(c+dx)(a+8b\tanh(c+dx))}{8d}
\end{aligned}$$

output

```
-3/16*(a+8*b)*ln(1-tanh(d*x+c))/d+3/16*(a-8*b)*ln(tanh(d*x+c)+1)/d-3/8*a*t
anh(d*x+c)/d-3/2*b*tanh(d*x+c)^2/d+1/4*sinh(d*x+c)^4*(b+a*tanh(d*x+c))/d-1
/8*sinh(d*x+c)^2*tanh(d*x+c)*(a+8*b*tanh(d*x+c))/d
```

3.49.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.70

$$\begin{aligned}
&\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx \\
&= \frac{3a(c+dx)}{8d} \\
&\quad + \frac{b\left(12\log(\cosh(c+dx)) + 2\operatorname{sech}^2(c+dx) - 4\sinh^2(c+dx) + \sinh^4(c+dx)\right)}{4d} \\
&\quad - \frac{a\sinh(2(c+dx))}{4d} + \frac{a\sinh(4(c+dx))}{32d}
\end{aligned}$$

input
```
Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3),x]
```

output
```
(3*a*(c + d*x))/(8*d) + (b*(12*Log[Cosh[c + d*x]] + 2*Sech[c + d*x]^2 - 4*
Sinh[c + d*x]^2 + Sinh[c + d*x]^4))/(4*d) - (a*Sinh[2*(c + d*x)])/(4*d) +
(a*Sinh[4*(c + d*x)])/(32*d)
```

3.49.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4146, 2335, 25, 2335, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \sin(ic+idx)^4\left(a+ib\tan(ic+idx)^3\right)dx$$

$$\downarrow 4146$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(b\tanh^3(c+dx)+a\right)}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\downarrow 2335$$

$$\frac{\frac{1}{4}\int -\frac{\tanh^3(c+dx)\left(4b\tanh^2(c+dx)+a\tanh(c+dx)+4b\right)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)+\frac{\tanh^4(c+dx)\left(a\tanh(c+dx)+b\right)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{\tanh^4(c+dx)\left(a\tanh(c+dx)+b\right)}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{1}{4}\int \frac{\tanh^3(c+dx)\left(4b\tanh^2(c+dx)+a\tanh(c+dx)+4b\right)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 2335$$

$$\frac{\frac{1}{4}\left(-\frac{1}{2}\int -\frac{3\tanh^2(c+dx)\left(a+8b\tanh(c+dx)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{\tanh^3(c+dx)\left(a+8b\tanh(c+dx)\right)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{\tanh^4(c+dx)\left(a\tanh(c+dx)+b\right)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 27$$

$$\frac{\frac{1}{4}\left(\frac{3}{2}\int \frac{\tanh^2(c+dx)(a+8b\tanh(c+dx))}{1-\tanh^2(c+dx)}d\tanh(c+dx) - \frac{\tanh^3(c+dx)(a+8b\tanh(c+dx))}{2(1-\tanh^2(c+dx))}\right) + \frac{\tanh^4(c+dx)(a\tanh(c+dx)+b)}{4(1-\tanh^2(c+dx))^2}}{d}$$

$$\downarrow 523$$

$$\frac{\frac{1}{4}\left(\frac{3}{2}\int \left(-a-8b\tanh(c+dx)+\frac{a+8b\tanh(c+dx)}{1-\tanh^2(c+dx)}\right)d\tanh(c+dx) - \frac{\tanh^3(c+dx)(a+8b\tanh(c+dx))}{2(1-\tanh^2(c+dx))}\right) + \frac{\tanh^4(c+dx)(a\tanh(c}{4(1-\tanh^2(c+d}}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{4}\left(\frac{3}{2}\left(a\operatorname{arctanh}(\tanh(c+dx)) - a\tanh(c+dx) - 4b\tanh^2(c+dx) - 4b\log\left(1-\tanh^2(c+dx)\right)\right) - \frac{\tanh^3(c+dx)(a+8}{2(1-\tanh^2}\right.}{d}$$

input
```
Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3),x]
```

output
```
((Tanh[c + d*x]^4*(b + a*Tanh[c + d*x]))/(4*(1 - Tanh[c + d*x]^2)^2) + (-1
/2*(Tanh[c + d*x]^3*(a + 8*b*Tanh[c + d*x]))/(1 - Tanh[c + d*x]^2) + (3*(a
*ArcTanh[Tanh[c + d*x]] - 4*b*Log[1 - Tanh[c + d*x]^2] - a*Tanh[c + d*x] -
 4*b*Tanh[c + d*x]^2))/2)/4)/d
```

3.49.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.49.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\sinh(dx+c)^6}{4\cosh(dx+c)^2}-\frac{3\sinh(dx+c)^4}{4\cosh(dx+c)^2}+3\ln(\cosh(dx+c))-\frac{3\tanh(dx}{2}}{d}$
default	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\sinh(dx+c)^6}{4\cosh(dx+c)^2}-\frac{3\sinh(dx+c)^4}{4\cosh(dx+c)^2}+3\ln(\cosh(dx+c))-\frac{3\tanh(dx}{2}}{d}$
risch	$\frac{3ax}{8}-3bx+\frac{e^{4dx+4c}a}{64d}+\frac{e^{4dx+4c}b}{64d}-\frac{e^{2dx+2c}a}{8d}-\frac{5e^{2dx+2c}b}{16d}+\frac{e^{-2dx-2c}a}{8d}-\frac{5e^{-2dx-2c}b}{16d}-\frac{e^{-4dx-4c}a}{64d}$

input
```
int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+b*(
1/4*sinh(d*x+c)^6/cosh(d*x+c)^2-3/4*sinh(d*x+c)^4/cosh(d*x+c)^2+3*ln(cosh(
d*x+c))-3/2*tanh(d*x+c)^2))
```

3.49.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1530 vs. 2(120) = 240.

Time = 0.28 (sec) , antiderivative size = 1530, normalized size of antiderivative = 11.59

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/64*((a + b)*cosh(d*x + c)^12 + 12*(a + b)*cosh(d*x + c)*sinh(d*x + c)^11
 + (a + b)*sinh(d*x + c)^12 - 6*(a + 3*b)*cosh(d*x + c)^10 + 6*(11*(a + b)
*cosh(d*x + c)^2 - a - 3*b)*sinh(d*x + c)^10 + 20*(11*(a + b)*cosh(d*x + c
)^3 - 3*(a + 3*b)*cosh(d*x + c))*sinh(d*x + c)^9 + 3*(8*(a - 8*b)*d*x - 5*
a - 13*b)*cosh(d*x + c)^8 + 3*(165*(a + b)*cosh(d*x + c)^4 + 8*(a - 8*b)*d
*x - 90*(a + 3*b)*cosh(d*x + c)^2 - 5*a - 13*b)*sinh(d*x + c)^8 + 24*(33*(
a + b)*cosh(d*x + c)^5 - 30*(a + 3*b)*cosh(d*x + c)^3 + (8*(a - 8*b)*d*x -
 5*a - 13*b)*cosh(d*x + c))*sinh(d*x + c)^7 + 8*(6*(a - 8*b)*d*x + 11*b)*c
osh(d*x + c)^6 + 4*(231*(a + b)*cosh(d*x + c)^6 - 315*(a + 3*b)*cosh(d*x +
 c)^4 + 12*(a - 8*b)*d*x + 21*(8*(a - 8*b)*d*x - 5*a - 13*b)*cosh(d*x + c)
^2 + 22*b)*sinh(d*x + c)^6 + 24*(33*(a + b)*cosh(d*x + c)^7 - 63*(a + 3*b)
*cosh(d*x + c)^5 + 7*(8*(a - 8*b)*d*x - 5*a - 13*b)*cosh(d*x + c)^3 + 2*(6
*(a - 8*b)*d*x + 11*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(8*(a - 8*b)*d*x
 + 5*a - 13*b)*cosh(d*x + c)^4 + 3*(165*(a + b)*cosh(d*x + c)^8 - 420*(a +
 3*b)*cosh(d*x + c)^6 + 70*(8*(a - 8*b)*d*x - 5*a - 13*b)*cosh(d*x + c)^4
+ 8*(a - 8*b)*d*x + 40*(6*(a - 8*b)*d*x + 11*b)*cosh(d*x + c)^2 + 5*a - 13
*b)*sinh(d*x + c)^4 + 4*(55*(a + b)*cosh(d*x + c)^9 - 180*(a + 3*b)*cosh(d
*x + c)^7 + 42*(8*(a - 8*b)*d*x - 5*a - 13*b)*cosh(d*x + c)^5 + 40*(6*(a -
 8*b)*d*x + 11*b)*cosh(d*x + c)^3 + 3*(8*(a - 8*b)*d*x + 5*a - 13*b)*cosh(
d*x + c))*sinh(d*x + c)^3 + 6*(a - 3*b)*cosh(d*x + c)^2 + 6*(11*(a + b)...
```

3.49.6 Sympy [F]

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \int \left(a+b\tanh^3\left(c+dx\right)\right)\sinh^4\left(c+dx\right)\,dx$$

input
```
integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)*sinh(c + d*x)**4, x)
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.47

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$= \frac{1}{64}a\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right)$$
$$+ \frac{1}{64}b\left(\frac{192(dx+c)}{d} - \frac{20e^{(-2dx-2c)} - e^{(-4dx-4c)}}{d} + \frac{192\log\left(e^{(-2dx-2c)}+1\right)}{d} - \frac{18e^{(-2dx-2c)} + 39e^{(-4dx}}{d(e^{(-4dx-4c)} + 2e^{(-}}\right.$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c
)/d - e^(-4*d*x - 4*c)/d) + 1/64*b*(192*(d*x + c)/d - (20*e^(-2*d*x - 2*c)
 - e^(-4*d*x - 4*c))/d + 192*log(e^(-2*d*x - 2*c) + 1)/d - (18*e^(-2*d*x -
 2*c) + 39*e^(-4*d*x - 4*c) - 108*e^(-6*d*x - 6*c) - 1)/(d*(e^(-4*d*x - 4*
c) + 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c))))
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.54

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$= \frac{24(dx+c)(a-8b) + ae^{(4dx+4c)} + be^{(4dx+4c)} - 8ae^{(2dx+2c)} - 20be^{(2dx+2c)} + 192b\log\left(e^{(2dx+2c)}+1\right) -}{64d}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
1/64*(24*(d*x + c)*(a - 8*b) + a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) - 8*a
*e^(2*d*x + 2*c) - 20*b*e^(2*d*x + 2*c) + 192*b*log(e^(2*d*x + 2*c) + 1) -
 (9*a*e^(8*d*x + 8*c) + 72*b*e^(8*d*x + 8*c) + 10*a*e^(6*d*x + 6*c) + 36*b
*e^(6*d*x + 6*c) - 6*a*e^(4*d*x + 4*c) + 111*b*e^(4*d*x + 4*c) - 6*a*e^(2*
d*x + 2*c) + 18*b*e^(2*d*x + 2*c) + a - b)/(e^(4*d*x + 4*c) + e^(2*d*x + 2
*c))^2)/d
```

3.49.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.18

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = x\left(\frac{3a}{8}-3b\right)+\frac{2b}{d\left(\mathrm{e}^{2c+2dx}+1\right)}+\frac{\mathrm{e}^{4c+4dx}(a+b)}{64d}$$
$$-\frac{2b}{d\left(2\,\mathrm{e}^{2c+2dx}+\mathrm{e}^{4c+4dx}+1\right)}$$
$$-\frac{\mathrm{e}^{-4c-4dx}(a-b)}{64d}+\frac{3b\ln\left(\mathrm{e}^{2c}\mathrm{e}^{2dx}+1\right)}{d}$$
$$+\frac{\mathrm{e}^{-2c-2dx}(2a-5b)}{16d}-\frac{\mathrm{e}^{2c+2dx}(2a+5b)}{16d}$$

input
```
int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^3),x)
```

output
```
x*((3*a)/8 - 3*b) + (2*b)/(d*(exp(2*c + 2*d*x) + 1)) + (exp(4*c + 4*d*x)*(
a + b))/(64*d) - (2*b)/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (
exp(- 4*c - 4*d*x)*(a - b))/(64*d) + (3*b*log(exp(2*c)*exp(2*d*x) + 1))/d
+ (exp(- 2*c - 2*d*x)*(2*a - 5*b))/(16*d) - (exp(2*c + 2*d*x)*(2*a + 5*b))
/(16*d)
```

3.50 $\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

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3.50.1 Optimal result

Integrand size = 21, antiderivative size = 98

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = {}& \frac{5b\arctan(\sinh(c+dx))}{2d} \\ & - \frac{a\cosh(c+dx)}{d} + \frac{a\cosh^3(c+dx)}{3d} \\ & - \frac{5b\sinh(c+dx)}{2d} + \frac{5b\sinh^3(c+dx)}{6d} \\ & - \frac{b\sinh^3(c+dx)\tanh^2(c+dx)}{2d}\end{aligned}$$

output

```
5/2*b*arctan(sinh(d*x+c))/d-a*cosh(d*x+c)/d+1/3*a*cosh(d*x+c)^3/d-5/2*b*si
nh(d*x+c)/d+5/6*b*sinh(d*x+c)^3/d-1/2*b*sinh(d*x+c)^3*tanh(d*x+c)^2/d
```

3.50.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{5b\arctan(\sinh(c+dx))}{2d} - \frac{3a\cosh(c+dx)}{4d} + \frac{a\cosh(3(c+dx))}{12d} - \frac{5b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d} - \frac{5b\sinh(c+dx)\tanh^2(c+dx)}{3d} + \frac{b\sinh^3(c+dx)\tanh^2(c+dx)}{3d}$$

input
```
Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3),x]
```

output
```
(5*b*ArcTan[Sinh[c + d*x]])/(2*d) - (3*a*Cosh[c + d*x])/(4*d) + (a*Cosh[3*
(c + d*x)])/(12*d) - (5*b*Sech[c + d*x]*Tanh[c + d*x])/(2*d) - (5*b*Sinh[c
 + d*x]*Tanh[c + d*x]^2)/(3*d) + (b*Sinh[c + d*x]^3*Tanh[c + d*x]^2)/(3*d)
```

3.50.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int i\sin(ic+idx)^3\left(a+ib\tan(ic+idx)^3\right)dx$$

$$\downarrow 26$$

$$i\int \sin(ic+idx)^3\left(ib\tan(ic+idx)^3+a\right)dx$$

$$\downarrow 4149$$

$$i \int \left(-ib \tanh^3(c+dx) \sinh^3(c+dx) - ia \sinh^3(c+dx)\right) dx$$

$$\downarrow 2009$$

$$i\left(-\frac{ia \cosh^3(c+dx)}{3d} + \frac{ia \cosh(c+dx)}{d} - \frac{5ib \arctan(\sinh(c+dx))}{2d} - \frac{5ib \sinh^3(c+dx)}{6d} + \frac{5ib \sinh(c+dx)}{2d} + \frac{ib \si}{}\right.$$

input
```
Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3),x]
```

output
```
I*((((-5*I)/2)*b*ArcTan[Sinh[c + d*x]])/d + (I*a*Cosh[c + d*x])/d - ((I/3)
*a*Cosh[c + d*x]^3)/d + (((5*I)/2)*b*Sinh[c + d*x])/d - (((5*I)/6)*b*Sinh[
c + d*x]^3)/d + ((I/2)*b*Sinh[c + d*x]^3*Tanh[c + d*x]^2)/d)
```

3.50.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

3.50.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^5}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)^3}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{5\ \mathrm{sech}(dx+c)\tanh(dx+c)}{2}+5\arctan\left(e^{d}\right.\right.}{d}$
default	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^5}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)^3}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{5\ \mathrm{sech}(dx+c)\tanh(dx+c)}{2}+5\arctan\left(e^{d}\right.\right.}{d}$
risch	$\frac{e^{3dx+3c}a}{24d}+\frac{e^{3dx+3c}b}{24d}-\frac{3e^{dx+c}a}{8d}-\frac{9e^{dx+c}b}{8d}-\frac{3e^{-dx-c}a}{8d}+\frac{9e^{-dx-c}b}{8d}+\frac{e^{-3dx-3c}a}{24d}-\frac{e^{-3dx-3c}b}{24d}-\frac{be^{d}}{d}$

input
```
int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+b*(1/3*sinh(d*x+c)^5/cosh(d*x+
c)^2-5/3*sinh(d*x+c)^3/cosh(d*x+c)^2-5*sinh(d*x+c)/cosh(d*x+c)^2+5/2*sech(
d*x+c)*tanh(d*x+c)+5*arctan(exp(d*x+c))))
```

3.50.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1070 vs. $2(88) = 176$.

Time = 0.28 (sec) , antiderivative size = 1070, normalized size of antiderivative = 10.92

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/24*((a + b)*cosh(d*x + c)^10 + 10*(a + b)*cosh(d*x + c)*sinh(d*x + c)^9
+ (a + b)*sinh(d*x + c)^10 - (7*a + 25*b)*cosh(d*x + c)^8 + (45*(a + b)*co
sh(d*x + c)^2 - 7*a - 25*b)*sinh(d*x + c)^8 + 8*(15*(a + b)*cosh(d*x + c)^
3 - (7*a + 25*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(13*a + 25*b)*cosh(d*x
 + c)^6 + 2*(105*(a + b)*cosh(d*x + c)^4 - 14*(7*a + 25*b)*cosh(d*x + c)^2
 - 13*a - 25*b)*sinh(d*x + c)^6 + 4*(63*(a + b)*cosh(d*x + c)^5 - 14*(7*a
+ 25*b)*cosh(d*x + c)^3 - 3*(13*a + 25*b)*cosh(d*x + c))*sinh(d*x + c)^5 -
 2*(13*a - 25*b)*cosh(d*x + c)^4 + 2*(105*(a + b)*cosh(d*x + c)^6 - 35*(7*
a + 25*b)*cosh(d*x + c)^4 - 15*(13*a + 25*b)*cosh(d*x + c)^2 - 13*a + 25*b
)*sinh(d*x + c)^4 + 8*(15*(a + b)*cosh(d*x + c)^7 - 7*(7*a + 25*b)*cosh(d*
x + c)^5 - 5*(13*a + 25*b)*cosh(d*x + c)^3 - (13*a - 25*b)*cosh(d*x + c))*
sinh(d*x + c)^3 - (7*a - 25*b)*cosh(d*x + c)^2 + (45*(a + b)*cosh(d*x + c)
^8 - 28*(7*a + 25*b)*cosh(d*x + c)^6 - 30*(13*a + 25*b)*cosh(d*x + c)^4 -
12*(13*a - 25*b)*cosh(d*x + c)^2 - 7*a + 25*b)*sinh(d*x + c)^2 + 120*(b*co
sh(d*x + c)^7 + 7*b*cosh(d*x + c)*sinh(d*x + c)^6 + b*sinh(d*x + c)^7 + 2*
b*cosh(d*x + c)^5 + (21*b*cosh(d*x + c)^2 + 2*b)*sinh(d*x + c)^5 + 5*(7*b*
cosh(d*x + c)^3 + 2*b*cosh(d*x + c))*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 +
 (35*b*cosh(d*x + c)^4 + 20*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^3 + (21*b
*cosh(d*x + c)^5 + 20*b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)
^2 + (7*b*cosh(d*x + c)^6 + 10*b*cosh(d*x + c)^4 + 3*b*cosh(d*x + c)^2)...
```

3.50.6 Sympy [F]

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \int \left(a+b\tanh^3\left(c+dx\right)\right)\sinh^3\left(c+dx\right)dx$$

input
```
integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)*sinh(c + d*x)**3, x)
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.78

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$= \frac{1}{24}b\left(\frac{27\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}-\frac{120\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{25\,e^{(-2\,dx-2\,c)}+77\,e^{(-4\,dx-4\,c)}+3\,e^{(-6\,dx-6\,c)}-1}{d\left(e^{(-3\,dx-3\,c)}+2\,e^{(-5\,dx-5\,c)}+e^{(-7\,dx-7\,c)}\right)}\right.$$
$$\left.+\frac{1}{24}a\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)\right.$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/24*b*((27*e^(-d*x - c) - e^(-3*d*x - 3*c))/d - 120*arctan(e^(-d*x - c))/
d - (25*e^(-2*d*x - 2*c) + 77*e^(-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) - 1)/(
d*(e^(-3*d*x - 3*c) + 2*e^(-5*d*x - 5*c) + e^(-7*d*x - 7*c)))) + 1/24*a*(e
^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d
)
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.35

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$= \frac{120\,b\arctan\left(e^{(dx+c)}\right)+ae^{(3\,dx+3\,c)}+be^{(3\,dx+3\,c)}-9\,ae^{(dx+c)}-27\,be^{(dx+c)}-\left(9\,ae^{(2\,dx+2\,c)}-27\,be^{(2\,dx+2\,c)}-\right.}{24\,d}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
1/24*(120*b*arctan(e^(d*x + c)) + a*e^(3*d*x + 3*c) + b*e^(3*d*x + 3*c) -
9*a*e^(d*x + c) - 27*b*e^(d*x + c) - (9*a*e^(2*d*x + 2*c) - 27*b*e^(2*d*x
+ 2*c) - a + b)*e^(-3*d*x - 3*c) - 24*(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/
(e^(2*d*x + 2*c) + 1)^2)/d
```

3.50.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.74

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{e^{3c+3dx}(a+b)}{24d} + \frac{5\operatorname{atan}\left(\frac{b e^{dx} e^{c}\sqrt{d^2}}{d\sqrt{b^2}}\right)\sqrt{b^2}}{\sqrt{d^2}} + \frac{e^{-3c-3dx}(a-b)}{24d} - \frac{e^{c+dx}(3a+9b)}{8d} - \frac{e^{-c-dx}(3a-9b)}{8d} - \frac{b e^{c+dx}}{d\left(e^{2c+2dx}+1\right)} + \frac{2b e^{c+dx}}{d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}$$

input
```
int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^3),x)
```

output
```
(exp(3*c + 3*d*x)*(a + b))/(24*d) + (5*atan((b*exp(d*x)*exp(c)*(d^2)^(1/2)
)/(d*(b^2)^(1/2)))*(b^2)^(1/2))/(d^2)^(1/2) + (exp(- 3*c - 3*d*x)*(a - b))
/(24*d) - (exp(c + d*x)*(3*a + 9*b))/(8*d) - (exp(- c - d*x)*(3*a - 9*b))/
(8*d) - (b*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)) + (2*b*exp(c + d*x))/(
d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.51 $\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

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3.51.1 Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{(a+4b)\log(1-\tanh(c+dx))}{4d} - \frac{(a-4b)\log(1+\tanh(c+dx))}{4d} + \frac{a\tanh(c+dx)}{2d} + \frac{b\tanh^2(c+dx)}{2d} + \frac{\sinh^2(c+dx)(b+a\tanh(c+dx))}{2d}$$

output
```
1/4*(a+4*b)*ln(1-tanh(d*x+c))/d-1/4*(a-4*b)*ln(tanh(d*x+c)+1)/d+1/2*a*tanh
(d*x+c)/d+1/2*b*tanh(d*x+c)^2/d+1/2*sinh(d*x+c)^2*(b+a*tanh(d*x+c))/d
```

3.51.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.69

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{a(-c-dx)}{2d} - \frac{b\left(4\log(\cosh(c+dx)) + \operatorname{sech}^2(c+dx) - \sinh^2(c+dx)\right)}{2d} + \frac{a\sinh(2(c+dx))}{4d}$$

input
```
Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3),x]
```

output
```
(a*(-c - d*x))/(2*d) - (b*(4*Log[Cosh[c + d*x]] + Sech[c + d*x]^2 - Sinh[c
 + d*x]^2))/(2*d) + (a*Sinh[2*(c + d*x)])/(4*d)
```

3.51.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 25, 4146, 2335, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int -\sin(ic+idx)^2\left(a+ib\tan(ic+idx)^3\right)dx$$

$$\downarrow 25$$

$$-\int \sin(ic+idx)^2\left(ib\tan(ic+idx)^3+a\right)dx$$

$$\downarrow 4146$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(b\tanh^3(c+dx)+a\right)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 2335$$

$$\frac{\frac{1}{2}\int -\frac{\tanh(c+dx)\left(2b\tanh^2(c+dx)+a\tanh(c+dx)+2b\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{\tanh^2(c+dx)\left(a\tanh(c+dx)+b\right)}{2\left(1-\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{\tanh^2(c+dx)\left(a\tanh(c+dx)+b\right)}{2\left(1-\tanh^2(c+dx)\right)}-\frac{1}{2}\int \frac{\tanh(c+dx)\left(2b\tanh^2(c+dx)+a\tanh(c+dx)+2b\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 2333$$

$$\frac{\frac{\tanh^2(c+dx)\left(a\tanh(c+dx)+b\right)}{2\left(1-\tanh^2(c+dx)\right)}-\frac{1}{2}\int \left(-a-2b\tanh(c+dx)+\frac{a+4b\tanh(c+dx)}{1-\tanh^2(c+dx)}\right)d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{2}\left(-a\operatorname{arctanh}(\tanh(c+dx))+a\tanh(c+dx)+b\tanh^2(c+dx)+2b\log\left(1-\tanh^2(c+dx)\right)\right)+\frac{\tanh^2(c+dx)(a\tanh}{2(1-\tanh^2(c+}}{d}$$

input
```
Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3),x]
```

output
```
((Tanh[c + d*x]^2*(b + a*Tanh[c + d*x]))/(2*(1 - Tanh[c + d*x]^2)) + (-(a*
ArcTanh[Tanh[c + d*x]]) + 2*b*Log[1 - Tanh[c + d*x]^2] + a*Tanh[c + d*x] +
 b*Tanh[c + d*x]^2)/2)/d
```

3.51.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.51.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.68

method	result
derivativedivides	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\left(\frac{\sinh(dx+c)^4}{2\cosh(dx+c)^2}-2\ln(\cosh(dx+c))+\tanh(dx+c)^2\right)}{d}$
default	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\left(\frac{\sinh(dx+c)^4}{2\cosh(dx+c)^2}-2\ln(\cosh(dx+c))+\tanh(dx+c)^2\right)}{d}$
risch	$-\frac{ax}{2}+2bx+\frac{e^{2dx+2c}a}{8d}+\frac{e^{2dx+2c}b}{8d}-\frac{e^{-2dx-2c}a}{8d}+\frac{e^{-2dx-2c}b}{8d}+\frac{4bc}{d}-\frac{2be^{2dx+2c}}{d\left(e^{2dx+2c}+1\right)^2}-\frac{2b\ln\left(e^{2dx+2c}\right.}{d}$

input

```
int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output

```
1/d*(a*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b*(1/2*sinh(d*x+c)^4/co
sh(d*x+c)^2-2*ln(cosh(d*x+c))+tanh(d*x+c)^2))
```

3.51.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(90) = 180$.

Time = 0.32 (sec) , antiderivative size = 924, normalized size of antiderivative = 9.24

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \text{Too large to display}$$

input

```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/8*((a + b)*cosh(d*x + c)^8 + 8*(a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (
a + b)*sinh(d*x + c)^8 - 2*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c)^6 - 2*(
2*(a - 4*b)*d*x - 14*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c)^6 + 4*
(14*(a + b)*cosh(d*x + c)^3 - 3*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c))*s
inh(d*x + c)^5 - 2*(4*(a - 4*b)*d*x + 7*b)*cosh(d*x + c)^4 + 2*(35*(a + b)
*cosh(d*x + c)^4 - 4*(a - 4*b)*d*x - 15*(2*(a - 4*b)*d*x - a - b)*cosh(d*x
 + c)^2 - 7*b)*sinh(d*x + c)^4 + 8*(7*(a + b)*cosh(d*x + c)^5 - 5*(2*(a -
4*b)*d*x - a - b)*cosh(d*x + c)^3 - (4*(a - 4*b)*d*x + 7*b)*cosh(d*x + c))
*sinh(d*x + c)^3 - 2*(2*(a - 4*b)*d*x + a - b)*cosh(d*x + c)^2 + 2*(14*(a
+ b)*cosh(d*x + c)^6 - 15*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c)^4 - 2*(a
 - 4*b)*d*x - 6*(4*(a - 4*b)*d*x + 7*b)*cosh(d*x + c)^2 - a + b)*sinh(d*x
+ c)^2 - 16*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sin
h(d*x + c)^6 + 2*b*cosh(d*x + c)^4 + (15*b*cosh(d*x + c)^2 + 2*b)*sinh(d*x
 + c)^4 + 4*(5*b*cosh(d*x + c)^3 + 2*b*cosh(d*x + c))*sinh(d*x + c)^3 + b*
cosh(d*x + c)^2 + (15*b*cosh(d*x + c)^4 + 12*b*cosh(d*x + c)^2 + b)*sinh(d
*x + c)^2 + 2*(3*b*cosh(d*x + c)^5 + 4*b*cosh(d*x + c)^3 + b*cosh(d*x + c)
)*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*
(2*(a + b)*cosh(d*x + c)^7 - 3*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c)^5 -
 2*(4*(a - 4*b)*d*x + 7*b)*cosh(d*x + c)^3 - (2*(a - 4*b)*d*x + a - b)*cos
h(d*x + c))*sinh(d*x + c) - a + b)/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + ...
```

3.51.6 Sympy [F]

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \int \left(a+b\tanh^3\left(c+dx\right)\right)\sinh^2\left(c+dx\right)dx$$

input
```
integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)*sinh(c + d*x)**2, x)
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.41

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = -\frac{1}{8}a\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)$$
$$-\frac{1}{8}b\left(\frac{16(dx+c)}{d}-\frac{e^{(-2dx-2c)}}{d}+\frac{16\log\left(e^{(-2dx-2c)}+1\right)}{d}-\frac{2e^{(-2dx-2c)}-15e^{(-4dx-4c)}+1}{d(e^{(-2dx-2c)}+2e^{(-4dx-4c)}+e^{(-6dx-6c)})}\right)$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/8*b*(16*(d*x + c
)/d - e^(-2*d*x - 2*c)/d + 16*log(e^(-2*d*x - 2*c) + 1)/d - (2*e^(-2*d*x -
 2*c) - 15*e^(-4*d*x - 4*c) + 1)/(d*(e^(-2*d*x - 2*c) + 2*e^(-4*d*x - 4*c)
 + e^(-6*d*x - 6*c))))
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.40

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx =$$
$$-\frac{4(dx+c)(a-4b)-ae^{(2dx+2c)}-be^{(2dx+2c)}-\left(2ae^{(2dx+2c)}-8be^{(2dx+2c)}-a+b\right)e^{(-2dx-2c)}+16b\log}{8d}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
-1/8*(4*(d*x + c)*(a - 4*b) - a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) - (2*a
*e^(2*d*x + 2*c) - 8*b*e^(2*d*x + 2*c) - a + b)*e^(-2*d*x - 2*c) + 16*b*lo
g(e^(2*d*x + 2*c) + 1) - 8*(3*b*e^(4*d*x + 4*c) + 4*b*e^(2*d*x + 2*c) + 3*
b)/(e^(2*d*x + 2*c) + 1)^2)/d
```

3.51.9 Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.15

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{e^{2c+2dx}(a+b)}{8d} - \frac{2b}{d\left(e^{2c+2dx}+1\right)} - x\left(\frac{a}{2}-2b\right) + \frac{2b}{d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)} - \frac{e^{-2c-2dx}(a-b)}{8d} - \frac{2b\ln\left(e^{2c}e^{2dx}+1\right)}{d}$$

input
```
int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^3),x)
```

output
```
(exp(2*c + 2*d*x)*(a + b))/(8*d) - (2*b)/(d*(exp(2*c + 2*d*x) + 1)) - x*(a
/2 - 2*b) + (2*b)/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (exp(-
 2*c - 2*d*x)*(a - b))/(8*d) - (2*b*log(exp(2*c)*exp(2*d*x) + 1))/d
```

3.52 $\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

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3.52.1 Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = -\frac{3b\arctan(\sinh(c+dx))}{2d}+\frac{a\cosh(c+dx)}{d}+\frac{3b\sinh(c+dx)}{2d}-\frac{b\sinh(c+dx)\tanh^2(c+dx)}{2d}$$

output
```
-3/2*b*arctan(sinh(d*x+c))/d+a*cosh(d*x+c)/d+3/2*b*sinh(d*x+c)/d-1/2*b*sin
h(d*x+c)*tanh(d*x+c)^2/d
```

3.52.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = -\frac{3b\arctan(\sinh(c+dx))}{2d}+\frac{a\cosh(c)\cosh(dx)}{d}+\frac{a\sinh(c)\sinh(dx)}{d}+\frac{3b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}+\frac{b\sinh(c+dx)\tanh^2(c+dx)}{d}$$

input
```
Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3),x]
```

output
```
(-3*b*ArcTan[Sinh[c + d*x]])/(2*d) + (a*Cosh[c]*Cosh[d*x])/d + (a*Sinh[c]*
Sinh[d*x])/d + (3*b*Sech[c + d*x]*Tanh[c + d*x])/(2*d) + (b*Sinh[c + d*x]*
Tanh[c + d*x]^2)/d
```

3.52.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int -i\sin(ic+idx)\left(a+ib\tan(ic+idx)^3\right)\,dx$$

$$\downarrow 26$$

$$-i\int \sin(ic+idx)\left(ib\tan(ic+idx)^3+a\right)\,dx$$

$$\downarrow 4149$$

$$-i\int \left(ib\sinh(c+dx)\tanh^3(c+dx)+ia\sinh(c+dx)\right)\,dx$$

$$\downarrow 2009$$

$$-i\left(\frac{ia\cosh(c+dx)}{d}-\frac{3ib\arctan(\sinh(c+dx))}{2d}+\frac{3ib\sinh(c+dx)}{2d}-\frac{ib\sinh(c+dx)\tanh^2(c+dx)}{2d}\right)$$

input
```
Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3),x]
```

output
```
(-I)*((((-3*I)/2)*b*ArcTan[Sinh[c + d*x]])/d + (I*a*Cosh[c + d*x])/d + (((
3*I)/2)*b*Sinh[c + d*x])/d - ((I/2)*b*Sinh[c + d*x]*Tanh[c + d*x]^2)/d)
```

3.52.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

3.52.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16

method	result	siz
derivativedivides	$\frac{a\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\ \mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan\left(\mathrm{e}^{dx+c}\right)\right)}{d}$	73
default	$\frac{a\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\ \mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan\left(\mathrm{e}^{dx+c}\right)\right)}{d}$	73
risch	$\frac{\mathrm{e}^{dx+c}a}{2d}+\frac{\mathrm{e}^{dx+c}b}{2d}+\frac{\mathrm{e}^{-dx-c}a}{2d}-\frac{\mathrm{e}^{-dx-c}b}{2d}+\frac{b\,\mathrm{e}^{dx+c}\left(\mathrm{e}^{2dx+2c}-1\right)}{d\left(\mathrm{e}^{2dx+2c}+1\right)^2}+\frac{3ib\ln\left(\mathrm{e}^{dx+c}-i\right)}{2d}-\frac{3ib\ln\left(\mathrm{e}^{dx+c}+i\right)}{2d}$	12

input
```
int(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*cosh(d*x+c)+b*(sinh(d*x+c)^3/cosh(d*x+c)^2+3*sinh(d*x+c)/cosh(d*x+c
)^2-3/2*sech(d*x+c)*tanh(d*x+c)-3*arctan(exp(d*x+c))))
```

3.52.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(57) = 114$.

Time $= 0.33$ (sec) , antiderivative size $= 528$, normalized size of antiderivative $= 8.38$

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$= \frac{(a+b)\cosh(dx+c)^6 + 6(a+b)\cosh(dx+c)\sinh(dx+c)^5 + (a+b)\sinh(dx+c)^6 + 3(a+b)\cosh(dx$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/2*((a + b)*cosh(d*x + c)^6 + 6*(a + b)*cosh(d*x + c)*sinh(d*x + c)^5 + (
a + b)*sinh(d*x + c)^6 + 3*(a + b)*cosh(d*x + c)^4 + 3*(5*(a + b)*cosh(d*x
 + c)^2 + a + b)*sinh(d*x + c)^4 + 4*(5*(a + b)*cosh(d*x + c)^3 + 3*(a + b
)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)^2 + 3*(5*(a + b
)*cosh(d*x + c)^4 + 6*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 - 6
*(b*cosh(d*x + c)^5 + 5*b*cosh(d*x + c)*sinh(d*x + c)^4 + b*sinh(d*x + c)^
5 + 2*b*cosh(d*x + c)^3 + 2*(5*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^3 + 2*
(5*b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^2 + b*cosh(d*x + c
) + (5*b*cosh(d*x + c)^4 + 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c))*arctan(
cosh(d*x + c) + sinh(d*x + c)) + 6*((a + b)*cosh(d*x + c)^5 + 2*(a + b)*co
sh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)/(d*cosh(d*x
+ c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 + 2*d*cosh(
d*x + c)^3 + 2*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + 2*(5*d*cosh(d*x
 + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*d*cosh
(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

3.52.6 Sympy [F]

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \int \left(a+b\tanh^3(c+dx)\right)\sinh(c+dx)\,dx$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)*sinh(c + d*x), x)
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.67

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$= \frac{1}{2}b\left(\frac{6\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)}}{d} + \frac{4\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} + 1}{d(e^{(-dx-c)} + 2\,e^{(-3\,dx-3\,c)} + e^{(-5\,dx-5\,c)})}\right)$$
$$+ \frac{a\cosh(dx+c)}{d}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/2*b*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) - e
^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x - 5
*c)))) + a*cosh(d*x + c)/d
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$= -\frac{6\,b\arctan\left(e^{(dx+c)}\right) - ae^{(dx+c)} - be^{(dx+c)} - (a-b)e^{(-dx-c)} - \frac{2\left(be^{(3\,dx+3\,c)} - be^{(dx+c)}\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^2}}{2\,d}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
-1/2*(6*b*arctan(e^(d*x + c)) - a*e^(d*x + c) - b*e^(d*x + c) - (a - b)*e^
(-d*x - c) - 2*(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^2
)/d
```

3.52.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.03

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{\mathrm{e}^{-c-dx}\,(a-b)}{2\,d} - \frac{3\,\mathrm{atan}\left(\frac{b\,\mathrm{e}^{dx}\,\mathrm{e}^{c}\,\sqrt{d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{d^2}} + \frac{\mathrm{e}^{c+dx}\,(a+b)}{2\,d} + \frac{b\,\mathrm{e}^{c+dx}}{d\,\left(\mathrm{e}^{2\,c+2\,dx}+1\right)} - \frac{2\,b\,\mathrm{e}^{c+dx}}{d\,\left(2\,\mathrm{e}^{2\,c+2\,dx}+\mathrm{e}^{4\,c+4\,dx}+1\right)}$$

input
```
int(sinh(c + d*x)*(a + b*tanh(c + d*x)^3),x)
```

output
```
(exp(- c - d*x)*(a - b))/(2*d) - (3*atan((b*exp(d*x)*exp(c)*(d^2)^(1/2))/(
d*(b^2)^(1/2)))*(b^2)^(1/2))/(d^2)^(1/2) + (exp(c + d*x)*(a + b))/(2*d) +
(b*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)) - (2*b*exp(c + d*x))/(d*(2*exp
(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.53 $\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

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3.53.1 Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{b\arctan(\sinh(c+dx))}{2d} - \frac{a\operatorname{arctanh}(\cosh(c+dx))}{d} - \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

output
```
1/2*b*arctan(sinh(d*x+c))/d-a*arctanh(cosh(d*x+c))/d-1/2*b*sech(d*x+c)*tan
h(d*x+c)/d
```

3.53.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.53

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{b\arctan(\sinh(c+dx))}{2d} - \frac{a\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{a\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

input
```
Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3),x]
```

output
```
(b*ArcTan[Sinh[c + d*x]])/(2*d) - (a*Log[Cosh[c/2 + (d*x)/2]])/d + (a*Log[
Sinh[c/2 + (d*x)/2]])/d - (b*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

3.53.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{i\left(a+ib\tan(ic+idx)^3\right)}{\sin(ic+idx)}dx$$

$$\downarrow 26$$

$$i\int \frac{ib\tan(ic+idx)^3+a}{\sin(ic+idx)}dx$$

$$\downarrow 4149$$

$$i\int \left(-ib\operatorname{sech}(c+dx)\tanh^2(c+dx)-ia\operatorname{csch}(c+dx)\right)dx$$

$$\downarrow 2009$$

$$i\left(\frac{ia\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{ib\arctan(\sinh(c+dx))}{2d}+\frac{ib\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}\right)$$

input
```
Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3),x]
```

output
```
I*(((-1/2*I)*b*ArcTan[Sinh[c + d*x]])/d + (I*a*ArcTanh[Cosh[c + d*x]])/d +
 ((I/2)*b*Sech[c + d*x]*Tanh[c + d*x])/d)
```

3.53.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

3.53.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$\frac{-2a\ \operatorname{arctanh}\left(e^{dx+c}\right)+b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)}{d}$	56
default	$\frac{-2a\ \operatorname{arctanh}\left(e^{dx+c}\right)+b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)}{d}$	56
risch	$-\frac{b\,e^{dx+c}\left(e^{2dx+2c}-1\right)}{d\left(e^{2dx+2c}+1\right)^2}+\frac{ib\ln\left(e^{dx+c}+i\right)}{2d}-\frac{ib\ln\left(e^{dx+c}-i\right)}{2d}+\frac{a\ln\left(e^{dx+c}-1\right)}{d}-\frac{a\ln\left(e^{dx+c}+1\right)}{d}$	101

input
```
int(csch(d*x+c)*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a*arctanh(exp(d*x+c))+b*(-sinh(d*x+c)/cosh(d*x+c)^2+1/2*sech(d*x+c
)*tanh(d*x+c)+arctan(exp(d*x+c))))
```

3.53.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(45) = 90$.

Time $= 0.28$ (sec) , antiderivative size $= 522$, normalized size of antiderivative $= 10.65$

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx =$$

$$-\frac{b\cosh(dx+c)^3+3\,b\cosh(dx+c)\sinh(dx+c)^2+b\sinh(dx+c)^3-\left(b\cosh(dx+c)^4+4\,b\cosh(dx+\right.}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
-(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^
3 - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x +
c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 +
 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b)*arctan(cosh(d*
x + c) + sinh(d*x + c)) - b*cosh(d*x + c) + (a*cosh(d*x + c)^4 + 4*a*cosh(
d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*a*cosh(d*x + c)^2 + 2*(3*
a*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + a*cosh(d*x
 + c))*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a*cosh
(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*a*
cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2 + 4*(a*cosh(
d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(
d*x + c) - 1) + (3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c))/(d*cosh(d*x + c)^
4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x +
 c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3
 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

3.53.6 Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \int \left(a+b\tanh^3(c+dx)\right)\operatorname{csch}(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x), x)
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$= -b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)}\right) + \frac{a\log\left(\tanh\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}{d}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2
*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a*log(tanh(1/2*d*x + 1/2*c))/d
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.51

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$= \frac{b\arctan\left(e^{(dx+c)}\right) - a\log\left(e^{(dx+c)}+1\right) + a\log\left(\left|e^{(dx+c)}-1\right|\right) - \frac{be^{(3\,dx+3\,c)} - be^{(dx+c)}}{\left(e^{(2\,dx+2\,c)}+1\right)^2}}{d}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
(b*arctan(e^(d*x + c)) - a*log(e^(d*x + c) + 1) + a*log(abs(e^(d*x + c) -
1)) - (b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^2)/d
```

3.53.9 Mupad [B] (verification not implemented)

Time = 3.24 (sec) , antiderivative size = 233, normalized size of antiderivative = 4.76

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$= \frac{2\,b\,\mathrm{e}^{c+dx}}{d+2\,d\,\mathrm{e}^{2\,c+2\,dx}+d\,\mathrm{e}^{4\,c+4\,dx}} - \frac{a\,\ln\left(-8\,a\,b^2-32\,a^3-32\,a^3\,\mathrm{e}^{dx}\,\mathrm{e}^{c}-8\,a\,b^2\,\mathrm{e}^{dx}\,\mathrm{e}^{c}\right)}{d}$$
$$+\frac{a\,\ln\left(8\,a\,b^2+32\,a^3-32\,a^3\,\mathrm{e}^{dx}\,\mathrm{e}^{c}-8\,a\,b^2\,\mathrm{e}^{dx}\,\mathrm{e}^{c}\right)}{d}$$
$$-\frac{b\left(\ln\left(4\,b^3\,\mathrm{e}^{dx}\,\mathrm{e}^{c}+16\,a^2\,b\,\mathrm{e}^{dx}\,\mathrm{e}^{c}-a^2\,b\,16\mathrm{i}-b^3\,4\mathrm{i}\right)\,1\mathrm{i}-\ln\left(4\,b^3\,\mathrm{e}^{dx}\,\mathrm{e}^{c}+16\,a^2\,b\,\mathrm{e}^{dx}\,\mathrm{e}^{c}+a^2\,b\,16\mathrm{i}+b^3\,4\mathrm{i}\right)\,1\right.}{2\,d}$$
$$-\frac{b\,\mathrm{e}^{c+dx}}{d+d\,\mathrm{e}^{2\,c+2\,dx}}$$

input
```
int((a + b*tanh(c + d*x)^3)/sinh(c + d*x),x)
```

output
```
(2*b*exp(c + d*x))/(d + 2*d*exp(2*c + 2*d*x) + d*exp(4*c + 4*d*x)) - (a*lo
g(- 8*a*b^2 - 32*a^3 - 32*a^3*exp(d*x)*exp(c) - 8*a*b^2*exp(d*x)*exp(c)))/
d + (a*log(8*a*b^2 + 32*a^3 - 32*a^3*exp(d*x)*exp(c) - 8*a*b^2*exp(d*x)*ex
p(c)))/d - (b*(log(4*b^3*exp(d*x)*exp(c) - b^3*4i - a^2*b*16i + 16*a^2*b*e
xp(d*x)*exp(c))*1i - log(a^2*b*16i + b^3*4i + 4*b^3*exp(d*x)*exp(c) + 16*a
^2*b*exp(d*x)*exp(c))*1i))/(2*d) - (b*exp(c + d*x))/(d + d*exp(2*c + 2*d*x
))
```

3.54 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

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3.54.1 Optimal result

Integrand size = 21, antiderivative size = 29

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = -\frac{a\coth(c+dx)}{d}+\frac{b\tanh^2(c+dx)}{2d}$$

output
```
-a*coth(d*x+c)/d+1/2*b*tanh(d*x+c)^2/d
```

3.54.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = -\frac{a\coth(c+dx)}{d}-\frac{b\operatorname{sech}^2(c+dx)}{2d}$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3),x]
```

output
```
-((a*Coth[c + d*x])/d) - (b*Sech[c + d*x]^2)/(2*d)
```

3.54.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 25, 4146, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int -\frac{a+ib\tan(ic+idx)^3}{\sin(ic+idx)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{ib\tan(ic+idx)^3+a}{\sin(ic+idx)^2}dx$$

$$\downarrow 4146$$

$$\frac{\int \coth^2(c+dx)\left(b\tanh^3(c+dx)+a\right)d\tanh(c+dx)}{d}$$

$$\downarrow 802$$

$$\frac{\int \left(a\coth^2(c+dx)+b\tanh(c+dx)\right)d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{2}b\tanh^2(c+dx)-a\coth(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3),x]
```

output
```
(-(a*Coth[c + d*x]) + (b*Tanh[c + d*x]^2)/2)/d
```

3.54.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.54.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{-\frac{a}{\tanh(dx+c)}+\frac{b\tanh(dx+c)^2}{2}}{d}$	28
default	$\frac{-\frac{a}{\tanh(dx+c)}+\frac{b\tanh(dx+c)^2}{2}}{d}$	28
risch	$-\frac{2(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-b\,e^{2dx+2c}+a)}{d(e^{2dx+2c}+1)^2(e^{2dx+2c}-1)}$	80

input
```
int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a/tanh(d*x+c)+1/2*b*tanh(d*x+c)^2)
```

3.54.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(27) = 54$.

Time $= 0.25$ (sec) , antiderivative size $= 141$, normalized size of antiderivative $= 4.86$

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = -\frac{2\left((2\,a+b)\cosh(dx+c)^2+2\,b\cosh(dx+c)\sinh(dx+c)+(2\,a+b)\right.}{d\cosh(dx+c)^4+6\,d\cosh(dx+c)^2\sinh(dx+c)^2+4\,d\cosh(dx+c)\sinh(dx+c)^3+d\sinh(dx+c)^4}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
-2*((2*a + b)*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + (2*a + b
)*sinh(d*x + c)^2 + 2*a - b)/(d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2*sinh
(d*x + c)^2 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 + 4*(d
*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) - d)
```

3.54.6 Sympy [F]

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \int \left(a+b\tanh^3(c+dx)\right)\operatorname{csch}^2(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x)**2, x)
```

3.54.7 Maxima [A] (verification not implemented)

Time $= 0.20$ (sec) , antiderivative size $= 44$, normalized size of antiderivative $= 1.52$

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{2\,a}{d(e^{(-2\,dx-2\,c)}-1)} - \frac{2\,b}{d(e^{(dx+c)}+e^{(-dx-c)})^2}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
2*a/(d*(e^(-2*d*x - 2*c) - 1)) - 2*b/(d*(e^(d*x + c) + e^(-d*x - c))^2)
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = -\frac{2\left(\frac{a}{e^{(2\,dx+2\,c)}-1}+\frac{be^{(2\,dx+2\,c)}}{\left(e^{(2\,dx+2\,c)}+1\right)^2}\right)}{d}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
-2*(a/(e^(2*d*x + 2*c) - 1) + b*e^(2*d*x + 2*c)/(e^(2*d*x + 2*c) + 1)^2)/d
```

3.54.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.72

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = -\frac{2\left(a+2\,a\,\mathrm{e}^{2\,c+2\,d\,x}+a\,\mathrm{e}^{4\,c+4\,d\,x}-b\,\mathrm{e}^{2\,c+2\,d\,x}+b\,\mathrm{e}^{4\,c+4\,d\,x}\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)^2}$$

input
```
int((a + b*tanh(c + d*x)^3)/sinh(c + d*x)^2,x)
```

output
```
-(2*(a + 2*a*exp(2*c + 2*d*x) + a*exp(4*c + 4*d*x) - b*exp(2*c + 2*d*x) +
b*exp(4*c + 4*d*x)))/(d*(exp(2*c + 2*d*x) - 1)*(exp(2*c + 2*d*x) + 1)^2)
```

3.55 $\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

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3.55.1 Optimal result

Integrand size = 21, antiderivative size = 71

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{b\arctan(\sinh(c+dx))}{2d}+\frac{a\operatorname{arctanh}(\cosh(c+dx))}{2d} - \frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d} + \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

output
```
1/2*b*arctan(sinh(d*x+c))/d+1/2*a*arctanh(cosh(d*x+c))/d-1/2*a*coth(d*x+c)
*csch(d*x+c)/d+1/2*b*sech(d*x+c)*tanh(d*x+c)/d
```

3.55.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.61

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{b\arctan(\sinh(c+dx))}{2d}-\frac{a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3),x]
```

output
```
(b*ArcTan[Sinh[c + d*x]])/(2*d) - (a*Csch[(c + d*x)/2]^2)/(8*d) + (a*Log[C
osh[(c + d*x)/2]])/(2*d) - (a*Log[Sinh[(c + d*x)/2]])/(2*d) - (a*Sech[(c +
 d*x)/2]^2)/(8*d) + (b*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

3.55.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\left(a+ib\tan(ic+idx)^3\right)}{\sin(ic+idx)^3}dx$$

$$\downarrow 26$$

$$-i\int \frac{ib\tan(ic+idx)^3+a}{\sin(ic+idx)^3}dx$$

$$\downarrow 4149$$

$$-i\int \left(ia\operatorname{csch}^3(c+dx)+ib\operatorname{sech}^3(c+dx)\right)dx$$

$$\downarrow 2009$$

$$-i\left(\frac{ia\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{ia\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{ib\arctan(\sinh(c+dx))}{2d}+\frac{ib\tanh(c+dx)\operatorname{sech}(c+\right.$$

input
```
Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3),x]
```

output
```
(-I)*(((I/2)*b*ArcTan[Sinh[c + d*x]])/d + ((I/2)*a*ArcTanh[Cosh[c + d*x]])
/d - ((I/2)*a*Coth[c + d*x]*Csch[c + d*x])/d + ((I/2)*b*Sech[c + d*x]*Tanh
[c + d*x])/d)
```

3.55.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

3.55.4 Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+b\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)}{d}$
default	$\frac{a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+b\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)}{d}$
risch	$-\frac{e^{dx+c}\left(a\,e^{6dx+6c}-b\,e^{6dx+6c}+3a\,e^{4dx+4c}+3b\,e^{4dx+4c}+3\,e^{2dx+2c}a-3b\,e^{2dx+2c}+a+b\right)}{d\left(e^{2dx+2c}+1\right)^2\left(e^{2dx+2c}-1\right)^2}+\frac{ib\ln\left(e^{dx+c}+i\right)}{2d}-\frac{ib\ln(e^{d}$

input
```
int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b*(1/2*sech(d*x+
c)*tanh(d*x+c)+arctan(exp(d*x+c))))
```

3.55.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1188 vs. 2(63) = 126.

Time = 0.27 (sec) , antiderivative size = 1188, normalized size of antiderivative = 16.73

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
-1/2*(2*(a - b)*cosh(d*x + c)^7 + 14*(a - b)*cosh(d*x + c)*sinh(d*x + c)^6
 + 2*(a - b)*sinh(d*x + c)^7 + 6*(a + b)*cosh(d*x + c)^5 + 6*(7*(a - b)*co
sh(d*x + c)^2 + a + b)*sinh(d*x + c)^5 + 10*(7*(a - b)*cosh(d*x + c)^3 + 3
*(a + b)*cosh(d*x + c))*sinh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)^3 + 2*(3
5*(a - b)*cosh(d*x + c)^4 + 30*(a + b)*cosh(d*x + c)^2 + 3*a - 3*b)*sinh(d
*x + c)^3 + 6*(7*(a - b)*cosh(d*x + c)^5 + 10*(a + b)*cosh(d*x + c)^3 + 3*
(a - b)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(b*cosh(d*x + c)^8 + 56*b*cosh(
d*x + c)^3*sinh(d*x + c)^5 + 28*b*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*b*co
sh(d*x + c)*sinh(d*x + c)^7 + b*sinh(d*x + c)^8 - 2*b*cosh(d*x + c)^4 + 2*
(35*b*cosh(d*x + c)^4 - b)*sinh(d*x + c)^4 + 8*(7*b*cosh(d*x + c)^5 - b*co
sh(d*x + c))*sinh(d*x + c)^3 + 4*(7*b*cosh(d*x + c)^6 - 3*b*cosh(d*x + c)^
2)*sinh(d*x + c)^2 + 8*(b*cosh(d*x + c)^7 - b*cosh(d*x + c)^3)*sinh(d*x +
c) + b)*arctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(a + b)*cosh(d*x + c) -
(a*cosh(d*x + c)^8 + 56*a*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*a*cosh(d*x
+ c)^2*sinh(d*x + c)^6 + 8*a*cosh(d*x + c)*sinh(d*x + c)^7 + a*sinh(d*x +
c)^8 - 2*a*cosh(d*x + c)^4 + 2*(35*a*cosh(d*x + c)^4 - a)*sinh(d*x + c)^4
+ 8*(7*a*cosh(d*x + c)^5 - a*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a*cosh(
d*x + c)^6 - 3*a*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a*cosh(d*x + c)^7 -
 a*cosh(d*x + c)^3)*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) +
 1) + (a*cosh(d*x + c)^8 + 56*a*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*a*...
```

3.55.6 Sympy [F]

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \int \left(a+b\tanh^3\left(c+dx\right)\right)\operatorname{csch}^3\left(c+dx\right)dx$$

input
```
integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x)**3, x)
```

3.55.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(63) = 126$.

Time $= 0.28$ (sec) , antiderivative size $= 156$, normalized size of antiderivative $= 2.20$

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$= -b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)}\right)$$
$$+ \frac{1}{2}\,a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(e^{(-dx-c)} + e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} - 1)}\right)$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2
*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 1/2*a*(log(e^(-d*x - c) + 1)/d - l
og(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d
*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

3.55.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(63) = 126$.

Time $= 0.31$ (sec) , antiderivative size $= 143$, normalized size of antiderivative $= 2.01$

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$= \frac{2\,b\arctan\left(e^{(dx+c)}\right) + a\log\left(e^{(dx+c)}+1\right) - a\log\left(\left|e^{(dx+c)}-1\right|\right) - \frac{2\left(ae^{(7\,dx+7\,c)} - be^{(7\,dx+7\,c)} + 3\,ae^{(5\,dx+5\,c)} + 3\,be^{(5\,dx+5}\right.}{\left(e^{(4\,dx+4}\right.}}{2\,d}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
1/2*(2*b*arctan(e^(d*x + c)) + a*log(e^(d*x + c) + 1) - a*log(abs(e^(d*x +
 c) - 1)) - 2*(a*e^(7*d*x + 7*c) - b*e^(7*d*x + 7*c) + 3*a*e^(5*d*x + 5*c)
 + 3*b*e^(5*d*x + 5*c) + 3*a*e^(3*d*x + 3*c) - 3*b*e^(3*d*x + 3*c) + a*e^(
d*x + c) + b*e^(d*x + c))/(e^(4*d*x + 4*c) - 1)^2)/d
```

3.55.9 Mupad [B] (verification not implemented)

Time = 3.30 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.44

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{a\,\ln\left(\mathrm{e}^{c+dx}+1\right)}{2\,d} - \frac{\frac{4\,\mathrm{e}^{3\,c+3\,dx}\,(a-b)}{d} + \frac{4\,\mathrm{e}^{c+dx}\,(a+b)}{d}}{\mathrm{e}^{8\,c+8\,dx} - 2\,\mathrm{e}^{4\,c+4\,dx} + 1} - \frac{a\,\ln\left(\mathrm{e}^{c+dx}-1\right)}{2\,d} - \frac{\frac{\mathrm{e}^{3\,c+3\,dx}\,(a-b)}{d} + \frac{3\,\mathrm{e}^{c+dx}\,(a+b)}{d}}{\mathrm{e}^{4\,c+4\,dx}-1} - \frac{b\,\ln\left(\mathrm{e}^{c+dx}-\mathrm{i}\right)\,1\mathrm{i}}{2\,d} + \frac{b\,\ln\left(\mathrm{e}^{c+dx}+1\mathrm{i}\right)\,1\mathrm{i}}{2\,d}$$

input
```
int((a + b*tanh(c + d*x)^3)/sinh(c + d*x)^3,x)
```

output
```
(a*log(exp(c + d*x) + 1))/(2*d) - ((4*exp(3*c + 3*d*x)*(a - b))/d + (4*exp
(c + d*x)*(a + b))/d)/(exp(8*c + 8*d*x) - 2*exp(4*c + 4*d*x) + 1) - (a*log
(exp(c + d*x) - 1))/(2*d) - ((exp(3*c + 3*d*x)*(a - b))/d + (3*exp(c + d*x
)*(a + b))/d)/(exp(4*c + 4*d*x) - 1) - (b*log(exp(c + d*x) - 1i)*1i)/(2*d)
 + (b*log(exp(c + d*x) + 1i)*1i)/(2*d)
```

3.56 $\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

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3.56.1 Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{a\coth(c+dx)}{d} - \frac{a\coth^3(c+dx)}{3d} + \frac{b\log(\tanh(c+dx))}{d} - \frac{b\tanh^2(c+dx)}{2d}$$

output
```
a*coth(d*x+c)/d-1/3*a*coth(d*x+c)^3/d+b*ln(tanh(d*x+c))/d-1/2*b*tanh(d*x+c
)^2/d
```

3.56.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.32

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{2a\coth(c+dx)}{3d} - \frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d} - \frac{b\left(2\log(\cosh(c+dx)) - 2\log(\sinh(c+dx)) - \operatorname{sech}^2(c+dx)\right)}{2d}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3),x]
```

output
```
(2*a*Coth[c + d*x])/(3*d) - (a*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) - (b*(
2*Log[Cosh[c + d*x]] - 2*Log[Sinh[c + d*x]] - Sech[c + d*x]^2))/(2*d)
```

3.56.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4146, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a+ib\tan(ic+idx)^3}{\sin(ic+idx)^4}dx$$

$$\downarrow 4146$$

$$\frac{\int \coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)\left(b\tanh^3(c+dx)+a\right)d\tanh(c+dx)}{d}$$

$$\downarrow 2333$$

$$\frac{\int \left(a\coth^4(c+dx)-a\coth^2(c+dx)+b\coth(c+dx)-b\tanh(c+dx)\right)d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{3}a\coth^3(c+dx)+a\coth(c+dx)-\frac{1}{2}b\tanh^2(c+dx)+b\log(\tanh(c+dx))}{d}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3),x]
```

output
```
(a*Coth[c + d*x] - (a*Coth[c + d*x]^3)/3 + b*Log[Tanh[c + d*x]] - (b*Tanh[
c + d*x]^2)/2)/d
```

3.56.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.56.4 Maple [A] (verified)

Time = 4.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{a\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+b\left(\frac{1}{2\cosh(dx+c)^2}+\ln(\tanh(dx+c))\right)}{d}$
default	$\frac{a\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+b\left(\frac{1}{2\cosh(dx+c)^2}+\ln(\tanh(dx+c))\right)}{d}$
risch	$-\frac{2\left(-3b\,\mathrm{e}^{8dx+8c}+6a\,\mathrm{e}^{6dx+6c}+9b\,\mathrm{e}^{6dx+6c}+10a\,\mathrm{e}^{4dx+4c}-9b\,\mathrm{e}^{4dx+4c}+2\,\mathrm{e}^{2dx+2c}a+3b\,\mathrm{e}^{2dx+2c}-2a\right)}{3d\left(\mathrm{e}^{2dx+2c}-1\right)^3\left(\mathrm{e}^{2dx+2c}+1\right)^2}-\frac{b\ln\left(\mathrm{e}^{2dx+2c}+1\right.}{d}$

input
```
int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+b*(1/2/cosh(d*x+c)^2+ln(tanh(d*
x+c))))
```

3.56.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1739 vs. $2(52) = 104$.

Time = 0.27 (sec) , antiderivative size = 1739, normalized size of antiderivative = 31.05

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/3*(6*b*cosh(d*x + c)^8 + 48*b*cosh(d*x + c)*sinh(d*x + c)^7 + 6*b*sinh(d
*x + c)^8 - 6*(2*a + 3*b)*cosh(d*x + c)^6 + 6*(28*b*cosh(d*x + c)^2 - 2*a
- 3*b)*sinh(d*x + c)^6 + 12*(28*b*cosh(d*x + c)^3 - 3*(2*a + 3*b)*cosh(d*x
 + c))*sinh(d*x + c)^5 - 2*(10*a - 9*b)*cosh(d*x + c)^4 + 2*(210*b*cosh(d*
x + c)^4 - 45*(2*a + 3*b)*cosh(d*x + c)^2 - 10*a + 9*b)*sinh(d*x + c)^4 +
8*(42*b*cosh(d*x + c)^5 - 15*(2*a + 3*b)*cosh(d*x + c)^3 - (10*a - 9*b)*co
sh(d*x + c))*sinh(d*x + c)^3 - 2*(2*a + 3*b)*cosh(d*x + c)^2 + 2*(84*b*cos
h(d*x + c)^6 - 45*(2*a + 3*b)*cosh(d*x + c)^4 - 6*(10*a - 9*b)*cosh(d*x +
c)^2 - 2*a - 3*b)*sinh(d*x + c)^2 - 3*(b*cosh(d*x + c)^10 + 10*b*cosh(d*x
+ c)*sinh(d*x + c)^9 + b*sinh(d*x + c)^10 - b*cosh(d*x + c)^8 + (45*b*cosh
(d*x + c)^2 - b)*sinh(d*x + c)^8 + 8*(15*b*cosh(d*x + c)^3 - b*cosh(d*x +
c))*sinh(d*x + c)^7 - 2*b*cosh(d*x + c)^6 + 2*(105*b*cosh(d*x + c)^4 - 14*
b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^6 + 4*(63*b*cosh(d*x + c)^5 - 14*b*co
sh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^5 + 2*b*cosh(d*x + c)^4 +
 2*(105*b*cosh(d*x + c)^6 - 35*b*cosh(d*x + c)^4 - 15*b*cosh(d*x + c)^2 +
b)*sinh(d*x + c)^4 + 8*(15*b*cosh(d*x + c)^7 - 7*b*cosh(d*x + c)^5 - 5*b*c
osh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)^3 + b*cosh(d*x + c)^2 + (4
5*b*cosh(d*x + c)^8 - 28*b*cosh(d*x + c)^6 - 30*b*cosh(d*x + c)^4 + 12*b*c
osh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 2*(5*b*cosh(d*x + c)^9 - 4*b*cosh(d*
x + c)^7 - 6*b*cosh(d*x + c)^5 + 4*b*cosh(d*x + c)^3 + b*cosh(d*x + c))...
```

3.56.6 Sympy [F]

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \int \left(a+b\tanh^3\left(c+dx\right)\right)\operatorname{csch}^4\left(c+dx\right)dx$$

input
```
integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x)**4, x)
```

3.56.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(52) = 104$.

Time = 0.29 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.29

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$= b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$
$$+\frac{4}{3}a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*c
) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) +
1))) + 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*
c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c
) + e^(-6*d*x - 6*c) - 1)))
```

3.56.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(52) = 104$.

Time = 0.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.66

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx =$$
$$-\frac{6\,b\log\left(e^{(2\,dx+2\,c)}+1\right)-6\,b\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)-\frac{3\left(3\,be^{(4\,dx+4\,c)}+10\,be^{(2\,dx+2\,c)}+3\,b\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^2}+\frac{11\,be^{(6\,dx+6\,c)}-33\,be^{(4\,dx+4\,c)}}{(e}}{6\,d}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
-1/6*(6*b*log(e^(2*d*x + 2*c) + 1) - 6*b*log(abs(e^(2*d*x + 2*c) - 1)) - 3
*(3*b*e^(4*d*x + 4*c) + 10*b*e^(2*d*x + 2*c) + 3*b)/(e^(2*d*x + 2*c) + 1)^
2 + (11*b*e^(6*d*x + 6*c) - 33*b*e^(4*d*x + 4*c) + 24*a*e^(2*d*x + 2*c) +
33*b*e^(2*d*x + 2*c) - 8*a - 11*b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

3.56.9 Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.89

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{2\,b}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)} - \frac{4\,a}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)} - \frac{2\,b}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)} - \frac{8\,a}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)} - \frac{2\,\operatorname{atan}\left(\frac{b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{-d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{-d^2}}$$

input
```
int((a + b*tanh(c + d*x)^3)/sinh(c + d*x)^4,x)
```

output
```
(2*b)/(d*(exp(2*c + 2*d*x) + 1)) - (4*a)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c
+ 2*d*x) + 1)) - (2*b)/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (
8*a)/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)
) - (2*atan((b*exp(2*c)*exp(2*d*x)*(-d^2)^(1/2))/(d*(b^2)^(1/2)))*(b^2)^(1
/2))/(-d^2)^(1/2)
```

3.57 $\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

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3.57.1 Optimal result

Integrand size = 23, antiderivative size = 170

$$\begin{aligned}
&\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx \\
&= \frac{3}{8}\left(a^2+21b^2\right)x + \frac{6ab\log(\cosh(c+dx))}{d} - \frac{6b^2\tanh(c+dx)}{d} \\
&\quad - \frac{ab\tanh^2(c+dx)}{d} - \frac{b^2\tanh^3(c+dx)}{d} - \frac{b^2\tanh^5(c+dx)}{5d} \\
&\quad + \frac{\cosh^3(c+dx)\sinh(c+dx)\left(a^2+b^2+2ab\tanh(c+dx)\right)}{4d} \\
&\quad - \frac{\cosh(c+dx)\sinh(c+dx)\left(5a^2+17b^2+20ab\tanh(c+dx)\right)}{8d}
\end{aligned}$$

output
```
3/8*(a^2+21*b^2)*x+6*a*b*ln(cosh(d*x+c))/d-6*b^2*tanh(d*x+c)/d-a*b*tanh(d*
x+c)^2/d-b^2*tanh(d*x+c)^3/d-1/5*b^2*tanh(d*x+c)^5/d+1/4*cosh(d*x+c)^3*sin
h(d*x+c)*(a^2+b^2+2*a*b*tanh(d*x+c))/d-1/8*cosh(d*x+c)*sinh(d*x+c)*(5*a^2+
17*b^2+20*a*b*tanh(d*x+c))/d
```

3.57.2 Mathematica [A] (verified)

Time = 7.06 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.92

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$=\frac{60(a^2+21b^2)(c+dx)-200ab\cosh(2(c+dx))+10ab\cosh(4(c+dx))+960ab\log(\cosh(c+dx))+160a$$

input
```
Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
(60*(a^2 + 21*b^2)*(c + d*x) - 200*a*b*Cosh[2*(c + d*x)] + 10*a*b*Cosh[4*(
c + d*x)] + 960*a*b*Log[Cosh[c + d*x]] + 160*a*b*Sech[c + d*x]^2 - 40*(a^2
 + 4*b^2)*Sinh[2*(c + d*x)] + 5*(a^2 + b^2)*Sinh[4*(c + d*x)] - 1152*b^2*T
anh[c + d*x] + 224*b^2*Sech[c + d*x]^2*Tanh[c + d*x] - 32*b^2*Sech[c + d*x
]^4*Tanh[c + d*x])/(160*d)
```

3.57.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4146, 2335, 25, 2335, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$\downarrow 3042$$
$$\int \sin(ic+idx)^4\left(a+ib\tan(ic+idx)^3\right)^2\,dx$$
$$\downarrow 4146$$
$$\frac{\int \frac{\tanh^4(c+dx)\left(b\tanh^3(c+dx)+a\right)^2}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$
$$\downarrow 2335$$
$$\frac{\frac{1}{4}\int -\frac{\tanh^3(c+dx)\left(4b^2\tanh^5(c+dx)+4b^2\tanh^3(c+dx)+8ab\tanh^2(c+dx)+(a^2+5b^2)\tanh(c+dx)+8ab\right)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)+\frac{\tanh^4(c+dx)((a}{4(1-t}}{d}$$

$$\downarrow 25$$

$$\frac{\tanh^4(c+dx)\left(\left(a^2+b^2\right)\tanh(c+dx)+2ab\right)}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{1}{4}\int\frac{\tanh^3(c+dx)\left(4b^2\tanh^5(c+dx)+4b^2\tanh^3(c+dx)+8ab\tanh^2(c+dx)+\left(a^2+5b^2\right)\tanh(c+dx)+8\right.}{\left(1-\tanh^2(c+dx)\right)^2}$$

$$d$$

$$\downarrow 2335$$

$$\frac{1}{4}\left(-\frac{1}{2}\int-\frac{\tanh^2(c+dx)\left(8b^2\tanh^4(c+dx)+16b^2\tanh^2(c+dx)+48ab\tanh(c+dx)+3\left(a^2+13b^2\right)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{\tanh^3(c+dx)\left(a^2+16ab\tanh(c\right.}{2\left(1-\tanh^2(c\right.}$$

$$d$$

$$\downarrow 25$$

$$\frac{1}{4}\left(\frac{1}{2}\int\frac{\tanh^2(c+dx)\left(8b^2\tanh^4(c+dx)+16b^2\tanh^2(c+dx)+48ab\tanh(c+dx)+3\left(a^2+13b^2\right)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{\tanh^3(c+dx)\left(a^2+16ab\tanh(c\right.}{2\left(1-\tanh^2(c+dx)\right.}$$

$$d$$

$$\downarrow 2333$$

$$\frac{1}{4}\left(\frac{1}{2}\int\left(-8b^2\tanh^4(c+dx)-24b^2\tanh^2(c+dx)-48ab\tanh(c+dx)-3\left(a^2+21b^2\right)+\frac{3\left(a^2+16b\tanh(c+dx)a+21b^2\right)}{1-\tanh^2(c+dx)}\right.\right.$$

$$d$$

$$\downarrow 2009$$

$$\frac{1}{4}\left(\frac{1}{2}\left(3\left(a^2+21b^2\right)\operatorname{arctanh}(\tanh(c+dx))-3\left(a^2+21b^2\right)\tanh(c+dx)-24ab\tanh^2(c+dx)-24ab\log\left(1-\tanh\right.\right.\right.$$

input
```
Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
((Tanh[c + d*x]^4*(2*a*b + (a^2 + b^2)*Tanh[c + d*x]))/(4*(1 - Tanh[c + d*
x]^2)^2) + (-1/2*(Tanh[c + d*x]^3*(a^2 + 13*b^2 + 16*a*b*Tanh[c + d*x]))/(
1 - Tanh[c + d*x]^2) + (3*(a^2 + 21*b^2)*ArcTanh[Tanh[c + d*x]] - 24*a*b*L
og[1 - Tanh[c + d*x]^2] - 3*(a^2 + 21*b^2)*Tanh[c + d*x] - 24*a*b*Tanh[c +
 d*x]^2 - 8*b^2*Tanh[c + d*x]^3 - (8*b^2*Tanh[c + d*x]^5)/5)/2)/4)/d
```

3.57.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.57.4 Maple [A] (verified)

Time = 7.30 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{a^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\sinh(dx+c)^6}{4\cosh(dx+c)^2}-\frac{3\sinh(dx+c)^4}{4\cosh(dx+c)^2}+3\ln(\cosh(dx+c))-\frac{3\tanh}{d}$
default	$\frac{a^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\sinh(dx+c)^6}{4\cosh(dx+c)^2}-\frac{3\sinh(dx+c)^4}{4\cosh(dx+c)^2}+3\ln(\cosh(dx+c))-\frac{3\tanh}{d}$
risch	$\frac{3a^2x}{8}-6abx+\frac{63b^2x}{8}+\frac{e^{4dx+4c}a^2}{64d}+\frac{e^{4dx+4c}ab}{32d}+\frac{e^{4dx+4c}b^2}{64d}-\frac{e^{2dx+2c}a^2}{8d}-\frac{5e^{2dx+2c}ab}{8d}-\frac{e^{2dx+2c}b^2}{2d}-$

input
```
int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+2
*a*b*(1/4*sinh(d*x+c)^6/cosh(d*x+c)^2-3/4*sinh(d*x+c)^4/cosh(d*x+c)^2+3*ln
(cosh(d*x+c))-3/2*tanh(d*x+c)^2)+b^2*(1/4*sinh(d*x+c)^9/cosh(d*x+c)^5-9/8*
sinh(d*x+c)^7/cosh(d*x+c)^5+63/8*d*x+63/8*c-63/8*tanh(d*x+c)-21/8*tanh(d*x
+c)^3-63/40*tanh(d*x+c)^5))
```

3.57.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5034 vs. 2(162) = 324.

Time = 0.31 (sec) , antiderivative size = 5034, normalized size of antiderivative = 29.61

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.57.6 Sympy [F]

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \int \left(a+b\tanh^3(c+dx)\right)^2 \sinh^4(c+dx)\,dx$$

input
```
integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**3)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**2*sinh(c + d*x)**4, x)
```

3.57.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(162) = 324.

Time = 0.28 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.23

$$\begin{aligned}
&\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx \\
&= \frac{1}{64}a^2\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) \\
&\quad + \frac{1}{320}b^2\left(\frac{2520(dx+c)}{d} + \frac{5\left(32e^{(-2dx-2c)} - e^{(-4dx-4c)}\right)}{d} - \frac{135e^{(-2dx-2c)} + 5358e^{(-4dx-4c)} + 18190e^{(-}}{d(e^{(-4dx-4c)} + 5e^{(-6dx-6c)} + 10}\right. \\
&\quad + \frac{1}{32}ab\left(\frac{192(dx+c)}{d} - \frac{20e^{(-2dx-2c)} - e^{(-4dx-4c)}}{d} + \frac{192\log\left(e^{(-2dx-2c)}+1\right)}{d} - \frac{18e^{(-2dx-2c)} + 39e^{(-4}}{d(e^{(-4dx-4c)} + 2e^{(}}\right.
\end{aligned}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
1/64*a^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) + 1/320*b^2*(2520*(d*x + c)/d + 5*(32*e^(-2*d*
x - 2*c) - e^(-4*d*x - 4*c))/d - (135*e^(-2*d*x - 2*c) + 5358*e^(-4*d*x -
4*c) + 18190*e^(-6*d*x - 6*c) + 28455*e^(-8*d*x - 8*c) + 19995*e^(-10*d*x
- 10*c) + 6560*e^(-12*d*x - 12*c) - 5)/(d*(e^(-4*d*x - 4*c) + 5*e^(-6*d*x
- 6*c) + 10*e^(-8*d*x - 8*c) + 10*e^(-10*d*x - 10*c) + 5*e^(-12*d*x - 12*c
) + e^(-14*d*x - 14*c)))) + 1/32*a*b*(192*(d*x + c)/d - (20*e^(-2*d*x - 2*
c) - e^(-4*d*x - 4*c))/d + 192*log(e^(-2*d*x - 2*c) + 1)/d - (18*e^(-2*d*x
 - 2*c) + 39*e^(-4*d*x - 4*c) - 108*e^(-6*d*x - 6*c) - 1)/(d*(e^(-4*d*x -
4*c) + 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c))))
```

3.57.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(162) = 324$.

Time $= 0.47$ (sec) , antiderivative size $= 374$, normalized size of antiderivative $= 2.20$

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$

$$= \frac{5\,a^2e^{(4\,dx+4\,c)} + 10\,abe^{(4\,dx+4\,c)} + 5\,b^2e^{(4\,dx+4\,c)} - 40\,a^2e^{(2\,dx+2\,c)} - 200\,abe^{(2\,dx+2\,c)} - 160\,b^2e^{(2\,dx+2\,c)} + 1920\,d}{}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
1/320*(5*a^2*e^(4*d*x + 4*c) + 10*a*b*e^(4*d*x + 4*c) + 5*b^2*e^(4*d*x + 4
*c) - 40*a^2*e^(2*d*x + 2*c) - 200*a*b*e^(2*d*x + 2*c) - 160*b^2*e^(2*d*x
+ 2*c) + 1920*a*b*log(e^(2*d*x + 2*c) + 1) + 120*(a^2 - 16*a*b + 21*b^2)*(
d*x + c) - 5*(18*a^2*e^(4*d*x + 4*c) - 288*a*b*e^(4*d*x + 4*c) + 378*b^2*e
^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) + 40*a*b*e^(2*d*x + 2*c) - 32*b^2*e
^(2*d*x + 2*c) + a^2 - 2*a*b + b^2)*e^(-4*d*x - 4*c) - 32*(137*a*b*e^(10*d
*x + 10*c) + 645*a*b*e^(8*d*x + 8*c) - 200*b^2*e^(8*d*x + 8*c) + 1250*a*b*
e^(6*d*x + 6*c) - 600*b^2*e^(6*d*x + 6*c) + 1250*a*b*e^(4*d*x + 4*c) - 840
*b^2*e^(4*d*x + 4*c) + 645*a*b*e^(2*d*x + 2*c) - 520*b^2*e^(2*d*x + 2*c) +
 137*a*b - 144*b^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.57.9 Mupad [B] (verification not implemented)

Time $= 0.48$ (sec) , antiderivative size $= 359$, normalized size of antiderivative $= 2.11$

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$

$$= x\left(\frac{3\,a^2}{8} - 6\,a\,b + \frac{63\,b^2}{8}\right) + \frac{4\,(5\,b^2+a\,b)}{d\,(\mathrm{e}^{2\,c+2\,d\,x}+1)} + \frac{\mathrm{e}^{-2\,c-2\,d\,x}\,(a^2-5\,a\,b+4\,b^2)}{8\,d}$$
$$- \frac{\mathrm{e}^{2\,c+2\,d\,x}\,(a^2+5\,a\,b+4\,b^2)}{8\,d} + \frac{\mathrm{e}^{4\,c+4\,d\,x}\,(a+b)^2}{64\,d} - \frac{4\,(5\,b^2+a\,b)}{d\,(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1)}$$
$$+ \frac{24\,b^2}{d\,(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1)} - \frac{\mathrm{e}^{-4\,c-4\,d\,x}\,(a-b)^2}{64\,d}$$
$$- \frac{16\,b^2}{d\,(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1)}$$
$$+ \frac{32\,b^2}{5\,d\,(5\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}+5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}+1)}$$
$$+ \frac{6\,a\,b\,\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+1\right)}{d}$$

input
```
int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^3)^2,x)
```

output
```
x*((3*a^2)/8 - 6*a*b + (63*b^2)/8) + (4*(a*b + 5*b^2))/(d*(exp(2*c + 2*d*x
) + 1)) + (exp(- 2*c - 2*d*x)*(a^2 - 5*a*b + 4*b^2))/(8*d) - (exp(2*c + 2*
d*x)*(5*a*b + a^2 + 4*b^2))/(8*d) + (exp(4*c + 4*d*x)*(a + b)^2)/(64*d) -
(4*(a*b + 5*b^2))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (24*b^
2)/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) -
(exp(- 4*c - 4*d*x)*(a - b)^2)/(64*d) - (16*b^2)/(d*(4*exp(2*c + 2*d*x) +
6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (32*b^2
)/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5
*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (6*a*b*log(exp(2*c)*exp(2*d
*x) + 1))/d
```

3.58 $\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

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3.58.1 Optimal result

Integrand size = 23, antiderivative size = 182

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = {}& \frac{5ab\arctan(\sinh(c+dx))}{d}-\frac{a^2\cosh(c+dx)}{d}\\ &-\frac{4b^2\cosh(c+dx)}{d}+\frac{a^2\cosh^3(c+dx)}{3d}\\ &+\frac{b^2\cosh^3(c+dx)}{3d}-\frac{6b^2\operatorname{sech}(c+dx)}{d}\\ &+\frac{4b^2\operatorname{sech}^3(c+dx)}{3d}-\frac{b^2\operatorname{sech}^5(c+dx)}{5d}\\ &-\frac{5ab\sinh(c+dx)}{d}+\frac{5ab\sinh^3(c+dx)}{3d}\\ &-\frac{ab\sinh^3(c+dx)\tanh^2(c+dx)}{d}\end{aligned}$$

output

```
5*a*b*arctan(sinh(d*x+c))/d-a^2*cosh(d*x+c)/d-4*b^2*cosh(d*x+c)/d+1/3*a^2*
cosh(d*x+c)^3/d+1/3*b^2*cosh(d*x+c)^3/d-6*b^2*sech(d*x+c)/d+4/3*b^2*sech(d
*x+c)^3/d-1/5*b^2*sech(d*x+c)^5/d-5*a*b*sinh(d*x+c)/d+5/3*a*b*sinh(d*x+c)^
3/d-a*b*sinh(d*x+c)^3*tanh(d*x+c)^2/d
```

3.58.2 Mathematica [A] (verified)

Time = 2.23 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.66

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$= \frac{-45(a^2+5b^2)\cosh(c+dx)+5(a^2+b^2)\cosh(3(c+dx))-2b\left(-40b\operatorname{sech}^3(c+dx)+6b\operatorname{sech}^5(c+dx)-5c\right.}{}$$

input
```
Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
(-45*(a^2 + 5*b^2)*Cosh[c + d*x] + 5*(a^2 + b^2)*Cosh[3*(c + d*x)] - 2*b*(
-40*b*Sech[c + d*x]^3 + 6*b*Sech[c + d*x]^5 - 5*a*(60*ArcTan[Tanh[(c + d*x
)/2]] - 27*Sinh[c + d*x] + Sinh[3*(c + d*x)]) + 30*Sech[c + d*x]*(6*b + a*
Tanh[c + d*x])))/(60*d)
```

3.58.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$\downarrow 3042$$
$$\int i\sin(ic+idx)^3\left(a+ib\tan(ic+idx)^3\right)^2\,dx$$
$$\downarrow 26$$
$$i\int \sin(ic+idx)^3\left(ib\tan(ic+idx)^3+a\right)^2\,dx$$
$$\downarrow 4149$$
$$i\int \left(-ib^2\sinh^3(c+dx)\tanh^6(c+dx)-2iab\sinh^3(c+dx)\tanh^3(c+dx)-ia^2\sinh^3(c+dx)\right)dx$$
$$\downarrow 2009$$

$$i\left(-\frac{ia^2\cosh^3(c+dx)}{3d}+\frac{ia^2\cosh(c+dx)}{d}-\frac{5iab\arctan(\sinh(c+dx))}{d}-\frac{5iab\sinh^3(c+dx)}{3d}+\frac{5iab\sinh(c+dx)}{d}\right.$$

input
```
Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
I*(((-5*I)*a*b*ArcTan[Sinh[c + d*x]])/d + (I*a^2*Cosh[c + d*x])/d + ((4*I)
*b^2*Cosh[c + d*x])/d - ((I/3)*a^2*Cosh[c + d*x]^3)/d - ((I/3)*b^2*Cosh[c
+ d*x]^3)/d + ((6*I)*b^2*Sech[c + d*x])/d - (((4*I)/3)*b^2*Sech[c + d*x]^3
)/d + ((I/5)*b^2*Sech[c + d*x]^5)/d + ((5*I)*a*b*Sinh[c + d*x])/d - (((5*I
)/3)*a*b*Sinh[c + d*x]^3)/d + (I*a*b*Sinh[c + d*x]^3*Tanh[c + d*x]^2)/d)
```

3.58.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

3.58.4 Maple [A] (verified)

Time = 3.70 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{a^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+2ab\left(\frac{\sinh(dx+c)^5}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)^3}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{5\ \mathrm{sech}(dx+c)\tanh(dx+c)}{2}+5\arctan}{d}$
default	$\frac{a^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+2ab\left(\frac{\sinh(dx+c)^5}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)^3}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{5\ \mathrm{sech}(dx+c)\tanh(dx+c)}{2}+5\arctan}{d}$
risch	$\frac{e^{3dx+3c}a^2}{24d}+\frac{e^{3dx+3c}ab}{12d}+\frac{e^{3dx+3c}b^2}{24d}-\frac{3e^{dx+c}a^2}{8d}-\frac{9e^{dx+c}ab}{4d}-\frac{15b^2e^{dx+c}}{8d}-\frac{3e^{-dx-c}a^2}{8d}+\frac{9e^{-dx-c}ab}{4d}-$

input
```
int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+2*a*b*(1/3*sinh(d*x+c)^5/cos
h(d*x+c)^2-5/3*sinh(d*x+c)^3/cosh(d*x+c)^2-5*sinh(d*x+c)/cosh(d*x+c)^2+5/2
*sech(d*x+c)*tanh(d*x+c)+5*arctan(exp(d*x+c)))+b^2*(1/3*sinh(d*x+c)^8/cosh
(d*x+c)^5-8/3*sinh(d*x+c)^6/cosh(d*x+c)^5-16*sinh(d*x+c)^4/cosh(d*x+c)^5-6
4/3*sinh(d*x+c)^2/cosh(d*x+c)^5-128/15/cosh(d*x+c)^5))
```

3.58.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3341 vs. $2(172) = 344$.

Time $= 0.29$ (sec) , antiderivative size $= 3341$, normalized size of antiderivative $= 18.36$

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
1/120*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^16 + 80*(a^2 + 2*a*b + b^2)*cos
h(d*x + c)*sinh(d*x + c)^15 + 5*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^16 - 20*
(a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^14 + 20*(30*(a^2 + 2*a*b + b^2)*cosh
(d*x + c)^2 - a^2 - 11*a*b - 10*b^2)*sinh(d*x + c)^14 + 280*(10*(a^2 + 2*a
*b + b^2)*cosh(d*x + c)^3 - (a^2 + 11*a*b + 10*b^2)*cosh(d*x + c))*sinh(d*
x + c)^13 - 20*(11*a^2 + 61*a*b + 137*b^2)*cosh(d*x + c)^12 + 20*(455*(a^2
 + 2*a*b + b^2)*cosh(d*x + c)^4 - 91*(a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)
^2 - 11*a^2 - 61*a*b - 137*b^2)*sinh(d*x + c)^12 + 80*(273*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^5 - 91*(a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^3 - 3*(11*
a^2 + 61*a*b + 137*b^2)*cosh(d*x + c))*sinh(d*x + c)^11 - 20*(31*a^2 + 87*
a*b + 390*b^2)*cosh(d*x + c)^10 + 20*(2002*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)^6 - 1001*(a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^4 - 66*(11*a^2 + 61*a*b
+ 137*b^2)*cosh(d*x + c)^2 - 31*a^2 - 87*a*b - 390*b^2)*sinh(d*x + c)^10 +
 40*(1430*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 1001*(a^2 + 11*a*b + 10*b^
2)*cosh(d*x + c)^5 - 110*(11*a^2 + 61*a*b + 137*b^2)*cosh(d*x + c)^3 - 5*(
31*a^2 + 87*a*b + 390*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 2*(425*a^2 + 5
649*b^2)*cosh(d*x + c)^8 + 2*(32175*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 -
30030*(a^2 + 11*a*b + 10*b^2)*cosh(d*x + c)^6 - 4950*(11*a^2 + 61*a*b + 13
7*b^2)*cosh(d*x + c)^4 - 450*(31*a^2 + 87*a*b + 390*b^2)*cosh(d*x + c)^2 -
 425*a^2 - 5649*b^2)*sinh(d*x + c)^8 + 16*(3575*(a^2 + 2*a*b + b^2)*cos...
```

3.58.6 Sympy [F]

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \int \left(a+b\tanh^3(c+dx)\right)^2 \sinh^3(c+dx)\,dx$$

input
```
integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**3)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**2*sinh(c + d*x)**3, x)
```

3.58.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(172) = 344.

Time = 0.29 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.91

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx =$$
$$-\frac{1}{120}b^2\left(\frac{5\left(45\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}\right)}{d}+\frac{200\,e^{(-2\,dx-2\,c)}+2515\,e^{(-4\,dx-4\,c)}+6680\,e^{(-6\,dx-6\,c)}+9073\,e^{(-8\,dx}}{d(e^{(-3\,dx-3\,c)}+5\,e^{(-5\,dx-5\,c)}+10\,e^{(-7\,dx-7\,c)}+10\,e^{(-9}}\right.$$
$$+\frac{1}{12}ab\left(\frac{27\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}-\frac{120\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{25\,e^{(-2\,dx-2\,c)}+77\,e^{(-4\,dx-4\,c)}+3\,e^{(-6\,dx-6\,c)}}{d(e^{(-3\,dx-3\,c)}+2\,e^{(-5\,dx-5\,c)}+e^{(-7\,dx-7\,c)})}\right.$$
$$+\frac{1}{24}a^2\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-1/120*b^2*(5*(45*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (200*e^(-2*d*x - 2*
c) + 2515*e^(-4*d*x - 4*c) + 6680*e^(-6*d*x - 6*c) + 9073*e^(-8*d*x - 8*c)
 + 5600*e^(-10*d*x - 10*c) + 1665*e^(-12*d*x - 12*c) - 5)/(d*(e^(-3*d*x -
3*c) + 5*e^(-5*d*x - 5*c) + 10*e^(-7*d*x - 7*c) + 10*e^(-9*d*x - 9*c) + 5*
e^(-11*d*x - 11*c) + e^(-13*d*x - 13*c)))) + 1/12*a*b*((27*e^(-d*x - c) -
e^(-3*d*x - 3*c))/d - 120*arctan(e^(-d*x - c))/d - (25*e^(-2*d*x - 2*c) +
77*e^(-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) - 1)/(d*(e^(-3*d*x - 3*c) + 2*e^(
-5*d*x - 5*c) + e^(-7*d*x - 7*c)))) + 1/24*a^2*(e^(3*d*x + 3*c)/d - 9*e^(d
*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.59

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$=\frac{1200\,ab\arctan\left(e^{(dx+c)}\right)+5\,a^2e^{(3\,dx+3\,c)}+10\,abe^{(3\,dx+3\,c)}+5\,b^2e^{(3\,dx+3\,c)}-45\,a^2e^{(dx+c)}-270\,abe^{(dx+c)}-22}{}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
1/120*(1200*a*b*arctan(e^(d*x + c)) + 5*a^2*e^(3*d*x + 3*c) + 10*a*b*e^(3*
d*x + 3*c) + 5*b^2*e^(3*d*x + 3*c) - 45*a^2*e^(d*x + c) - 270*a*b*e^(d*x +
 c) - 225*b^2*e^(d*x + c) - 5*(9*a^2*e^(2*d*x + 2*c) - 54*a*b*e^(2*d*x + 2
*c) + 45*b^2*e^(2*d*x + 2*c) - a^2 + 2*a*b - b^2)*e^(-3*d*x - 3*c) - 16*(1
5*a*b*e^(9*d*x + 9*c) + 90*b^2*e^(9*d*x + 9*c) + 30*a*b*e^(7*d*x + 7*c) +
280*b^2*e^(7*d*x + 7*c) + 428*b^2*e^(5*d*x + 5*c) - 30*a*b*e^(3*d*x + 3*c)
 + 280*b^2*e^(3*d*x + 3*c) - 15*a*b*e^(d*x + c) + 90*b^2*e^(d*x + c))/(e^(
2*d*x + 2*c) + 1)^5)/d
```

3.58.9 Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.18

$$
\begin{aligned}
&\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx \\
&= \frac{\mathrm{e}^{3c+3dx}(a+b)^2}{24d} - \frac{\mathrm{e}^{c+dx}(3a^2+18ab+15b^2)}{8d} \\
&\quad - \frac{\mathrm{e}^{-c-dx}(3a^2-18ab+15b^2)}{8d} + \frac{\mathrm{e}^{-3c-3dx}(a-b)^2}{24d} \\
&\quad + \frac{10\,\mathrm{atan}\left(\frac{ab\mathrm{e}^{dx}\mathrm{e}^{c}\sqrt{d^2}}{d\sqrt{a^2b^2}}\right)\sqrt{a^2b^2}}{\sqrt{d^2}} - \frac{256b^2\mathrm{e}^{c+dx}}{15d\left(3\mathrm{e}^{2c+2dx}+3\mathrm{e}^{4c+4dx}+\mathrm{e}^{6c+6dx}+1\right)} \\
&\quad + \frac{64b^2\mathrm{e}^{c+dx}}{5d\left(4\mathrm{e}^{2c+2dx}+6\mathrm{e}^{4c+4dx}+4\mathrm{e}^{6c+6dx}+\mathrm{e}^{8c+8dx}+1\right)} \\
&\quad - \frac{32b^2\mathrm{e}^{c+dx}}{5d\left(5\mathrm{e}^{2c+2dx}+10\mathrm{e}^{4c+4dx}+10\mathrm{e}^{6c+6dx}+5\mathrm{e}^{8c+8dx}+\mathrm{e}^{10c+10dx}+1\right)} \\
&\quad - \frac{2\mathrm{e}^{c+dx}(6b^2+ab)}{d\left(\mathrm{e}^{2c+2dx}+1\right)} + \frac{4\mathrm{e}^{c+dx}(8b^2+3ab)}{3d\left(2\mathrm{e}^{2c+2dx}+\mathrm{e}^{4c+4dx}+1\right)}
\end{aligned}
$$

input
```
int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^3)^2,x)
```

output

```
(exp(3*c + 3*d*x)*(a + b)^2)/(24*d) - (exp(c + d*x)*(18*a*b + 3*a^2 + 15*b
^2))/(8*d) - (exp(- c - d*x)*(3*a^2 - 18*a*b + 15*b^2))/(8*d) + (exp(- 3*c
 - 3*d*x)*(a - b)^2)/(24*d) + (10*atan((a*b*exp(d*x)*exp(c)*(d^2)^(1/2))/(
d*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(d^2)^(1/2) - (256*b^2*exp(c + d*x))/
(15*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) +
(64*b^2*exp(c + d*x))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*ex
p(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (32*b^2*exp(c + d*x))/(5*d*(5*ex
p(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8
*d*x) + exp(10*c + 10*d*x) + 1)) - (2*exp(c + d*x)*(a*b + 6*b^2))/(d*(exp(
2*c + 2*d*x) + 1)) + (4*exp(c + d*x)*(3*a*b + 8*b^2))/(3*d*(2*exp(2*c + 2*
d*x) + exp(4*c + 4*d*x) + 1))
```

3.59 $\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

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3.59.1 Optimal result

Integrand size = 23, antiderivative size = 129

$$\begin{aligned}&\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx\\&=-\frac{1}{2}\left(a^2+7b^2\right)x-\frac{4ab\log(\cosh(c+dx))}{d}+\frac{3b^2\tanh(c+dx)}{d}\\&\quad+\frac{ab\tanh^2(c+dx)}{d}+\frac{2b^2\tanh^3(c+dx)}{3d}+\frac{b^2\tanh^5(c+dx)}{5d}\\&\quad+\frac{\cosh(c+dx)\sinh(c+dx)\left(a^2+b^2+2ab\tanh(c+dx)\right)}{2d}\end{aligned}$$

output

```
-1/2*(a^2+7*b^2)*x-4*a*b*ln(cosh(d*x+c))/d+3*b^2*tanh(d*x+c)/d+a*b*tanh(d*
x+c)^2/d+2/3*b^2*tanh(d*x+c)^3/d+1/5*b^2*tanh(d*x+c)^5/d+1/2*cosh(d*x+c)*s
inh(d*x+c)*(a^2+b^2+2*a*b*tanh(d*x+c))/d
```

3.59.2 Mathematica [A] (verified)

Time = 2.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.06

$$\begin{aligned}&\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx\\&=\frac{-30a^2c-210b^2c-30a^2dx-210b^2dx+30ab\cosh(2(c+dx))-240ab\log(\cosh(c+dx))+15a^2\sinh(2(c\end{aligned}$$

input
```
Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
(-30*a^2*c - 210*b^2*c - 30*a^2*d*x - 210*b^2*d*x + 30*a*b*Cosh[2*(c + d*x
)] - 240*a*b*Log[Cosh[c + d*x]] + 15*a^2*Sinh[2*(c + d*x)] + 15*b^2*Sinh[2
*(c + d*x)] + 232*b^2*Tanh[c + d*x] + 12*b^2*Sech[c + d*x]^4*Tanh[c + d*x]
 - 4*b*Sech[c + d*x]^2*(15*a + 16*b*Tanh[c + d*x]))/(60*d)
```

3.59.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 25, 4146, 2335, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int -\sin(ic+idx)^2\left(a+ib\tan(ic+idx)^3\right)^2 dx$$

$$\downarrow 25$$

$$-\int \sin(ic+idx)^2\left(ib\tan(ic+idx)^3+a\right)^2 dx$$

$$\downarrow 4146$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(b\tanh^3(c+dx)+a\right)^2}{\left(1-\tanh^2(c+dx)\right)^2} d\tanh(c+dx)}{d}$$

$$\downarrow 2335$$

$$\frac{\frac{1}{2}\int -\frac{\tanh(c+dx)\left(2b^2\tanh^5(c+dx)+2b^2\tanh^3(c+dx)+4ab\tanh^2(c+dx)+\left(a^2+3b^2\right)\tanh(c+dx)+4ab\right)}{1-\tanh^2(c+dx)} d\tanh(c+dx) + \frac{\tanh^2(c+dx)\left(\left(a^2}{2\left(1-\tan\right.}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{\tanh^2(c+dx)\left(\left(a^2+b^2\right)\tanh(c+dx)+2ab\right)}{2\left(1-\tanh^2(c+dx)\right)} - \frac{1}{2}\int \frac{\tanh(c+dx)\left(2b^2\tanh^5(c+dx)+2b^2\tanh^3(c+dx)+4ab\tanh^2(c+dx)+\left(a^2+3b^2\right)\tanh(c+dx)+4a\right.}{1-\tanh^2(c+dx)}}{d}$$

$$\downarrow 2333$$

$$\frac{\tanh^2(c+dx)\left(\left(a^2+b^2\right)\tanh(c+dx)+2ab\right)}{2\left(1-\tanh^2(c+dx)\right)} - \frac{1}{2}\int \frac{\left(-2b^2\tanh^4(c+dx) - 4b^2\tanh^2(c+dx) - 4ab\tanh(c+dx) - a^2 - 7b^2 +}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{2}\left(-\left(a^2+7b^2\right)\operatorname{arctanh}(\tanh(c+dx)) + \left(a^2+7b^2\right)\tanh(c+dx) + 2ab\tanh^2(c+dx) + 4ab\log\left(1-\tanh^2(c+dx\right.\right.}{d}$$

input
```
Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
((Tanh[c + d*x]^2*(2*a*b + (a^2 + b^2)*Tanh[c + d*x]))/(2*(1 - Tanh[c + d*
x]^2)) + (-((a^2 + 7*b^2)*ArcTanh[Tanh[c + d*x]]) + 4*a*b*Log[1 - Tanh[c +
 d*x]^2] + (a^2 + 7*b^2)*Tanh[c + d*x] + 2*a*b*Tanh[c + d*x]^2 + (4*b^2*Ta
nh[c + d*x]^3)/3 + (2*b^2*Tanh[c + d*x]^5)/5)/2)/d
```

3.59.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.59.4 Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\left(\frac{\sinh(dx+c)^4}{2\cosh(dx+c)^2}-2\ln(\cosh(dx+c))+\tanh(dx+c)^2\right)+b^2\left(\frac{\sinh(dx+c)^7}{2\cosh(dx+c)^5}-\frac{7dx}{2}-\frac{7c}{2}\right.}{d}$
default	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\left(\frac{\sinh(dx+c)^4}{2\cosh(dx+c)^2}-2\ln(\cosh(dx+c))+\tanh(dx+c)^2\right)+b^2\left(\frac{\sinh(dx+c)^7}{2\cosh(dx+c)^5}-\frac{7dx}{2}-\frac{7c}{2}\right.}{d}$
risch	$-\frac{a^2x}{2}+4abx-\frac{7b^2x}{2}+\frac{e^{2dx+2c}a^2}{8d}+\frac{e^{2dx+2c}ab}{4d}+\frac{e^{2dx+2c}b^2}{8d}-\frac{e^{-2dx-2c}a^2}{8d}+\frac{e^{-2dx-2c}ab}{4d}-\frac{e^{-2dx-2c}}{8d}$

input
```
int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+2*a*b*(1/2*sinh(d*x+c
)^4/cosh(d*x+c)^2-2*ln(cosh(d*x+c))+tanh(d*x+c)^2)+b^2*(1/2*sinh(d*x+c)^7/
cosh(d*x+c)^5-7/2*d*x-7/2*c+7/2*tanh(d*x+c)+7/6*tanh(d*x+c)^3+7/10*tanh(d*
x+c)^5))
```

3.59.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3649 vs. $2(121) = 242$.

Time = 0.28 (sec) , antiderivative size = 3649, normalized size of antiderivative = 28.29

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

output

```
1/120*(15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^14 + 210*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)*sinh(d*x + c)^13 + 15*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^14 -
15*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*x + c)^12
 - 15*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 91*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^
2 - 5*a^2 - 10*a*b - 5*b^2)*sinh(d*x + c)^12 + 60*(91*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^3 - 3*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)
*cosh(d*x + c))*sinh(d*x + c)^11 - 15*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^
2 + 10*a*b + 87*b^2)*cosh(d*x + c)^10 + 15*(1001*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)^4 - 20*(a^2 - 8*a*b + 7*b^2)*d*x - 66*(4*(a^2 - 8*a*b + 7*b^2)*d*
x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*x + c)^2 + 9*a^2 - 10*a*b - 87*b^2)*sin
h(d*x + c)^10 + 30*(1001*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 - 110*(4*(a^2
 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*x + c)^3 - 5*(20*(a
^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2)*cosh(d*x + c))*sinh(d*x
 + c)^9 - 15*(40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + 251*b^2)*cos
h(d*x + c)^8 + 15*(3003*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 - 495*(4*(a^2
- 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*x + c)^4 - 40*(a^2 -
 8*a*b + 7*b^2)*d*x - 45*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b +
87*b^2)*cosh(d*x + c)^2 + 5*a^2 - 66*a*b - 251*b^2)*sinh(d*x + c)^8 + 120*
(429*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 99*(4*(a^2 - 8*a*b + 7*b^2)*d*x
 - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*x + c)^5 - 15*(20*(a^2 - 8*a*b + 7*b^...
```

3.59.6 Sympy [F]

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \int \left(a+b\tanh^3\left(c+dx\right)\right)^2\sinh^2\left(c+dx\right)dx$$

input

```
integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**3)**2,x)
```

output

```
Integral((a + b*tanh(c + d*x)**3)**2*sinh(c + d*x)**2, x)
```

3.59.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(121) = 242$.

Time = 0.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.33

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = -\frac{1}{8}a^2\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right)$$
$$-\frac{1}{120}b^2\left(\frac{420\,(dx+c)}{d} + \frac{15\,e^{(-2dx-2c)}}{d} - \frac{1003\,e^{(-2dx-2c)} + 3350\,e^{(-4dx-4c)} + 5590\,e^{(-6dx-6c)} + 3915\,e^{(-}}{d(e^{(-2dx-2c)} + 5\,e^{(-4dx-4c)} + 10\,e^{(-6dx-6c)} + 10\,e^{(-8dx-8c)} +}\right.$$
$$\left.-\frac{1}{4}ab\left(\frac{16\,(dx+c)}{d} - \frac{e^{(-2dx-2c)}}{d} + \frac{16\,\log\left(e^{(-2dx-2c)}+1\right)}{d} - \frac{2\,e^{(-2dx-2c)} - 15\,e^{(-4dx-4c)} + 1}{d(e^{(-2dx-2c)} + 2\,e^{(-4dx-4c)} + e^{(-6dx-6c)})}\right)\right.$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/120*b^2*(420*(
d*x + c)/d + 15*e^(-2*d*x - 2*c)/d - (1003*e^(-2*d*x - 2*c) + 3350*e^(-4*d
*x - 4*c) + 5590*e^(-6*d*x - 6*c) + 3915*e^(-8*d*x - 8*c) + 1455*e^(-10*d*
x - 10*c) + 15)/(d*(e^(-2*d*x - 2*c) + 5*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
 6*c) + 10*e^(-8*d*x - 8*c) + 5*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c))))
 - 1/4*a*b*(16*(d*x + c)/d - e^(-2*d*x - 2*c)/d + 16*log(e^(-2*d*x - 2*c)
+ 1)/d - (2*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 1)/(d*(e^(-2*d*x - 2*
c) + 2*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))))
```

3.59.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(121) = 242$.

Time = 0.42 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.29

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$= \frac{15\,a^2e^{(2dx+2c)} + 30\,abe^{(2dx+2c)} + 15\,b^2e^{(2dx+2c)} - 480\,ab\log\left(e^{(2dx+2c)}+1\right) - 60\,(a^2 - 8\,ab + 7\,b^2)(dx+c}{}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
1/120*(15*a^2*e^(2*d*x + 2*c) + 30*a*b*e^(2*d*x + 2*c) + 15*b^2*e^(2*d*x +
 2*c) - 480*a*b*log(e^(2*d*x + 2*c) + 1) - 60*(a^2 - 8*a*b + 7*b^2)*(d*x +
 c) + 15*(2*a^2*e^(2*d*x + 2*c) - 16*a*b*e^(2*d*x + 2*c) + 14*b^2*e^(2*d*x
 + 2*c) - a^2 + 2*a*b - b^2)*e^(-2*d*x - 2*c) + 8*(137*a*b*e^(10*d*x + 10*
c) + 625*a*b*e^(8*d*x + 8*c) - 180*b^2*e^(8*d*x + 8*c) + 1190*a*b*e^(6*d*x
 + 6*c) - 480*b^2*e^(6*d*x + 6*c) + 1190*a*b*e^(4*d*x + 4*c) - 680*b^2*e^(
4*d*x + 4*c) + 625*a*b*e^(2*d*x + 2*c) - 400*b^2*e^(2*d*x + 2*c) + 137*a*b
 - 116*b^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.59.9 Mupad [B] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.37

$$
\begin{aligned}
&\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx \\
&= \frac{\mathrm{e}^{2c+2dx}(a+b)^2}{8d} - \frac{4(3b^2+ab)}{d\left(\mathrm{e}^{2c+2dx}+1\right)} - x\left(\frac{a^2}{2} - 4ab + \frac{7b^2}{2}\right) \\
&\quad + \frac{4(4b^2+ab)}{d\left(2\,\mathrm{e}^{2c+2dx}+\mathrm{e}^{4c+4dx}+1\right)} - \frac{64b^2}{3d\left(3\,\mathrm{e}^{2c+2dx}+3\,\mathrm{e}^{4c+4dx}+\mathrm{e}^{6c+6dx}+1\right)} \\
&\quad - \frac{\mathrm{e}^{-2c-2dx}(a-b)^2}{8d} + \frac{16b^2}{d\left(4\,\mathrm{e}^{2c+2dx}+6\,\mathrm{e}^{4c+4dx}+4\,\mathrm{e}^{6c+6dx}+\mathrm{e}^{8c+8dx}+1\right)} \\
&\quad - \frac{32b^2}{5d\left(5\,\mathrm{e}^{2c+2dx}+10\,\mathrm{e}^{4c+4dx}+10\,\mathrm{e}^{6c+6dx}+5\,\mathrm{e}^{8c+8dx}+\mathrm{e}^{10c+10dx}+1\right)} \\
&\quad - \frac{4ab\ln\left(\mathrm{e}^{2c}\mathrm{e}^{2dx}+1\right)}{d}
\end{aligned}
$$

input
```
int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^3)^2,x)
```

output
```
(exp(2*c + 2*d*x)*(a + b)^2)/(8*d) - (4*(a*b + 3*b^2))/(d*(exp(2*c + 2*d*x
) + 1)) - x*(a^2/2 - 4*a*b + (7*b^2)/2) + (4*(a*b + 4*b^2))/(d*(2*exp(2*c
+ 2*d*x) + exp(4*c + 4*d*x) + 1)) - (64*b^2)/(3*d*(3*exp(2*c + 2*d*x) + 3*
exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (exp(- 2*c - 2*d*x)*(a - b)^2)
/(8*d) + (16*b^2)/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c
+ 6*d*x) + exp(8*c + 8*d*x) + 1)) - (32*b^2)/(5*d*(5*exp(2*c + 2*d*x) + 10
*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c +
10*d*x) + 1)) - (4*a*b*log(exp(2*c)*exp(2*d*x) + 1))/d
```

3.60 $\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

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3.60.1 Optimal result

Integrand size = 21, antiderivative size = 123

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = &-\frac{3ab\arctan(\sinh(c+dx))}{d}\\ &+\frac{a^2\cosh(c+dx)}{d}+\frac{b^2\cosh(c+dx)}{d}\\ &+\frac{3b^2\operatorname{sech}(c+dx)}{d}-\frac{b^2\operatorname{sech}^3(c+dx)}{d}\\ &+\frac{b^2\operatorname{sech}^5(c+dx)}{5d}+\frac{3ab\sinh(c+dx)}{d}\\ &-\frac{ab\sinh(c+dx)\tanh^2(c+dx)}{d}\end{aligned}$$

output

```
-3*a*b*arctan(sinh(d*x+c))/d+a^2*cosh(d*x+c)/d+b^2*cosh(d*x+c)/d+3*b^2*sec
h(d*x+c)/d-b^2*sech(d*x+c)^3/d+1/5*b^2*sech(d*x+c)^5/d+3*a*b*sinh(d*x+c)/d
-a*b*sinh(d*x+c)*tanh(d*x+c)^2/d
```

3.60.2 Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.73

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$=\frac{5(a^2+b^2)\cosh(c+dx)+b\left(-5b\operatorname{sech}^3(c+dx)+b\operatorname{sech}^5(c+dx)+10a\left(-3\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+\si\right.\right.}{5d}$$

input
```
Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
(5*(a^2 + b^2)*Cosh[c + d*x] + b*(-5*b*Sech[c + d*x]^3 + b*Sech[c + d*x]^5
 + 10*a*(-3*ArcTan[Tanh[(c + d*x)/2]] + Sinh[c + d*x]) + 5*Sech[c + d*x]*(
3*b + a*Tanh[c + d*x])))/(5*d)
```

3.60.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$\downarrow 3042$$
$$\int -i\sin(ic+idx)\left(a+ib\tan(ic+idx)^3\right)^2\,dx$$
$$\downarrow 26$$
$$-i\int \sin(ic+idx)\left(ib\tan(ic+idx)^3+a\right)^2\,dx$$
$$\downarrow 4149$$
$$-i\int \left(ib^2\sinh(c+dx)\tanh^6(c+dx)+2iab\sinh(c+dx)\tanh^3(c+dx)+ia^2\sinh(c+dx)\right)dx$$
$$\downarrow 2009$$

$$-i\left(\frac{ia^2\cosh(c+dx)}{d}-\frac{3iab\arctan(\sinh(c+dx))}{d}+\frac{3iab\sinh(c+dx)}{d}-\frac{iab\sinh(c+dx)\tanh^2(c+dx)}{d}+\frac{ib^2\cos}{}\right.$$

input
```
Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
(-I)*(((-3*I)*a*b*ArcTan[Sinh[c + d*x]])/d + (I*a^2*Cosh[c + d*x])/d + (I*
b^2*Cosh[c + d*x])/d + ((3*I)*b^2*Sech[c + d*x])/d - (I*b^2*Sech[c + d*x]^
3)/d + ((I/5)*b^2*Sech[c + d*x]^5)/d + ((3*I)*a*b*Sinh[c + d*x])/d - (I*a*
b*Sinh[c + d*x]*Tanh[c + d*x]^2)/d)
```

3.60.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

3.60.4 Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{\cosh(dx+c)a^2+2ab\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\ \mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan\left(\mathrm{e}^{dx+c}\right)\right)+b^2\left(\frac{\sinh(dx+c)^6}{\cosh(dx+c)^5}+\frac{6\sinh(dx}{\cosh(dx}\right.$
default	$\frac{\cosh(dx+c)a^2+2ab\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\ \mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan\left(\mathrm{e}^{dx+c}\right)\right)+b^2\left(\frac{\sinh(dx+c)^6}{\cosh(dx+c)^5}+\frac{6\sinh(dx}{\cosh(dx}\right.$
risch	$\frac{\mathrm{e}^{dx+c}a^2}{2d}+\frac{\mathrm{e}^{dx+c}ab}{d}+\frac{b^2\mathrm{e}^{dx+c}}{2d}+\frac{\mathrm{e}^{-dx-c}a^2}{2d}-\frac{\mathrm{e}^{-dx-c}ab}{d}+\frac{\mathrm{e}^{-dx-c}b^2}{2d}+\frac{2b\,\mathrm{e}^{dx+c}(5a\,\mathrm{e}^{8dx+8c}+15b\,\mathrm{e}^{8dx+8c}+1}$

input
```
int(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(cosh(d*x+c)*a^2+2*a*b*(sinh(d*x+c)^3/cosh(d*x+c)^2+3*sinh(d*x+c)/cosh
(d*x+c)^2-3/2*sech(d*x+c)*tanh(d*x+c)-3*arctan(exp(d*x+c)))+b^2*(sinh(d*x+
c)^6/cosh(d*x+c)^5+6*sinh(d*x+c)^4/cosh(d*x+c)^5+8*sinh(d*x+c)^2/cosh(d*x+
c)^5+16/5/cosh(d*x+c)^5))
```

3.60.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2230 vs. 2(121) = 242.

Time = 0.28 (sec) , antiderivative size = 2230, normalized size of antiderivative = 18.13

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

output

```
1/10*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^12 + 60*(a^2 + 2*a*b + b^2)*cosh
(d*x + c)*sinh(d*x + c)^11 + 5*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^12 + 30*(
a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^10 + 30*(11*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)^2 + a^2 + 2*a*b + 3*b^2)*sinh(d*x + c)^10 + 100*(11*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c
)^9 + 5*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*x + c)^8 + 5*(495*(a^2 + 2*a*b +
 b^2)*cosh(d*x + c)^4 + 270*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^2 + 15*a^2
 + 18*a*b + 47*b^2)*sinh(d*x + c)^8 + 40*(99*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^5 + 90*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^3 + (15*a^2 + 18*a*b + 47*
b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(25*a^2 + 91*b^2)*cosh(d*x + c)^6
+ 4*(1155*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 1575*(a^2 + 2*a*b + 3*b^2)
*cosh(d*x + c)^4 + 35*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*x + c)^2 + 25*a^2
+ 91*b^2)*sinh(d*x + c)^6 + 8*(495*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 9
45*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^5 + 35*(15*a^2 + 18*a*b + 47*b^2)*c
osh(d*x + c)^3 + 3*(25*a^2 + 91*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 5*(1
5*a^2 - 18*a*b + 47*b^2)*cosh(d*x + c)^4 + 5*(495*(a^2 + 2*a*b + b^2)*cosh
(d*x + c)^8 + 1260*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^6 + 70*(15*a^2 + 18
*a*b + 47*b^2)*cosh(d*x + c)^4 + 12*(25*a^2 + 91*b^2)*cosh(d*x + c)^2 + 15
*a^2 - 18*a*b + 47*b^2)*sinh(d*x + c)^4 + 20*(55*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)^9 + 180*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^7 + 14*(15*a^2 + 1...
```

3.60.6 Sympy [F]

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \int \left(a+b\tanh^3(c+dx)\right)^2 \sinh(c+dx)\,dx$$

input

```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**3)**2,x)
```

output

```
Integral((a + b*tanh(c + d*x)**3)**2*sinh(c + d*x), x)
```

3.60.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(121) = 242$.

Time = 0.28 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.06

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$= ab\left(\frac{6\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)}}{d} + \frac{4\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} + 1}{d(e^{(-dx-c)} + 2\,e^{(-3\,dx-3\,c)} + e^{(-5\,dx-5\,c)})}\right)$$
$$+ \frac{1}{10}\,b^2\left(\frac{5\,e^{(-dx-c)}}{d} + \frac{85\,e^{(-2\,dx-2\,c)} + 210\,e^{(-4\,dx-4\,c)} + 314\,e^{(-6\,dx-6\,c)} + 185\,e^{(-8\,dx-8\,c)} + 65\,e^{(-10\,dx-10\,c)} +}{d(e^{(-dx-c)} + 5\,e^{(-3\,dx-3\,c)} + 10\,e^{(-5\,dx-5\,c)} + 10\,e^{(-7\,dx-7\,c)} + 5\,e^{(-9\,dx-9\,c)} + e^{(-11\,dx-11}\right.$$
$$+ \frac{a^2\cosh(dx+c)}{d}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
a*b*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) - e^(
-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c
)))) + 1/10*b^2*(5*e^(-d*x - c)/d + (85*e^(-2*d*x - 2*c) + 210*e^(-4*d*x -
 4*c) + 314*e^(-6*d*x - 6*c) + 185*e^(-8*d*x - 8*c) + 65*e^(-10*d*x - 10*c
) + 5)/(d*(e^(-d*x - c) + 5*e^(-3*d*x - 3*c) + 10*e^(-5*d*x - 5*c) + 10*e^
(-7*d*x - 7*c) + 5*e^(-9*d*x - 9*c) + e^(-11*d*x - 11*c)))) + a^2*cosh(d*x
 + c)/d
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.65

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx =$$
$$-\frac{60\,ab\arctan\left(e^{(dx+c)}\right) - 5\,a^2e^{(dx+c)} - 10\,abe^{(dx+c)} - 5\,b^2e^{(dx+c)} - 5\,(a^2 - 2\,ab + b^2)e^{(-dx-c)} - \frac{4\,(5\,abe^{(9\,dx+}}{10\,d}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
-1/10*(60*a*b*arctan(e^(d*x + c)) - 5*a^2*e^(d*x + c) - 10*a*b*e^(d*x + c)
 - 5*b^2*e^(d*x + c) - 5*(a^2 - 2*a*b + b^2)*e^(-d*x - c) - 4*(5*a*b*e^(9*
d*x + 9*c) + 15*b^2*e^(9*d*x + 9*c) + 10*a*b*e^(7*d*x + 7*c) + 40*b^2*e^(7
*d*x + 7*c) + 66*b^2*e^(5*d*x + 5*c) - 10*a*b*e^(3*d*x + 3*c) + 40*b^2*e^(
3*d*x + 3*c) - 5*a*b*e^(d*x + c) + 15*b^2*e^(d*x + c))/(e^(2*d*x + 2*c) +
1)^5)/d
```

3.60.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.75

$$
\begin{aligned}
&\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx \\
&= \frac{\mathrm{e}^{c+dx}\,(a+b)^2}{2\,d} + \frac{\mathrm{e}^{-c-dx}\,(a-b)^2}{2\,d} - \frac{6\,\mathrm{atan}\left(\frac{a\,b\,\mathrm{e}^{dx}\,\mathrm{e}^{c}\,\sqrt{d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{d^2}} \\
&\quad + \frac{72\,b^2\,\mathrm{e}^{c+dx}}{5\,d\,\left(3\,\mathrm{e}^{2\,c+2\,dx}+3\,\mathrm{e}^{4\,c+4\,dx}+\mathrm{e}^{6\,c+6\,dx}+1\right)} \\
&\quad - \frac{64\,b^2\,\mathrm{e}^{c+dx}}{5\,d\,\left(4\,\mathrm{e}^{2\,c+2\,dx}+6\,\mathrm{e}^{4\,c+4\,dx}+4\,\mathrm{e}^{6\,c+6\,dx}+\mathrm{e}^{8\,c+8\,dx}+1\right)} \\
&\quad + \frac{32\,b^2\,\mathrm{e}^{c+dx}}{5\,d\,\left(5\,\mathrm{e}^{2\,c+2\,dx}+10\,\mathrm{e}^{4\,c+4\,dx}+10\,\mathrm{e}^{6\,c+6\,dx}+5\,\mathrm{e}^{8\,c+8\,dx}+\mathrm{e}^{10\,c+10\,dx}+1\right)} \\
&\quad + \frac{2\,\mathrm{e}^{c+dx}\,(3\,b^2+a\,b)}{d\,\left(\mathrm{e}^{2\,c+2\,dx}+1\right)} - \frac{4\,\mathrm{e}^{c+dx}\,(2\,b^2+a\,b)}{d\,\left(2\,\mathrm{e}^{2\,c+2\,dx}+\mathrm{e}^{4\,c+4\,dx}+1\right)}
\end{aligned}
$$

input
```
int(sinh(c + d*x)*(a + b*tanh(c + d*x)^3)^2,x)
```

output
```
(exp(c + d*x)*(a + b)^2)/(2*d) + (exp(- c - d*x)*(a - b)^2)/(2*d) - (6*ata
n((a*b*exp(d*x)*exp(c)*(d^2)^(1/2))/(d*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/
(d^2)^(1/2) + (72*b^2*exp(c + d*x))/(5*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c +
 4*d*x) + exp(6*c + 6*d*x) + 1)) - (64*b^2*exp(c + d*x))/(5*d*(4*exp(2*c +
 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1))
 + (32*b^2*exp(c + d*x))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) +
10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (2*e
xp(c + d*x)*(a*b + 3*b^2))/(d*(exp(2*c + 2*d*x) + 1)) - (4*exp(c + d*x)*(a
*b + 2*b^2))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.61 $\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

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3.61.1 Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \frac{ab\arctan(\sinh(c+dx))}{d} - \frac{a^2\operatorname{arctanh}(\cosh(c+dx))}{d} - \frac{b^2\operatorname{sech}(c+dx)}{d} + \frac{2b^2\operatorname{sech}^3(c+dx)}{3d} - \frac{b^2\operatorname{sech}^5(c+dx)}{5d} - \frac{ab\operatorname{sech}(c+dx)\tanh(c+dx)}{d}$$

output

```
a*b*arctan(sinh(d*x+c))/d-a^2*arctanh(cosh(d*x+c))/d-b^2*sech(d*x+c)/d+2/3
*b^2*sech(d*x+c)^3/d-1/5*b^2*sech(d*x+c)^5/d-a*b*sech(d*x+c)*tanh(d*x+c)/d
```

3.61.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.28

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \frac{2ab\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a^2\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{a^2\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{b^2\operatorname{sech}(c+dx)}{d} + \frac{2b^2\operatorname{sech}^3(c+dx)}{3d} - \frac{b^2\operatorname{sech}^5(c+dx)}{5d} - \frac{ab\operatorname{sech}(c+dx)\tanh(c+dx)}{d}$$

input
```
Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
(2*a*b*ArcTan[Tanh[(c + d*x)/2]])/d - (a^2*Log[Cosh[(c + d*x)/2]])/d + (a^
2*Log[Sinh[(c + d*x)/2]])/d - (b^2*Sech[c + d*x])/d + (2*b^2*Sech[c + d*x]
^3)/(3*d) - (b^2*Sech[c + d*x]^5)/(5*d) - (a*b*Sech[c + d*x]*Tanh[c + d*x]
)/d
```

3.61.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{i\left(a+ib\tan(ic+idx)^3\right)^2}{\sin(ic+idx)}dx$$

$$\downarrow 26$$

$$i \int \frac{\left(ib \tan(ic + idx)^3 + a\right)^2}{\sin(ic + idx)} dx$$

$$\downarrow 4149$$

$$i \int \left(-ib^2 \operatorname{sech}(c + dx) \tanh^5(c + dx) - 2iab \operatorname{sech}(c + dx) \tanh^2(c + dx) - ia^2 \operatorname{csch}(c + dx)\right) dx$$

$$\downarrow 2009$$

$$i\left(\frac{ia^2 \operatorname{arctanh}(\cosh(c + dx))}{d} - \frac{iab \arctan(\sinh(c + dx))}{d} + \frac{iab \tanh(c + dx) \operatorname{sech}(c + dx)}{d} + \frac{ib^2 \operatorname{sech}^5(c + dx)}{5d} - \frac{2i}{}\right.$$

input
```
Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
I*(((-I)*a*b*ArcTan[Sinh[c + d*x]])/d + (I*a^2*ArcTanh[Cosh[c + d*x]])/d +
 (I*b^2*Sech[c + d*x])/d - (((2*I)/3)*b^2*Sech[c + d*x]^3)/d + ((I/5)*b^2*
Sech[c + d*x]^5)/d + (I*a*b*Sech[c + d*x]*Tanh[c + d*x])/d)
```

3.61.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

3.61.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{-2a^2 \operatorname{arctanh}\left(e^{dx+c}\right)+2ab\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b^2\left(-\frac{\sinh(dx+c)^4}{\cosh(dx+c)^5}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)^5}-\right.}{d}$
default	$\frac{-2a^2 \operatorname{arctanh}\left(e^{dx+c}\right)+2ab\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b^2\left(-\frac{\sinh(dx+c)^4}{\cosh(dx+c)^5}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)^5}-\right.}{d}$
risch	$-\frac{2b\,e^{dx+c}\left(15a\,e^{8dx+8c}+15b\,e^{8dx+8c}+30a\,e^{6dx+6c}+20b\,e^{6dx+6c}+58b\,e^{4dx+4c}-30\,e^{2dx+2c}a+20b\,e^{2dx+2c}-15a+15b\right)}{15d\left(e^{2dx+2c}+1\right)^5}$

input
```
int(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a^2*arctanh(exp(d*x+c))+2*a*b*(-sinh(d*x+c)/cosh(d*x+c)^2+1/2*sech
(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+b^2*(-sinh(d*x+c)^4/cosh(d*x+c)^5-
4/3*sinh(d*x+c)^2/cosh(d*x+c)^5-8/15/cosh(d*x+c)^5))
```

3.61.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2498 vs. 2(94) = 188.

Time = 0.29 (sec) , antiderivative size = 2498, normalized size of antiderivative = 25.49

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
-1/15*(30*(a*b + b^2)*cosh(d*x + c)^9 + 270*(a*b + b^2)*cosh(d*x + c)*sinh
(d*x + c)^8 + 30*(a*b + b^2)*sinh(d*x + c)^9 + 20*(3*a*b + 2*b^2)*cosh(d*x
 + c)^7 + 20*(54*(a*b + b^2)*cosh(d*x + c)^2 + 3*a*b + 2*b^2)*sinh(d*x + c
)^7 + 116*b^2*cosh(d*x + c)^5 + 140*(18*(a*b + b^2)*cosh(d*x + c)^3 + (3*a
*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 4*(945*(a*b + b^2)*cosh(d*x +
 c)^4 + 105*(3*a*b + 2*b^2)*cosh(d*x + c)^2 + 29*b^2)*sinh(d*x + c)^5 + 20
*(189*(a*b + b^2)*cosh(d*x + c)^5 + 35*(3*a*b + 2*b^2)*cosh(d*x + c)^3 + 2
9*b^2*cosh(d*x + c))*sinh(d*x + c)^4 - 20*(3*a*b - 2*b^2)*cosh(d*x + c)^3
+ 20*(126*(a*b + b^2)*cosh(d*x + c)^6 + 35*(3*a*b + 2*b^2)*cosh(d*x + c)^4
 + 58*b^2*cosh(d*x + c)^2 - 3*a*b + 2*b^2)*sinh(d*x + c)^3 + 20*(54*(a*b +
 b^2)*cosh(d*x + c)^7 + 21*(3*a*b + 2*b^2)*cosh(d*x + c)^5 + 58*b^2*cosh(d
*x + c)^3 - 3*(3*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 30*(a*b*cos
h(d*x + c)^10 + 10*a*b*cosh(d*x + c)*sinh(d*x + c)^9 + a*b*sinh(d*x + c)^1
0 + 5*a*b*cosh(d*x + c)^8 + 5*(9*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^
8 + 10*a*b*cosh(d*x + c)^6 + 40*(3*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c)
)*sinh(d*x + c)^7 + 10*(21*a*b*cosh(d*x + c)^4 + 14*a*b*cosh(d*x + c)^2 +
a*b)*sinh(d*x + c)^6 + 10*a*b*cosh(d*x + c)^4 + 4*(63*a*b*cosh(d*x + c)^5
+ 70*a*b*cosh(d*x + c)^3 + 15*a*b*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(21*
a*b*cosh(d*x + c)^6 + 35*a*b*cosh(d*x + c)^4 + 15*a*b*cosh(d*x + c)^2 + a*
b)*sinh(d*x + c)^4 + 5*a*b*cosh(d*x + c)^2 + 40*(3*a*b*cosh(d*x + c)^7 ...
```

3.61.6 Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \int \left(a+b\tanh^3(c+dx)\right)^2 \operatorname{csch}(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**3)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x), x)
```

3.61.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(94) = 188$.

Time = 0.29 (sec) , antiderivative size = 447, normalized size of antiderivative = 4.56

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2 dx$$
$$= -2ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)}\right)$$
$$- \frac{2}{15}b^2\left(\frac{15\,e^{(-dx-c)}}{d(5\,e^{(-2\,dx-2\,c)} + 10\,e^{(-4\,dx-4\,c)} + 10\,e^{(-6\,dx-6\,c)} + 5\,e^{(-8\,dx-8\,c)} + e^{(-10\,dx-10\,c)} + 1)} + \frac{}{d(5\,e^{(-2\,dx-2\,c}\right.$$
$$\left. + \frac{a^2\log\left(\tanh\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}{d}\right.$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-2*a*b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e
^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - 2/15*b^2*(15*e^(-d*x - c)/(d*(
5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d
*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 20*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x
 - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) +
 e^(-10*d*x - 10*c) + 1)) + 58*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
 - 10*c) + 1)) + 20*e^(-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x
 - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) +
1)) + 15*e^(-9*d*x - 9*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + a^2*
log(tanh(1/2*d*x + 1/2*c))/d
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.82

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2 dx$$
$$= \frac{30\,ab\arctan\left(e^{(dx+c)}\right) - 15\,a^2\log\left(e^{(dx+c)}+1\right) + 15\,a^2\log\left(\left|e^{(dx+c)}-1\right|\right) - \frac{2\left(15\,abe^{(9\,dx+9\,c)}+15\,b^2e^{(9\,dx+9\,c)}+30\,ab\right.}{}}{15\,d}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
1/15*(30*a*b*arctan(e^(d*x + c)) - 15*a^2*log(e^(d*x + c) + 1) + 15*a^2*lo
g(abs(e^(d*x + c) - 1)) - 2*(15*a*b*e^(9*d*x + 9*c) + 15*b^2*e^(9*d*x + 9*
c) + 30*a*b*e^(7*d*x + 7*c) + 20*b^2*e^(7*d*x + 7*c) + 58*b^2*e^(5*d*x + 5
*c) - 30*a*b*e^(3*d*x + 3*c) + 20*b^2*e^(3*d*x + 3*c) - 15*a*b*e^(d*x + c)
 + 15*b^2*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.61.9 Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 522, normalized size of antiderivative = 5.33

$$
\begin{aligned}
&\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx \\
&= \frac{a^2\,\ln\left(32\,a^6+32\,a^4\,b^2-32\,a^6\,\mathrm{e}^{dx}\,\mathrm{e}^{c}-32\,a^4\,b^2\,\mathrm{e}^{dx}\,\mathrm{e}^{c}\right)}{d} \\
&\quad-\frac{176\,b^2\,\mathrm{e}^{c+dx}}{15\,\left(d+3\,d\,\mathrm{e}^{2\,c+2\,dx}+3\,d\,\mathrm{e}^{4\,c+4\,dx}+d\,\mathrm{e}^{6\,c+6\,dx}\right)} \\
&\quad-\frac{32\,b^2\,\mathrm{e}^{c+dx}}{5\,\left(d+5\,d\,\mathrm{e}^{2\,c+2\,dx}+10\,d\,\mathrm{e}^{4\,c+4\,dx}+10\,d\,\mathrm{e}^{6\,c+6\,dx}+5\,d\,\mathrm{e}^{8\,c+8\,dx}+d\,\mathrm{e}^{10\,c+10\,dx}\right)} \\
&\quad-\frac{a^2\,\ln\left(-32\,a^6-32\,a^4\,b^2-32\,a^6\,\mathrm{e}^{dx}\,\mathrm{e}^{c}-32\,a^4\,b^2\,\mathrm{e}^{dx}\,\mathrm{e}^{c}\right)}{d} \\
&\quad-\frac{2\,b^2\,\mathrm{e}^{c+dx}}{d+d\,\mathrm{e}^{2\,c+2\,dx}}+\frac{16\,b^2\,\mathrm{e}^{c+dx}}{3\,\left(d+2\,d\,\mathrm{e}^{2\,c+2\,dx}+d\,\mathrm{e}^{4\,c+4\,dx}\right)} \\
&\quad+\frac{64\,b^2\,\mathrm{e}^{c+dx}}{5\,\left(d+4\,d\,\mathrm{e}^{2\,c+2\,dx}+6\,d\,\mathrm{e}^{4\,c+4\,dx}+4\,d\,\mathrm{e}^{6\,c+6\,dx}+d\,\mathrm{e}^{8\,c+8\,dx}\right)} \\
&\quad-\frac{2\,a\,b\,\mathrm{e}^{c+dx}}{d+d\,\mathrm{e}^{2\,c+2\,dx}}+\frac{4\,a\,b\,\mathrm{e}^{c+dx}}{d+2\,d\,\mathrm{e}^{2\,c+2\,dx}+d\,\mathrm{e}^{4\,c+4\,dx}} \\
&\quad-\frac{a\,b\left(\ln\left(32\,a^3\,b^3\,\mathrm{e}^{dx}\,\mathrm{e}^{c}+32\,a^5\,b\,\mathrm{e}^{dx}\,\mathrm{e}^{c}-a^5\,b\,32\mathrm{i}-a^3\,b^3\,32\mathrm{i}\right)\,1\mathrm{i}-\ln\left(32\,a^3\,b^3\,\mathrm{e}^{dx}\,\mathrm{e}^{c}+32\,a^5\,b\,\mathrm{e}^{dx}\,\mathrm{e}^{c}+a^5\,b\right.\right.}{d}
\end{aligned}
$$

input
```
int((a + b*tanh(c + d*x)^3)^2/sinh(c + d*x),x)
```

output

```
(a^2*log(32*a^6 + 32*a^4*b^2 - 32*a^6*exp(d*x)*exp(c) - 32*a^4*b^2*exp(d*x
)*exp(c)))/d - (176*b^2*exp(c + d*x))/(15*(d + 3*d*exp(2*c + 2*d*x) + 3*d*
exp(4*c + 4*d*x) + d*exp(6*c + 6*d*x))) - (32*b^2*exp(c + d*x))/(5*(d + 5*
d*exp(2*c + 2*d*x) + 10*d*exp(4*c + 4*d*x) + 10*d*exp(6*c + 6*d*x) + 5*d*e
xp(8*c + 8*d*x) + d*exp(10*c + 10*d*x))) - (a^2*log(- 32*a^6 - 32*a^4*b^2
- 32*a^6*exp(d*x)*exp(c) - 32*a^4*b^2*exp(d*x)*exp(c)))/d - (2*b^2*exp(c +
 d*x))/(d + d*exp(2*c + 2*d*x)) + (16*b^2*exp(c + d*x))/(3*(d + 2*d*exp(2*
c + 2*d*x) + d*exp(4*c + 4*d*x))) + (64*b^2*exp(c + d*x))/(5*(d + 4*d*exp(
2*c + 2*d*x) + 6*d*exp(4*c + 4*d*x) + 4*d*exp(6*c + 6*d*x) + d*exp(8*c + 8
*d*x))) - (2*a*b*exp(c + d*x))/(d + d*exp(2*c + 2*d*x)) + (4*a*b*exp(c + d
*x))/(d + 2*d*exp(2*c + 2*d*x) + d*exp(4*c + 4*d*x)) - (a*b*(log(32*a^3*b^
3*exp(d*x)*exp(c) - a^3*b^3*32i - a^5*b*32i + 32*a^5*b*exp(d*x)*exp(c))*1i
 - log(a^5*b*32i + a^3*b^3*32i + 32*a^3*b^3*exp(d*x)*exp(c) + 32*a^5*b*exp
(d*x)*exp(c))*1i))/d
```

3.62 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

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3.62.1 Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = -\frac{a^2\coth(c+dx)}{d}+\frac{ab\tanh^2(c+dx)}{d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

output
```
-a^2*coth(d*x+c)/d+a*b*tanh(d*x+c)^2/d+1/5*b^2*tanh(d*x+c)^5/d
```

3.62.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.00

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = -\frac{a^2\coth(c+dx)}{d}-\frac{ab\operatorname{sech}^2(c+dx)}{d}+\frac{b^2\tanh(c+dx)}{5d}-\frac{2b^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{5d}+\frac{b^2\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
-((a^2*Coth[c + d*x])/d) - (a*b*Sech[c + d*x]^2)/d + (b^2*Tanh[c + d*x])/(
5*d) - (2*b^2*Sech[c + d*x]^2*Tanh[c + d*x])/(5*d) + (b^2*Sech[c + d*x]^4*
Tanh[c + d*x])/(5*d)
```

3.62.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 25, 4146, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a+ib\tan(ic+idx)^3\right)^2}{\sin(ic+idx)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(ib\tan(ic+idx)^3+a\right)^2}{\sin(ic+idx)^2}dx$$

$$\downarrow 4146$$

$$\frac{\int \coth^2(c+dx)\left(b\tanh^3(c+dx)+a\right)^2 d\tanh(c+dx)}{d}$$

$$\downarrow 802$$

$$\frac{\int \left(b^2\tanh^4(c+dx)+2ab\tanh(c+dx)+a^2\coth^2(c+dx)\right) d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{-a^2\coth(c+dx)+ab\tanh^2(c+dx)+\frac{1}{5}b^2\tanh^5(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
(-(a^2*Coth[c + d*x]) + a*b*Tanh[c + d*x]^2 + (b^2*Tanh[c + d*x]^5)/5)/d
```

3.62.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.62.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(45) = 90$.

Time = 4.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.09

method	result
derivativedivides	$\frac{-a^2\coth(dx+c)-\frac{ab}{\cosh(dx+c)^2}+b^2\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{8}\right)}{d}$
default	$\frac{-a^2\coth(dx+c)-\frac{ab}{\cosh(dx+c)^2}+b^2\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{8}\right)}{d}$
risch	$-\frac{2\left(5a^2e^{10dx+10c}+10ab\,e^{10dx+10c}+5b^2e^{10dx+10c}+25a^2e^{8dx+8c}+20ab\,e^{8dx+8c}-5b^2e^{8dx+8c}+50a^2e^{6dx+6c}+10b^2e^{6dx+}\right.}{5d\left(e^{2dx+2c}+1\right)^5\left(e^{2dx+}\right.}$

input
```
int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2*coth(d*x+c)-a*b/cosh(d*x+c)^2+b^2*(-1/2*sinh(d*x+c)^3/cosh(d*x+c
)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*
x+c)^2)*tanh(d*x+c)))
```

3.62.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(45) = 90$.

Time $= 0.25$ (sec) , antiderivative size $= 518$, normalized size of antiderivative $= 11.02$

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx =$$

$$-\frac{4\left(\left(5\,a^2+5\,ab+2\,b^2\right)\cosh\left(dx+c\right)^5+5\left(5\,a^2+5\,ab+2\,b^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4+\right.}{5\left(d\cosh\left(dx+c\right)^7+7\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^6+d\sinh\left(dx+c\right)^7+3\,d\cosh\left(dx+c\right)^5+\left(21\,d\cos\right.}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
-4/5*((5*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^5 + 5*(5*a^2 + 5*a*b + 2*b^2)*
cosh(d*x + c)*sinh(d*x + c)^4 + (5*a*b + 3*b^2)*sinh(d*x + c)^5 + (25*a^2
+ 5*a*b - 2*b^2)*cosh(d*x + c)^3 + (10*(5*a*b + 3*b^2)*cosh(d*x + c)^2 + 1
5*a*b - 3*b^2)*sinh(d*x + c)^3 + (10*(5*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)
^3 + 3*(25*a^2 + 5*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^2
 - a*b)*cosh(d*x + c) + (5*(5*a*b + 3*b^2)*cosh(d*x + c)^4 + 9*(5*a*b - b^
2)*cosh(d*x + c)^2 + 10*a*b + 10*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^7 +
7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 3*d*cosh(d*x + c)^
5 + (21*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3
+ 3*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (35*d*cosh(d*x
+ c)^4 + 50*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3 + 3*(7*d*cosh(d*x + c
)^5 + 10*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^2 - 5*d*cosh(d
*x + c) + (7*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 27*d*cosh(d*x + c)
^2 + 5*d)*sinh(d*x + c))
```

3.62.6 Sympy [F]

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \int \left(a+b\tanh^3(c+dx)\right)^2 \operatorname{csch}^2(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**3)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x)**2, x)
```

3.62.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(45) = 90$.

Time $= 0.21$ (sec) , antiderivative size $= 256$, normalized size of antiderivative $= 5.45$

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$= \frac{2}{5}b^2\left(\frac{10\,e^{(-4\,dx-4\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{}{d(5\,e^{(-2\,dx-2\,c)}+}\right.$$
$$\left.+\frac{2\,a^2}{d(e^{(-2\,dx-2\,c)}-1)}-\frac{4\,ab}{d(e^{(dx+c)}+e^{(-dx-c)})^2}\right.$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
2/5*b^2*(10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
+ 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*
e^(-8*d*x - 8*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d
*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*
d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c
) + e^(-10*d*x - 10*c) + 1))) + 2*a^2/(d*(e^(-2*d*x - 2*c) - 1)) - 4*a*b/(
d*(e^(d*x + c) + e^(-d*x - c))^2)
```

3.62.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(45) = 90$.

Time $= 0.40$ (sec) , antiderivative size $= 122$, normalized size of antiderivative $= 2.60$

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = -\frac{2\left(\frac{5a^2}{e^{(2dx+2c)}-1}+\frac{10abe^{(8dx+8c)}+5b^2e^{(8dx+8c)}+30abe^{(6dx+6c)}+30abe^{(4dx+4c)}+10b^2e^{(4dx+4c)}+10abe^{(2dx+2c)}+b^2}{\left(e^{(2dx+2c)}+1\right)^5}\right)}{5d}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
-2/5*(5*a^2/(e^(2*d*x + 2*c) - 1) + (10*a*b*e^(8*d*x + 8*c) + 5*b^2*e^(8*d
*x + 8*c) + 30*a*b*e^(6*d*x + 6*c) + 30*a*b*e^(4*d*x + 4*c) + 10*b^2*e^(4*
d*x + 4*c) + 10*a*b*e^(2*d*x + 2*c) + b^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.62.9 Mupad [B] (verification not implemented)

Time $= 2.00$ (sec) , antiderivative size $= 483$, normalized size of antiderivative $= 10.28$

$$\begin{aligned}
&\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx \\
&= -\frac{\frac{2e^{8c+8dx}(b^2+2ab)}{5d}-\frac{8e^{2c+2dx}(b^2+ab)}{5d}-\frac{2(2ab-b^2)}{5d}+\frac{8e^{6c+6dx}(ab-b^2)}{5d}+\frac{12b^2e^{4c+4dx}}{5d}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1} \\
&\quad -\frac{\frac{2b^2}{5d}+\frac{2e^{4c+4dx}(b^2+2ab)}{5d}+\frac{4e^{2c+2dx}(ab-b^2)}{5d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}-\frac{\frac{2(ab-b^2)}{5d}+\frac{2e^{2c+2dx}(b^2+2ab)}{5d}}{2e^{2c+2dx}+e^{4c+4dx}+1} \\
&\quad -\frac{\frac{2e^{6c+6dx}(b^2+2ab)}{5d}-\frac{2(b^2+ab)}{5d}+\frac{6e^{4c+4dx}(ab-b^2)}{5d}+\frac{6b^2e^{2c+2dx}}{5d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1} \\
&\quad -\frac{2a^2}{d\left(e^{2c+2dx}-1\right)}-\frac{2(b^2+2ab)}{5d\left(e^{2c+2dx}+1\right)}
\end{aligned}$$

input
```
int((a + b*tanh(c + d*x)^3)^2/sinh(c + d*x)^2,x)
```

output

```
- ((2*exp(8*c + 8*d*x)*(2*a*b + b^2))/(5*d) - (8*exp(2*c + 2*d*x)*(a*b + b
^2))/(5*d) - (2*(2*a*b - b^2))/(5*d) + (8*exp(6*c + 6*d*x)*(a*b - b^2))/(5
*d) + (12*b^2*exp(4*c + 4*d*x))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c +
4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1
) - ((2*b^2)/(5*d) + (2*exp(4*c + 4*d*x)*(2*a*b + b^2))/(5*d) + (4*exp(2*c
 + 2*d*x)*(a*b - b^2))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + e
xp(6*c + 6*d*x) + 1) - ((2*(a*b - b^2))/(5*d) + (2*exp(2*c + 2*d*x)*(2*a*b
 + b^2))/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*exp(6*c
+ 6*d*x)*(2*a*b + b^2))/(5*d) - (2*(a*b + b^2))/(5*d) + (6*exp(4*c + 4*d*x
)*(a*b - b^2))/(5*d) + (6*b^2*exp(2*c + 2*d*x))/(5*d))/(4*exp(2*c + 2*d*x)
 + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - (2*a^
2)/(d*(exp(2*c + 2*d*x) - 1)) - (2*(2*a*b + b^2))/(5*d*(exp(2*c + 2*d*x) +
 1))
```

3.63 $\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

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3.63.1 Optimal result

Integrand size = 23, antiderivative size = 107

$$\begin{aligned}\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = {}& \frac{ab\arctan(\sinh(c+dx))}{d}\\ &+\frac{a^2\operatorname{arctanh}(\cosh(c+dx))}{2d}\\ &-\frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d}\\ &-\frac{b^2\operatorname{sech}^3(c+dx)}{3d}+\frac{b^2\operatorname{sech}^5(c+dx)}{5d}\\ &+\frac{ab\operatorname{sech}(c+dx)\tanh(c+dx)}{d}\end{aligned}$$

output

```
a*b*arctan(sinh(d*x+c))/d+1/2*a^2*arctanh(cosh(d*x+c))/d-1/2*a^2*coth(d*x+
c)*csch(d*x+c)/d-1/3*b^2*sech(d*x+c)^3/d+1/5*b^2*sech(d*x+c)^5/d+a*b*sech(
d*x+c)*tanh(d*x+c)/d
```

3.63.2 Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.49

$$\begin{aligned}
&\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx \\
&= \frac{2ab\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} \\
&\quad + \frac{a^2\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a^2\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} \\
&\quad - \frac{b^2\operatorname{sech}^3(c+dx)}{3d} + \frac{b^2\operatorname{sech}^5(c+dx)}{5d} + \frac{ab\operatorname{sech}(c+dx)\tanh(c+dx)}{d}
\end{aligned}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
(2*a*b*ArcTan[Tanh[(c + d*x)/2]])/d - (a^2*Csch[(c + d*x)/2]^2)/(8*d) + (a
^2*Log[Cosh[(c + d*x)/2]])/(2*d) - (a^2*Log[Sinh[(c + d*x)/2]])/(2*d) - (a
^2*Sech[(c + d*x)/2]^2)/(8*d) - (b^2*Sech[c + d*x]^3)/(3*d) + (b^2*Sech[c
+ d*x]^5)/(5*d) + (a*b*Sech[c + d*x]*Tanh[c + d*x])/d
```

3.63.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\left(a+ib\tan(ic+idx)^3\right)^2}{\sin(ic+idx)^3}\,dx$$

$$\downarrow 26$$

$$-i\int \frac{\left(ib\tan(ic+idx)^3+a\right)^2}{\sin(ic+idx)^3}\,dx$$

$$\downarrow 4149$$

$$-i\int \left(ia^2\text{csch}^3(c+dx)+2iab\text{sech}^3(c+dx)+ib^2\text{sech}^3(c+dx)\tanh^3(c+dx)\right)dx$$

$$\downarrow 2009$$

$$-i\left(\frac{ia^2\text{arctanh}(\cosh(c+dx))}{2d}-\frac{ia^2\coth(c+dx)\text{csch}(c+dx)}{2d}+\frac{iab\arctan(\sinh(c+dx))}{d}+\frac{iab\tanh(c+dx)\text{sech}}{d}\right.$$

input
```
Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
(-I)*((I*a*b*ArcTan[Sinh[c + d*x]])/d + ((I/2)*a^2*ArcTanh[Cosh[c + d*x]])
/d - ((I/2)*a^2*Coth[c + d*x]*Csch[c + d*x])/d - ((I/3)*b^2*Sech[c + d*x]^
3)/d + ((I/5)*b^2*Sech[c + d*x]^5)/d + (I*a*b*Sech[c + d*x]*Tanh[c + d*x])
/d)
```

3.63.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

3.63.4 Maple [A] (verified)

Time = 8.68 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{a^2\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+2ab\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b^2\left(-\frac{\sinh(dx+c)^2}{3\cosh(dx+c)^5}-\frac{}{15\,co}\right.}{d}$
default	$\frac{a^2\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+2ab\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b^2\left(-\frac{\sinh(dx+c)^2}{3\cosh(dx+c)^5}-\frac{}{15\,co}\right.}{d}$
risch	$-\frac{e^{dx+c}\left(15a^2e^{12dx+12c}-30ab\,e^{12dx+12c}+90a^2e^{10dx+10c}+40b^2e^{10dx+10c}+225a^2e^{8dx+8c}+90ab\,e^{8dx+8c}-96b^2e^{8dx+8c}\right.}{15d\left(e^{2dx+2c}-\right.}$

input
```
int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+2*a*b*(1/2*sec
h(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+b^2*(-1/3*sinh(d*x+c)^2/cosh(d*x+
c)^5-2/15/cosh(d*x+c)^5))
```

3.63.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4642 vs. 2(99) = 198.

Time = 0.32 (sec) , antiderivative size = 4642, normalized size of antiderivative = 43.38

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
-1/30*(30*(a^2 - 2*a*b)*cosh(d*x + c)^13 + 390*(a^2 - 2*a*b)*cosh(d*x + c)
*sinh(d*x + c)^12 + 30*(a^2 - 2*a*b)*sinh(d*x + c)^13 + 20*(9*a^2 + 4*b^2)
*cosh(d*x + c)^11 + 20*(117*(a^2 - 2*a*b)*cosh(d*x + c)^2 + 9*a^2 + 4*b^2)
*sinh(d*x + c)^11 + 220*(39*(a^2 - 2*a*b)*cosh(d*x + c)^3 + (9*a^2 + 4*b^2
)*cosh(d*x + c))*sinh(d*x + c)^10 + 6*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x
+ c)^9 + 2*(10725*(a^2 - 2*a*b)*cosh(d*x + c)^4 + 550*(9*a^2 + 4*b^2)*cosh
(d*x + c)^2 + 225*a^2 + 90*a*b - 96*b^2)*sinh(d*x + c)^9 + 6*(6435*(a^2 -
2*a*b)*cosh(d*x + c)^5 + 550*(9*a^2 + 4*b^2)*cosh(d*x + c)^3 + 9*(75*a^2 +
 30*a*b - 32*b^2)*cosh(d*x + c))*sinh(d*x + c)^8 + 8*(75*a^2 + 28*b^2)*cos
h(d*x + c)^7 + 8*(6435*(a^2 - 2*a*b)*cosh(d*x + c)^6 + 825*(9*a^2 + 4*b^2)
*cosh(d*x + c)^4 + 27*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^2 + 75*a^2
+ 28*b^2)*sinh(d*x + c)^7 + 8*(6435*(a^2 - 2*a*b)*cosh(d*x + c)^7 + 1155*(
9*a^2 + 4*b^2)*cosh(d*x + c)^5 + 63*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x +
c)^3 + 7*(75*a^2 + 28*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 6*(75*a^2 - 30
*a*b - 32*b^2)*cosh(d*x + c)^5 + 6*(6435*(a^2 - 2*a*b)*cosh(d*x + c)^8 + 1
540*(9*a^2 + 4*b^2)*cosh(d*x + c)^6 + 126*(75*a^2 + 30*a*b - 32*b^2)*cosh(
d*x + c)^4 + 28*(75*a^2 + 28*b^2)*cosh(d*x + c)^2 + 75*a^2 - 30*a*b - 32*b
^2)*sinh(d*x + c)^5 + 2*(10725*(a^2 - 2*a*b)*cosh(d*x + c)^9 + 3300*(9*a^2
 + 4*b^2)*cosh(d*x + c)^7 + 378*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^5
 + 140*(75*a^2 + 28*b^2)*cosh(d*x + c)^3 + 15*(75*a^2 - 30*a*b - 32*b^2...
```

3.63.6 Sympy [F]

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \int \left(a+b\tanh^3(c+dx)\right)^2 \operatorname{csch}^3(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**3)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x)**3, x)
```

3.63.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(99) = 198$.

Time $= 0.29$ (sec) , antiderivative size $= 378$, normalized size of antiderivative $= 3.53$

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$= -2ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)}\right)$$
$$+ \frac{1}{2}a^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(e^{(-dx-c)} + e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} - 1)}\right)$$
$$- \frac{8}{15}b^2\left(\frac{5\,e^{(-3\,dx-3\,c)}}{d(5\,e^{(-2\,dx-2\,c)} + 10\,e^{(-4\,dx-4\,c)} + 10\,e^{(-6\,dx-6\,c)} + 5\,e^{(-8\,dx-8\,c)} + e^{(-10\,dx-10\,c)} + 1)} - \frac{}{d(5\,e^{(-2\,dx-2\,c}}\right.$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-2*a*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e
^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 1/2*a^2*(log(e^(-d*x - c) + 1)
/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e
^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 8/15*b^2*(5*e^(-3*d*x - 3*c)/(
d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-
8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 2*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d
*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c)
 + e^(-10*d*x - 10*c) + 1)) + 5*e^(-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) +
10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*
x - 10*c) + 1)))
```

3.63.8 Giac [A] (verification not implemented)

Time $= 0.39$ (sec) , antiderivative size $= 192$, normalized size of antiderivative $= 1.79$

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$= \frac{60\,ab\arctan\left(e^{(dx+c)}\right) + 15\,a^2\log\left(e^{(dx+c)}+1\right) - 15\,a^2\log\left(\left|e^{(dx+c)}-1\right|\right) - \frac{30\left(a^2e^{(3\,dx+3\,c)} + a^2e^{(dx+c)}\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^2} + \frac{4\left(15\,ab}{}}{30\,d}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
1/30*(60*a*b*arctan(e^(d*x + c)) + 15*a^2*log(e^(d*x + c) + 1) - 15*a^2*lo
g(abs(e^(d*x + c) - 1)) - 30*(a^2*e^(3*d*x + 3*c) + a^2*e^(d*x + c))/(e^(2
*d*x + 2*c) - 1)^2 + 4*(15*a*b*e^(9*d*x + 9*c) + 30*a*b*e^(7*d*x + 7*c) -
20*b^2*e^(7*d*x + 7*c) + 8*b^2*e^(5*d*x + 5*c) - 30*a*b*e^(3*d*x + 3*c) -
20*b^2*e^(3*d*x + 3*c) - 15*a*b*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.63.9 Mupad [B] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 561, normalized size of antiderivative = 5.24

$$
\begin{aligned}
&\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx \\
&= \frac{a^2\,\mathrm{e}^{c+dx}}{d-d\,\mathrm{e}^{2c+2dx}} + \frac{136\,b^2\,\mathrm{e}^{c+dx}}{15\,\left(d+3\,d\,\mathrm{e}^{2c+2dx}+3\,d\,\mathrm{e}^{4c+4dx}+d\,\mathrm{e}^{6c+6dx}\right)} \\
&\quad + \frac{32\,b^2\,\mathrm{e}^{c+dx}}{5\,\left(d+5\,d\,\mathrm{e}^{2c+2dx}+10\,d\,\mathrm{e}^{4c+4dx}+10\,d\,\mathrm{e}^{6c+6dx}+5\,d\,\mathrm{e}^{8c+8dx}+d\,\mathrm{e}^{10c+10dx}\right)} \\
&\quad - \frac{a^2\,\ln\left(4\,a^6\,\mathrm{e}^{dx}\,\mathrm{e}^{c}-16\,a^4\,b^2-4\,a^6+16\,a^4\,b^2\,\mathrm{e}^{dx}\,\mathrm{e}^{c}\right)}{2\,d} \\
&\quad + \frac{a^2\,\ln\left(4\,a^6+16\,a^4\,b^2+4\,a^6\,\mathrm{e}^{dx}\,\mathrm{e}^{c}+16\,a^4\,b^2\,\mathrm{e}^{dx}\,\mathrm{e}^{c}\right)}{2\,d} \\
&\quad - \frac{2\,a^2\,\mathrm{e}^{c+dx}}{d-2\,d\,\mathrm{e}^{2c+2dx}+d\,\mathrm{e}^{4c+4dx}} - \frac{8\,b^2\,\mathrm{e}^{c+dx}}{3\,\left(d+2\,d\,\mathrm{e}^{2c+2dx}+d\,\mathrm{e}^{4c+4dx}\right)} \\
&\quad - \frac{64\,b^2\,\mathrm{e}^{c+dx}}{5\,\left(d+4\,d\,\mathrm{e}^{2c+2dx}+6\,d\,\mathrm{e}^{4c+4dx}+4\,d\,\mathrm{e}^{6c+6dx}+d\,\mathrm{e}^{8c+8dx}\right)} \\
&\quad + \frac{2\,a\,b\,\mathrm{e}^{c+dx}}{d+d\,\mathrm{e}^{2c+2dx}} - \frac{4\,a\,b\,\mathrm{e}^{c+dx}}{d+2\,d\,\mathrm{e}^{2c+2dx}+d\,\mathrm{e}^{4c+4dx}} \\
&\quad - \frac{a\,b\left(\ln\left(32\,a^3\,b^3\,\mathrm{e}^{dx}\,\mathrm{e}^{c}+8\,a^5\,b\,\mathrm{e}^{dx}\,\mathrm{e}^{c}-a^5\,b\,8\mathrm{i}-a^3\,b^3\,32\mathrm{i}\right)\,1\mathrm{i}-\ln\left(32\,a^3\,b^3\,\mathrm{e}^{dx}\,\mathrm{e}^{c}+8\,a^5\,b\,\mathrm{e}^{dx}\,\mathrm{e}^{c}+a^5\,b\,8\mathrm{i}+\right.\right.}{d}
\end{aligned}
$$

input
```
int((a + b*tanh(c + d*x)^3)^2/sinh(c + d*x)^3,x)
```

output

```
(a^2*exp(c + d*x))/(d - d*exp(2*c + 2*d*x)) + (136*b^2*exp(c + d*x))/(15*(
d + 3*d*exp(2*c + 2*d*x) + 3*d*exp(4*c + 4*d*x) + d*exp(6*c + 6*d*x))) + (
32*b^2*exp(c + d*x))/(5*(d + 5*d*exp(2*c + 2*d*x) + 10*d*exp(4*c + 4*d*x)
+ 10*d*exp(6*c + 6*d*x) + 5*d*exp(8*c + 8*d*x) + d*exp(10*c + 10*d*x))) -
(a^2*log(4*a^6*exp(d*x)*exp(c) - 16*a^4*b^2 - 4*a^6 + 16*a^4*b^2*exp(d*x)*
exp(c)))/(2*d) + (a^2*log(4*a^6 + 16*a^4*b^2 + 4*a^6*exp(d*x)*exp(c) + 16*
a^4*b^2*exp(d*x)*exp(c)))/(2*d) - (2*a^2*exp(c + d*x))/(d - 2*d*exp(2*c +
2*d*x) + d*exp(4*c + 4*d*x)) - (8*b^2*exp(c + d*x))/(3*(d + 2*d*exp(2*c +
2*d*x) + d*exp(4*c + 4*d*x))) - (64*b^2*exp(c + d*x))/(5*(d + 4*d*exp(2*c
+ 2*d*x) + 6*d*exp(4*c + 4*d*x) + 4*d*exp(6*c + 6*d*x) + d*exp(8*c + 8*d*x
))) + (2*a*b*exp(c + d*x))/(d + d*exp(2*c + 2*d*x)) - (4*a*b*exp(c + d*x))
/(d + 2*d*exp(2*c + 2*d*x) + d*exp(4*c + 4*d*x)) - (a*b*(log(32*a^3*b^3*ex
p(d*x)*exp(c) - a^3*b^3*32i - a^5*b*8i + 8*a^5*b*exp(d*x)*exp(c))*1i - log
(a^5*b*8i + a^3*b^3*32i + 32*a^3*b^3*exp(d*x)*exp(c) + 8*a^5*b*exp(d*x)*ex
p(c))*1i))/d
```

3.64 $\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

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3.64.1 Optimal result

Integrand size = 23, antiderivative size = 97

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \frac{a^2\coth(c+dx)}{d} - \frac{a^2\coth^3(c+dx)}{3d} + \frac{2ab\log(\tanh(c+dx))}{d} - \frac{ab\tanh^2(c+dx)}{d} + \frac{b^2\tanh^3(c+dx)}{3d} - \frac{b^2\tanh^5(c+dx)}{5d}$$

output
```
a^2*coth(d*x+c)/d-1/3*a^2*coth(d*x+c)^3/d+2*a*b*ln(tanh(d*x+c))/d-a*b*tanh
(d*x+c)^2/d+1/3*b^2*tanh(d*x+c)^3/d-1/5*b^2*tanh(d*x+c)^5/d
```

3.64.2 Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.52

$$\begin{aligned}&\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx \\ &= \frac{2a^2\coth(c+dx)}{3d} - \frac{a^2\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d} - \frac{2ab\log(\cosh(c+dx))}{d} \\ &\quad + \frac{2ab\log(\sinh(c+dx))}{d} + \frac{ab\operatorname{sech}^2(c+dx)}{d} + \frac{2b^2\tanh(c+dx)}{15d} \\ &\quad + \frac{b^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{15d} - \frac{b^2\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}\end{aligned}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
(2*a^2*Coth[c + d*x])/(3*d) - (a^2*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) -
(2*a*b*Log[Cosh[c + d*x]])/d + (2*a*b*Log[Sinh[c + d*x]])/d + (a*b*Sech[c
+ d*x]^2)/d + (2*b^2*Tanh[c + d*x])/(15*d) + (b^2*Sech[c + d*x]^2*Tanh[c +
 d*x])/(15*d) - (b^2*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d)
```

3.64.3 Rubi [A] (verified)

Time $= 0.34$ (sec) , antiderivative size $= 83$, normalized size of antiderivative $= 0.86$, number of steps used $= 5$, number of rules used $= 4$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used $= \{3042, 4146, 2333, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+ib\tan(ic+idx)^3\right)^2}{\sin(ic+idx)^4}dx$$

$$\downarrow 4146$$

$$\frac{\int \coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)\left(b\tanh^3(c+dx)+a\right)^2 d\tanh(c+dx)}{d}$$

$$\downarrow 2333$$

$$\frac{\int \left(a^2\coth^4(c+dx)-a^2\coth^2(c+dx)+2ab\coth(c+dx)-b^2\tanh^4(c+dx)+b^2\tanh^2(c+dx)-2ab\tanh(c+\right.}{d}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{3}a^2\coth^3(c+dx)+a^2\coth(c+dx)-ab\tanh^2(c+dx)+2ab\log(\tanh(c+dx))-\frac{1}{5}b^2\tanh^5(c+dx)+\frac{1}{3}b^2\tanh}{d}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
(a^2*Coth[c + d*x] - (a^2*Coth[c + d*x]^3)/3 + 2*a*b*Log[Tanh[c + d*x]] -
a*b*Tanh[c + d*x]^2 + (b^2*Tanh[c + d*x]^3)/3 - (b^2*Tanh[c + d*x]^5)/5)/d
```

3.64.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.64.4 Maple [A] (verified)

Time = 20.51 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{a^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+2ab\left(\frac{1}{2\cosh(dx+c)^2}+\ln(\tanh(dx+c))\right)+b^2\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sec}}{4}\right.\right.}{d}$
default	$\frac{a^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+2ab\left(\frac{1}{2\cosh(dx+c)^2}+\ln(\tanh(dx+c))\right)+b^2\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sec}}{4}\right.\right.}{d}$
risch	$-\frac{4\left(-15ab\,e^{14dx+14c}+15a^2e^{12dx+12c}+15b^2e^{12dx+12c}+70a^2e^{10dx+10c}+45ab\,e^{10dx+10c}-50b^2e^{10dx+10c}+125a^2e^{8dx+8}\right.}{15d}$

input
```
int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+2*a*b*(1/2/cosh(d*x+c)^2+ln(t
anh(d*x+c)))+b^2*(-1/4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)
^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)))
```

3.64.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4125 vs. 2(91) = 182.

Time = 0.28 (sec) , antiderivative size = 4125, normalized size of antiderivative = 42.53

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
2/15*(30*a*b*cosh(d*x + c)^14 + 420*a*b*cosh(d*x + c)*sinh(d*x + c)^13 + 3
0*a*b*sinh(d*x + c)^14 - 30*(a^2 + b^2)*cosh(d*x + c)^12 + 30*(91*a*b*cosh
(d*x + c)^2 - a^2 - b^2)*sinh(d*x + c)^12 + 120*(91*a*b*cosh(d*x + c)^3 -
3*(a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c)^11 - 10*(14*a^2 + 9*a*b - 10*b^
2)*cosh(d*x + c)^10 + 10*(3003*a*b*cosh(d*x + c)^4 - 198*(a^2 + b^2)*cosh(
d*x + c)^2 - 14*a^2 - 9*a*b + 10*b^2)*sinh(d*x + c)^10 + 20*(3003*a*b*cosh
(d*x + c)^5 - 330*(a^2 + b^2)*cosh(d*x + c)^3 - 5*(14*a^2 + 9*a*b - 10*b^2
)*cosh(d*x + c))*sinh(d*x + c)^9 - 10*(25*a^2 + 13*b^2)*cosh(d*x + c)^8 +
10*(9009*a*b*cosh(d*x + c)^6 - 1485*(a^2 + b^2)*cosh(d*x + c)^4 - 45*(14*a
^2 + 9*a*b - 10*b^2)*cosh(d*x + c)^2 - 25*a^2 - 13*b^2)*sinh(d*x + c)^8 +
80*(1287*a*b*cosh(d*x + c)^7 - 297*(a^2 + b^2)*cosh(d*x + c)^5 - 15*(14*a^
2 + 9*a*b - 10*b^2)*cosh(d*x + c)^3 - (25*a^2 + 13*b^2)*cosh(d*x + c))*sin
h(d*x + c)^7 - 2*(100*a^2 - 45*a*b - 44*b^2)*cosh(d*x + c)^6 + 2*(45045*a*
b*cosh(d*x + c)^8 - 13860*(a^2 + b^2)*cosh(d*x + c)^6 - 1050*(14*a^2 + 9*a
*b - 10*b^2)*cosh(d*x + c)^4 - 140*(25*a^2 + 13*b^2)*cosh(d*x + c)^2 - 100
*a^2 + 45*a*b + 44*b^2)*sinh(d*x + c)^6 + 4*(15015*a*b*cosh(d*x + c)^9 - 5
940*(a^2 + b^2)*cosh(d*x + c)^7 - 630*(14*a^2 + 9*a*b - 10*b^2)*cosh(d*x +
 c)^5 - 140*(25*a^2 + 13*b^2)*cosh(d*x + c)^3 - 3*(100*a^2 - 45*a*b - 44*b
^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(25*a^2 + 17*b^2)*cosh(d*x + c)^4 +
 2*(15015*a*b*cosh(d*x + c)^10 - 7425*(a^2 + b^2)*cosh(d*x + c)^8 - 105...
```

3.64.6 Sympy [F]

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \int \left(a+b\tanh^3(c+dx)\right)^2 \operatorname{csch}^4(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**3)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x)**4, x)
```

3.64.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(91) = 182$.

Time = 0.30 (sec) , antiderivative size = 468, normalized size of antiderivative = 4.82

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$

$$= 2\,ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$
$$+\frac{4}{15}\,b^2\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}-\frac{}{d(5\,e^{(-2\,dx-2\,c}\right.$$
$$+\frac{4}{3}\,a^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right.$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
2*a*b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x -
 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c
) + 1))) + 4/15*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*
d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c)
 + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*
e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d
*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*
d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c
) + e^(-10*d*x - 10*c) + 1))) + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*
x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x
 - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

3.64.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(91) = 182$.

Time = 0.38 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.57

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx =$$
$$-\frac{60\,ab\log\left(e^{(2\,dx+2\,c)}+1\right)-60\,ab\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{10\left(11\,abe^{(6\,dx+6\,c)}-33\,abe^{(4\,dx+4\,c)}+12\,a^2e^{(2\,dx+2\,c)}+33\,abe^{(2\,d}\right.}{\left(e^{(2\,dx+2\,c)}-1\right)^3}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
-1/30*(60*a*b*log(e^(2*d*x + 2*c) + 1) - 60*a*b*log(abs(e^(2*d*x + 2*c) -
1)) + 10*(11*a*b*e^(6*d*x + 6*c) - 33*a*b*e^(4*d*x + 4*c) + 12*a^2*e^(2*d*
x + 2*c) + 33*a*b*e^(2*d*x + 2*c) - 4*a^2 - 11*a*b)/(e^(2*d*x + 2*c) - 1)^
3 - (137*a*b*e^(10*d*x + 10*c) + 805*a*b*e^(8*d*x + 8*c) + 1730*a*b*e^(6*d
*x + 6*c) - 120*b^2*e^(6*d*x + 6*c) + 1730*a*b*e^(4*d*x + 4*c) + 40*b^2*e^
(4*d*x + 4*c) + 805*a*b*e^(2*d*x + 2*c) - 40*b^2*e^(2*d*x + 2*c) + 137*a*b
 - 8*b^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.64.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 344, normalized size of antiderivative = 3.55

$$\begin{aligned}
&\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx \\
&= \frac{40\,b^2}{3\,d\,\left(3\,\mathrm{e}^{2c+2dx}+3\,\mathrm{e}^{4c+4dx}+\mathrm{e}^{6c+6dx}+1\right)} - \frac{4\,a^2}{d\,\left(\mathrm{e}^{4c+4dx}-2\,\mathrm{e}^{2c+2dx}+1\right)} \\
&\quad - \frac{8\,a^2}{3\,d\,\left(3\,\mathrm{e}^{2c+2dx}-3\,\mathrm{e}^{4c+4dx}+\mathrm{e}^{6c+6dx}-1\right)} - \frac{4\,(b^2+a\,b)}{d\,\left(2\,\mathrm{e}^{2c+2dx}+\mathrm{e}^{4c+4dx}+1\right)} \\
&\quad - \frac{16\,b^2}{d\,\left(4\,\mathrm{e}^{2c+2dx}+6\,\mathrm{e}^{4c+4dx}+4\,\mathrm{e}^{6c+6dx}+\mathrm{e}^{8c+8dx}+1\right)} \\
&\quad + \frac{32\,b^2}{5\,d\,\left(5\,\mathrm{e}^{2c+2dx}+10\,\mathrm{e}^{4c+4dx}+10\,\mathrm{e}^{6c+6dx}+5\,\mathrm{e}^{8c+8dx}+\mathrm{e}^{10c+10dx}+1\right)} \\
&\quad - \frac{4\,\operatorname{atan}\left(\frac{a\,b\,\mathrm{e}^{2c}\,\mathrm{e}^{2dx}\,\sqrt{-d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{-d^2}} + \frac{4\,a\,b}{d\,\left(\mathrm{e}^{2c+2dx}+1\right)}
\end{aligned}$$

input
```
int((a + b*tanh(c + d*x)^3)^2/sinh(c + d*x)^4,x)
```

output
```
(40*b^2)/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x)
+ 1)) - (4*a^2)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*a^2)/
(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (
4*(a*b + b^2))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (16*b^2)/
(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c
 + 8*d*x) + 1)) + (32*b^2)/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x)
+ 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (4
*atan((a*b*exp(2*c)*exp(2*d*x)*(-d^2)^(1/2))/(d*(a^2*b^2)^(1/2)))*(a^2*b^2
)^(1/2))/(-d^2)^(1/2) + (4*a*b)/(d*(exp(2*c + 2*d*x) + 1))
```

3.65 $\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

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3.65.1 Optimal result

Integrand size = 23, antiderivative size = 275

$$\begin{aligned}
&\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx \\
&= \frac{3}{8}a\left(a^2+63b^2\right)x + \frac{3b(3a^2+5b^2)\log(\cosh(c+dx))}{d} - \frac{18ab^2\tanh(c+dx)}{d} \\
&\quad - \frac{b(3a^2+10b^2)\tanh^2(c+dx)}{2d} - \frac{3ab^2\tanh^3(c+dx)}{d} - \frac{3b^3\tanh^4(c+dx)}{2d} \\
&\quad - \frac{3ab^2\tanh^5(c+dx)}{5d} - \frac{b^3\tanh^6(c+dx)}{2d} - \frac{b^3\tanh^8(c+dx)}{8d} \\
&\quad + \frac{\cosh^3(c+dx)\sinh(c+dx)\left(a(a^2+3b^2)+b(3a^2+b^2)\tanh(c+dx)\right)}{4d} \\
&\quad - \frac{\cosh(c+dx)\sinh(c+dx)\left(a(5a^2+51b^2)+2b(15a^2+11b^2)\tanh(c+dx)\right)}{8d}
\end{aligned}$$

output

```
3/8*a*(a^2+63*b^2)*x+3*b*(3*a^2+5*b^2)*ln(cosh(d*x+c))/d-18*a*b^2*tanh(d*x
+c)/d-1/2*b*(3*a^2+10*b^2)*tanh(d*x+c)^2/d-3*a*b^2*tanh(d*x+c)^3/d-3/2*b^3
*tanh(d*x+c)^4/d-3/5*a*b^2*tanh(d*x+c)^5/d-1/2*b^3*tanh(d*x+c)^6/d-1/8*b^3
*tanh(d*x+c)^8/d+1/4*cosh(d*x+c)^3*sinh(d*x+c)*(a*(a^2+3*b^2)+b*(3*a^2+b^2
)*tanh(d*x+c))/d-1/8*cosh(d*x+c)*sinh(d*x+c)*(a*(5*a^2+51*b^2)+2*b*(15*a^2
+11*b^2)*tanh(d*x+c))/d
```

3.65.2 Mathematica [A] (verified)

Time = 7.37 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.07

$$\begin{aligned}
&\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx \\
&= \frac{3a(a^2+63b^2)(c+dx)}{8d} - \frac{b(15a^2+11b^2)\cosh(2(c+dx))}{8d} + \frac{b(3a^2+b^2)\cosh(4(c+dx))}{32d} \\
&\quad + \frac{3(3a^2b+5b^3)\log(\cosh(c+dx))}{d} + \frac{b(3a^2+20b^2)\operatorname{sech}^2(c+dx)}{2d} \\
&\quad - \frac{15b^3\operatorname{sech}^4(c+dx)}{4d} + \frac{b^3\operatorname{sech}^6(c+dx)}{d} - \frac{b^3\operatorname{sech}^8(c+dx)}{8d} \\
&\quad - \frac{a(a^2+12b^2)\sinh(2(c+dx))}{4d} + \frac{a(a^2+3b^2)\sinh(4(c+dx))}{32d} - \frac{108ab^2\tanh(c+dx)}{5d} \\
&\quad + \frac{21ab^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{5d} - \frac{3ab^2\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}
\end{aligned}$$

input
```
Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
(3*a*(a^2 + 63*b^2)*(c + d*x))/(8*d) - (b*(15*a^2 + 11*b^2)*Cosh[2*(c + d*
x)])/(8*d) + (b*(3*a^2 + b^2)*Cosh[4*(c + d*x)])/(32*d) + (3*(3*a^2*b + 5*
b^3)*Log[Cosh[c + d*x]])/d + (b*(3*a^2 + 20*b^2)*Sech[c + d*x]^2)/(2*d) -
(15*b^3*Sech[c + d*x]^4)/(4*d) + (b^3*Sech[c + d*x]^6)/d - (b^3*Sech[c + d
*x]^8)/(8*d) - (a*(a^2 + 12*b^2)*Sinh[2*(c + d*x)])/(4*d) + (a*(a^2 + 3*b^
2)*Sinh[4*(c + d*x)])/(32*d) - (108*a*b^2*Tanh[c + d*x])/(5*d) + (21*a*b^2
*Sech[c + d*x]^2*Tanh[c + d*x])/(5*d) - (3*a*b^2*Sech[c + d*x]^4*Tanh[c +
d*x])/(5*d)
```

3.65.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4146, 2335, 25, 2335, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$

$$\downarrow \text{3042}$$

$$\int \sin(ic+idx)^4 \left(a+ib\tan(ic+idx)^3\right)^3 dx$$

$$\downarrow 4146$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(b\tanh^3(c+dx)+a\right)^3}{\left(1-\tanh^2(c+dx)\right)^3} d\tanh(c+dx)}{d}$$

$$\downarrow 2335$$

$$\frac{1}{4}\int -\frac{\tanh^3(c+dx)\left(4b^3\tanh^8(c+dx)+4b^3\tanh^6(c+dx)+12ab^2\tanh^5(c+dx)+4b^3\tanh^4(c+dx)+12ab^2\tanh^3(c+dx)+4b\left(3a^2+b^2\right)\tanh^2(c+dx)+\right.}{\left(1-\tanh^2(c+dx)\right)^2}$$

$$d$$

$$\downarrow 25$$

$$\frac{\tanh^4(c+dx)\left(a\left(a^2+3b^2\right)\tanh(c+dx)+b\left(3a^2+b^2\right)\right)}{4\left(1-\tanh^2(c+dx)\right)^2} - \frac{1}{4}\int \frac{\tanh^3(c+dx)\left(4b^3\tanh^8(c+dx)+4b^3\tanh^6(c+dx)+12ab^2\tanh^5(c+dx)+4b^3\tanh^4(c+\right.}{(1-\text{ta}}$$

$$d$$

$$\downarrow 2335$$

$$\frac{1}{4}\left(-\frac{1}{2}\int -\frac{\tanh^2(c+dx)\left(8b^3\tanh^7(c+dx)+16b^3\tanh^5(c+dx)+24ab^2\tanh^4(c+dx)+24b^3\tanh^3(c+dx)+48ab^2\tanh^2(c+dx)+72b\left(a^2+b^2\right)\tanh(\right.}{1-\tanh^2(c+dx)}$$

$$\downarrow 25$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{\tanh^2(c+dx)\left(8b^3\tanh^7(c+dx)+16b^3\tanh^5(c+dx)+24ab^2\tanh^4(c+dx)+24b^3\tanh^3(c+dx)+48ab^2\tanh^2(c+dx)+72b\left(a^2+b^2\right)\tanh(c+dx\right.}{1-\tanh^2(c+dx)}$$

$$\downarrow 2333$$

$$\frac{1}{4}\left(\frac{1}{2}\int \left(-8b^3\tanh^7(c+dx)-24b^3\tanh^5(c+dx)-24ab^2\tanh^4(c+dx)-48b^3\tanh^3(c+dx)-72ab^2\tanh^2(c+\right.\right.$$

$$\downarrow 2009$$

$$\frac{1}{4}\left(\frac{1}{2}\left(3a\left(a^2+63b^2\right)\operatorname{arctanh}(\tanh(c+dx))-12b\left(3a^2+5b^2\right)\tanh^2(c+dx)-3a\left(a^2+63b^2\right)\tanh(c+dx)-12b(3\right.\right.$$

input
```
Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]
```

3.65. $\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3 dx$

output
```
((Tanh[c + d*x]^4*(b*(3*a^2 + b^2) + a*(a^2 + 3*b^2)*Tanh[c + d*x]))/(4*(1
 - Tanh[c + d*x]^2)^2) + (-1/2*(Tanh[c + d*x]^3*(a*(a^2 + 39*b^2) + 4*b*(6
*a^2 + 5*b^2)*Tanh[c + d*x]))/(1 - Tanh[c + d*x]^2) + (3*a*(a^2 + 63*b^2)*
ArcTanh[Tanh[c + d*x]] - 12*b*(3*a^2 + 5*b^2)*Log[1 - Tanh[c + d*x]^2] - 3
*a*(a^2 + 63*b^2)*Tanh[c + d*x] - 12*b*(3*a^2 + 5*b^2)*Tanh[c + d*x]^2 - 2
4*a*b^2*Tanh[c + d*x]^3 - 12*b^3*Tanh[c + d*x]^4 - (24*a*b^2*Tanh[c + d*x]
^5)/5 - 4*b^3*Tanh[c + d*x]^6 - b^3*Tanh[c + d*x]^8)/2)/4)/d
```

3.65.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.65.4 Maple [A] (verified)

Time = 41.60 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{a^3\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\sinh(dx+c)^6}{4\cosh(dx+c)^2}-\frac{3\sinh(dx+c)^4}{4\cosh(dx+c)^2}+3\ln(\cosh(dx+c))-\frac{3\tan}{}$
default	$\frac{a^3\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\sinh(dx+c)^6}{4\cosh(dx+c)^2}-\frac{3\sinh(dx+c)^4}{4\cosh(dx+c)^2}+3\ln(\cosh(dx+c))-\frac{3\tan}{}$
risch	$-\frac{e^{2dx+2c}a^3}{8d}-\frac{11e^{2dx+2c}b^3}{16d}+\frac{3a^3x}{8}-9ba^2x-\frac{18bca^2}{d}+\frac{9b\ln(e^{2dx+2c}+1)a^2}{d}+\frac{15b^3\ln(e^{2dx+2c}+1)}{d}+$

input
```
int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+3
*a^2*b*(1/4*sinh(d*x+c)^6/cosh(d*x+c)^2-3/4*sinh(d*x+c)^4/cosh(d*x+c)^2+3*
ln(cosh(d*x+c))-3/2*tanh(d*x+c)^2)+3*a*b^2*(1/4*sinh(d*x+c)^9/cosh(d*x+c)^
5-9/8*sinh(d*x+c)^7/cosh(d*x+c)^5+63/8*d*x+63/8*c-63/8*tanh(d*x+c)-21/8*ta
nh(d*x+c)^3-63/40*tanh(d*x+c)^5)+b^3*(1/4*sinh(d*x+c)^12/cosh(d*x+c)^8-3/2
*sinh(d*x+c)^10/cosh(d*x+c)^8+15*ln(cosh(d*x+c))-15/2*tanh(d*x+c)^2-15/4*t
anh(d*x+c)^4-5/2*tanh(d*x+c)^6-15/8*tanh(d*x+c)^8))
```

3.65.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12323 vs. 2(259) = 518.

Time = 0.40 (sec) , antiderivative size = 12323, normalized size of antiderivative = 44.81

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.65.6 Sympy [F(-1)]

Timed out.

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**3)**3,x)
```

output
```
Timed out
```

3.65.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 647 vs. 2(259) = 518.

Time = 0.30 (sec) , antiderivative size = 647, normalized size of antiderivative = 2.35

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$= \frac{1}{64}a^3\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)$$
$$+\frac{3}{320}ab^2\left(\frac{2520(dx+c)}{d}+\frac{5\left(32e^{(-2dx-2c)}-e^{(-4dx-4c)}\right)}{d}-\frac{135e^{(-2dx-2c)}+5358e^{(-4dx-4c)}+18190e^{(}}{d(e^{(-4dx-4c)}+5e^{(-6dx-6c)}+10}\right.$$
$$+\frac{1}{64}b^3\left(\frac{960(dx+c)}{d}-\frac{44e^{(-2dx-2c)}-e^{(-4dx-4c)}}{d}+\frac{960\log\left(e^{(-2dx-2c)}+1\right)}{d}-\frac{36e^{(-2dx-2c)}+324e^{(-}}{d(e^{(-4dx}}\right.$$
$$+\frac{3}{64}a^2b\left(\frac{192(dx+c)}{d}-\frac{20e^{(-2dx-2c)}-e^{(-4dx-4c)}}{d}+\frac{192\log\left(e^{(-2dx-2c)}+1\right)}{d}-\frac{18e^{(-2dx-2c)}+39e^{(-}}{d(e^{(-4dx-4c)}+2e}\right.$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

output

```
1/64*a^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) + 3/320*a*b^2*(2520*(d*x + c)/d + 5*(32*e^(-2*
d*x - 2*c) - e^(-4*d*x - 4*c))/d - (135*e^(-2*d*x - 2*c) + 5358*e^(-4*d*x
- 4*c) + 18190*e^(-6*d*x - 6*c) + 28455*e^(-8*d*x - 8*c) + 19995*e^(-10*d*
x - 10*c) + 6560*e^(-12*d*x - 12*c) - 5)/(d*(e^(-4*d*x - 4*c) + 5*e^(-6*d*
x - 6*c) + 10*e^(-8*d*x - 8*c) + 10*e^(-10*d*x - 10*c) + 5*e^(-12*d*x - 12
*c) + e^(-14*d*x - 14*c)))) + 1/64*b^3*(960*(d*x + c)/d - (44*e^(-2*d*x -
2*c) - e^(-4*d*x - 4*c))/d + 960*log(e^(-2*d*x - 2*c) + 1)/d - (36*e^(-2*d
*x - 2*c) + 324*e^(-4*d*x - 4*c) - 1384*e^(-6*d*x - 6*c) - 9126*e^(-8*d*x
- 8*c) - 24112*e^(-10*d*x - 10*c) - 31868*e^(-12*d*x - 12*c) - 25912*e^(-1
4*d*x - 14*c) - 11169*e^(-16*d*x - 16*c) - 2516*e^(-18*d*x - 18*c) - 1)/(d
*(e^(-4*d*x - 4*c) + 8*e^(-6*d*x - 6*c) + 28*e^(-8*d*x - 8*c) + 56*e^(-10*
d*x - 10*c) + 70*e^(-12*d*x - 12*c) + 56*e^(-14*d*x - 14*c) + 28*e^(-16*d*
x - 16*c) + 8*e^(-18*d*x - 18*c) + e^(-20*d*x - 20*c))) + 3/64*a^2*b*(192
*(d*x + c)/d - (20*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d + 192*log(e^(-2*
d*x - 2*c) + 1)/d - (18*e^(-2*d*x - 2*c) + 39*e^(-4*d*x - 4*c) - 108*e^(-6
*d*x - 6*c) - 1)/(d*(e^(-4*d*x - 4*c) + 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8
*c))))
```

3.65.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(259) = 518$.

Time $= 0.89$ (sec) , antiderivative size $= 694$, normalized size of antiderivative $= 2.52$

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$

$$= \frac{35\,a^3e^{(4\,dx+4\,c)} + 105\,a^2be^{(4\,dx+4\,c)} + 105\,ab^2e^{(4\,dx+4\,c)} + 35\,b^3e^{(4\,dx+4\,c)} - 280\,a^3e^{(2\,dx+2\,c)} - 2100\,a^2be^{(2\,dx+2\,c)}}{}$$

input

```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

output

```
1/2240*(35*a^3*e^(4*d*x + 4*c) + 105*a^2*b*e^(4*d*x + 4*c) + 105*a*b^2*e^(
4*d*x + 4*c) + 35*b^3*e^(4*d*x + 4*c) - 280*a^3*e^(2*d*x + 2*c) - 2100*a^2
*b*e^(2*d*x + 2*c) - 3360*a*b^2*e^(2*d*x + 2*c) - 1540*b^3*e^(2*d*x + 2*c)
 + 840*(a^3 - 24*a^2*b + 63*a*b^2 - 40*b^3)*(d*x + c) - 35*(18*a^3*e^(4*d*
x + 4*c) - 432*a^2*b*e^(4*d*x + 4*c) + 1134*a*b^2*e^(4*d*x + 4*c) - 720*b^
3*e^(4*d*x + 4*c) - 8*a^3*e^(2*d*x + 2*c) + 60*a^2*b*e^(2*d*x + 2*c) - 96*
a*b^2*e^(2*d*x + 2*c) + 44*b^3*e^(2*d*x + 2*c) + a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*e^(-4*d*x - 4*c) + 6720*(3*a^2*b + 5*b^3)*log(e^(2*d*x + 2*c) + 1) -
 8*(6849*a^2*b*e^(16*d*x + 16*c) + 11415*b^3*e^(16*d*x + 16*c) + 53112*a^2
*b*e^(14*d*x + 14*c) - 16800*a*b^2*e^(14*d*x + 14*c) + 80120*b^3*e^(14*d*x
 + 14*c) + 181692*a^2*b*e^(12*d*x + 12*c) - 100800*a*b^2*e^(12*d*x + 12*c)
 + 269220*b^3*e^(12*d*x + 12*c) + 358344*a^2*b*e^(10*d*x + 10*c) - 272160*
a*b^2*e^(10*d*x + 10*c) + 520520*b^3*e^(10*d*x + 10*c) + 445830*a^2*b*e^(8
*d*x + 8*c) - 423360*a*b^2*e^(8*d*x + 8*c) + 648970*b^3*e^(8*d*x + 8*c) +
358344*a^2*b*e^(6*d*x + 6*c) - 405216*a*b^2*e^(6*d*x + 6*c) + 520520*b^3*e
^(6*d*x + 6*c) + 181692*a^2*b*e^(4*d*x + 4*c) - 237888*a*b^2*e^(4*d*x + 4*
c) + 269220*b^3*e^(4*d*x + 4*c) + 53112*a^2*b*e^(2*d*x + 2*c) - 79968*a*b^
2*e^(2*d*x + 2*c) + 80120*b^3*e^(2*d*x + 2*c) + 6849*a^2*b - 12096*a*b^2 +
 11415*b^3)/(e^(2*d*x + 2*c) + 1)^8)/d
```

3.65.9 Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 682, normalized size of antiderivative = 2.48

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$

$$= x\left(\frac{3a^3}{8}-9a^2b+\frac{189ab^2}{8}-15b^3\right)-\frac{4\left(71b^3+12ab^2\right)}{d\left(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1\right)}$$
$$-\frac{256b^3}{d\left(6e^{2c+2dx}+15e^{4c+4dx}+20e^{6c+6dx}+15e^{8c+8dx}+6e^{10c+10dx}+e^{12c+12dx}+1\right)}$$
$$+\frac{\ln\left(e^{2c}e^{2dx}+1\right)\left(9a^2b+15b^3\right)}{d}+\frac{2\left(3a^2b+30ab^2+20b^3\right)}{d\left(e^{2c+2dx}+1\right)}$$
$$+\frac{32\left(50b^3+3ab^2\right)}{5d\left(5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1\right)}$$
$$+\frac{128b^3}{d\left(7e^{2c+2dx}+21e^{4c+4dx}+35e^{6c+6dx}+35e^{8c+8dx}+21e^{10c+10dx}+7e^{12c+12dx}+e^{14c+14dx}+1\right)}$$
$$-\frac{2\left(3a^2b+30ab^2+50b^3\right)}{d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}$$
$$-\frac{32b^3}{d\left(8e^{2c+2dx}+28e^{4c+4dx}+56e^{6c+6dx}+70e^{8c+8dx}+56e^{10c+10dx}+28e^{12c+12dx}+8e^{14c+14dx}+e^{16c}\right.}$$
$$+\frac{e^{4c+4dx}(a+b)^3}{64d}-\frac{e^{-4c-4dx}(a-b)^3}{64d}+\frac{8\left(23b^3+9ab^2\right)}{d\left(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1\right)}$$
$$-\frac{e^{2c+2dx}(a+b)^2(2a+11b)}{16d}+\frac{e^{-2c-2dx}(a-b)^2(2a-11b)}{16d}$$

input
```
int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^3)^3,x)
```

output

```
x*((189*a*b^2)/8 - 9*a^2*b + (3*a^3)/8 - 15*b^3) - (4*(12*a*b^2 + 71*b^3))
/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*
c + 8*d*x) + 1)) - (256*b^3)/(d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x)
+ 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(1
2*c + 12*d*x) + 1)) + (log(exp(2*c)*exp(2*d*x) + 1)*(9*a^2*b + 15*b^3))/d
+ (2*(30*a*b^2 + 3*a^2*b + 20*b^3))/(d*(exp(2*c + 2*d*x) + 1)) + (32*(3*a*
b^2 + 50*b^3))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c
 + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (128*b^3)/(d*(
7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*
c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*
d*x) + 1)) - (2*(30*a*b^2 + 3*a^2*b + 50*b^3))/(d*(2*exp(2*c + 2*d*x) + ex
p(4*c + 4*d*x) + 1)) - (32*b^3)/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*
x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 2
8*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1)) + (
exp(4*c + 4*d*x)*(a + b)^3)/(64*d) - (exp(- 4*c - 4*d*x)*(a - b)^3)/(64*d)
 + (8*(9*a*b^2 + 23*b^3))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + ex
p(6*c + 6*d*x) + 1)) - (exp(2*c + 2*d*x)*(a + b)^2*(2*a + 11*b))/(16*d) +
(exp(- 2*c - 2*d*x)*(a - b)^2*(2*a - 11*b))/(16*d)
```

3.66 $\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

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3.66.1 Optimal result

Integrand size = 23, antiderivative size = 351

$$
\begin{aligned}
&\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx \\
&= \frac{15a^2b\arctan(\sinh(c+dx))}{2d} + \frac{1155b^3\arctan(\sinh(c+dx))}{128d} - \frac{a^3\cosh(c+dx)}{d} \\
&\quad - \frac{12ab^2\cosh(c+dx)}{d} + \frac{a^3\cosh^3(c+dx)}{3d} + \frac{ab^2\cosh^3(c+dx)}{d} - \frac{18ab^2\operatorname{sech}(c+dx)}{d} \\
&\quad + \frac{4ab^2\operatorname{sech}^3(c+dx)}{d} - \frac{3ab^2\operatorname{sech}^5(c+dx)}{5d} - \frac{15a^2b\sinh(c+dx)}{2d} - \frac{1155b^3\sinh(c+dx)}{128d} \\
&\quad + \frac{5a^2b\sinh^3(c+dx)}{2d} + \frac{385b^3\sinh^3(c+dx)}{128d} - \frac{3a^2b\sinh^3(c+dx)\tanh^2(c+dx)}{2d} \\
&\quad - \frac{231b^3\sinh^3(c+dx)\tanh^2(c+dx)}{128d} - \frac{33b^3\sinh^3(c+dx)\tanh^4(c+dx)}{64d} \\
&\quad - \frac{11b^3\sinh^3(c+dx)\tanh^6(c+dx)}{48d} - \frac{b^3\sinh^3(c+dx)\tanh^8(c+dx)}{8d}
\end{aligned}
$$

output

```
15/2*a^2*b*arctan(sinh(d*x+c))/d+1155/128*b^3*arctan(sinh(d*x+c))/d-a^3*co
sh(d*x+c)/d-12*a*b^2*cosh(d*x+c)/d+1/3*a^3*cosh(d*x+c)^3/d+a*b^2*cosh(d*x+
c)^3/d-18*a*b^2*sech(d*x+c)/d+4*a*b^2*sech(d*x+c)^3/d-3/5*a*b^2*sech(d*x+c
)^5/d-15/2*a^2*b*sinh(d*x+c)/d-1155/128*b^3*sinh(d*x+c)/d+5/2*a^2*b*sinh(d
*x+c)^3/d+385/128*b^3*sinh(d*x+c)^3/d-3/2*a^2*b*sinh(d*x+c)^3*tanh(d*x+c)^
2/d-231/128*b^3*sinh(d*x+c)^3*tanh(d*x+c)^2/d-33/64*b^3*sinh(d*x+c)^3*tanh
(d*x+c)^4/d-11/48*b^3*sinh(d*x+c)^3*tanh(d*x+c)^6/d-1/8*b^3*sinh(d*x+c)^3*
tanh(d*x+c)^8/d
```

3.66.2 Mathematica [A] (verified)

Time = 8.23 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.83

$$\begin{aligned}
&\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx \\
&= \frac{15(64a^2b+77b^3)\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{64d} - \frac{3a(a^2+15b^2)\cosh(c+dx)}{4d} \\
&\quad + \frac{a(a^2+3b^2)\cosh(3(c+dx))}{12d} - \frac{18ab^2\operatorname{sech}(c+dx)}{d} \\
&\quad + \frac{4ab^2\operatorname{sech}^3(c+dx)}{d} - \frac{3ab^2\operatorname{sech}^5(c+dx)}{5d} - \frac{3b(9a^2+7b^2)\sinh(c+dx)}{4d} \\
&\quad - \frac{3\operatorname{sech}^2(c+dx)\left(64a^2b\sinh(c+dx)+255b^3\sinh(c+dx)\right)}{128d} \\
&\quad + \frac{b(3a^2+b^2)\sinh(3(c+dx))}{12d} + \frac{515b^3\operatorname{sech}^3(c+dx)\tanh(c+dx)}{192d} \\
&\quad - \frac{41b^3\operatorname{sech}^5(c+dx)\tanh(c+dx)}{48d} + \frac{b^3\operatorname{sech}^7(c+dx)\tanh(c+dx)}{8d}
\end{aligned}$$

input
```
Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
(15*(64*a^2*b + 77*b^3)*ArcTan[Tanh[(c + d*x)/2]])/(64*d) - (3*a*(a^2 + 15
*b^2)*Cosh[c + d*x])/(4*d) + (a*(a^2 + 3*b^2)*Cosh[3*(c + d*x)])/(12*d) -
(18*a*b^2*Sech[c + d*x])/d + (4*a*b^2*Sech[c + d*x]^3)/d - (3*a*b^2*Sech[c
 + d*x]^5)/(5*d) - (3*b*(9*a^2 + 7*b^2)*Sinh[c + d*x])/(4*d) - (3*Sech[c +
 d*x]^2*(64*a^2*b*Sinh[c + d*x] + 255*b^3*Sinh[c + d*x]))/(128*d) + (b*(3*
a^2 + b^2)*Sinh[3*(c + d*x)])/(12*d) + (515*b^3*Sech[c + d*x]^3*Tanh[c + d
*x])/(192*d) - (41*b^3*Sech[c + d*x]^5*Tanh[c + d*x])/(48*d) + (b^3*Sech[c
 + d*x]^7*Tanh[c + d*x])/(8*d)
```

3.66.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$

$$\downarrow 3042$$

$$\int i\sin(ic+idx)^3\left(a+ib\tan(ic+idx)^3\right)^3\,dx$$

$$\downarrow 26$$

$$i\int \sin(ic+idx)^3\left(ib\tan(ic+idx)^3+a\right)^3\,dx$$

$$\downarrow 4149$$

$$i\int \left(-ib^3\sinh^3(c+dx)\tanh^9(c+dx)-3iab^2\sinh^3(c+dx)\tanh^6(c+dx)-3ia^2b\sinh^3(c+dx)\tanh^3(c+dx)-\right.$$

$$\downarrow 2009$$

$$i\left(-\frac{ia^3\cosh^3(c+dx)}{3d}+\frac{ia^3\cosh(c+dx)}{d}-\frac{15ia^2b\arctan(\sinh(c+dx))}{2d}-\frac{5ia^2b\sinh^3(c+dx)}{2d}+\frac{15ia^2b\sinh(c+\right.$$

input
```
Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
I*((((-15*I)/2)*a^2*b*ArcTan[Sinh[c + d*x]])/d - (((1155*I)/128)*b^3*ArcTa
n[Sinh[c + d*x]])/d + (I*a^3*Cosh[c + d*x])/d + ((12*I)*a*b^2*Cosh[c + d*x
])/d - ((I/3)*a^3*Cosh[c + d*x]^3)/d - (I*a*b^2*Cosh[c + d*x]^3)/d + ((18*
I)*a*b^2*Sech[c + d*x])/d - ((4*I)*a*b^2*Sech[c + d*x]^3)/d + (((3*I)/5)*a
*b^2*Sech[c + d*x]^5)/d + (((15*I)/2)*a^2*b*Sinh[c + d*x])/d + (((1155*I)/
128)*b^3*Sinh[c + d*x])/d - (((5*I)/2)*a^2*b*Sinh[c + d*x]^3)/d - (((385*I
)/128)*b^3*Sinh[c + d*x]^3)/d + (((3*I)/2)*a^2*b*Sinh[c + d*x]^3*Tanh[c +
d*x]^2)/d + (((231*I)/128)*b^3*Sinh[c + d*x]^3*Tanh[c + d*x]^2)/d + (((33*
I)/64)*b^3*Sinh[c + d*x]^3*Tanh[c + d*x]^4)/d + (((11*I)/48)*b^3*Sinh[c +
d*x]^3*Tanh[c + d*x]^6)/d + ((I/8)*b^3*Sinh[c + d*x]^3*Tanh[c + d*x]^8)/d)
```

3.66.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

3.66.4 Maple [A] (verified)

Time = 23.77 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{a^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+3a^2b\left(\frac{\sinh(dx+c)^5}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)^3}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{5\ \mathrm{sech}(dx+c)\tanh(dx+c)}{2}+5\arcta$
default	$\frac{a^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+3a^2b\left(\frac{\sinh(dx+c)^5}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)^3}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{5\ \mathrm{sech}(dx+c)\tanh(dx+c)}{2}+5\arcta$
risch	$\frac{e^{3dx+3c}a^3}{24d}+\frac{e^{3dx+3c}a^2b}{8d}+\frac{e^{3dx+3c}ab^2}{8d}+\frac{b^3e^{3dx+3c}}{24d}-\frac{3e^{dx+c}a^3}{8d}-\frac{27e^{dx+c}a^2b}{8d}-\frac{45e^{dx+c}ab^2}{8d}-\frac{21b^3e^{dx}}{8d}$

input
```
int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+3*a^2*b*(1/3*sinh(d*x+c)^5/c
osh(d*x+c)^2-5/3*sinh(d*x+c)^3/cosh(d*x+c)^2-5*sinh(d*x+c)/cosh(d*x+c)^2+5
/2*sech(d*x+c)*tanh(d*x+c)+5*arctan(exp(d*x+c)))+3*a*b^2*(1/3*sinh(d*x+c)^
8/cosh(d*x+c)^5-8/3*sinh(d*x+c)^6/cosh(d*x+c)^5-16*sinh(d*x+c)^4/cosh(d*x+
c)^5-64/3*sinh(d*x+c)^2/cosh(d*x+c)^5-128/15/cosh(d*x+c)^5)+b^3*(1/3*sinh(
d*x+c)^11/cosh(d*x+c)^8-11/3*sinh(d*x+c)^9/cosh(d*x+c)^8-33*sinh(d*x+c)^7/
cosh(d*x+c)^8-77*sinh(d*x+c)^5/cosh(d*x+c)^8-77*sinh(d*x+c)^3/cosh(d*x+c)^
8-33*sinh(d*x+c)/cosh(d*x+c)^8+33*(1/8*sech(d*x+c)^7+7/48*sech(d*x+c)^5+35
/192*sech(d*x+c)^3+35/128*sech(d*x+c))*tanh(d*x+c)+1155/64*arctan(exp(d*x+
c))))
```

3.66.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8462 vs. $2(325) = 650$.

Time $= 0.36$ (sec) , antiderivative size $= 8462$, normalized size of antiderivative $= 24.11$

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.66.6 Sympy [F(-1)]

Timed out.

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**3)**3,x)
```

output
```
Timed out
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.72

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$

$$= \frac{1}{192}b^3\left(\frac{8\left(63\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}\right)}{d}-\frac{3465\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{440\,e^{(-2\,dx-2\,c)}+6103\,e^{(-4\,dx-4\,c)}+21019}{d(e^{(-3\,dx-3\,c)}+8\,e^{(-5\,dx-}}\right.$$

$$-\frac{1}{40}ab^2\left(\frac{5\left(45\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}\right)}{d}+\frac{200\,e^{(-2\,dx-2\,c)}+2515\,e^{(-4\,dx-4\,c)}+6680\,e^{(-6\,dx-6\,c)}+9073\,e^{(-8\,d}}{d(e^{(-3\,dx-3\,c)}+5\,e^{(-5\,dx-5\,c)}+10\,e^{(-7\,dx-7\,c)}+10\,e^{(-9}}\right.$$

$$+\frac{1}{8}a^2b\left(\frac{27\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}-\frac{120\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{25\,e^{(-2\,dx-2\,c)}+77\,e^{(-4\,dx-4\,c)}+3\,e^{(-6\,dx-6\,c)}}{d(e^{(-3\,dx-3\,c)}+2\,e^{(-5\,dx-5\,c)}+e^{(-7\,dx-7\,c)})}\right.$$

$$+\frac{1}{24}a^3\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
1/192*b^3*(8*(63*e^(-d*x - c) - e^(-3*d*x - 3*c))/d - 3465*arctan(e^(-d*x
- c))/d - (440*e^(-2*d*x - 2*c) + 6103*e^(-4*d*x - 4*c) + 21019*e^(-6*d*x
- 6*c) + 41207*e^(-8*d*x - 8*c) + 40243*e^(-10*d*x - 10*c) + 22589*e^(-12*
d*x - 12*c) + 505*e^(-14*d*x - 14*c) - 3331*e^(-16*d*x - 16*c) - 1791*e^(-
18*d*x - 18*c) - 8)/(d*(e^(-3*d*x - 3*c) + 8*e^(-5*d*x - 5*c) + 28*e^(-7*d
*x - 7*c) + 56*e^(-9*d*x - 9*c) + 70*e^(-11*d*x - 11*c) + 56*e^(-13*d*x -
13*c) + 28*e^(-15*d*x - 15*c) + 8*e^(-17*d*x - 17*c) + e^(-19*d*x - 19*c))
)) - 1/40*a*b^2*(5*(45*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (200*e^(-2*d*x
 - 2*c) + 2515*e^(-4*d*x - 4*c) + 6680*e^(-6*d*x - 6*c) + 9073*e^(-8*d*x -
 8*c) + 5600*e^(-10*d*x - 10*c) + 1665*e^(-12*d*x - 12*c) - 5)/(d*(e^(-3*d
*x - 3*c) + 5*e^(-5*d*x - 5*c) + 10*e^(-7*d*x - 7*c) + 10*e^(-9*d*x - 9*c)
 + 5*e^(-11*d*x - 11*c) + e^(-13*d*x - 13*c)))) + 1/8*a^2*b*((27*e^(-d*x -
 c) - e^(-3*d*x - 3*c))/d - 120*arctan(e^(-d*x - c))/d - (25*e^(-2*d*x - 2
*c) + 77*e^(-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) - 1)/(d*(e^(-3*d*x - 3*c) +
 2*e^(-5*d*x - 5*c) + e^(-7*d*x - 7*c)))) + 1/24*a^3*(e^(3*d*x + 3*c)/d -
9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.65

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$

$$= \frac{40\,a^3e^{(3\,dx+3\,c)} + 120\,a^2be^{(3\,dx+3\,c)} + 120\,ab^2e^{(3\,dx+3\,c)} + 40\,b^3e^{(3\,dx+3\,c)} - 360\,a^3e^{(dx+c)} - 3240\,a^2be^{(dx+c)} - 5}{}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
1/960*(40*a^3*e^(3*d*x + 3*c) + 120*a^2*b*e^(3*d*x + 3*c) + 120*a*b^2*e^(3
*d*x + 3*c) + 40*b^3*e^(3*d*x + 3*c) - 360*a^3*e^(d*x + c) - 3240*a^2*b*e^
(d*x + c) - 5400*a*b^2*e^(d*x + c) - 2520*b^3*e^(d*x + c) + 225*(64*a^2*b
+ 77*b^3)*arctan(e^(d*x + c)) - 40*(9*a^3*e^(2*d*x + 2*c) - 81*a^2*b*e^(2*
d*x + 2*c) + 135*a*b^2*e^(2*d*x + 2*c) - 63*b^3*e^(2*d*x + 2*c) - a^3 + 3*
a^2*b - 3*a*b^2 + b^3)*e^(-3*d*x - 3*c) - (2880*a^2*b*e^(15*d*x + 15*c) +
34560*a*b^2*e^(15*d*x + 15*c) + 11475*b^3*e^(15*d*x + 15*c) + 14400*a^2*b*
e^(13*d*x + 13*c) + 211200*a*b^2*e^(13*d*x + 13*c) + 36775*b^3*e^(13*d*x +
 13*c) + 25920*a^2*b*e^(11*d*x + 11*c) + 590592*a*b^2*e^(11*d*x + 11*c) +
67715*b^3*e^(11*d*x + 11*c) + 14400*a^2*b*e^(9*d*x + 9*c) + 957696*a*b^2*e
^(9*d*x + 9*c) + 27055*b^3*e^(9*d*x + 9*c) - 14400*a^2*b*e^(7*d*x + 7*c) +
 957696*a*b^2*e^(7*d*x + 7*c) - 27055*b^3*e^(7*d*x + 7*c) - 25920*a^2*b*e^
(5*d*x + 5*c) + 590592*a*b^2*e^(5*d*x + 5*c) - 67715*b^3*e^(5*d*x + 5*c) -
 14400*a^2*b*e^(3*d*x + 3*c) + 211200*a*b^2*e^(3*d*x + 3*c) - 36775*b^3*e^
(3*d*x + 3*c) - 2880*a^2*b*e^(d*x + c) + 34560*a*b^2*e^(d*x + c) - 11475*b
^3*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^8)/d
```

3.66.9 Mupad [B] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 757, normalized size of antiderivative = 2.16

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \frac{\mathrm{e}^{3c+3dx}(a+b)^3}{24d} + \frac{\mathrm{e}^{-3c-3dx}(a-b)^3}{24d}$$
$$+ \frac{15\,\mathrm{atan}\left(\frac{\mathrm{e}^{dx}\,\mathrm{e}^{c}\left(77b^3\sqrt{d^2}+64a^2b\sqrt{d^2}\right)}{d\sqrt{4096a^4b^2+9856a^2b^4+5929b^6}}\right)\sqrt{4096a^4b^2+9856a^2b^4+5929b^6}}{64\sqrt{d^2}}$$
$$- \frac{3\,\mathrm{e}^{-c-dx}(a-b)^2(a-7b)}{8d} - \frac{\mathrm{e}^{c+dx}(11005b^3+6144ab^2)}{120d\,(3\,\mathrm{e}^{2c+2dx}+3\,\mathrm{e}^{4c+4dx}+\mathrm{e}^{6c+6dx}+1)}$$
$$+ \frac{\mathrm{e}^{c+dx}(3365b^3+768ab^2)}{20d\,(4\,\mathrm{e}^{2c+2dx}+6\,\mathrm{e}^{4c+4dx}+4\,\mathrm{e}^{6c+6dx}+\mathrm{e}^{8c+8dx}+1)}$$
$$+ \frac{596b^3\,\mathrm{e}^{c+dx}}{3d\,(6\,\mathrm{e}^{2c+2dx}+15\,\mathrm{e}^{4c+4dx}+20\,\mathrm{e}^{6c+6dx}+15\,\mathrm{e}^{8c+8dx}+6\,\mathrm{e}^{10c+10dx}+\mathrm{e}^{12c+12dx}+1)}$$
$$- \frac{3\,\mathrm{e}^{c+dx}(a+b)^2(a+7b)}{8d} - \frac{3\,\mathrm{e}^{c+dx}(64a^2b+768ab^2+255b^3)}{64d\,(\mathrm{e}^{2c+2dx}+1)}$$
$$- \frac{2\,\mathrm{e}^{c+dx}(1625b^3+144ab^2)}{15d\,(5\,\mathrm{e}^{2c+2dx}+10\,\mathrm{e}^{4c+4dx}+10\,\mathrm{e}^{6c+6dx}+5\,\mathrm{e}^{8c+8dx}+\mathrm{e}^{10c+10dx}+1)}$$
$$- \frac{112b^3\,\mathrm{e}^{c+dx}}{d\,(7\,\mathrm{e}^{2c+2dx}+21\,\mathrm{e}^{4c+4dx}+35\,\mathrm{e}^{6c+6dx}+35\,\mathrm{e}^{8c+8dx}+21\,\mathrm{e}^{10c+10dx}+7\,\mathrm{e}^{12c+12dx}+\mathrm{e}^{14c+14dx}+1)}$$
$$+ \frac{\mathrm{e}^{c+dx}(576a^2b+3072ab^2+4355b^3)}{96d\,(2\,\mathrm{e}^{2c+2dx}+\mathrm{e}^{4c+4dx}+1)}$$
$$+ \frac{32b^3\,\mathrm{e}^{c+dx}}{d\,(8\,\mathrm{e}^{2c+2dx}+28\,\mathrm{e}^{4c+4dx}+56\,\mathrm{e}^{6c+6dx}+70\,\mathrm{e}^{8c+8dx}+56\,\mathrm{e}^{10c+10dx}+28\,\mathrm{e}^{12c+12dx}+8\,\mathrm{e}^{14c+14dx}+\mathrm{e}^{16c}$$

input
```
int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^3)^3,x)
```

output

```
(exp(3*c + 3*d*x)*(a + b)^3)/(24*d) + (exp(- 3*c - 3*d*x)*(a - b)^3)/(24*d
) + (15*atan((exp(d*x)*exp(c)*(77*b^3*(d^2)^(1/2) + 64*a^2*b*(d^2)^(1/2)))
/(d*(5929*b^6 + 9856*a^2*b^4 + 4096*a^4*b^2)^(1/2)))*(5929*b^6 + 9856*a^2*
b^4 + 4096*a^4*b^2)^(1/2))/(64*(d^2)^(1/2)) - (3*exp(- c - d*x)*(a - b)^2*
(a - 7*b))/(8*d) - (exp(c + d*x)*(6144*a*b^2 + 11005*b^3))/(120*d*(3*exp(2
*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (exp(c + d*x)*
(768*a*b^2 + 3365*b^3))/(20*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4
*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (596*b^3*exp(c + d*x))/(3*d*(
6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*
c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (3*exp(c +
d*x)*(a + b)^2*(a + 7*b))/(8*d) - (3*exp(c + d*x)*(768*a*b^2 + 64*a^2*b +
255*b^3))/(64*d*(exp(2*c + 2*d*x) + 1)) - (2*exp(c + d*x)*(144*a*b^2 + 162
5*b^3))/(15*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d
*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (112*b^3*exp(c + d*x
))/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35
*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*
c + 14*d*x) + 1)) + (exp(c + d*x)*(3072*a*b^2 + 576*a^2*b + 4355*b^3))/(96
*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (32*b^3*exp(c + d*x))/(d
*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(
8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14...
```

3.67 $\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

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3.67.1 Optimal result

Integrand size = 23, antiderivative size = 220

$$\begin{aligned}
&\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx \\
&= -\frac{1}{2}a\left(a^2+21b^2\right)x - \frac{b(6a^2+5b^2)\log(\cosh(c+dx))}{d} + \frac{9ab^2\tanh(c+dx)}{d} \\
&\quad + \frac{b(3a^2+4b^2)\tanh^2(c+dx)}{2d} + \frac{2ab^2\tanh^3(c+dx)}{d} + \frac{3b^3\tanh^4(c+dx)}{4d} \\
&\quad + \frac{3ab^2\tanh^5(c+dx)}{5d} + \frac{b^3\tanh^6(c+dx)}{3d} + \frac{b^3\tanh^8(c+dx)}{8d} \\
&\quad + \frac{\cosh(c+dx)\sinh(c+dx)\left(a(a^2+3b^2)+b(3a^2+b^2)\tanh(c+dx)\right)}{2d}
\end{aligned}$$

output

```
-1/2*a*(a^2+21*b^2)*x-b*(6*a^2+5*b^2)*ln(cosh(d*x+c))/d+9*a*b^2*tanh(d*x+c
)/d+1/2*b*(3*a^2+4*b^2)*tanh(d*x+c)^2/d+2*a*b^2*tanh(d*x+c)^3/d+3/4*b^3*ta
nh(d*x+c)^4/d+3/5*a*b^2*tanh(d*x+c)^5/d+1/3*b^3*tanh(d*x+c)^6/d+1/8*b^3*ta
nh(d*x+c)^8/d+1/2*cosh(d*x+c)*sinh(d*x+c)*(a*(a^2+3*b^2)+b*(3*a^2+b^2)*tan
h(d*x+c))/d
```

3.67.2 Mathematica [A] (verified)

Time = 7.38 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.11

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$= -\frac{a(a^2+21b^2)(c+dx)}{2d} + \frac{b(3a^2+b^2)\cosh(2(c+dx))}{4d} + \frac{(-6a^2b-5b^3)\log(\cosh(c+dx))}{d}$$
$$- \frac{b(3a^2+10b^2)\operatorname{sech}^2(c+dx)}{2d} + \frac{5b^3\operatorname{sech}^4(c+dx)}{2d} - \frac{5b^3\operatorname{sech}^6(c+dx)}{6d}$$
$$+ \frac{b^3\operatorname{sech}^8(c+dx)}{8d} + \frac{a(a^2+3b^2)\sinh(2(c+dx))}{4d} + \frac{58ab^2\tanh(c+dx)}{5d}$$
$$- \frac{16ab^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{5d} + \frac{3ab^2\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}$$

input
```
Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
-1/2*(a*(a^2 + 21*b^2)*(c + d*x))/d + (b*(3*a^2 + b^2)*Cosh[2*(c + d*x)])/
(4*d) + ((-6*a^2*b - 5*b^3)*Log[Cosh[c + d*x]])/d - (b*(3*a^2 + 10*b^2)*Se
ch[c + d*x]^2)/(2*d) + (5*b^3*Sech[c + d*x]^4)/(2*d) - (5*b^3*Sech[c + d*x
]^6)/(6*d) + (b^3*Sech[c + d*x]^8)/(8*d) + (a*(a^2 + 3*b^2)*Sinh[2*(c + d*
x)])/(4*d) + (58*a*b^2*Tanh[c + d*x])/(5*d) - (16*a*b^2*Sech[c + d*x]^2*Ta
nh[c + d*x])/(5*d) + (3*a*b^2*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d)
```

3.67.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 25, 4146, 2335, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$\downarrow 3042$$
$$\int -\sin(ic+idx)^2\left(a+ib\tan(ic+idx)^3\right)^3\,dx$$
$$\downarrow 25$$

$$-\int \sin(ic+idx)^2\left(ib\tan(ic+idx)^3+a\right)^3 dx$$

$$\downarrow 4146$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(b\tanh^3(c+dx)+a\right)^3}{\left(1-\tanh^2(c+dx)\right)^2} d\tanh(c+dx)}{d}$$

$$\downarrow 2335$$

$$\frac{\frac{1}{2}\int -\frac{\tanh(c+dx)\left(2b^3\tanh^8(c+dx)+2b^3\tanh^6(c+dx)+6ab^2\tanh^5(c+dx)+2b^3\tanh^4(c+dx)+6ab^2\tanh^3(c+dx)+2b\left(3a^2+b^2\right)\tanh^2(c+dx)+a(}{1-\tanh^2(c+dx)}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{\tanh^2(c+dx)\left(a\left(a^2+3b^2\right)\tanh(c+dx)+b\left(3a^2+b^2\right)\right)}{2\left(1-\tanh^2(c+dx)\right)} - \frac{1}{2}\int \frac{\tanh(c+dx)\left(2b^3\tanh^8(c+dx)+2b^3\tanh^6(c+dx)+6ab^2\tanh^5(c+dx)+2b^3\tanh^4(c+dx)\right.}{1-\tan}}{d}$$

$$\downarrow 2333$$

$$\frac{\frac{\tanh^2(c+dx)\left(a\left(a^2+3b^2\right)\tanh(c+dx)+b\left(3a^2+b^2\right)\right)}{2\left(1-\tanh^2(c+dx)\right)} - \frac{1}{2}\int \left(-2b^3\tanh^7(c+dx)-4b^3\tanh^5(c+dx)-6ab^2\tanh^4(c+dx)-6\right.}$$

$$\downarrow 2009$$

$$\frac{1}{2}\left(-a\left(a^2+21b^2\right)\operatorname{arctanh}(\tanh(c+dx))+b\left(3a^2+4b^2\right)\tanh^2(c+dx)+a\left(a^2+21b^2\right)\tanh(c+dx)+b\left(6a^2+5b^2\right.\right.$$

input
```
Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
((Tanh[c + d*x]^2*(b*(3*a^2 + b^2) + a*(a^2 + 3*b^2)*Tanh[c + d*x]))/(2*(1
 - Tanh[c + d*x]^2)) + (-(a*(a^2 + 21*b^2)*ArcTanh[Tanh[c + d*x]]) + b*(6*
a^2 + 5*b^2)*Log[1 - Tanh[c + d*x]^2] + a*(a^2 + 21*b^2)*Tanh[c + d*x] + b
*(3*a^2 + 4*b^2)*Tanh[c + d*x]^2 + 4*a*b^2*Tanh[c + d*x]^3 + (3*b^3*Tanh[c
 + d*x]^4)/2 + (6*a*b^2*Tanh[c + d*x]^5)/5 + (2*b^3*Tanh[c + d*x]^6)/3 + (
b^3*Tanh[c + d*x]^8)/4)/2)/d
```

3.67.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.67.4 Maple [A] (verified)

Time = 8.88 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b\left(\frac{\sinh(dx+c)^4}{2\cosh(dx+c)^2}-2\ln(\cosh(dx+c))+\tanh(dx+c)^2\right)+3ab^2\left(\frac{\sinh(dx+c)^7}{2\cosh(dx+c)^5}-\frac{7dx}{2}\right.$
default	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b\left(\frac{\sinh(dx+c)^4}{2\cosh(dx+c)^2}-2\ln(\cosh(dx+c))+\tanh(dx+c)^2\right)+3ab^2\left(\frac{\sinh(dx+c)^7}{2\cosh(dx+c)^5}-\frac{7dx}{2}\right.$
risch	$-\frac{a^3x}{2}+6ba^2x-\frac{21ab^2x}{2}+5b^3x+\frac{e^{2dx+2c}a^3}{8d}+\frac{3e^{2dx+2c}a^2b}{8d}+\frac{3e^{2dx+2c}ab^2}{8d}+\frac{e^{2dx+2c}b^3}{8d}-\frac{e^{-2dx-}}{8d}$

input
```
int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+3*a^2*b*(1/2*sinh(d*x
+c)^4/cosh(d*x+c)^2-2*ln(cosh(d*x+c))+tanh(d*x+c)^2)+3*a*b^2*(1/2*sinh(d*x
+c)^7/cosh(d*x+c)^5-7/2*d*x-7/2*c+7/2*tanh(d*x+c)+7/6*tanh(d*x+c)^3+7/10*t
anh(d*x+c)^5)+b^3*(1/2*sinh(d*x+c)^10/cosh(d*x+c)^8-5*ln(cosh(d*x+c))+5/2*
tanh(d*x+c)^2+5/4*tanh(d*x+c)^4+5/6*tanh(d*x+c)^6+5/8*tanh(d*x+c)^8))
```

3.67.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9862 vs. 2(206) = 412.

Time = 0.37 (sec) , antiderivative size = 9862, normalized size of antiderivative = 44.83

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.67.6 Sympy [F]

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \int \left(a+b\tanh^3(c+dx)\right)^3 \sinh^2(c+dx)\,dx$$

input
```
integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**3)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**3*sinh(c + d*x)**2, x)
```

3.67.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(206) = 412$.

Time $= 0.32$ (sec) , antiderivative size $= 544$, normalized size of antiderivative $= 2.47$

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = -\frac{1}{8}a^3\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right)$$
$$-\frac{1}{40}ab^2\left(\frac{420\,(dx+c)}{d} + \frac{15\,e^{(-2dx-2c)}}{d} - \frac{1003\,e^{(-2dx-2c)} + 3350\,e^{(-4dx-4c)} + 5590\,e^{(-6dx-6c)} + 3915\,e^{(-}}{d(e^{(-2dx-2c)} + 5\,e^{(-4dx-4c)} + 10\,e^{(-6dx-6c)} + 10\,e^{(-8dx-8c)} +}\right.$$
$$-\frac{1}{24}b^3\left(\frac{120\,(dx+c)}{d} - \frac{3\,e^{(-2dx-2c)}}{d} + \frac{120\,\log\left(e^{(-2dx-2c)}+1\right)}{d} - \frac{24\,e^{(-2dx-2c)} - 396\,e^{(-4dx-4c)} - 1752}{d(e^{(-2dx-2c)} + 8\,e^{(-4dx-4c)} + 28\,e^{(}}\right.$$
$$-\frac{3}{8}a^2b\left(\frac{16\,(dx+c)}{d} - \frac{e^{(-2dx-2c)}}{d} + \frac{16\,\log\left(e^{(-2dx-2c)}+1\right)}{d} - \frac{2\,e^{(-2dx-2c)} - 15\,e^{(-4dx-4c)} + 1}{d(e^{(-2dx-2c)} + 2\,e^{(-4dx-4c)} + e^{(-6dx-6c)})}\right.$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
-1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/40*a*b^2*(420*
(d*x + c)/d + 15*e^(-2*d*x - 2*c)/d - (1003*e^(-2*d*x - 2*c) + 3350*e^(-4*
d*x - 4*c) + 5590*e^(-6*d*x - 6*c) + 3915*e^(-8*d*x - 8*c) + 1455*e^(-10*d
*x - 10*c) + 15)/(d*(e^(-2*d*x - 2*c) + 5*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
 - 6*c) + 10*e^(-8*d*x - 8*c) + 5*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)))
) - 1/24*b^3*(120*(d*x + c)/d - 3*e^(-2*d*x - 2*c)/d + 120*log(e^(-2*d*x -
 2*c) + 1)/d - (24*e^(-2*d*x - 2*c) - 396*e^(-4*d*x - 4*c) - 1752*e^(-6*d*
x - 6*c) - 4430*e^(-8*d*x - 8*c) - 5464*e^(-10*d*x - 10*c) - 4556*e^(-12*d
*x - 12*c) - 1896*e^(-14*d*x - 14*c) - 477*e^(-16*d*x - 16*c) + 3)/(d*(e^(
-2*d*x - 2*c) + 8*e^(-4*d*x - 4*c) + 28*e^(-6*d*x - 6*c) + 56*e^(-8*d*x -
8*c) + 70*e^(-10*d*x - 10*c) + 56*e^(-12*d*x - 12*c) + 28*e^(-14*d*x - 14*
c) + 8*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c)))) - 3/8*a^2*b*(16*(d*x + c
)/d - e^(-2*d*x - 2*c)/d + 16*log(e^(-2*d*x - 2*c) + 1)/d - (2*e^(-2*d*x -
 2*c) - 15*e^(-4*d*x - 4*c) + 1)/(d*(e^(-2*d*x - 2*c) + 2*e^(-4*d*x - 4*c)
 + e^(-6*d*x - 6*c))))
```

3.67.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(206) = 412$.

Time $= 0.71$ (sec) , antiderivative size $= 577$, normalized size of antiderivative $= 2.62$

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$

$$= \frac{105\,a^3e^{(2\,dx+2\,c)} + 315\,a^2be^{(2\,dx+2\,c)} + 315\,ab^2e^{(2\,dx+2\,c)} + 105\,b^3e^{(2\,dx+2\,c)} - 420\,(a^3 - 12\,a^2b + 21\,ab^2 - 10\,b^3}{}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

output

```
1/840*(105*a^3*e^(2*d*x + 2*c) + 315*a^2*b*e^(2*d*x + 2*c) + 315*a*b^2*e^(
2*d*x + 2*c) + 105*b^3*e^(2*d*x + 2*c) - 420*(a^3 - 12*a^2*b + 21*a*b^2 -
10*b^3)*(d*x + c) + 105*(2*a^3*e^(2*d*x + 2*c) - 24*a^2*b*e^(2*d*x + 2*c)
+ 42*a*b^2*e^(2*d*x + 2*c) - 20*b^3*e^(2*d*x + 2*c) - a^3 + 3*a^2*b - 3*a*
b^2 + b^3)*e^(-2*d*x - 2*c) - 840*(6*a^2*b + 5*b^3)*log(e^(2*d*x + 2*c) +
1) + (13698*a^2*b*e^(16*d*x + 16*c) + 11415*b^3*e^(16*d*x + 16*c) + 104544
*a^2*b*e^(14*d*x + 14*c) - 30240*a*b^2*e^(14*d*x + 14*c) + 74520*b^3*e^(14
*d*x + 14*c) + 353304*a^2*b*e^(12*d*x + 12*c) - 171360*a*b^2*e^(12*d*x + 1
2*c) + 252420*b^3*e^(12*d*x + 12*c) + 691488*a^2*b*e^(10*d*x + 10*c) - 446
880*a*b^2*e^(10*d*x + 10*c) + 476840*b^3*e^(10*d*x + 10*c) + 858060*a^2*b*
e^(8*d*x + 8*c) - 682080*a*b^2*e^(8*d*x + 8*c) + 601930*b^3*e^(8*d*x + 8*c
) + 691488*a^2*b*e^(6*d*x + 6*c) - 644448*a*b^2*e^(6*d*x + 6*c) + 476840*b
^3*e^(6*d*x + 6*c) + 353304*a^2*b*e^(4*d*x + 4*c) - 374304*a*b^2*e^(4*d*x
+ 4*c) + 252420*b^3*e^(4*d*x + 4*c) + 104544*a^2*b*e^(2*d*x + 2*c) - 12566
4*a*b^2*e^(2*d*x + 2*c) + 74520*b^3*e^(2*d*x + 2*c) + 13698*a^2*b - 19488*
a*b^2 + 11415*b^3)/(e^(2*d*x + 2*c) + 1)^8)/d
```

3.67.9 Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 617, normalized size of antiderivative = 2.80

$$
\begin{aligned}
&\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx \\
&= \frac{8\left(29\,b^3+6\,a\,b^2\right)}{d\left(4\,\mathrm{e}^{2c+2dx}+6\,\mathrm{e}^{4c+4dx}+4\,\mathrm{e}^{6c+6dx}+\mathrm{e}^{8c+8dx}+1\right)} \\
&\quad+\frac{736\,b^3}{3\,d\left(6\,\mathrm{e}^{2c+2dx}+15\,\mathrm{e}^{4c+4dx}+20\,\mathrm{e}^{6c+6dx}+15\,\mathrm{e}^{8c+8dx}+6\,\mathrm{e}^{10c+10dx}+\mathrm{e}^{12c+12dx}+1\right)} \\
&\quad-\frac{\ln\left(\mathrm{e}^{2c}\,\mathrm{e}^{2dx}+1\right)\left(6\,a^2\,b+5\,b^3\right)}{d}-\frac{2\left(3\,a^2\,b+18\,a\,b^2+10\,b^3\right)}{d\left(\mathrm{e}^{2c+2dx}+1\right)} \\
&\quad-\frac{96\left(15\,b^3+a\,b^2\right)}{5\,d\left(5\,\mathrm{e}^{2c+2dx}+10\,\mathrm{e}^{4c+4dx}+10\,\mathrm{e}^{6c+6dx}+5\,\mathrm{e}^{8c+8dx}+\mathrm{e}^{10c+10dx}+1\right)} \\
&\quad-\frac{128\,b^3}{d\left(7\,\mathrm{e}^{2c+2dx}+21\,\mathrm{e}^{4c+4dx}+35\,\mathrm{e}^{6c+6dx}+35\,\mathrm{e}^{8c+8dx}+21\,\mathrm{e}^{10c+10dx}+7\,\mathrm{e}^{12c+12dx}+\mathrm{e}^{14c+14dx}+1\right)} \\
&\quad-\frac{x\left(a-b\right)^2\left(a-10\,b\right)}{2}+\frac{6\left(a^2\,b+8\,a\,b^2+10\,b^3\right)}{d\left(2\,\mathrm{e}^{2c+2dx}+\mathrm{e}^{4c+4dx}+1\right)} \\
&\quad+\frac{32\,b^3}{d\left(8\,\mathrm{e}^{2c+2dx}+28\,\mathrm{e}^{4c+4dx}+56\,\mathrm{e}^{6c+6dx}+70\,\mathrm{e}^{8c+8dx}+56\,\mathrm{e}^{10c+10dx}+28\,\mathrm{e}^{12c+12dx}+8\,\mathrm{e}^{14c+14dx}+\mathrm{e}^{16c}\right.} \\
&\quad+\frac{\mathrm{e}^{2c+2dx}\left(a+b\right)^3}{8\,d}-\frac{\mathrm{e}^{-2c-2dx}\left(a-b\right)^3}{8\,d}-\frac{16\left(25\,b^3+12\,a\,b^2\right)}{3\,d\left(3\,\mathrm{e}^{2c+2dx}+3\,\mathrm{e}^{4c+4dx}+\mathrm{e}^{6c+6dx}+1\right)}
\end{aligned}
$$

input
```
int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^3)^3,x)
```

output
```
(8*(6*a*b^2 + 29*b^3))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp
(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (736*b^3)/(3*d*(6*exp(2*c + 2*d*x
) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*ex
p(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (log(exp(2*c)*exp(2*d*x) + 1
)*(6*a^2*b + 5*b^3))/d - (2*(18*a*b^2 + 3*a^2*b + 10*b^3))/(d*(exp(2*c + 2
*d*x) + 1)) - (96*(a*b^2 + 15*b^3))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c
+ 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) +
 1)) - (128*b^3)/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c
 + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*
d*x) + exp(14*c + 14*d*x) + 1)) - (x*(a - b)^2*(a - 10*b))/2 + (6*(8*a*b^2
 + a^2*b + 10*b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (32*
b^3)/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) +
70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*ex
p(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1)) + (exp(2*c + 2*d*x)*(a + b)^3)
/(8*d) - (exp(- 2*c - 2*d*x)*(a - b)^3)/(8*d) - (16*(12*a*b^2 + 25*b^3))/(
3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1))
```

3.68 $\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

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3.68.1 Optimal result

Integrand size = 21, antiderivative size = 269

$$\begin{aligned}
&\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx \\
&= -\frac{9a^2b\arctan(\sinh(c+dx))}{2d} - \frac{315b^3\arctan(\sinh(c+dx))}{128d} + \frac{a^3\cosh(c+dx)}{d} \\
&\quad + \frac{3ab^2\cosh(c+dx)}{d} + \frac{9ab^2\operatorname{sech}(c+dx)}{d} - \frac{3ab^2\operatorname{sech}^3(c+dx)}{d} + \frac{3ab^2\operatorname{sech}^5(c+dx)}{5d} \\
&\quad + \frac{9a^2b\sinh(c+dx)}{2d} + \frac{315b^3\sinh(c+dx)}{128d} - \frac{3a^2b\sinh(c+dx)\tanh^2(c+dx)}{2d} \\
&\quad - \frac{105b^3\sinh(c+dx)\tanh^2(c+dx)}{128d} - \frac{21b^3\sinh(c+dx)\tanh^4(c+dx)}{64d} \\
&\quad - \frac{3b^3\sinh(c+dx)\tanh^6(c+dx)}{16d} - \frac{b^3\sinh(c+dx)\tanh^8(c+dx)}{8d}
\end{aligned}$$

output

```
-9/2*a^2*b*arctan(sinh(d*x+c))/d-315/128*b^3*arctan(sinh(d*x+c))/d+a^3*cos
h(d*x+c)/d+3*a*b^2*cosh(d*x+c)/d+9*a*b^2*sech(d*x+c)/d-3*a*b^2*sech(d*x+c)
^3/d+3/5*a*b^2*sech(d*x+c)^5/d+9/2*a^2*b*sinh(d*x+c)/d+315/128*b^3*sinh(d*
x+c)/d-3/2*a^2*b*sinh(d*x+c)*tanh(d*x+c)^2/d-105/128*b^3*sinh(d*x+c)*tanh(
d*x+c)^2/d-21/64*b^3*sinh(d*x+c)*tanh(d*x+c)^4/d-3/16*b^3*sinh(d*x+c)*tanh
(d*x+c)^6/d-1/8*b^3*sinh(d*x+c)*tanh(d*x+c)^8/d
```

3.68.2 Mathematica [A] (verified)

Time = 7.69 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.87

$$\begin{aligned}
&\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx \\
&= -\frac{9(64a^2b+35b^3)\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{64d} + \frac{a(a^2+3b^2)\cosh(c+dx)}{d} \\
&\quad + \frac{9ab^2\operatorname{sech}(c+dx)}{d} - \frac{3ab^2\operatorname{sech}^3(c+dx)}{d} + \frac{3ab^2\operatorname{sech}^5(c+dx)}{5d} \\
&\quad + \frac{b(3a^2+b^2)\sinh(c+dx)}{d} + \frac{\operatorname{sech}^2(c+dx)\left(192a^2b\sinh(c+dx)+325b^3\sinh(c+dx)\right)}{128d} \\
&\quad - \frac{105b^3\operatorname{sech}^3(c+dx)\tanh(c+dx)}{64d} \\
&\quad + \frac{11b^3\operatorname{sech}^5(c+dx)\tanh(c+dx)}{16d} - \frac{b^3\operatorname{sech}^7(c+dx)\tanh(c+dx)}{8d}
\end{aligned}$$

input
```
Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
(-9*(64*a^2*b + 35*b^3)*ArcTan[Tanh[(c + d*x)/2]])/(64*d) + (a*(a^2 + 3*b^
2)*Cosh[c + d*x])/d + (9*a*b^2*Sech[c + d*x])/d - (3*a*b^2*Sech[c + d*x]^3
)/d + (3*a*b^2*Sech[c + d*x]^5)/(5*d) + (b*(3*a^2 + b^2)*Sinh[c + d*x])/d
+ (Sech[c + d*x]^2*(192*a^2*b*Sinh[c + d*x] + 325*b^3*Sinh[c + d*x]))/(128
*d) - (105*b^3*Sech[c + d*x]^3*Tanh[c + d*x])/(64*d) + (11*b^3*Sech[c + d*
x]^5*Tanh[c + d*x])/(16*d) - (b^3*Sech[c + d*x]^7*Tanh[c + d*x])/(8*d)
```

3.68.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$

$$\downarrow \text{3042}$$

$$\int -i\sin(ic+idx)\left(a+ib\tan(ic+idx)^3\right)^3\,dx$$

$$\downarrow 26$$

$$-i\int \sin(ic+idx)\left(ib\tan(ic+idx)^3+a\right)^3\,dx$$

$$\downarrow 4149$$

$$-i\int \left(ib^3\sinh(c+dx)\tanh^9(c+dx)+3iab^2\sinh(c+dx)\tanh^6(c+dx)+3ia^2b\sinh(c+dx)\tanh^3(c+dx)+ia^3\right.$$

$$\downarrow 2009$$

$$-i\left(\frac{ia^3\cosh(c+dx)}{d}-\frac{9ia^2b\arctan(\sinh(c+dx))}{2d}+\frac{9ia^2b\sinh(c+dx)}{2d}-\frac{3ia^2b\sinh(c+dx)\tanh^2(c+dx)}{2d}+\frac{3i}{}\right.$$

input
```
Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
(-I)*((((-9*I)/2)*a^2*b*ArcTan[Sinh[c + d*x]])/d - (((315*I)/128)*b^3*ArcT
an[Sinh[c + d*x]])/d + (I*a^3*Cosh[c + d*x])/d + ((3*I)*a*b^2*Cosh[c + d*x
])/d + ((9*I)*a*b^2*Sech[c + d*x])/d - ((3*I)*a*b^2*Sech[c + d*x]^3)/d + (
((3*I)/5)*a*b^2*Sech[c + d*x]^5)/d + (((9*I)/2)*a^2*b*Sinh[c + d*x])/d + (
((315*I)/128)*b^3*Sinh[c + d*x])/d - (((3*I)/2)*a^2*b*Sinh[c + d*x]*Tanh[c
 + d*x]^2)/d - (((105*I)/128)*b^3*Sinh[c + d*x]*Tanh[c + d*x]^2)/d - (((21
*I)/64)*b^3*Sinh[c + d*x]*Tanh[c + d*x]^4)/d - (((3*I)/16)*b^3*Sinh[c + d*
x]*Tanh[c + d*x]^6)/d - ((I/8)*b^3*Sinh[c + d*x]*Tanh[c + d*x]^8)/d)
```

3.68.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

3.68.4 Maple [A] (verified)

Time = 5.72 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{a^3\cosh(dx+c)+3a^2b\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan\left(\mathrm{e}^{dx+c}\right)\right)+3ab^2\left(\frac{\sinh(dx+c)^6}{\cosh(dx+c)^5}+\frac{6\sin}{\cos}\right.}$
default	$\frac{a^3\cosh(dx+c)+3a^2b\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan\left(\mathrm{e}^{dx+c}\right)\right)+3ab^2\left(\frac{\sinh(dx+c)^6}{\cosh(dx+c)^5}+\frac{6\sin}{\cos}\right.}$
risch	$\frac{\mathrm{e}^{dx+c}a^3}{2d}+\frac{3\,\mathrm{e}^{dx+c}a^2b}{2d}+\frac{3\,\mathrm{e}^{dx+c}ab^2}{2d}+\frac{b^3\mathrm{e}^{dx+c}}{2d}+\frac{\mathrm{e}^{-dx-c}a^3}{2d}-\frac{3\,\mathrm{e}^{-dx-c}a^2b}{2d}+\frac{3\,\mathrm{e}^{-dx-c}ab^2}{2d}-\frac{\mathrm{e}^{-dx-c}b^3}{2d}+$

input
```
int(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*cosh(d*x+c)+3*a^2*b*(sinh(d*x+c)^3/cosh(d*x+c)^2+3*sinh(d*x+c)/co
sh(d*x+c)^2-3/2*sech(d*x+c)*tanh(d*x+c)-3*arctan(exp(d*x+c)))+3*a*b^2*(sin
h(d*x+c)^6/cosh(d*x+c)^5+6*sinh(d*x+c)^4/cosh(d*x+c)^5+8*sinh(d*x+c)^2/cos
h(d*x+c)^5+16/5/cosh(d*x+c)^5)+b^3*(sinh(d*x+c)^9/cosh(d*x+c)^8+9*sinh(d*x
+c)^7/cosh(d*x+c)^8+21*sinh(d*x+c)^5/cosh(d*x+c)^8+21*sinh(d*x+c)^3/cosh(d
*x+c)^8+9*sinh(d*x+c)/cosh(d*x+c)^8-9*(1/8*sech(d*x+c)^7+7/48*sech(d*x+c)^
5+35/192*sech(d*x+c)^3+35/128*sech(d*x+c))*tanh(d*x+c)-315/64*arctan(exp(d
*x+c))))
```

3.68.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6410 vs. 2(249) = 498.

Time = 0.32 (sec) , antiderivative size = 6410, normalized size of antiderivative = 23.83

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.68.6 Sympy [F]

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \int \left(a+b\tanh^3(c+dx)\right)^3 \sinh(c+dx)\,dx$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**3)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**3*sinh(c + d*x), x)
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.80

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$= \frac{1}{64}b^3\left(\frac{315\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{32\,e^{(-dx-c)}}{d} + \frac{581\,e^{(-2\,dx-2\,c)} + 1681\,e^{(-4\,dx-4\,c)} + 3605\,e^{(-6\,dx-6\,c)} + 2569\,e}{d(e^{(-dx-c)} + 8\,e^{(-3\,dx-3\,c)} + 28\,e^{(-5\,dx-5\,c)} + 56\,e^{(-7\,dx-7}\right.$$
$$+ \frac{3}{2}a^2b\left(\frac{6\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)}}{d} + \frac{4\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} + 1}{d(e^{(-dx-c)} + 2\,e^{(-3\,dx-3\,c)} + e^{(-5\,dx-5\,c)})}\right)$$
$$+ \frac{3}{10}ab^2\left(\frac{5\,e^{(-dx-c)}}{d} + \frac{85\,e^{(-2\,dx-2\,c)} + 210\,e^{(-4\,dx-4\,c)} + 314\,e^{(-6\,dx-6\,c)} + 185\,e^{(-8\,dx-8\,c)} + 65\,e^{(-10\,dx-10\,c)}}{d(e^{(-dx-c)} + 5\,e^{(-3\,dx-3\,c)} + 10\,e^{(-5\,dx-5\,c)} + 10\,e^{(-7\,dx-7\,c)} + 5\,e^{(-9\,dx-9\,c)} + e^{(-11\,dx-}\right.$$
$$+ \frac{a^3\cosh(dx+c)}{d}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
1/64*b^3*(315*arctan(e^(-d*x - c))/d - 32*e^(-d*x - c)/d + (581*e^(-2*d*x
- 2*c) + 1681*e^(-4*d*x - 4*c) + 3605*e^(-6*d*x - 6*c) + 2569*e^(-8*d*x -
8*c) + 1463*e^(-10*d*x - 10*c) - 917*e^(-12*d*x - 12*c) - 529*e^(-14*d*x -
 14*c) - 293*e^(-16*d*x - 16*c) + 32)/(d*(e^(-d*x - c) + 8*e^(-3*d*x - 3*c
) + 28*e^(-5*d*x - 5*c) + 56*e^(-7*d*x - 7*c) + 70*e^(-9*d*x - 9*c) + 56*e
^(-11*d*x - 11*c) + 28*e^(-13*d*x - 13*c) + 8*e^(-15*d*x - 15*c) + e^(-17*
d*x - 17*c)))) + 3/2*a^2*b*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4
*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x -
 3*c) + e^(-5*d*x - 5*c)))) + 3/10*a*b^2*(5*e^(-d*x - c)/d + (85*e^(-2*d*x
 - 2*c) + 210*e^(-4*d*x - 4*c) + 314*e^(-6*d*x - 6*c) + 185*e^(-8*d*x - 8*
c) + 65*e^(-10*d*x - 10*c) + 5)/(d*(e^(-d*x - c) + 5*e^(-3*d*x - 3*c) + 10
*e^(-5*d*x - 5*c) + 10*e^(-7*d*x - 7*c) + 5*e^(-9*d*x - 9*c) + e^(-11*d*x
- 11*c)))) + a^3*cosh(d*x + c)/d
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.72

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$= \frac{160\,a^3e^{(dx+c)} + 480\,a^2be^{(dx+c)} + 480\,ab^2e^{(dx+c)} + 160\,b^3e^{(dx+c)} - 45\left(64\,a^2b+35\,b^3\right)\arctan\left(e^{(dx+c)}\right) + 160}{}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
1/320*(160*a^3*e^(d*x + c) + 480*a^2*b*e^(d*x + c) + 480*a*b^2*e^(d*x + c)
 + 160*b^3*e^(d*x + c) - 45*(64*a^2*b + 35*b^3)*arctan(e^(d*x + c)) + 160*
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*e^(-d*x - c) + (960*a^2*b*e^(15*d*x + 15*c
) + 5760*a*b^2*e^(15*d*x + 15*c) + 1625*b^3*e^(15*d*x + 15*c) + 4800*a^2*b
*e^(13*d*x + 13*c) + 32640*a*b^2*e^(13*d*x + 13*c) + 3925*b^3*e^(13*d*x +
13*c) + 8640*a^2*b*e^(11*d*x + 11*c) + 88704*a*b^2*e^(11*d*x + 11*c) + 906
5*b^3*e^(11*d*x + 11*c) + 4800*a^2*b*e^(9*d*x + 9*c) + 143232*a*b^2*e^(9*d
*x + 9*c) + 1645*b^3*e^(9*d*x + 9*c) - 4800*a^2*b*e^(7*d*x + 7*c) + 143232
*a*b^2*e^(7*d*x + 7*c) - 1645*b^3*e^(7*d*x + 7*c) - 8640*a^2*b*e^(5*d*x +
5*c) + 88704*a*b^2*e^(5*d*x + 5*c) - 9065*b^3*e^(5*d*x + 5*c) - 4800*a^2*b
*e^(3*d*x + 3*c) + 32640*a*b^2*e^(3*d*x + 3*c) - 3925*b^3*e^(3*d*x + 3*c)
- 960*a^2*b*e^(d*x + c) + 5760*a*b^2*e^(d*x + c) - 1625*b^3*e^(d*x + c))/(
e^(2*d*x + 2*c) + 1)^8)/d
```

3.68.9 Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 707, normalized size of antiderivative = 2.63

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \frac{\mathrm{e}^{c+dx}(a+b)^3}{2d} + \frac{\mathrm{e}^{-c-dx}(a-b)^3}{2d}$$
$$- \frac{9\,\mathrm{atan}\left(\frac{\mathrm{e}^{dx}\,\mathrm{e}^{c}\left(35\,b^3\sqrt{d^2}+64\,a^2\,b\sqrt{d^2}\right)}{d\sqrt{4096\,a^4\,b^2+4480\,a^2\,b^4+1225\,b^6}}\right)\sqrt{4096\,a^4\,b^2+4480\,a^2\,b^4+1225\,b^6}}{64\sqrt{d^2}}$$
$$+ \frac{\mathrm{e}^{c+dx}\left(2455\,b^3+1728\,a\,b^2\right)}{40\,d\left(3\,\mathrm{e}^{2c+2dx}+3\,\mathrm{e}^{4c+4dx}+\mathrm{e}^{6c+6dx}+1\right)}$$
$$- \frac{\mathrm{e}^{c+dx}\left(2605\,b^3+768\,a\,b^2\right)}{20\,d\left(4\,\mathrm{e}^{2c+2dx}+6\,\mathrm{e}^{4c+4dx}+4\,\mathrm{e}^{6c+6dx}+\mathrm{e}^{8c+8dx}+1\right)}$$
$$- \frac{188\,b^3\,\mathrm{e}^{c+dx}}{d\left(6\,\mathrm{e}^{2c+2dx}+15\,\mathrm{e}^{4c+4dx}+20\,\mathrm{e}^{6c+6dx}+15\,\mathrm{e}^{8c+8dx}+6\,\mathrm{e}^{10c+10dx}+\mathrm{e}^{12c+12dx}+1\right)}$$
$$+ \frac{\mathrm{e}^{c+dx}\left(192\,a^2\,b+1152\,a\,b^2+325\,b^3\right)}{64\,d\left(\mathrm{e}^{2c+2dx}+1\right)}$$
$$+ \frac{2\,\mathrm{e}^{c+dx}\left(475\,b^3+48\,a\,b^2\right)}{5\,d\left(5\,\mathrm{e}^{2c+2dx}+10\,\mathrm{e}^{4c+4dx}+10\,\mathrm{e}^{6c+6dx}+5\,\mathrm{e}^{8c+8dx}+\mathrm{e}^{10c+10dx}+1\right)}$$
$$+ \frac{112\,b^3\,\mathrm{e}^{c+dx}}{d\left(7\,\mathrm{e}^{2c+2dx}+21\,\mathrm{e}^{4c+4dx}+35\,\mathrm{e}^{6c+6dx}+35\,\mathrm{e}^{8c+8dx}+21\,\mathrm{e}^{10c+10dx}+7\,\mathrm{e}^{12c+12dx}+\mathrm{e}^{14c+14dx}+1\right)}$$
$$- \frac{\mathrm{e}^{c+dx}\left(192\,a^2\,b+768\,a\,b^2+745\,b^3\right)}{32\,d\left(2\,\mathrm{e}^{2c+2dx}+\mathrm{e}^{4c+4dx}+1\right)}$$
$$- \frac{32\,b^3\,\mathrm{e}^{c+dx}}{d\left(8\,\mathrm{e}^{2c+2dx}+28\,\mathrm{e}^{4c+4dx}+56\,\mathrm{e}^{6c+6dx}+70\,\mathrm{e}^{8c+8dx}+56\,\mathrm{e}^{10c+10dx}+28\,\mathrm{e}^{12c+12dx}+8\,\mathrm{e}^{14c+14dx}+\mathrm{e}^{16c}\right.$$

input
```
int(sinh(c + d*x)*(a + b*tanh(c + d*x)^3)^3,x)
```

output

```
(exp(c + d*x)*(a + b)^3)/(2*d) + (exp(- c - d*x)*(a - b)^3)/(2*d) - (9*ata
n((exp(d*x)*exp(c)*(35*b^3*(d^2)^(1/2) + 64*a^2*b*(d^2)^(1/2)))/(d*(1225*b
^6 + 4480*a^2*b^4 + 4096*a^4*b^2)^(1/2)))*(1225*b^6 + 4480*a^2*b^4 + 4096*
a^4*b^2)^(1/2))/(64*(d^2)^(1/2)) + (exp(c + d*x)*(1728*a*b^2 + 2455*b^3))/
(40*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) -
(exp(c + d*x)*(768*a*b^2 + 2605*b^3))/(20*d*(4*exp(2*c + 2*d*x) + 6*exp(4*
c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (188*b^3*exp(c
+ d*x))/(d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x)
 + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) +
 (exp(c + d*x)*(1152*a*b^2 + 192*a^2*b + 325*b^3))/(64*d*(exp(2*c + 2*d*x)
 + 1)) + (2*exp(c + d*x)*(48*a*b^2 + 475*b^3))/(5*d*(5*exp(2*c + 2*d*x) +
10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c
+ 10*d*x) + 1)) + (112*b^3*exp(c + d*x))/(d*(7*exp(2*c + 2*d*x) + 21*exp(4
*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10
*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) - (exp(c + d*x)*(7
68*a*b^2 + 192*a^2*b + 745*b^3))/(32*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d
*x) + 1)) - (32*b^3*exp(c + d*x))/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*
d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) +
 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1))
```

3.69 $\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

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3.69.1 Optimal result

Integrand size = 21, antiderivative size = 219

$$\begin{aligned}
&\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx \\
&= \frac{3a^2b\arctan(\sinh(c+dx))}{2d} + \frac{35b^3\arctan(\sinh(c+dx))}{128d} \\
&\quad - \frac{a^3\operatorname{arctanh}(\cosh(c+dx))}{d} - \frac{3ab^2\operatorname{sech}(c+dx)}{d} + \frac{2ab^2\operatorname{sech}^3(c+dx)}{d} \\
&\quad - \frac{3ab^2\operatorname{sech}^5(c+dx)}{5d} - \frac{3a^2b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d} \\
&\quad - \frac{35b^3\operatorname{sech}(c+dx)\tanh(c+dx)}{128d} - \frac{35b^3\operatorname{sech}(c+dx)\tanh^3(c+dx)}{192d} \\
&\quad - \frac{7b^3\operatorname{sech}(c+dx)\tanh^5(c+dx)}{48d} - \frac{b^3\operatorname{sech}(c+dx)\tanh^7(c+dx)}{8d}
\end{aligned}$$

output

```
3/2*a^2*b*arctan(sinh(d*x+c))/d+35/128*b^3*arctan(sinh(d*x+c))/d-a^3*arcta
nh(cosh(d*x+c))/d-3*a*b^2*sech(d*x+c)/d+2*a*b^2*sech(d*x+c)^3/d-3/5*a*b^2*
sech(d*x+c)^5/d-3/2*a^2*b*sech(d*x+c)*tanh(d*x+c)/d-35/128*b^3*sech(d*x+c)
*tanh(d*x+c)/d-35/192*b^3*sech(d*x+c)*tanh(d*x+c)^3/d-7/48*b^3*sech(d*x+c)
*tanh(d*x+c)^5/d-1/8*b^3*sech(d*x+c)*tanh(d*x+c)^7/d
```

3.69.2 Mathematica [A] (verified)

Time = 8.52 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.77

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$=\frac{30\left(b(192a^2+35b^2)\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+64a^3\left(-\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\right)\right.$$

input
```
Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
(30*(b*(192*a^2 + 35*b^2)*ArcTan[Tanh[(c + d*x)/2]] + 64*a^3*(-Log[Cosh[(c
 + d*x)/2]] + Log[Sinh[(c + d*x)/2]])) + 240*b^3*Sech[c + d*x]^7*Tanh[c +
d*x] - 8*b^2*Sech[c + d*x]^5*(144*a + 125*b*Tanh[c + d*x]) + 10*b^2*Sech[c
 + d*x]^3*(384*a + 163*b*Tanh[c + d*x]) - 45*b*Sech[c + d*x]*(128*a*b + (6
4*a^2 + 31*b^2)*Tanh[c + d*x]))/(1920*d)
```

3.69.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$\downarrow 3042$$
$$\int \frac{i\left(a+ib\tan(ic+idx)^3\right)^3}{\sin(ic+idx)}dx$$
$$\downarrow 26$$
$$i\int \frac{\left(ib\tan(ic+idx)^3+a\right)^3}{\sin(ic+idx)}dx$$
$$\downarrow 4149$$

$$i\int\left(-ib^3\text{sech}(c+dx)\tanh^8(c+dx)-3iab^2\text{sech}(c+dx)\tanh^5(c+dx)-3ia^2b\text{sech}(c+dx)\tanh^2(c+dx)-ia^3\text{c}\right.$$

$$\downarrow 2009$$

$$i\left(\frac{ia^3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3ia^2b\arctan(\sinh(c+dx))}{2d}+\frac{3ia^2b\tanh(c+dx)\text{sech}(c+dx)}{2d}+\frac{3iab^2\text{sech}^5(c+dx)}{5d}\right.$$

input
```
Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
I*((((-3*I)/2)*a^2*b*ArcTan[Sinh[c + d*x]])/d - (((35*I)/128)*b^3*ArcTan[S
inh[c + d*x]])/d + (I*a^3*ArcTanh[Cosh[c + d*x]])/d + ((3*I)*a*b^2*Sech[c
+ d*x])/d - ((2*I)*a*b^2*Sech[c + d*x]^3)/d + (((3*I)/5)*a*b^2*Sech[c + d*
x]^5)/d + (((3*I)/2)*a^2*b*Sech[c + d*x]*Tanh[c + d*x])/d + (((35*I)/128)*
b^3*Sech[c + d*x]*Tanh[c + d*x])/d + (((35*I)/192)*b^3*Sech[c + d*x]*Tanh[
c + d*x]^3)/d + (((7*I)/48)*b^3*Sech[c + d*x]*Tanh[c + d*x]^5)/d + ((I/8)*
b^3*Sech[c + d*x]*Tanh[c + d*x]^7)/d)
```

3.69.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

3.69.4 Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{-2a^3 \operatorname{arctanh}\left(e^{dx+c}\right)+3a^2b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3a\,b^2\left(-\frac{\sinh(dx+c)^4}{\cosh(dx+c)^5}-\frac{4\sinh(dx+c}{3\cosh(dx+}\right.$
default	$\frac{-2a^3 \operatorname{arctanh}\left(e^{dx+c}\right)+3a^2b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3a\,b^2\left(-\frac{\sinh(dx+c)^4}{\cosh(dx+c)^5}-\frac{4\sinh(dx+c}{3\cosh(dx+}\right.$
risch	$-\frac{b\,e^{dx+c}\left(2880a^2e^{14dx+14c}+5760ab\,e^{14dx+14c}+1395b^2e^{14dx+14c}+14400a^2e^{12dx+12c}+24960ab\,e^{12dx+12c}+455b^2e^{12dx}\right.}{}$

input
```
int(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a^3*arctanh(exp(d*x+c))+3*a^2*b*(-sinh(d*x+c)/cosh(d*x+c)^2+1/2*se
ch(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+3*a*b^2*(-sinh(d*x+c)^4/cosh(d*x
+c)^5-4/3*sinh(d*x+c)^2/cosh(d*x+c)^5-8/15/cosh(d*x+c)^5)+b^3*(-sinh(d*x+c
)^7/cosh(d*x+c)^8-7/3*sinh(d*x+c)^5/cosh(d*x+c)^8-7/3*sinh(d*x+c)^3/cosh(d
*x+c)^8-sinh(d*x+c)/cosh(d*x+c)^8+(1/8*sech(d*x+c)^7+7/48*sech(d*x+c)^5+35
/192*sech(d*x+c)^3+35/128*sech(d*x+c))*tanh(d*x+c)+35/64*arctan(exp(d*x+c)
)))
```

3.69.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7127 vs. $2(203) = 406$.

Time = 0.34 (sec) , antiderivative size = 7127, normalized size of antiderivative = 32.54

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.69.6 Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \int \left(a+b\tanh^3(c+dx)\right)^3 \operatorname{csch}(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**3)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x), x)
```

3.69.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(203) = 406$.

Time $= 0.29$ (sec) , antiderivative size $= 654$, normalized size of antiderivative $= 2.99$

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx =$$
$$-\frac{1}{192}b^3\left(\frac{105\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{279\,e^{(-dx-c)}+91\,e^{(-3\,dx-3\,c)}+1799\,e^{(-5\,dx-5\,c)}-1085\,e^{(-7\,dx-7\,c)}+108}{d(8\,e^{(-2\,dx-2\,c)}+28\,e^{(-4\,dx-4\,c)}+56\,e^{(-6\,dx-6\,c)}+70\,e^{(-8\,dx-8\,c)}+56}\right.$$
$$-3\,a^2b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$
$$-\frac{2}{5}ab^2\left(\frac{15\,e^{(-dx-c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{}{d(5\,e^{(-2\,dx-2\,c}\right.$$
$$+\frac{a^3\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
-1/192*b^3*(105*arctan(e^(-d*x - c))/d + (279*e^(-d*x - c) + 91*e^(-3*d*x
- 3*c) + 1799*e^(-5*d*x - 5*c) - 1085*e^(-7*d*x - 7*c) + 1085*e^(-9*d*x -
9*c) - 1799*e^(-11*d*x - 11*c) - 91*e^(-13*d*x - 13*c) - 279*e^(-15*d*x -
15*c))/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c)
+ 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*
e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1))) - 3*a^2*b*(arctan(e^(-d*x -
 c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*
d*x - 4*c) + 1))) - 2/5*a*b^2*(15*e^(-d*x - c)/(d*(5*e^(-2*d*x - 2*c) + 10
*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
- 10*c) + 1)) + 20*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x
- 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1
)) + 58*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10
*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 20*e^(
-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-9*d*x - 9*
c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*
e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + a^3*log(tanh(1/2*d*x + 1/2*
c))/d
```

3.69.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(203) = 406$.

Time $= 0.55$ (sec) , antiderivative size $= 414$, normalized size of antiderivative $= 1.89$

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx =$$

$$-\frac{960\,a^3\log\left(e^{(dx+c)}+1\right)-960\,a^3\log\left(\left|e^{(dx+c)}-1\right|\right)-15\left(192\,a^2b+35\,b^3\right)\arctan\left(e^{(dx+c)}\right)+\frac{2880\,a^2be^{(15\,dx}}{}}{}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

output

```
-1/960*(960*a^3*log(e^(d*x + c) + 1) - 960*a^3*log(abs(e^(d*x + c) - 1)) -
 15*(192*a^2*b + 35*b^3)*arctan(e^(d*x + c)) + (2880*a^2*b*e^(15*d*x + 15*
c) + 5760*a*b^2*e^(15*d*x + 15*c) + 1395*b^3*e^(15*d*x + 15*c) + 14400*a^2
*b*e^(13*d*x + 13*c) + 24960*a*b^2*e^(13*d*x + 13*c) + 455*b^3*e^(13*d*x +
 13*c) + 25920*a^2*b*e^(11*d*x + 11*c) + 62592*a*b^2*e^(11*d*x + 11*c) + 8
995*b^3*e^(11*d*x + 11*c) + 14400*a^2*b*e^(9*d*x + 9*c) + 103296*a*b^2*e^(
9*d*x + 9*c) - 5425*b^3*e^(9*d*x + 9*c) - 14400*a^2*b*e^(7*d*x + 7*c) + 10
3296*a*b^2*e^(7*d*x + 7*c) + 5425*b^3*e^(7*d*x + 7*c) - 25920*a^2*b*e^(5*d
*x + 5*c) + 62592*a*b^2*e^(5*d*x + 5*c) - 8995*b^3*e^(5*d*x + 5*c) - 14400
*a^2*b*e^(3*d*x + 3*c) + 24960*a*b^2*e^(3*d*x + 3*c) - 455*b^3*e^(3*d*x +
3*c) - 2880*a^2*b*e^(d*x + c) + 5760*a*b^2*e^(d*x + c) - 1395*b^3*e^(d*x +
 c))/(e^(2*d*x + 2*c) + 1)^8)/d
```

3.69.9 Mupad [B] (verification not implemented)

Time = 7.23 (sec) , antiderivative size = 671, normalized size of antiderivative = 3.06

$$
\begin{aligned}
&\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx \\
&= \frac{a^3\,\ln\left(\mathrm{e}^{c+dx}-1\right)}{d} - \frac{a^3\,\ln\left(\mathrm{e}^{c+dx}+1\right)}{d} - \frac{\mathrm{e}^{c+dx}\left(4445\,b^3+4224\,a\,b^2\right)}{120\,d\,\left(3\,\mathrm{e}^{2\,c+2\,dx}+3\,\mathrm{e}^{4\,c+4\,dx}+\mathrm{e}^{6\,c+6\,dx}+1\right)} \\
&\quad + \frac{\mathrm{e}^{c+dx}\left(1925\,b^3+768\,a\,b^2\right)}{20\,d\,\left(4\,\mathrm{e}^{2\,c+2\,dx}+6\,\mathrm{e}^{4\,c+4\,dx}+4\,\mathrm{e}^{6\,c+6\,dx}+\mathrm{e}^{8\,c+8\,dx}+1\right)} \\
&\quad + \frac{532\,b^3\,\mathrm{e}^{c+dx}}{3\,d\,\left(6\,\mathrm{e}^{2\,c+2\,dx}+15\,\mathrm{e}^{4\,c+4\,dx}+20\,\mathrm{e}^{6\,c+6\,dx}+15\,\mathrm{e}^{8\,c+8\,dx}+6\,\mathrm{e}^{10\,c+10\,dx}+\mathrm{e}^{12\,c+12\,dx}+1\right)} \\
&\quad - \frac{3\,\mathrm{e}^{c+dx}\left(64\,a^2\,b+128\,a\,b^2+31\,b^3\right)}{64\,d\,\left(\mathrm{e}^{2\,c+2\,dx}+1\right)} \\
&\quad - \frac{2\,\mathrm{e}^{c+dx}\left(1225\,b^3+144\,a\,b^2\right)}{15\,d\,\left(5\,\mathrm{e}^{2\,c+2\,dx}+10\,\mathrm{e}^{4\,c+4\,dx}+10\,\mathrm{e}^{6\,c+6\,dx}+5\,\mathrm{e}^{8\,c+8\,dx}+\mathrm{e}^{10\,c+10\,dx}+1\right)} \\
&\quad - \frac{112\,b^3\,\mathrm{e}^{c+dx}}{d\,\left(7\,\mathrm{e}^{2\,c+2\,dx}+21\,\mathrm{e}^{4\,c+4\,dx}+35\,\mathrm{e}^{6\,c+6\,dx}+35\,\mathrm{e}^{8\,c+8\,dx}+21\,\mathrm{e}^{10\,c+10\,dx}+7\,\mathrm{e}^{12\,c+12\,dx}+\mathrm{e}^{14\,c+14\,dx}+1\right)} \\
&\quad + \frac{\mathrm{e}^{c+dx}\left(576\,a^2\,b+1536\,a\,b^2+931\,b^3\right)}{96\,d\,\left(2\,\mathrm{e}^{2\,c+2\,dx}+\mathrm{e}^{4\,c+4\,dx}+1\right)} \\
&\quad + \frac{32\,b^3\,\mathrm{e}^{c+dx}}{d\,\left(8\,\mathrm{e}^{2\,c+2\,dx}+28\,\mathrm{e}^{4\,c+4\,dx}+56\,\mathrm{e}^{6\,c+6\,dx}+70\,\mathrm{e}^{8\,c+8\,dx}+56\,\mathrm{e}^{10\,c+10\,dx}+28\,\mathrm{e}^{12\,c+12\,dx}+8\,\mathrm{e}^{14\,c+14\,dx}+\mathrm{e}^{16\,c} \\
&\quad - \frac{b\,\ln\left(\mathrm{e}^{c+dx}-\mathrm{i}\right)\,\left(192\,a^2+35\,b^2\right)\,1\mathrm{i}}{128\,d} + \frac{b\,\ln\left(\mathrm{e}^{c+dx}+1\mathrm{i}\right)\,\left(192\,a^2+35\,b^2\right)\,1\mathrm{i}}{128\,d}
\end{aligned}
$$

input

```
int((a + b*tanh(c + d*x)^3)^3/sinh(c + d*x),x)
```

output

```
(a^3*log(exp(c + d*x) - 1))/d - (a^3*log(exp(c + d*x) + 1))/d - (exp(c + d
*x)*(4224*a*b^2 + 4445*b^3))/(120*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*
x) + exp(6*c + 6*d*x) + 1)) - (b*log(exp(c + d*x) - 1i)*(192*a^2 + 35*b^2)
*1i)/(128*d) + (b*log(exp(c + d*x) + 1i)*(192*a^2 + 35*b^2)*1i)/(128*d) +
(exp(c + d*x)*(768*a*b^2 + 1925*b^3))/(20*d*(4*exp(2*c + 2*d*x) + 6*exp(4*
c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (532*b^3*exp(c
+ d*x))/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*
x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1))
 - (3*exp(c + d*x)*(128*a*b^2 + 64*a^2*b + 31*b^3))/(64*d*(exp(2*c + 2*d*x
) + 1)) - (2*exp(c + d*x)*(144*a*b^2 + 1225*b^3))/(15*d*(5*exp(2*c + 2*d*x
) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(1
0*c + 10*d*x) + 1)) - (112*b^3*exp(c + d*x))/(d*(7*exp(2*c + 2*d*x) + 21*e
xp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c
+ 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) + (exp(c + d*x
)*(1536*a*b^2 + 576*a^2*b + 931*b^3))/(96*d*(2*exp(2*c + 2*d*x) + exp(4*c
+ 4*d*x) + 1)) + (32*b^3*exp(c + d*x))/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c
 + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d
*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) +
1))
```

3.70 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

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3.70.1 Optimal result

Integrand size = 23, antiderivative size = 71

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = -\frac{a^3\coth(c+dx)}{d}+\frac{3a^2b\tanh^2(c+dx)}{2d}+\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^8(c+dx)}{8d}$$

output
```
-a^3*coth(d*x+c)/d+3/2*a^2*b*tanh(d*x+c)^2/d+3/5*a*b^2*tanh(d*x+c)^5/d+1/8
*b^3*tanh(d*x+c)^8/d
```

3.70.2 Mathematica [A] (verified)

Time = 2.39 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.59

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \frac{-40a^3\coth(c+dx)+b\left(-20b^2\operatorname{sech}^6(c+dx)+5b^2\operatorname{sech}^8(c+dx)+24ab\tanh(c+dx)+6b\operatorname{sech}^4(c+dx)(5\right.}{40d}$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
(-40*a^3*Coth[c + d*x] + b*(-20*b^2*Sech[c + d*x]^6 + 5*b^2*Sech[c + d*x]^
8 + 24*a*b*Tanh[c + d*x] + 6*b*Sech[c + d*x]^4*(5*b + 4*a*Tanh[c + d*x]) -
 4*Sech[c + d*x]^2*(15*a^2 + 5*b^2 + 12*a*b*Tanh[c + d*x])))/(40*d)
```

3.70.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 25, 4146, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a+ib\tan(ic+idx)^3\right)^3}{\sin(ic+idx)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(ib\tan(ic+idx)^3+a\right)^3}{\sin(ic+idx)^2}dx$$

$$\downarrow 4146$$

$$\frac{\int \coth^2(c+dx)\left(b\tanh^3(c+dx)+a\right)^3 d\tanh(c+dx)}{d}$$

$$\downarrow 802$$

$$\frac{\int \left(b^3\tanh^7(c+dx)+3ab^2\tanh^4(c+dx)+3a^2b\tanh(c+dx)+a^3\coth^2(c+dx)\right) d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{-a^3\coth(c+dx)+\frac{3}{2}a^2b\tanh^2(c+dx)+\frac{3}{5}ab^2\tanh^5(c+dx)+\frac{1}{8}b^3\tanh^8(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
(-(a^3*Coth[c + d*x]) + (3*a^2*b*Tanh[c + d*x]^2)/2 + (3*a*b^2*Tanh[c + d*
x]^5)/5 + (b^3*Tanh[c + d*x]^8)/8)/d
```

3.70.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.70.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(65) = 130$.

Time = 20.42 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.41

method	result
derivativedivides	$\frac{-a^3\coth(dx+c)-\frac{3a^2b}{2\cosh(dx+c)^2}+3ab^2\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{8}\right.}{d}$
default	$\frac{-a^3\coth(dx+c)-\frac{3a^2b}{2\cosh(dx+c)^2}+3ab^2\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{8}\right.}{d}$
risch	$-\frac{2(-3ab^2+5a^3e^{16dx+16c}+5b^3e^{16dx+16c}+35b^3e^{12dx+12c}+5a^3+5e^{4dx+4c}b^3+40a^3e^{14dx+14c}-5b^3e^{14dx+14c}-6e^{2dx+2}}{}$

input
```
int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^3*coth(d*x+c)-3/2*a^2*b/cosh(d*x+c)^2+3*a*b^2*(-1/2*sinh(d*x+c)^3/
cosh(d*x+c)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/
15*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(-1/2*sinh(d*x+c)^6/cosh(d*x+c)^8-3/4*s
inh(d*x+c)^4/cosh(d*x+c)^8-1/2*sinh(d*x+c)^2/cosh(d*x+c)^8-1/8/cosh(d*x+c)
^8))
```

3.70.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1192 vs. $2(65) = 130$.

Time $= 0.27$ (sec) , antiderivative size $= 1192$, normalized size of antiderivative $= 16.79$

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
-2/5*((10*a^3 + 15*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 8*(15*a^2*b
 + 18*a*b^2 + 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (10*a^3 + 15*a^2*b +
12*a*b^2 + 5*b^3)*sinh(d*x + c)^8 + 2*(40*a^3 + 30*a^2*b + 12*a*b^2 - 5*b^
3)*cosh(d*x + c)^6 + 2*(40*a^3 + 30*a^2*b + 12*a*b^2 - 5*b^3 + 14*(10*a^3
+ 15*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(14*(1
5*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 27*(5*a^2*b + 2*a*b^2)*cosh(
d*x + c))*sinh(d*x + c)^5 + 20*(14*a^3 + 3*a^2*b + 2*b^3)*cosh(d*x + c)^4
+ 10*(7*(10*a^3 + 15*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 28*a^3 +
6*a^2*b + 4*b^3 + 3*(40*a^3 + 30*a^2*b + 12*a*b^2 - 5*b^3)*cosh(d*x + c)^2
)*sinh(d*x + c)^4 + 8*(7*(15*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 4
5*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + 15*(7*a^2*b + 2*a*b^2 + b^3)*cosh(
d*x + c))*sinh(d*x + c)^3 + 350*a^3 - 75*a^2*b - 12*a*b^2 + 35*b^3 + 2*(28
0*a^3 - 30*a^2*b - 12*a*b^2 - 35*b^3)*cosh(d*x + c)^2 + 2*(14*(10*a^3 + 15
*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 15*(40*a^3 + 30*a^2*b + 12*a*
b^2 - 5*b^3)*cosh(d*x + c)^4 + 280*a^3 - 30*a^2*b - 12*a*b^2 - 35*b^3 + 60
*(14*a^3 + 3*a^2*b + 2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(2*(15*a^
2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 27*(5*a^2*b + 2*a*b^2)*cosh(d*x
+ c)^5 + 30*(7*a^2*b + 2*a*b^2 + b^3)*cosh(d*x + c)^3 + 21*(5*a^2*b + 2*a*
b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*d*cosh(d*x + c
)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + 6*d*cosh(d*x + c)^8 + 3*(15*d*...
```

3.70.6 Sympy [F]

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \int \left(a+b\tanh^3(c+dx)\right)^3 \operatorname{csch}^2(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**3)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x)**2, x)
```

3.70.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 679 vs. $2(65) = 130$.

Time $= 0.21$ (sec) , antiderivative size $= 679$, normalized size of antiderivative $= 9.56$

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx =$$
$$-2b^3\left(\frac{e^{(-2dx-2c)}}{d(8e^{(-2dx-2c)}+28e^{(-4dx-4c)}+56e^{(-6dx-6c)}+70e^{(-8dx-8c)}+56e^{(-10dx-10c)}+28e^{(-12dx-12c)}+}\right.$$
$$+\frac{6}{5}ab^2\left(\frac{10e^{(-4dx-4c)}}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}+\frac{}{d(5e^{(-2dx-2c}\right.$$
$$+\frac{2a^3}{d(e^{(-2dx-2c)}-1)}-\frac{6a^2b}{d(e^{(dx+c)}+e^{(-dx-c)})^2}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
-2*b^3*(e^(-2*d*x - 2*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56
*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-1
2*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) + 7*e^(-6*
d*x - 6*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6
*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c)
+ 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) + 7*e^(-10*d*x - 10*c)/(
d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(
-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d
*x - 14*c) + e^(-16*d*x - 16*c) + 1)) + e^(-14*d*x - 14*c)/(d*(8*e^(-2*d*x
 - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c)
+ 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e
^(-16*d*x - 16*c) + 1))) + 6/5*a*b^2*(10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x
- 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) +
e^(-10*d*x - 10*c) + 1)) + 5*e^(-8*d*x - 8*c)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d
*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 2*a^3/(d*(e^(
-2*d*x - 2*c) - 1)) - 6*a^2*b/(d*(e^(d*x + c) + e^(-d*x - c))^2)
```

3.70.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(65) = 130$.

Time $= 0.55$ (sec) , antiderivative size $= 311$, normalized size of antiderivative $= 4.38$

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx =$$

$$-\frac{2\left(\frac{5\,a^3}{e^{(2\,dx+2\,c)}-1}+\frac{15\,a^2be^{(14\,dx+14\,c)}+15\,ab^2e^{(14\,dx+14\,c)}+5\,b^3e^{(14\,dx+14\,c)}+90\,a^2be^{(12\,dx+12\,c)}+45\,ab^2e^{(12\,dx+12\,c)}+225\,a^2be^{(10\,dx+10\,c)}}{}\right.}{}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
-2/5*(5*a^3/(e^(2*d*x + 2*c) - 1) + (15*a^2*b*e^(14*d*x + 14*c) + 15*a*b^2
*e^(14*d*x + 14*c) + 5*b^3*e^(14*d*x + 14*c) + 90*a^2*b*e^(12*d*x + 12*c)
+ 45*a*b^2*e^(12*d*x + 12*c) + 225*a^2*b*e^(10*d*x + 10*c) + 75*a*b^2*e^(1
0*d*x + 10*c) + 35*b^3*e^(10*d*x + 10*c) + 300*a^2*b*e^(8*d*x + 8*c) + 105
*a*b^2*e^(8*d*x + 8*c) + 225*a^2*b*e^(6*d*x + 6*c) + 93*a*b^2*e^(6*d*x + 6
*c) + 35*b^3*e^(6*d*x + 6*c) + 90*a^2*b*e^(4*d*x + 4*c) + 39*a*b^2*e^(4*d*
x + 4*c) + 15*a^2*b*e^(2*d*x + 2*c) + 9*a*b^2*e^(2*d*x + 2*c) + 5*b^3*e^(2
*d*x + 2*c) + 3*a*b^2)/(e^(2*d*x + 2*c) + 1)^8)/d
```

3.70.9 Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 1515, normalized size of antiderivative = 21.34

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int((a + b*tanh(c + d*x)^3)^3/sinh(c + d*x)^2,x)
```

output
```
((3*a*b^2 - 15*a^2*b + 7*b^3)/(28*d) - (exp(2*c + 2*d*x)*(3*a*b^2 + 3*a^2*
b + b^3))/(4*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((9*a*b^2 +
 15*a^2*b - 35*b^3)/(140*d) + (exp(6*c + 6*d*x)*(3*a*b^2 + 3*a^2*b + b^3))
/(4*d) + (3*exp(2*c + 2*d*x)*(9*a^2*b - 3*a*b^2 + 7*b^3))/(28*d) - (3*exp(
4*c + 4*d*x)*(3*a*b^2 - 15*a^2*b + 7*b^3))/(28*d))/(4*exp(2*c + 2*d*x) + 6
*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((exp(10*
c + 10*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(4*d) - (3*a*b^2 + 9*a^2*b + 7*b^3)
/(28*d) + (5*exp(6*c + 6*d*x)*(9*a^2*b - 3*a*b^2 + 7*b^3))/(14*d) - (5*exp
(8*c + 8*d*x)*(3*a*b^2 - 15*a^2*b + 7*b^3))/(28*d) + (exp(2*c + 2*d*x)*(9*
a*b^2 - 15*a^2*b + 35*b^3))/(28*d) + (exp(4*c + 4*d*x)*(9*a*b^2 + 15*a^2*b
 - 35*b^3))/(14*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c
 + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x
) + 1) - ((9*a^2*b - 3*a*b^2 + 7*b^3)/(28*d) + (exp(4*c + 4*d*x)*(3*a*b^2
+ 3*a^2*b + b^3))/(4*d) - (exp(2*c + 2*d*x)*(3*a*b^2 - 15*a^2*b + 7*b^3))/
(14*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) -
 ((9*a*b^2 - 15*a^2*b + 35*b^3)/(140*d) + (exp(8*c + 8*d*x)*(3*a*b^2 + 3*a
^2*b + b^3))/(4*d) + (3*exp(4*c + 4*d*x)*(9*a^2*b - 3*a*b^2 + 7*b^3))/(14*
d) - (exp(6*c + 6*d*x)*(3*a*b^2 - 15*a^2*b + 7*b^3))/(7*d) + (exp(2*c + 2*
d*x)*(9*a*b^2 + 15*a^2*b - 35*b^3))/(35*d))/(5*exp(2*c + 2*d*x) + 10*exp(4
*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*...
```

3.71 $\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

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3.71.1 Optimal result

Integrand size = 23, antiderivative size = 232

$$\begin{aligned}
&\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx \\
&= \frac{3a^2b\arctan(\sinh(c+dx))}{2d} + \frac{5b^3\arctan(\sinh(c+dx))}{128d} \\
&\quad + \frac{a^3\operatorname{arctanh}(\cosh(c+dx))}{2d} - \frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d} \\
&\quad - \frac{ab^2\operatorname{sech}^3(c+dx)}{d} + \frac{3ab^2\operatorname{sech}^5(c+dx)}{5d} + \frac{3a^2b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d} \\
&\quad + \frac{5b^3\operatorname{sech}(c+dx)\tanh(c+dx)}{128d} - \frac{5b^3\operatorname{sech}^3(c+dx)\tanh(c+dx)}{64d} \\
&\quad - \frac{5b^3\operatorname{sech}^3(c+dx)\tanh^3(c+dx)}{48d} - \frac{b^3\operatorname{sech}^3(c+dx)\tanh^5(c+dx)}{8d}
\end{aligned}$$

output

```
3/2*a^2*b*arctan(sinh(d*x+c))/d+5/128*b^3*arctan(sinh(d*x+c))/d+1/2*a^3*ar
ctanh(cosh(d*x+c))/d-1/2*a^3*coth(d*x+c)*csch(d*x+c)/d-a*b^2*sech(d*x+c)^3
/d+3/5*a*b^2*sech(d*x+c)^5/d+3/2*a^2*b*sech(d*x+c)*tanh(d*x+c)/d+5/128*b^3
*sech(d*x+c)*tanh(d*x+c)/d-5/64*b^3*sech(d*x+c)^3*tanh(d*x+c)/d-5/48*b^3*s
ech(d*x+c)^3*tanh(d*x+c)^3/d-1/8*b^3*sech(d*x+c)^3*tanh(d*x+c)^5/d
```

3.71.2 Mathematica [A] (verified)

Time = 13.34 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.14

$$\begin{aligned}
&\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx \\
&= \frac{(192a^2b+5b^3)\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{64d} - \frac{a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} \\
&\quad + \frac{a^3\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a^3\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} \\
&\quad - \frac{a^3\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{ab^2\operatorname{sech}^3(c+dx)}{d} + \frac{3ab^2\operatorname{sech}^5(c+dx)}{5d} \\
&\quad + \frac{\operatorname{sech}^2(c+dx)\left(192a^2b\sinh(c+dx)+5b^3\sinh(c+dx)\right)}{128d} \\
&\quad - \frac{59b^3\operatorname{sech}^3(c+dx)\tanh(c+dx)}{192d} \\
&\quad + \frac{17b^3\operatorname{sech}^5(c+dx)\tanh(c+dx)}{48d} - \frac{b^3\operatorname{sech}^7(c+dx)\tanh(c+dx)}{8d}
\end{aligned}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
((192*a^2*b + 5*b^3)*ArcTan[Tanh[(c + d*x)/2]])/(64*d) - (a^3*Csch[(c + d*
x)/2]^2)/(8*d) + (a^3*Log[Cosh[(c + d*x)/2]])/(2*d) - (a^3*Log[Sinh[(c + d
*x)/2]])/(2*d) - (a^3*Sech[(c + d*x)/2]^2)/(8*d) - (a*b^2*Sech[c + d*x]^3)
/d + (3*a*b^2*Sech[c + d*x]^5)/(5*d) + (Sech[c + d*x]^2*(192*a^2*b*Sinh[c
+ d*x] + 5*b^3*Sinh[c + d*x]))/(128*d) - (59*b^3*Sech[c + d*x]^3*Tanh[c +
d*x])/(192*d) + (17*b^3*Sech[c + d*x]^5*Tanh[c + d*x])/(48*d) - (b^3*Sech[
c + d*x]^7*Tanh[c + d*x])/(8*d)
```

3.71.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\left(a+ib\tan(ic+idx)^3\right)^3}{\sin(ic+idx)^3}dx$$

$$\downarrow 26$$

$$-i\int \frac{\left(ib\tan(ic+idx)^3+a\right)^3}{\sin(ic+idx)^3}dx$$

$$\downarrow 4149$$

$$-i\int \left(ib^3\operatorname{sech}^3(c+dx)\tanh^6(c+dx)+3iab^2\operatorname{sech}^3(c+dx)\tanh^3(c+dx)+ia^3\operatorname{csch}^3(c+dx)+3ia^2b\operatorname{sech}^3(c+dx\right.$$

$$\downarrow 2009$$

$$-i\left(\frac{ia^3\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{ia^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{3ia^2b\arctan(\sinh(c+dx))}{2d}+\frac{3ia^2b\tanh(c+dx)}{2d}\right.$$

input
```
Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
(-I)*((((3*I)/2)*a^2*b*ArcTan[Sinh[c + d*x]])/d + (((5*I)/128)*b^3*ArcTan[
Sinh[c + d*x]])/d + ((I/2)*a^3*ArcTanh[Cosh[c + d*x]])/d - ((I/2)*a^3*Coth
[c + d*x]*Csch[c + d*x])/d - (I*a*b^2*Sech[c + d*x]^3)/d + (((3*I)/5)*a*b^
2*Sech[c + d*x]^5)/d + (((3*I)/2)*a^2*b*Sech[c + d*x]*Tanh[c + d*x])/d + (
((5*I)/128)*b^3*Sech[c + d*x]*Tanh[c + d*x])/d - (((5*I)/64)*b^3*Sech[c +
d*x]^3*Tanh[c + d*x])/d - (((5*I)/48)*b^3*Sech[c + d*x]^3*Tanh[c + d*x]^3)
/d - ((I/8)*b^3*Sech[c + d*x]^3*Tanh[c + d*x]^5)/d)
```

3.71.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

3.71.4 Maple [A] (verified)

Time = 38.16 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{a^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3a^2b\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3ab^2\left(-\frac{\sinh(dx+c)^2}{3\cosh(dx+c)^5}-\right.}{}$
default	$\frac{a^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3a^2b\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3ab^2\left(-\frac{\sinh(dx+c)^2}{3\cosh(dx+c)^5}-\right.}{}$
risch	$-\frac{e^{dx+c}\left(8640a^3e^{16dx+16c}+2135b^3e^{16dx+16c}+19760b^3e^{12dx+12c}+960a^3+8520e^{4dx+4c}b^3+34560a^3e^{14dx+14c}-8520b^3\right.}{}$

input
```
int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+3*a^2*b*(1/2*s
ech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+3*a*b^2*(-1/3*sinh(d*x+c)^2/cos
h(d*x+c)^5-2/15/cosh(d*x+c)^5)+b^3*(-1/3*sinh(d*x+c)^5/cosh(d*x+c)^8-1/3*s
inh(d*x+c)^3/cosh(d*x+c)^8-1/7*sinh(d*x+c)/cosh(d*x+c)^8+1/7*(1/8*sech(d*x
+c)^7+7/48*sech(d*x+c)^5+35/192*sech(d*x+c)^3+35/128*sech(d*x+c))*tanh(d*x
+c)+5/64*arctan(exp(d*x+c))))
```

3.71.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10985 vs. 2(212) = 424.

Time = 0.38 (sec) , antiderivative size = 10985, normalized size of antiderivative = 47.35

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.71.6 Sympy [F]

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \int \left(a+b\tanh^3(c+dx)\right)^3 \operatorname{csch}^3(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**3)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x)**3, x)
```

3.71.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. 2(212) = 424.

Time = 0.29 (sec) , antiderivative size = 586, normalized size of antiderivative = 2.53

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx =$$

$$-\frac{1}{192}b^3\left(\frac{15\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{15\,e^{(-dx-c)}-397\,e^{(-3\,dx-3\,c)}+895\,e^{(-5\,dx-5\,c)}-1765\,e^{(-7\,dx-7\,c)}+1765}{d(8\,e^{(-2\,dx-2\,c)}+28\,e^{(-4\,dx-4\,c)}+56\,e^{(-6\,dx-6\,c)}+70\,e^{(-8\,dx-8\,c)}+56\,e^{(}\right.$$

$$-3\,a^2b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$

$$+\frac{1}{2}a^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$

$$-\frac{8}{5}ab^2\left(\frac{5\,e^{(-3\,dx-3\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}-\frac{}{d(5\,e^{(-2\,dx-2\,c}\right.$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
-1/192*b^3*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) - 397*e^(-3*d*x -
 3*c) + 895*e^(-5*d*x - 5*c) - 1765*e^(-7*d*x - 7*c) + 1765*e^(-9*d*x - 9*
c) - 895*e^(-11*d*x - 11*c) + 397*e^(-13*d*x - 13*c) - 15*e^(-15*d*x - 15*
c))/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 7
0*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(
-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1))) - 3*a^2*b*(arctan(e^(-d*x - c)
)/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x
 - 4*c) + 1))) + 1/2*a^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/
d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x
 - 4*c) - 1))) - 8/5*a*b^2*(5*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2*c) + 10
*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
- 10*c) + 1)) - 2*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x -
 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)
) + 5*e^(-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e
^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)))
```

3.71.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(212) = 424$.

Time $= 0.56$ (sec) , antiderivative size $= 426$, normalized size of antiderivative $= 1.84$

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$

$$= \frac{480\,a^3\log\left(e^{(dx+c)}+1\right) - 480\,a^3\log\left(\left|e^{(dx+c)}-1\right|\right) + 15\left(192\,a^2b+5\,b^3\right)\arctan\left(e^{(dx+c)}\right) - \frac{960\left(a^3e^{(3\,dx+3\,c)}+a\right.}{\left(e^{(2\,dx+2\,c)}-\right.}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
1/960*(480*a^3*log(e^(d*x + c) + 1) - 480*a^3*log(abs(e^(d*x + c) - 1)) +
15*(192*a^2*b + 5*b^3)*arctan(e^(d*x + c)) - 960*(a^3*e^(3*d*x + 3*c) + a^
3*e^(d*x + c))/(e^(2*d*x + 2*c) - 1)^2 + (2880*a^2*b*e^(15*d*x + 15*c) + 7
5*b^3*e^(15*d*x + 15*c) + 14400*a^2*b*e^(13*d*x + 13*c) - 7680*a*b^2*e^(13
*d*x + 13*c) - 1985*b^3*e^(13*d*x + 13*c) + 25920*a^2*b*e^(11*d*x + 11*c)
- 19968*a*b^2*e^(11*d*x + 11*c) + 4475*b^3*e^(11*d*x + 11*c) + 14400*a^2*b
*e^(9*d*x + 9*c) - 21504*a*b^2*e^(9*d*x + 9*c) - 8825*b^3*e^(9*d*x + 9*c)
- 14400*a^2*b*e^(7*d*x + 7*c) - 21504*a*b^2*e^(7*d*x + 7*c) + 8825*b^3*e^(
7*d*x + 7*c) - 25920*a^2*b*e^(5*d*x + 5*c) - 19968*a*b^2*e^(5*d*x + 5*c) -
 4475*b^3*e^(5*d*x + 5*c) - 14400*a^2*b*e^(3*d*x + 3*c) - 7680*a*b^2*e^(3*
d*x + 3*c) + 1985*b^3*e^(3*d*x + 3*c) - 2880*a^2*b*e^(d*x + c) - 75*b^3*e^
(d*x + c))/(e^(2*d*x + 2*c) + 1)^8)/d
```

3.71.9 Mupad [B] (verification not implemented)

Time = 8.79 (sec) , antiderivative size = 731, normalized size of antiderivative = 3.15

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \frac{a^3\,\ln\left(\mathrm{e}^{c+dx}+1\right)}{2\,d} - \frac{a^3\,\ln\left(\mathrm{e}^{c+dx}-1\right)}{2\,d}$$
$$+ \frac{\mathrm{e}^{c+dx}\,(192\,a^2\,b+5\,b^3)}{64\,d\,\left(\mathrm{e}^{2\,c+2\,dx}+1\right)} + \frac{\mathrm{e}^{c+dx}\,(2245\,b^3+3264\,a\,b^2)}{120\,d\,\left(3\,\mathrm{e}^{2\,c+2\,dx}+3\,\mathrm{e}^{4\,c+4\,dx}+\mathrm{e}^{6\,c+6\,dx}+1\right)}$$
$$- \frac{\mathrm{e}^{c+dx}\,(1325\,b^3+768\,a\,b^2)}{20\,d\,\left(4\,\mathrm{e}^{2\,c+2\,dx}+6\,\mathrm{e}^{4\,c+4\,dx}+4\,\mathrm{e}^{6\,c+6\,dx}+\mathrm{e}^{8\,c+8\,dx}+1\right)}$$
$$- \frac{500\,b^3\,\mathrm{e}^{c+dx}}{3\,d\,\left(6\,\mathrm{e}^{2\,c+2\,dx}+15\,\mathrm{e}^{4\,c+4\,dx}+20\,\mathrm{e}^{6\,c+6\,dx}+15\,\mathrm{e}^{8\,c+8\,dx}+6\,\mathrm{e}^{10\,c+10\,dx}+\mathrm{e}^{12\,c+12\,dx}+1\right)}$$
$$+ \frac{2\,\mathrm{e}^{c+dx}\,(1025\,b^3+144\,a\,b^2)}{15\,d\,\left(5\,\mathrm{e}^{2\,c+2\,dx}+10\,\mathrm{e}^{4\,c+4\,dx}+10\,\mathrm{e}^{6\,c+6\,dx}+5\,\mathrm{e}^{8\,c+8\,dx}+\mathrm{e}^{10\,c+10\,dx}+1\right)}$$
$$+ \frac{112\,b^3\,\mathrm{e}^{c+dx}}{d\,\left(7\,\mathrm{e}^{2\,c+2\,dx}+21\,\mathrm{e}^{4\,c+4\,dx}+35\,\mathrm{e}^{6\,c+6\,dx}+35\,\mathrm{e}^{8\,c+8\,dx}+21\,\mathrm{e}^{10\,c+10\,dx}+7\,\mathrm{e}^{12\,c+12\,dx}+\mathrm{e}^{14\,c+14\,dx}+1\right)}$$
$$- \frac{\mathrm{e}^{c+dx}\,(576\,a^2\,b+768\,a\,b^2+251\,b^3)}{96\,d\,\left(2\,\mathrm{e}^{2\,c+2\,dx}+\mathrm{e}^{4\,c+4\,dx}+1\right)}$$
$$- \frac{32\,b^3\,\mathrm{e}^{c+dx}}{d\,\left(8\,\mathrm{e}^{2\,c+2\,dx}+28\,\mathrm{e}^{4\,c+4\,dx}+56\,\mathrm{e}^{6\,c+6\,dx}+70\,\mathrm{e}^{8\,c+8\,dx}+56\,\mathrm{e}^{10\,c+10\,dx}+28\,\mathrm{e}^{12\,c+12\,dx}+8\,\mathrm{e}^{14\,c+14\,dx}+\mathrm{e}^{16\,c}\right.}$$
$$- \frac{a^3\,\mathrm{e}^{c+dx}}{d\,\left(\mathrm{e}^{2\,c+2\,dx}-1\right)} - \frac{2\,a^3\,\mathrm{e}^{c+dx}}{d\,\left(\mathrm{e}^{4\,c+4\,dx}-2\,\mathrm{e}^{2\,c+2\,dx}+1\right)}$$
$$- \frac{b\,\ln\left(\mathrm{e}^{c+dx}-\mathrm{i}\right)\,\left(192\,a^2+5\,b^2\right)\,1\mathrm{i}}{128\,d} + \frac{b\,\ln\left(\mathrm{e}^{c+dx}+1\mathrm{i}\right)\,\left(192\,a^2+5\,b^2\right)\,1\mathrm{i}}{128\,d}$$

input
```
int((a + b*tanh(c + d*x)^3)^3/sinh(c + d*x)^3,x)
```

output

```
(a^3*log(exp(c + d*x) + 1))/(2*d) - (a^3*log(exp(c + d*x) - 1))/(2*d) + (e
xp(c + d*x)*(192*a^2*b + 5*b^3))/(64*d*(exp(2*c + 2*d*x) + 1)) + (exp(c +
d*x)*(3264*a*b^2 + 2245*b^3))/(120*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d
*x) + exp(6*c + 6*d*x) + 1)) - (b*log(exp(c + d*x) - 1i)*(192*a^2 + 5*b^2)
*1i)/(128*d) + (b*log(exp(c + d*x) + 1i)*(192*a^2 + 5*b^2)*1i)/(128*d) - (
exp(c + d*x)*(768*a*b^2 + 1325*b^3))/(20*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c
 + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (500*b^3*exp(c +
 d*x))/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x
) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1))
+ (2*exp(c + d*x)*(144*a*b^2 + 1025*b^3))/(15*d*(5*exp(2*c + 2*d*x) + 10*e
xp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10
*d*x) + 1)) + (112*b^3*exp(c + d*x))/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c +
 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x
) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) - (exp(c + d*x)*(768*a
*b^2 + 576*a^2*b + 251*b^3))/(96*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x)
+ 1)) - (32*b^3*exp(c + d*x))/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x)
 + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*
exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1)) - (a^
3*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a^3*exp(c + d*x))/(d*(exp(
4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

3.72 $\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

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3.72.1 Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \frac{a^3\coth(c+dx)}{d} - \frac{a^3\coth^3(c+dx)}{3d} + \frac{3a^2b\log(\tanh(c+dx))}{d} - \frac{3a^2b\tanh^2(c+dx)}{2d} + \frac{ab^2\tanh^3(c+dx)}{d} - \frac{3ab^2\tanh^5(c+dx)}{5d} + \frac{b^3\tanh^6(c+dx)}{6d} - \frac{b^3\tanh^8(c+dx)}{8d}$$

output

```
a^3*coth(d*x+c)/d-1/3*a^3*coth(d*x+c)^3/d+3*a^2*b*ln(tanh(d*x+c))/d-3/2*a^
2*b*tanh(d*x+c)^2/d+a*b^2*tanh(d*x+c)^3/d-3/5*a*b^2*tanh(d*x+c)^5/d+1/6*b^
3*tanh(d*x+c)^6/d-1/8*b^3*tanh(d*x+c)^8/d
```

3.72.2 Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.54

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \frac{2a^3\coth(c+dx)}{3d} - \frac{a^3\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d} - \frac{3a^2b\log(\cosh(c+dx))}{d} + \frac{3a^2b\log(\sinh(c+dx))}{d} + \frac{3a^2b\operatorname{sech}^2(c+dx)}{2d} - \frac{b^3\operatorname{sech}^4(c+dx)}{4d} + \frac{b^3\operatorname{sech}^6(c+dx)}{3d} - \frac{b^3\operatorname{sech}^8(c+dx)}{8d} + \frac{2ab^2\tanh(c+dx)}{5d} + \frac{ab^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{5d} - \frac{3ab^2\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
(2*a^3*Coth[c + d*x])/(3*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) -
(3*a^2*b*Log[Cosh[c + d*x]])/d + (3*a^2*b*Log[Sinh[c + d*x]])/d + (3*a^2*b
*Sech[c + d*x]^2)/(2*d) - (b^3*Sech[c + d*x]^4)/(4*d) + (b^3*Sech[c + d*x]
^6)/(3*d) - (b^3*Sech[c + d*x]^8)/(8*d) + (2*a*b^2*Tanh[c + d*x])/(5*d) +
(a*b^2*Sech[c + d*x]^2*Tanh[c + d*x])/(5*d) - (3*a*b^2*Sech[c + d*x]^4*Tan
h[c + d*x])/(5*d)
```

3.72.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4146, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a+ib\tan(ic+idx)^3\right)^3}{\sin(ic+idx)^4}dx$$

$$\downarrow 4146$$

$$\frac{\int \coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)\left(b\tanh^3(c+dx)+a\right)^3 d\tanh(c+dx)}{d}$$

$$\downarrow 2333$$

$$\frac{\int \left(-b^3\tanh^7(c+dx)+b^3\tanh^5(c+dx)-3ab^2\tanh^4(c+dx)+3ab^2\tanh^2(c+dx)-3a^2b\tanh(c+dx)+a^3\cot\right.}{d}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{3}a^3\coth^3(c+dx)+a^3\coth(c+dx)-\frac{3}{2}a^2b\tanh^2(c+dx)+3a^2b\log(\tanh(c+dx))-\frac{3}{5}ab^2\tanh^5(c+dx)+ab^2}{d}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
(a^3*Coth[c + d*x] - (a^3*Coth[c + d*x]^3)/3 + 3*a^2*b*Log[Tanh[c + d*x]]
- (3*a^2*b*Tanh[c + d*x]^2)/2 + a*b^2*Tanh[c + d*x]^3 - (3*a*b^2*Tanh[c +
d*x]^5)/5 + (b^3*Tanh[c + d*x]^6)/6 - (b^3*Tanh[c + d*x]^8)/8)/d
```

3.72.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.72.4 Maple [A] (verified)

Time = 62.88 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{a^3\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+3a^2b\left(\frac{1}{2\cosh(dx+c)^2}+\ln(\tanh(dx+c))\right)+3ab^2\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\right.}{d}$
default	$\frac{a^3\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+3a^2b\left(\frac{1}{2\cosh(dx+c)^2}+\ln(\tanh(dx+c))\right)+3ab^2\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\right.}{d}$
risch	$-\frac{2(-6ab^2+90ab^2e^{18dx+18c}+230a^3e^{16dx+16c}-130b^3e^{16dx+16c}-490b^3e^{12dx+12c}-10a^3-30e^{4dx+4c}b^3+760a^3e^{14dx+1}$

input

```
int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output

```
1/d*(a^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(1/2/cosh(d*x+c)^2+ln
(tanh(d*x+c)))+3*a*b^2*(-1/4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(
d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(-1/4*sinh(d*x+c)^4/cosh(d*x
+c)^8-1/6*sinh(d*x+c)^2/cosh(d*x+c)^8-1/24/cosh(d*x+c)^8))
```

3.72.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9459 vs. $2(128) = 256$.

Time = 0.35 (sec) , antiderivative size = 9459, normalized size of antiderivative = 68.54

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input

```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

output

```
Too large to include
```

3.72.6 Sympy [F]

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \int \left(a+b\tanh^3(c+dx)\right)^3 \operatorname{csch}^4(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**3)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x)**4, x)
```

3.72.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 997 vs. $2(128) = 256$.

Time $= 0.29$ (sec) , antiderivative size $= 997$, normalized size of antiderivative $= 7.22$

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
3*a^2*b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x
 - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4
*c) + 1))) + 4/5*a*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c
) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) +
15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-
6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(
-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x -
8*c) + e^(-10*d*x - 10*c) + 1))) + 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2
*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*
d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) - 4/3*b^3*(3*e^(
-4*d*x - 4*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x
- 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*
c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) - 4*e^(-6*d*x - 6*c)/
(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^
(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*
d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) + 10*e^(-8*d*x - 8*c)/(d*(8*e^(-2*d
*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c
) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c...
```

3.72.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(128) = 256$.

Time $= 0.58$ (sec) , antiderivative size $= 437$, normalized size of antiderivative $= 3.17$

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx =$$

$$-\frac{2520\,a^2b\log\left(e^{(2\,dx+2\,c)}+1\right)-2520\,a^2b\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{140\left(33\,a^2be^{(6\,dx+6\,c)}-99\,a^2be^{(4\,dx+4\,c)}+24\,a^3e^{(2\,dx+2\,c}\right.}{\left(e^{(2\,dx+2\,c)}-1\right)^3}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
-1/840*(2520*a^2*b*log(e^(2*d*x + 2*c) + 1) - 2520*a^2*b*log(abs(e^(2*d*x
+ 2*c) - 1)) + 140*(33*a^2*b*e^(6*d*x + 6*c) - 99*a^2*b*e^(4*d*x + 4*c) +
24*a^3*e^(2*d*x + 2*c) + 99*a^2*b*e^(2*d*x + 2*c) - 8*a^3 - 33*a^2*b)/(e^(
2*d*x + 2*c) - 1)^3 - (6849*a^2*b*e^(16*d*x + 16*c) + 59832*a^2*b*e^(14*d*
x + 14*c) + 222012*a^2*b*e^(12*d*x + 12*c) - 10080*a*b^2*e^(12*d*x + 12*c)
 - 3360*b^3*e^(12*d*x + 12*c) + 459144*a^2*b*e^(10*d*x + 10*c) - 26880*a*b
^2*e^(10*d*x + 10*c) + 4480*b^3*e^(10*d*x + 10*c) + 580230*a^2*b*e^(8*d*x
+ 8*c) - 23520*a*b^2*e^(8*d*x + 8*c) - 11200*b^3*e^(8*d*x + 8*c) + 459144*
a^2*b*e^(6*d*x + 6*c) - 10752*a*b^2*e^(6*d*x + 6*c) + 4480*b^3*e^(6*d*x +
6*c) + 222012*a^2*b*e^(4*d*x + 4*c) - 8736*a*b^2*e^(4*d*x + 4*c) - 3360*b^
3*e^(4*d*x + 4*c) + 59832*a^2*b*e^(2*d*x + 2*c) - 5376*a*b^2*e^(2*d*x + 2*
c) + 6849*a^2*b - 672*a*b^2)/(e^(2*d*x + 2*c) + 1)^8)/d
```

3.72.9 Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 646, normalized size of antiderivative = 4.68

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$

$$= \frac{96\,(10\,b^3+a\,b^2)}{5\,d\,(5\,\mathrm{e}^{2\,c+2\,dx}+10\,\mathrm{e}^{4\,c+4\,dx}+10\,\mathrm{e}^{6\,c+6\,dx}+5\,\mathrm{e}^{8\,c+8\,dx}+\mathrm{e}^{10\,c+10\,dx}+1)}$$
$$- \frac{640\,b^3}{3\,d\,(6\,\mathrm{e}^{2\,c+2\,dx}+15\,\mathrm{e}^{4\,c+4\,dx}+20\,\mathrm{e}^{6\,c+6\,dx}+15\,\mathrm{e}^{8\,c+8\,dx}+6\,\mathrm{e}^{10\,c+10\,dx}+\mathrm{e}^{12\,c+12\,dx}+1)}$$
$$- \frac{4\,(25\,b^3+12\,a\,b^2)}{d\,(4\,\mathrm{e}^{2\,c+2\,dx}+6\,\mathrm{e}^{4\,c+4\,dx}+4\,\mathrm{e}^{6\,c+6\,dx}+\mathrm{e}^{8\,c+8\,dx}+1)}$$
$$+ \frac{128\,b^3}{d\,(7\,\mathrm{e}^{2\,c+2\,dx}+21\,\mathrm{e}^{4\,c+4\,dx}+35\,\mathrm{e}^{6\,c+6\,dx}+35\,\mathrm{e}^{8\,c+8\,dx}+21\,\mathrm{e}^{10\,c+10\,dx}+7\,\mathrm{e}^{12\,c+12\,dx}+\mathrm{e}^{14\,c+14\,dx}+1)}$$
$$- \frac{2\,(3\,a^2\,b+6\,a\,b^2+2\,b^3)}{d\,(2\,\mathrm{e}^{2\,c+2\,dx}+\mathrm{e}^{4\,c+4\,dx}+1)} - \frac{6\operatorname{atan}\left(\frac{a^2\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,dx}\,\sqrt{-d^2}}{d\,\sqrt{a^4\,b^2}}\right)\sqrt{a^4\,b^2}}{\sqrt{-d^2}}$$
$$- \frac{32\,b^3}{d\,(8\,\mathrm{e}^{2\,c+2\,dx}+28\,\mathrm{e}^{4\,c+4\,dx}+56\,\mathrm{e}^{6\,c+6\,dx}+70\,\mathrm{e}^{8\,c+8\,dx}+56\,\mathrm{e}^{10\,c+10\,dx}+28\,\mathrm{e}^{12\,c+12\,dx}+8\,\mathrm{e}^{14\,c+14\,dx}+\mathrm{e}^{16\,c}$$
$$- \frac{4\,a^3}{d\,(\mathrm{e}^{4\,c+4\,dx}-2\,\mathrm{e}^{2\,c+2\,dx}+1)} - \frac{8\,a^3}{3\,d\,(3\,\mathrm{e}^{2\,c+2\,dx}-3\,\mathrm{e}^{4\,c+4\,dx}+\mathrm{e}^{6\,c+6\,dx}-1)}$$
$$+ \frac{8\,(11\,b^3+15\,a\,b^2)}{3\,d\,(3\,\mathrm{e}^{2\,c+2\,dx}+3\,\mathrm{e}^{4\,c+4\,dx}+\mathrm{e}^{6\,c+6\,dx}+1)} + \frac{6\,a^2\,b}{d\,(\mathrm{e}^{2\,c+2\,dx}+1)}$$

input
```
int((a + b*tanh(c + d*x)^3)^3/sinh(c + d*x)^4,x)
```

output

```
(96*(a*b^2 + 10*b^3))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*
exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (640*b^
3)/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) +
15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (4
*(12*a*b^2 + 25*b^3))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(
6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (128*b^3)/(d*(7*exp(2*c + 2*d*x) +
 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(
10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) - (2*(6*a
*b^2 + 3*a^2*b + 2*b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) -
 (6*atan((a^2*b*exp(2*c)*exp(2*d*x)*(-d^2)^(1/2))/(d*(a^4*b^2)^(1/2)))*(a^
4*b^2)^(1/2))/(-d^2)^(1/2) - (32*b^3)/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c
+ 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*
x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1
)) - (4*a^3)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*a^3)/(3*
d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) + (8*(
15*a*b^2 + 11*b^3))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*
c + 6*d*x) + 1)) + (6*a^2*b)/(d*(exp(2*c + 2*d*x) + 1))
```

3.73 $\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

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3.73.1 Optimal result

Integrand size = 23, antiderivative size = 491

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = -\frac{a^{2/3}\sqrt[3]{b}\left(a^2+3a^{4/3}b^{2/3}-b^2\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\left(a^{4/3}+a^{2/3}b^{2/3}+b^{4/3}\right)^3 d}$$
$$-\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3 d}+\frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3 d}$$
$$-\frac{a^{2/3}\sqrt[3]{b}\left(a^4+7a^2b^2+b^4+3a^{2/3}b^{4/3}(2a^2+b^2)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3\left(a^2-b^2\right)^3 d}$$
$$+\frac{a^{2/3}\sqrt[3]{b}\left(a^4+7a^2b^2+b^4+3a^{2/3}b^{4/3}(2a^2+b^2)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6\left(a^2-b^2\right)^3 d}$$
$$-\frac{a^2b(a^2+2b^2)\log\left(a+b\tanh^3(c+dx)\right)}{\left(a^2-b^2\right)^3 d}$$
$$+\frac{1}{16(a+b)d(1-\tanh(c+dx))^2}-\frac{5a-b}{16(a+b)^2 d(1-\tanh(c+dx))}$$
$$-\frac{1}{16(a-b)d(1+\tanh(c+dx))^2}+\frac{5a+b}{16(a-b)^2 d(1+\tanh(c+dx))}$$

output
```
-3/16*a*(a-5*b)*ln(1-tanh(d*x+c))/(a+b)^3/d+3/16*a*(a+5*b)*ln(tanh(d*x+c)+
1)/(a-b)^3/d-1/3*a^(2/3)*b^(1/3)*(a^4+7*a^2*b^2+b^4+3*a^(2/3)*b^(4/3)*(2*a
^2+b^2))*ln(a^(1/3)+b^(1/3)*tanh(d*x+c))/(a^2-b^2)^3/d+1/6*a^(2/3)*b^(1/3)
*(a^4+7*a^2*b^2+b^4+3*a^(2/3)*b^(4/3)*(2*a^2+b^2))*ln(a^(2/3)-a^(1/3)*b^(1
/3)*tanh(d*x+c)+b^(2/3)*tanh(d*x+c)^2)/(a^2-b^2)^3/d-a^2*b*(a^2+2*b^2)*ln(
a+b*tanh(d*x+c)^3)/(a^2-b^2)^3/d-1/3*a^(2/3)*b^(1/3)*(a^2+3*a^(4/3)*b^(2/3
)-b^2)*arctan(1/3*(a^(1/3)-2*b^(1/3)*tanh(d*x+c))/a^(1/3)*3^(1/2))/(a^(4/3
)+a^(2/3)*b^(2/3)+b^(4/3))^3/d*3^(1/2)+1/16/(a+b)/d/(1-tanh(d*x+c))^2+1/16
*(-5*a+b)/(a+b)^2/d/(1-tanh(d*x+c))-1/16/(a-b)/d/(tanh(d*x+c)+1)^2+1/16*(5
*a+b)/(a-b)^2/d/(tanh(d*x+c)+1)
```

3.73.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.15 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.31

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$
$$= \frac{-32ab\text{RootSum}\left[a-b+3a\#1+3b\#1+3a\#1^2-3b\#1^2+a\#1^3+b\#1^3\&,\ \frac{-6a^3c-12ab^2c-6a^3dx-12ab^2dx+3a^3}{}\right.}{}$$

input
```
Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^3),x]
```

output
```
(-32*a*b*RootSum[a - b + 3*a*#1 + 3*b*#1 + 3*a*#1^2 - 3*b*#1^2 + a*#1^3 +
b*#1^3 & , (-6*a^3*c - 12*a*b^2*c - 6*a^3*d*x - 12*a*b^2*d*x + 3*a^3*Log[E
^(2*(c + d*x)) - #1] + 6*a*b^2*Log[E^(2*(c + d*x)) - #1] - 8*a^3*c*#1 + 4*
a^2*b*c*#1 + 8*a*b^2*c*#1 - 4*b^3*c*#1 - 8*a^3*d*x*#1 + 4*a^2*b*d*x*#1 + 8
*a*b^2*d*x*#1 - 4*b^3*d*x*#1 + 4*a^3*Log[E^(2*(c + d*x)) - #1]*#1 - 2*a^2*
b*Log[E^(2*(c + d*x)) - #1]*#1 - 4*a*b^2*Log[E^(2*(c + d*x)) - #1]*#1 + 2*
b^3*Log[E^(2*(c + d*x)) - #1]*#1 - 10*a^3*c*#1^2 + 20*a^2*b*c*#1^2 - 20*a*
b^2*c*#1^2 + 4*b^3*c*#1^2 - 10*a^3*d*x*#1^2 + 20*a^2*b*d*x*#1^2 - 20*a*b^2
*d*x*#1^2 + 4*b^3*d*x*#1^2 + 5*a^3*Log[E^(2*(c + d*x)) - #1]*#1^2 - 10*a^2
*b*Log[E^(2*(c + d*x)) - #1]*#1^2 + 10*a*b^2*Log[E^(2*(c + d*x)) - #1]*#1^
2 - 2*b^3*Log[E^(2*(c + d*x)) - #1]*#1^2)/(a - b + 2*a*#1 + 2*b*#1 + a*#1^
2 - b*#1^2) & ] + 3*(4*b*(5*a^3 + 5*a^2*b + a*b^2 + b^3)*Cosh[2*(c + d*x)]
 - (a - b)*b*(a + b)^2*Cosh[4*(c + d*x)] - 8*a*(a^3 + a^2*b + 2*a*b^2 + 2*
b^3)*Sinh[2*(c + d*x)] + a*(a - b)*(12*(a^2 - 6*a*b + 5*b^2)*(c + d*x) + (
a + b)^2*Sinh[4*(c + d*x)])))/(96*(a - b)^2*(a + b)^3*d)
```

3.73.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 465, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4146, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sin(ic+idx)^4}{a+ib\tan(ic+idx)^3}\,dx$$

$$\downarrow 4146$$

$$\frac{\int \frac{\tanh^4(c+dx)}{(1-\tanh^2(c+dx))^3(b\tanh^3(c+dx)+a)}\,d\tanh(c+dx)}{d}$$

$$\downarrow 7276$$

$$\frac{\int \left(-\frac{3a(a-5b)}{16(a+b)^3(\tanh(c+dx)-1)}+\frac{3a(a+5b)}{16(a-b)^3(\tanh(c+dx)+1)}+\frac{ab\left(-3ab\left(a^2+2b^2\right)\tanh^2(c+dx)+\left(a^4+7b^2a^2+b^4\right)\tanh(c+dx)-3ab\left(2a^2+b^2\right)\right.}{\left(a^2-b^2\right)^3(b\tanh^3(c+dx)+a)}\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{a^2b\left(a^2+2b^2\right)\log\left(a+b\tanh^3(c+dx)\right)}{\left(a^2-b^2\right)^3}-\frac{a^{2/3}\sqrt[3]{b}\left(3a^{4/3}b^{2/3}+a^2-b^2\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\left(a^{2/3}b^{2/3}+a^{4/3}+b^{4/3}\right)^3}+\frac{a^{2/3}\sqrt[3]{b}\left(a^4+7a^2b^2+3a^{2/3}b^{4/3}\left(2a^2\right.\right.}{}$$

input
```
Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^3),x]
```

output
```
(-((a^(2/3)*b^(1/3)*(a^2 + 3*a^(4/3)*b^(2/3) - b^2)*ArcTan[(a^(1/3) - 2*b^
(1/3)*Tanh[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*(a^(4/3) + a^(2/3)*b^(2/
3) + b^(4/3))^3)) - (3*a*(a - 5*b)*Log[1 - Tanh[c + d*x]])/(16*(a + b)^3)
+ (3*a*(a + 5*b)*Log[1 + Tanh[c + d*x]])/(16*(a - b)^3) - (a^(2/3)*b^(1/3)
*(a^4 + 7*a^2*b^2 + b^4 + 3*a^(2/3)*b^(4/3)*(2*a^2 + b^2))*Log[a^(1/3) + b
^(1/3)*Tanh[c + d*x]])/(3*(a^2 - b^2)^3) + (a^(2/3)*b^(1/3)*(a^4 + 7*a^2*b
^2 + b^4 + 3*a^(2/3)*b^(4/3)*(2*a^2 + b^2))*Log[a^(2/3) - a^(1/3)*b^(1/3)*
Tanh[c + d*x] + b^(2/3)*Tanh[c + d*x]^2])/(6*(a^2 - b^2)^3) - (a^2*b*(a^2
+ 2*b^2)*Log[a + b*Tanh[c + d*x]^3])/(a^2 - b^2)^3 + 1/(16*(a + b)*(1 - Ta
nh[c + d*x])^2) - (5*a - b)/(16*(a + b)^2*(1 - Tanh[c + d*x])) - 1/(16*(a
- b)*(1 + Tanh[c + d*x])^2) + (5*a + b)/(16*(a - b)^2*(1 + Tanh[c + d*x]))
)/d
```

3.73.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.73.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 18.73 (sec) , antiderivative size = 452, normalized size of antiderivative = 0.92

method	result
derivativedivides	$-\frac{8}{(32a-32b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{32}{(64a-64b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{-a-5b}{8(a-b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{3a+3b}{8(a-b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+$
default	$-\frac{8}{(32a-32b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{32}{(64a-64b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{-a-5b}{8(a-b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{3a+3b}{8(a-b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+$
risch	Expression too large to display

input
```
int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-8/(32*a-32*b)/(1+tanh(1/2*d*x+1/2*c))^4+32/(64*a-64*b)/(1+tanh(1/2*d
*x+1/2*c))^3-1/8*(-a-5*b)/(a-b)^2/(1+tanh(1/2*d*x+1/2*c))^2-1/8*(3*a+3*b)/
(a-b)^2/(1+tanh(1/2*d*x+1/2*c))+3/8*a*(a+5*b)/(a-b)^3*ln(1+tanh(1/2*d*x+1/
2*c))+8/(32*a+32*b)/(tanh(1/2*d*x+1/2*c)-1)^4+32/(64*a+64*b)/(tanh(1/2*d*x
+1/2*c)-1)^3-1/8*(a-5*b)/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2-1/8*(3*a-3*b)/(
a+b)^2/(tanh(1/2*d*x+1/2*c)-1)-3/8*a*(a-5*b)/(a+b)^3*ln(tanh(1/2*d*x+1/2*c
)-1)-1/3*a*b/(a-b)^3/(a+b)^3*sum((3*a^2*(a^2+2*b^2)*_R^5+3*a*b*(-2*a^2-b^2
)*_R^4+2*(4*a^4+13*a^2*b^2+b^4)*_R^3+12*a*b*(a^2+2*b^2)*_R^2+(a^4-8*a^2*b^
2-2*b^4)*_R+6*a^3*b+3*a*b^3)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d
*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a)))
```

3.73.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.24 (sec) , antiderivative size = 17123, normalized size of antiderivative = 34.87

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.73.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**3),x)
```

output
```
Timed out
```

3.73.7 Maxima [F]

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh\left(dx+c\right)^4}{b\tanh\left(dx+c\right)^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-6*a^4*b*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) +
 3*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x
+ 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6) - (d*x + c)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d)) - 12*a^2*b^
3*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) + 3*a +
3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c)
+ 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6) - (d*x + c)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d)) + 10*a^4*b*integra
te(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) +
3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^
3 + a*b^4 + b^5) - 20*a^3*b^2*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x
+ 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x
)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5) + 20*a^2*b^3*integra
te(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) +
3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^
3 + a*b^4 + b^5) - 4*a*b^4*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6
*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(
a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5) + 8*a^4*b*integrate(e^(
2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a +
 b)*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 +...
```

3.73.8 Giac [F]

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh\left(dx+c\right)^4}{b\tanh\left(dx+c\right)^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

3.73.9 Mupad [B] (verification not implemented)

Time = 5.25 (sec) , antiderivative size = 3313, normalized size of antiderivative = 6.75

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^3),x)
```

output
```
symsum(log(- root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3
 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*
z - a^2*b, z, k)*((96*(a^2*b^10*d + 20*a^3*b^9*d - 89*a^4*b^8*d + 270*a^5*
b^7*d - 417*a^6*b^6*d + 408*a^7*b^5*d - 190*a^8*b^4*d + 58*a^9*b^3*d - 7*a
^10*b^2*d - a^2*b^10*d*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3
+ 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2
 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) - 52*a^3*b^9*d*exp(2*root(81*
a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 1
62*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp
(2*d*x) + 59*a^4*b^8*d*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3
+ 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2
 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) - 218*a^5*b^7*d*exp(2*root(81
*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 -
162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*ex
p(2*d*x) + 241*a^6*b^6*d*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^
3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z
^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) + 220*a^7*b^5*d*exp(2*root(
81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3
- 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*
exp(2*d*x) - 298*a^8*b^4*d*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d...
```

3.74 $\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

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3.74.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = i\text{Int}\left(-\frac{i\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}, x\right)$$

output
```
I*Unintegrable(-I*sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x)
```

3.74.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 826 vs. 2(33) = 66.

Time = 1.06 (sec) , antiderivative size = 826, normalized size of antiderivative = 35.91

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$
$$= \frac{-9a(a^2+3b^2)\cosh(c+dx)+a^3\cosh(3(c+dx))-ab^2\cosh(3(c+dx))-2ab\text{RootSum}\left[a-b+3a\#1^2+\right.}{}$$

input
```
Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^3),x]
```

output

```
(-9*a*(a^2 + 3*b^2)*Cosh[c + d*x] + a^3*Cosh[3*(c + d*x)] - a*b^2*Cosh[3*(
c + d*x)] - 2*a*b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*b*#1^
4 + a*#1^6 + b*#1^6 & , (3*a^2*c + 3*a*b*c + 3*b^2*c + 3*a^2*d*x + 3*a*b*d
*x + 3*b^2*d*x + 6*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(
c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 6*a*b*Log[-Cosh[(c + d*x)/2] - Si
nh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 6*b^2*Log
[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c +
 d*x)/2]*#1] + 2*a^2*c*#1^2 - 2*b^2*c*#1^2 + 2*a^2*d*x*#1^2 - 2*b^2*d*x*#1
^2 + 4*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*
#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 4*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c +
 d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 3*a^2*c*#1^
4 - 3*a*b*c*#1^4 + 3*b^2*c*#1^4 + 3*a^2*d*x*#1^4 - 3*a*b*d*x*#1^4 + 3*b^2*
d*x*#1^4 + 6*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*
x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 6*a*b*Log[-Cosh[(c + d*x)/2] - Sin
h[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 6*b^2
*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[
(c + d*x)/2]*#1]*#1^4)/(a*#1 + b*#1 + 2*a*#1^3 - 2*b*#1^3 + a*#1^5 + b*#1^
5) & ] + 27*a^2*b*Sinh[c + d*x] + 9*b^3*Sinh[c + d*x] - a^2*b*Sinh[3*(c +
d*x)] + b^3*Sinh[3*(c + d*x)])/(12*(a - b)^2*(a + b)^2*d)
```

3.74.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 26, 4151}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{i\sin(ic+idx)^3}{a+ib\tan(ic+idx)^3}dx$$

$$\downarrow 26$$

$$i\int \frac{\sin(ic+idx)^3}{ib\tan(ic+idx)^3+a}dx$$

$$\downarrow 4151$$

$$i \int -\frac{i \sinh^3(c+dx)}{b \tanh^3(c+dx)+a} dx$$

input
```
Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^3),x]
```

output
```
$Aborted
```

3.74.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4151
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Sin[e + f*x])^m*(a +
 b*(c*Tan[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.74.4 Maple [N/A] (verified)

Time = 7.25 (sec) , antiderivative size = 289, normalized size of antiderivative = 12.57

method	result
derivativedivides	$-\frac{ab\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)} \frac{\left(\left(2a^2+b^2\right)\_R^4-6ab\_R^3+2\left(4a^2+5b^2\right)\_R^2-6ab\_R+2a^2+b^2\right)}{\_R^5 a+2\_R^3 a+4\_R^2 b+\_R a}\right)}{3(a+b)^2(a-b)^2}$
default	$-\frac{ab\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)} \frac{\left(\left(2a^2+b^2\right)\_R^4-6ab\_R^3+2\left(4a^2+5b^2\right)\_R^2-6ab\_R+2a^2+b^2\right)}{\_R^5 a+2\_R^3 a+4\_R^2 b+\_R a}\right)}{3(a+b)^2(a-b)^2}$
risch	Expression too large to display

input
```
int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*a*b/(a+b)^2/(a-b)^2*sum(((2*a^2+b^2)*_R^4-6*a*b*_R^3+2*(4*a^2+5*
b^2)*_R^2-6*a*b*_R+2*a^2+b^2)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*
d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-16/3/(tanh(
1/2*d*x+1/2*c)-1)^3/(16*a+16*b)-8/(16*a+16*b)/(tanh(1/2*d*x+1/2*c)-1)^2-1/
2/(a+b)^2*(-a+2*b)/(tanh(1/2*d*x+1/2*c)-1)-8/(16*a-16*b)/(1+tanh(1/2*d*x+1
/2*c))^2+16/3/(1+tanh(1/2*d*x+1/2*c))^3/(16*a-16*b)-1/2*(a+2*b)/(a-b)^2/(1
+tanh(1/2*d*x+1/2*c)))
```

3.74.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 4.17 (sec) , antiderivative size = 62017, normalized size of antiderivative = 2696.39

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.74.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**3/(a+b*tanh(d*x+c)**3),x)
```

output
```
Timed out
```

3.74.7 Maxima [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 533, normalized size of antiderivative = 23.17

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh\left(dx+c\right)^3}{b\tanh\left(dx+c\right)^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/24*(a^3 + a^2*b - a*b^2 - b^3 + (a^3*e^(6*c) - a^2*b*e^(6*c) - a*b^2*e^(
6*c) + b^3*e^(6*c))*e^(6*d*x) - 9*(a^3*e^(4*c) - 3*a^2*b*e^(4*c) + 3*a*b^2
*e^(4*c) - b^3*e^(4*c))*e^(4*d*x) - 9*(a^3*e^(2*c) + 3*a^2*b*e^(2*c) + 3*a
*b^2*e^(2*c) + b^3*e^(2*c))*e^(2*d*x))*e^(-3*d*x)/(a^4*d*e^(3*c) - 2*a^2*b
^2*d*e^(3*c) + b^4*d*e^(3*c)) - 1/8*integrate(16*(3*(a^3*b*e^(5*c) - a^2*b
^2*e^(5*c) + a*b^3*e^(5*c))*e^(5*d*x) + 2*(a^3*b*e^(3*c) - a*b^3*e^(3*c))*
e^(3*d*x) + 3*(a^3*b*e^c + a^2*b^2*e^c + a*b^3*e^c)*e^(d*x))/(a^5 - a^4*b
- 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5*e^(6*c) + a^4*b*e^(6*c) - 2*a
^3*b^2*e^(6*c) - 2*a^2*b^3*e^(6*c) + a*b^4*e^(6*c) + b^5*e^(6*c))*e^(6*d*x
) + 3*(a^5*e^(4*c) - a^4*b*e^(4*c) - 2*a^3*b^2*e^(4*c) + 2*a^2*b^3*e^(4*c)
 + a*b^4*e^(4*c) - b^5*e^(4*c))*e^(4*d*x) + 3*(a^5*e^(2*c) + a^4*b*e^(2*c)
 - 2*a^3*b^2*e^(2*c) - 2*a^2*b^3*e^(2*c) + a*b^4*e^(2*c) + b^5*e^(2*c))*e^
(2*d*x)), x)
```

3.74.8 Giac [N/A]

Not integrable

Time = 3.88 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh\left(dx+c\right)^3}{b\tanh\left(dx+c\right)^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Hanged}$$

input
```
int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^3),x)
```

output
```
\text{Hanged}
```

3.75 $\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

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3.75.1 Optimal result

Integrand size = 23, antiderivative size = 384

$$
\begin{aligned}
&\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx \\
&= \frac{a^{2/3}\sqrt[3]{b}\left(a^2-3a^{2/3}b^{4/3}+2b^2\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\left(a^2-b^2\right)^2 d} \\
&\quad+\frac{(a-2b)\log(1-\tanh(c+dx))}{4(a+b)^2 d}-\frac{(a+2b)\log(1+\tanh(c+dx))}{4(a-b)^2 d} \\
&\quad+\frac{a^{2/3}\sqrt[3]{b}\left(a^2+3a^{2/3}b^{4/3}+2b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} \\
&\quad-\frac{a^{2/3}\sqrt[3]{b}\left(a^2+3a^{2/3}b^{4/3}+2b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6\left(a^2-b^2\right)^2 d} \\
&\quad+\frac{b(2a^2+b^2)\log\left(a+b\tanh^3(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} \\
&\quad+\frac{1}{4(a+b)d(1-\tanh(c+dx))}-\frac{1}{4(a-b)d(1+\tanh(c+dx))}
\end{aligned}
$$

output
```
1/4*(a-2*b)*ln(1-tanh(d*x+c))/(a+b)^2/d-1/4*(a+2*b)*ln(tanh(d*x+c)+1)/(a-b
)^2/d+1/3*a^(2/3)*b^(1/3)*(a^2+3*a^(2/3)*b^(4/3)+2*b^2)*ln(a^(1/3)+b^(1/3)
*tanh(d*x+c))/(a^2-b^2)^2/d-1/6*a^(2/3)*b^(1/3)*(a^2+3*a^(2/3)*b^(4/3)+2*b
^2)*ln(a^(2/3)-a^(1/3)*b^(1/3)*tanh(d*x+c)+b^(2/3)*tanh(d*x+c)^2)/(a^2-b^2
)^2/d+1/3*b*(2*a^2+b^2)*ln(a+b*tanh(d*x+c)^3)/(a^2-b^2)^2/d+1/3*a^(2/3)*b^
(1/3)*(a^2-3*a^(2/3)*b^(4/3)+2*b^2)*arctan(1/3*(a^(1/3)-2*b^(1/3)*tanh(d*x
+c))/a^(1/3)*3^(1/2))/(a^2-b^2)^2/d*3^(1/2)+1/4/(a+b)/d/(1-tanh(d*x+c))-1/
4/(a-b)/d/(tanh(d*x+c)+1)
```

3.75.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.32 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.10

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx =$$

$$-\frac{6(a^2-3ab+2b^2)(c+dx)+3b(a+b)\cosh(2(c+dx))+4b\text{RootSum}\left[a-b+3a\#1+3b\#1+3a\#1^2\right.}{}$$

input
```
Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^3),x]
```

output
```
-1/12*(6*(a^2 - 3*a*b + 2*b^2)*(c + d*x) + 3*b*(a + b)*Cosh[2*(c + d*x)] +
 4*b*RootSum[a - b + 3*a*#1 + 3*b*#1 + 3*a*#1^2 - 3*b*#1^2 + a*#1^3 + b*#1
^3 & , (4*a^2*c + 2*b^2*c + 4*a^2*d*x + 2*b^2*d*x - 2*a^2*Log[E^(2*(c + d*
x)) - #1] - b^2*Log[E^(2*(c + d*x)) - #1] + 4*a^2*c*#1 - 4*b^2*c*#1 + 4*a^
2*d*x*#1 - 4*b^2*d*x*#1 - 2*a^2*Log[E^(2*(c + d*x)) - #1]*#1 + 2*b^2*Log[E
^(2*(c + d*x)) - #1]*#1 + 8*a^2*c*#1^2 - 8*a*b*c*#1^2 + 2*b^2*c*#1^2 + 8*a
^2*d*x*#1^2 - 8*a*b*d*x*#1^2 + 2*b^2*d*x*#1^2 - 4*a^2*Log[E^(2*(c + d*x))
 - #1]*#1^2 + 4*a*b*Log[E^(2*(c + d*x)) - #1]*#1^2 - b^2*Log[E^(2*(c + d*x)
) - #1]*#1^2)/(a - b + 2*a*#1 + 2*b*#1 + a*#1^2 - b*#1^2) & ] - 3*a*(a + b
)*Sinh[2*(c + d*x)])/((a - b)*(a + b)^2*d)
```

3.75.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 25, 4146, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\sin(ic+idx)^2}{a+ib\tan(ic+idx)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{\sin(ic+idx)^2}{ib\tan(ic+idx)^3+a}dx$$

$$\downarrow 4146$$

$$\frac{\int \frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^2\left(b\tanh^3(c+dx)+a\right)}d\tanh(c+dx)}{d}$$

$$\downarrow 7276$$

$$\frac{\int \left(\frac{-a-2b}{4(a-b)^2(\tanh(c+dx)+1)}+\frac{a-2b}{4(a+b)^2(\tanh(c+dx)-1)}+\frac{b\left(3ba^2-\left(a^2+2b^2\right)\tanh(c+dx)a+b\left(2a^2+b^2\right)\tanh^2(c+dx)\right)}{\left(a^2-b^2\right)^2\left(b\tanh^3(c+dx)+a\right)}+\frac{1}{4(a+b)(\tanh(c+dx)}\right.}{d}$$

$$\downarrow 2009$$

$$\frac{b\left(2a^2+b^2\right)\log\left(a+b\tanh^3(c+dx)\right)}{3\left(a^2-b^2\right)^2}+\frac{a^{2/3}\sqrt[3]{b}\left(-3a^{2/3}b^{4/3}+a^2+2b^2\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\left(a^2-b^2\right)^2}-\frac{a^{2/3}\sqrt[3]{b}\left(3a^{2/3}b^{4/3}+a^2+2b^2\right)\log\left(a^2\right.}{6}$$

input
```
Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^3),x]
```

output
```
((a^(2/3)*b^(1/3)*(a^2 - 3*a^(2/3)*b^(4/3) + 2*b^2)*ArcTan[(a^(1/3) - 2*b^
(1/3)*Tanh[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*(a^2 - b^2)^2) + ((a - 2
*b)*Log[1 - Tanh[c + d*x]])/(4*(a + b)^2) - ((a + 2*b)*Log[1 + Tanh[c + d*
x]])/(4*(a - b)^2) + (a^(2/3)*b^(1/3)*(a^2 + 3*a^(2/3)*b^(4/3) + 2*b^2)*Lo
g[a^(1/3) + b^(1/3)*Tanh[c + d*x]])/(3*(a^2 - b^2)^2) - (a^(2/3)*b^(1/3)*(
a^2 + 3*a^(2/3)*b^(4/3) + 2*b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tanh[c + d*
x] + b^(2/3)*Tanh[c + d*x]^2])/(6*(a^2 - b^2)^2) + (b*(2*a^2 + b^2)*Log[a
+ b*Tanh[c + d*x]^3])/(3*(a^2 - b^2)^2) + 1/(4*(a + b)*(1 - Tanh[c + d*x])
) - 1/(4*(a - b)*(1 + Tanh[c + d*x])))/d
```

3.75.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.75.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.07 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{4}{(8a+8b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{8}{(16a+16b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(a-2b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2(a+b)^2}-\frac{4}{(8a-8b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{}{(16a-}$
default	$\frac{4}{(8a+8b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{8}{(16a+16b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(a-2b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2(a+b)^2}-\frac{4}{(8a-8b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{}{(16a-}$
risch	$-\frac{ax}{2(a+b)^2}+\frac{xb}{(a+b)^2}+\frac{e^{2dx+2c}}{8d(a+b)}-\frac{e^{-2dx-2c}}{8(a-b)d}-\frac{4a^2bd^3x}{a^4d^3-2a^2b^2d^3+b^4d^3}-\frac{2b^3d^3x}{a^4d^3-2a^2b^2d^3+b^4d^3}-\frac{4a^2bc}{a^4d^3-2a^2b^2}$

input
```
int(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(4/(8*a+8*b)/(tanh(1/2*d*x+1/2*c)-1)^2+8/(16*a+16*b)/(tanh(1/2*d*x+1/2
*c)-1)+1/2*(a-2*b)/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)-1)-4/(8*a-8*b)/(1+tanh(1
/2*d*x+1/2*c))^2+8/(16*a-16*b)/(1+tanh(1/2*d*x+1/2*c))+1/2/(a-b)^2*(-a-2*b
)*ln(1+tanh(1/2*d*x+1/2*c))+1/3*b/(a-b)^2/(a+b)^2*sum((a*(2*a^2+b^2)*_R^5-
3*a^2*b*_R^4+6*a*(a^2+b^2)*_R^3+4*b*(2*a^2+b^2)*_R^2-3*a*_R*b^2+3*a^2*b)/(
_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a
+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a)))
```

3.75.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 10695, normalized size of antiderivative = 27.85

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.75.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**2/(a+b*tanh(d*x+c)**3),x)
```

output
```
Timed out
```

3.75.7 Maxima [F]

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh\left(dx+c\right)^2}{b\tanh\left(dx+c\right)^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
4*a^2*b*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) +
3*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x +
 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^4 - 2*a^2*b^2 + b^4) - (
d*x + c)/((a^4 - 2*a^2*b^2 + b^4)*d)) + 2*b^3*(integrate(((a + b)*e^(4*d*x
 + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) + 3*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*
e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) +
a - b), x)/(a^4 - 2*a^2*b^2 + b^4) - (d*x + c)/((a^4 - 2*a^2*b^2 + b^4)*d)
) - 8*a^2*b*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)
*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 + a^2*b - a
*b^2 - b^3) + 8*a*b^2*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) +
 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 +
 a^2*b - a*b^2 - b^3) - 2*b^3*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x
+ 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x
)/(a^3 + a^2*b - a*b^2 - b^3) - 4*a^2*b*integrate(e^(2*d*x + 2*c)/((a + b)
*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) +
 a - b), x)/(a^3 + a^2*b - a*b^2 - b^3) + 4*b^3*integrate(e^(2*d*x + 2*c)/
((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x
+ 2*c) + a - b), x)/(a^3 + a^2*b - a*b^2 - b^3) - 1/8*(4*(a^2*d*e^(2*c) -
3*a*b*d*e^(2*c) + 2*b^2*d*e^(2*c))*x*e^(2*d*x) + a^2 + 2*a*b + b^2 - (a^2*
e^(4*c) - b^2*e^(4*c))*e^(4*d*x))*e^(-2*d*x)/(a^3*d*e^(2*c) + a^2*b*d*e...
```

3.75.8 Giac [F]

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh(dx+c)^2}{b\tanh(dx+c)^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

3.75.9 Mupad [B] (verification not implemented)

Time = 3.33 (sec) , antiderivative size = 2100, normalized size of antiderivative = 5.47

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^3),x)
```

output
```
symsum(log(root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*
a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k)*((2304*root(54*a^2*b
^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d
^2*z^2 - 9*b^2*d*z + b, z, k)*(146*a^5*b^5*d^2 - 133*a^4*b^6*d^2 - 24*a^3*
b^7*d^2 - 12*a^6*b^4*d^2 + 22*a^7*b^3*d^2 + a^8*b^2*d^2 + 32*a^3*b^7*d^2*e
xp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*
d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 577*a^4*b^6*
d^2*exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a
^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 548*a^5
*b^5*d^2*exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 +
 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 70
*a^6*b^4*d^2*exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z
^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x)
+ 68*a^7*b^3*d^2*exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d
^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d
*x) + a^8*b^2*d^2*exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*
d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*
d*x)))/((a + b)^8*(a - b)^2*(a^2 - 2*a*b + b^2)) + (1536*(24*a^3*b^8*d + 1
05*a^4*b^7*d - 156*a^5*b^6*d + 51*a^6*b^5*d - 30*a^7*b^4*d + 6*a^8*b^3*d -
 32*a^3*b^8*d*exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d...
```

3.76 $\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

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3.76.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx = -i\text{Int}\left(\frac{i\sinh(c+dx)}{a+b\tanh^3(c+dx)},x\right)$$

output
```
-I*Unintegrable(I*sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x)
```

3.76.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 409 vs. 2(31) = 62.

Time = 0.72 (sec) , antiderivative size = 409, normalized size of antiderivative = 19.48

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$
$$= \frac{6a\cosh(c+dx)+b\text{RootSum}\left[a-b+3a\#1^2+3b\#1^2+3a\#1^4-3b\#1^4+a\#1^6+b\#1^6\&,\ \frac{2ac+bc+2adx+bd}{}\right.}{}$$

input
```
Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^3),x]
```

output
```
(6*a*Cosh[c + d*x] + b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*
b*#1^4 + a*#1^6 + b*#1^6 & , (2*a*c + b*c + 2*a*d*x + b*d*x + 4*a*Log[-Cos
h[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)
/2]*#1] + 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/
2]*#1 - Sinh[(c + d*x)/2]*#1] + 2*a*c*#1^4 - b*c*#1^4 + 2*a*d*x*#1^4 - b*d
*x*#1^4 + 4*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/
2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c
+ d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(a*#1 + b*#
1 + 2*a*#1^3 - 2*b*#1^3 + a*#1^5 + b*#1^5) & ] - 6*b*Sinh[c + d*x])/(6*(a
- b)*(a + b)*d)
```

3.76.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 26, 4151}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{i\sin(ic+idx)}{a+ib\tan(ic+idx)^3}dx$$
$$\downarrow 26$$
$$-i\int \frac{\sin(ic+idx)}{ib\tan(ic+idx)^3+a}dx$$
$$\downarrow 4151$$
$$-i\int \frac{i\sinh(c+dx)}{b\tanh^3(c+dx)+a}dx$$

input
```
Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^3),x]
```

output
```
$Aborted
```

3.76.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4151
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Sin[e + f*x])^m*(a +
 b*(c*Tan[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.76.4 Maple [N/A] (verified)

Time = 1.62 (sec) , antiderivative size = 159, normalized size of antiderivative = 7.57

method	result
derivativedivides	$-\frac{4}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4{}_{a-2}\_R^3{}_{b+6}\_R^2{}_{a-2}\_R_{b+a}\right.}{\_R^5{}_{a+2}\_R^3{}_{a+4}\_R}\right.}{3(a-b)(a+b)}}{d}$
default	$-\frac{4}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4{}_{a-2}\_R^3{}_{b+6}\_R^2{}_{a-2}\_R_{b+a}\right.}{\_R^5{}_{a+2}\_R^3{}_{a+4}\_R}\right.}{3(a-b)(a+b)}}{d}$
risch	Expression too large to display

input
```
int(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-4/(4*a+4*b)/(tanh(1/2*d*x+1/2*c)-1)+1/3*b/(a-b)/(a+b)*sum((_R^4*a-2*
_R^3*b+6*_R^2*a-2*_R*b+a)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+
1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))+4/(4*a-4*b)/(1+
tanh(1/2*d*x+1/2*c)))
```

3.76.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 1.74 (sec) , antiderivative size = 40923, normalized size of antiderivative = 1948.71

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.76.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**3),x)
```

output
```
Timed out
```

3.76.7 Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 250, normalized size of antiderivative = 11.90

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh(dx+c)}{b\tanh(dx+c)^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/2*((a*e^(2*c) - b*e^(2*c))*e^(2*d*x) + a + b)*e^(-d*x)/(a^2*d*e^c - b^2*
d*e^c) + 1/2*integrate(4*((2*a*b*e^(5*c) - b^2*e^(5*c))*e^(5*d*x) + (2*a*b
*e^c + b^2*e^c)*e^(d*x))/(a^3 - a^2*b - a*b^2 + b^3 + (a^3*e^(6*c) + a^2*b
*e^(6*c) - a*b^2*e^(6*c) - b^3*e^(6*c))*e^(6*d*x) + 3*(a^3*e^(4*c) - a^2*b
*e^(4*c) - a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 3*(a^3*e^(2*c) + a^2*b
*e^(2*c) - a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

3.76.8 Giac [N/A]

Not integrable

Time = 3.42 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh(dx+c)}{b\tanh(dx+c)^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

3.76.9 Mupad [B] (verification not implemented)

Time = 83.41 (sec) , antiderivative size = 4474, normalized size of antiderivative = 213.05

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sinh(c + d*x)/(a + b*tanh(c + d*x)^3),x)
```

output

```
exp(- c - d*x)/(2*(a*d - b*d)) + symsum(log((81920*a^2*b^5*exp(d*x)*exp(ro
ot(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729
*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2
*z^2 - b^2, z, k)) + 221184*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z
^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^
2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^2*b^8*d^3 - 3538944*ro
ot(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729
*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2
*z^2 - b^2, z, k)^3*a^3*b^7*d^3 + 1990656*root(2187*a^6*b^2*d^6*z^6 - 2187
*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^
4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^4*b^6*d^
3 + 3538944*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6
*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 +
81*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^5*b^5*d^3 - 2211840*root(2187*a^6*b^2*
d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1
458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k
)^3*a^6*b^4*d^3 + 7962624*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6
 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*
b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^3*b^9*d^5 + 15925248*roo
t(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 7...
```

3.77 $\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

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3.77.9 Mupad [B] (verification not implemented) 656

3.77.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx = i\operatorname{Int}\left(-\frac{i\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}, x\right)$$

output
```
I*Unintegrable(-I*csch(d*x+c)/(a+b*tanh(d*x+c)^3),x)
```

3.77.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 331 vs. $2(31) = 62$.

Time = 0.63 (sec) , antiderivative size = 331, normalized size of antiderivative = 15.76

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx =$$

$$-\frac{6\log\left(\cosh\left(\tfrac{1}{2}(c+dx)\right)\right) - 6\log\left(\sinh\left(\tfrac{1}{2}(c+dx)\right)\right) + b\operatorname{RootSum}\left[a-b+3a\#1^2+3b\#1^2+3a\#1^4-3\right.}{}$$

input
```
Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^3),x]
```

output
```
-1/6*(6*Log[Cosh[(c + d*x)/2]] - 6*Log[Sinh[(c + d*x)/2]] + b*RootSum[a -
b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*b*#1^4 + a*#1^6 + b*#1^6 & , (c + d
*x + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 -
 Sinh[(c + d*x)/2]*#1] - 2*c*#1^2 - 2*d*x*#1^2 - 4*Log[-Cosh[(c + d*x)/2]
- Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 +
c*#1^4 + d*x*#1^4 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c
 + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(a*#1 + b*#1 + 2*a*#1^3 - 2*b*
#1^3 + a*#1^5 + b*#1^5) & ])/(a*d)
```

3.77.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 26, 4151}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{i}{\sin(ic+idx)\,(a+ib\tan(ic+idx)^3)}\,dx$$
$$\downarrow 26$$
$$i\int \frac{1}{\sin(ic+idx)\,(ib\tan(ic+idx)^3+a)}\,dx$$
$$\downarrow 4151$$
$$i\int -\frac{i\operatorname{csch}(c+dx)}{b\tanh^3(c+dx)+a}\,dx$$

input
```
Int[Csch[c + d*x]/(a + b*Tanh[c + d*x]^3),x]
```

output
```
$Aborted
```

3.77.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4151
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Sin[e + f*x])^m*(a +
 b*(c*Tan[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.77.4 Maple [N/A] (verified)

Time = 0.63 (sec) , antiderivative size = 96, normalized size of antiderivative = 4.57

method	result
derivativedivides	$\frac{-\frac{4b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
default	$\frac{-\frac{4b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
risch	$-\frac{\ln\left(\mathrm{e}^{dx+c}+1\right)}{da}+2\left(\sum_{\_R=\mathrm{RootOf}\left(\left(46656a^8d^6-46656a^6b^2d^6\right)\_Z^6+3888a^4b^2d^4\_Z^4-108a^2b^2d^2\_Z^2+b^2\right)}\_R\ln$

input
```
int(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-4/3*b/a*sum(_R^2/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2
*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))+1/a*ln(tanh(1/2*d*
x+1/2*c)))
```

3.77.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 1.73 (sec) , antiderivative size = 20085, normalized size of antiderivative = 956.43

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.77.6 Sympy [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**3), x)
```

3.77.7 Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 160, normalized size of antiderivative = 7.62

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)}{b\tanh(dx+c)^3+a}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-log((e^(d*x + c) + 1)*e^(-c))/(a*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d)
 - 2*integrate((b*e^(5*d*x + 5*c) - 2*b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(
a^2 - a*b + (a^2*e^(6*c) + a*b*e^(6*c))*e^(6*d*x) + 3*(a^2*e^(4*c) - a*b*e
^(4*c))*e^(4*d*x) + 3*(a^2*e^(2*c) + a*b*e^(2*c))*e^(2*d*x)), x)
```

3.77.8 Giac [N/A]

Not integrable

Time = 2.97 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)}{b\tanh(dx+c)^3+a}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

3.77.9 Mupad [B] (verification not implemented)

Time = 15.27 (sec) , antiderivative size = 3679, normalized size of antiderivative = 175.19

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^3)),x)
```

output

```
symsum(log(-(1409286144*b^6*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^
8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) + 13421
7728*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27
*a^2*b^2*d^2*z^2 - b^2, z, k)*b^7*d + 1879048192*root(729*a^6*b^2*d^6*z^6
- 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)*
a*b^6*d - 2818572288*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*
b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^2*b^7*d^3 - 40869298176*
root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*
b^2*d^2*z^2 - b^2, z, k)^3*a^3*b^6*d^3 + 28185722880*root(729*a^6*b^2*d^6*
z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z,
 k)^3*a^4*b^5*d^3 + 15502147584*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6
 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^5*b^4*d^3 + 1
8119393280*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^
4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^4*b^7*d^5 + 235552112640*root(729*
a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z
^2 - b^2, z, k)^5*a^5*b^6*d^5 + 14495514624*root(729*a^6*b^2*d^6*z^6 - 729
*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^6
*b^5*d^5 - 219244658688*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a
^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^7*b^4*d^5 - 489223618
56*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 2...
```

3.78 $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

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3.78.1 Optimal result

Integrand size = 23, antiderivative size = 157

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}d} - \frac{\coth(c+dx)}{ad} + \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d} - \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6a^{4/3}d}$$

output

```
-coth(d*x+c)/a/d+1/3*b^(1/3)*ln(a^(1/3)+b^(1/3)*tanh(d*x+c))/a^(4/3)/d-1/6
*b^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*tanh(d*x+c)+b^(2/3)*tanh(d*x+c)^2)/a^(
4/3)/d+1/3*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*tanh(d*x+c))/a^(1/3)*3^(1
/2))/a^(4/3)/d*3^(1/2)
```

3.78.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.66 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.21

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx =$$

$$-\frac{3\coth(c+dx)+2b\text{RootSum}\left[a-b+3a\#1+3b\#1+3a\#1^2-3b\#1^2+a\#1^3+b\#1^3\&,\ \frac{-c-dx-\log(-c}{}\right.}{}$$

input
```
Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^3),x]
```

output
```
-1/3*(3*Coth[c + d*x] + 2*b*RootSum[a - b + 3*a*#1 + 3*b*#1 + 3*a*#1^2 - 3
*b*#1^2 + a*#1^3 + b*#1^3 & , (-c - d*x - Log[-Cosh[c + d*x] - Sinh[c + d*
x] + Cosh[c + d*x]*#1 - Sinh[c + d*x]*#1] + c*#1 + d*x*#1 + Log[-Cosh[c +
d*x] - Sinh[c + d*x] + Cosh[c + d*x]*#1 - Sinh[c + d*x]*#1]*#1)/(a + b + 2
*a*#1 - 2*b*#1 + a*#1^2 + b*#1^2) & ])/(a*d)
```

3.78.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 25, 4146, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\sin(ic+idx)^2\,(a+ib\tan(ic+idx)^3)}\,dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\sin(ic+idx)^2\,(ib\tan(ic+idx)^3+a)}\,dx$$

$$\downarrow 4146$$

$$\frac{\int \frac{\coth^2(c+dx)}{b\tanh^3(c+dx)+a}\,d\tanh(c+dx)}{d}$$

$$\downarrow 847$$

$$\frac{-\frac{b\int \frac{\tanh(c+dx)}{b\tanh^3(c+dx)+a}\,d\tanh(c+dx)}{a}-\frac{\coth(c+dx)}{a}}{d}$$

$$\downarrow 821$$

$$\frac{-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}\tanh(c+dx)+\sqrt[3]{a}}{b^{2/3}\tanh^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+a^{2/3}}\,d\tanh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}\tanh(c+dx)+\sqrt[3]{a}}\,d\tanh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{\coth(c+dx)}{a}}{d}$$

$$\downarrow 16$$

$$\frac{-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}\tanh(c+dx)+\sqrt[3]{a}}{b^{2/3}\tanh^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+a^{2/3}}\,d\tanh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{\coth(c+dx)}{a}}{d}$$

$$\downarrow 1142$$

$$\frac{-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}\tanh^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+a^{2/3}}\,d\tanh(c+dx)+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)\right)}{b^{2/3}\tanh^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+a^{2/3}}\,d\tanh(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{}\right.}{3\sqrt[3]{}}\right.}{a}}{d}$$

$$\downarrow 25$$

$$\frac{-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}\tanh^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+a^{2/3}}\,d\tanh(c+dx)-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)\right)}{b^{2/3}\tanh^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+a^{2/3}}\,d\tanh(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{}\right.}{3\sqrt[3]{a}}\right.}{a}}{d}$$

$$\downarrow 27$$

3.78. $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

$$-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}\tanh^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+a^{2/3}}d\tanh(c+dx)-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{b^{2/3}\tanh^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+a^{2/3}}d\tanh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{}\right.}{3\sqrt[3]{}}\right)}{a}}{d}$$

$$\downarrow 1082$$

$$-\frac{b\left(\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{b^{2/3}\tanh^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+a^{2/3}}d\tanh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c\right.}{3\sqrt[3]{a}b^{2/3}}\right)}{a}}{d}$$

$$\downarrow 217$$

$$-\frac{b\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{b^{2/3}\tanh^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+a^{2/3}}d\tanh(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{\coth(c+d}{a}}{d}$$

$$\downarrow 1103$$

$$-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{\coth(c+dx)}{a}}{d}$$

3.78. $\int\frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}dx$

input
```
Int[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^3),x]
```

output
```
(-(Coth[c + d*x]/a) - (b*(-1/3*Log[a^(1/3) + b^(1/3)*Tanh[c + d*x]]/(a^(1/
3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*Tanh[c + d*x])/a^(1/3))/S
qrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*Tanh[c + d*x] + b^(2/3)*
Tanh[c + d*x]^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/a)/d
```

3.78.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.78.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.67 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{2b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^3-\_R\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{5}a+2\_R^{3}a+4\_R^{2}b+\_Ra}\right)}{3a}-\frac{}{2a\tanh}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{2b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^3-\_R\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{5}a+2\_R^{3}a+4\_R^{2}b+\_Ra}\right)}{3a}-\frac{}{2a\tanh}}{d}$
risch	$-\frac{2}{da\left(\mathrm{e}^{2dx+2c}-1\right)}+4\left(\sum_{\_R=\mathrm{RootOf}\left(1728a^4d^3\_Z^3-b\right)}\_R\ln\left(\mathrm{e}^{2dx+2c}+\frac{288a^3d^2\_R^2}{(a+b)\left(\frac{b}{a+b}+\frac{a}{a+b}\right)}-\frac{24a^2d\_}{(a+b)\left(\frac{b}{a+b}\right.}\right.\right.$

input
```
int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/a*tanh(1/2*d*x+1/2*c)+2/3*b/a*sum((_R^3-_R)/(_R^5*a+2*_R^3*a+4*_
R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+
3*_Z^2*a+a))-1/2/a/tanh(1/2*d*x+1/2*c))
```

3.78.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(123) = 246$.

Time $= 0.28$ (sec) , antiderivative size $= 640$, normalized size of antiderivative $= 4.08$

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx =$$

$$-\frac{2\left(\sqrt{3}\cosh\left(dx+c\right)^2+2\sqrt{3}\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sqrt{3}\sinh\left(dx+c\right)^2-\sqrt{3}\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b}{}\right.}{}$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
-1/6*(2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c) +
 sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(b/a)^(1/3)*arctan(-1/3*(sqrt(3)*b*cos
h(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh(d*
x + c)^2 - (sqrt(3)*a*cosh(d*x + c)^2 + 2*sqrt(3)*a*cosh(d*x + c)*sinh(d*x
 + c) + sqrt(3)*a*sinh(d*x + c)^2 + sqrt(3)*a)*(b/a)^(2/3) - (sqrt(3)*b*co
sh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh(d
*x + c)^2 - sqrt(3)*b)*(b/a)^(1/3))/b) + (cosh(d*x + c)^2 + 2*cosh(d*x + c
)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b/a)^(1/3)*log((a + b)*cosh(d*x +
c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 +
 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*
x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x +
c) - 2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x +
 c)^2 - a)*(b/a)^(2/3) + 2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x
 + c) + a*sinh(d*x + c)^2 + a)*(b/a)^(1/3) + a + b) - 2*(cosh(d*x + c)^2 +
 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b/a)^(1/3)*log((a +
 b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh
(d*x + c)^2 + 2*a*(b/a)^(2/3) - 2*a*(b/a)^(1/3) + a - b) + 12)/(a*d*cosh(d
*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2 - a*d)
```

3.78.6 Sympy [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**3), x)
```

3.78.7 Maxima [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}^2(dx+c)}{b\tanh(dx+c)^3+a}\,dx$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-2/(a*d*e^(2*d*x + 2*c) - a*d) - 4*integrate((b*e^(4*d*x + 4*c) - b*e^(2*d
*x + 2*c))/(a^2 - a*b + (a^2*e^(6*c) + a*b*e^(6*c))*e^(6*d*x) + 3*(a^2*e^(
4*c) - a*b*e^(4*c))*e^(4*d*x) + 3*(a^2*e^(2*c) + a*b*e^(2*c))*e^(2*d*x)),
x)
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.13

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = -\frac{2}{ad(e^{(2\,dx+2\,c)}-1)}$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
-2/(a*d*(e^(2*d*x + 2*c) - 1))
```

3.78.9 Mupad [B] (verification not implemented)

Time = 8.73 (sec) , antiderivative size = 669, normalized size of antiderivative = 4.26

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$
$$= \frac{b^{1/3}\ln\left(a^{1/3}-b^{1/3}+a^{1/3}\,\mathrm{e}^{2c+2dx}+b^{1/3}\,\mathrm{e}^{2c+2dx}\right)}{3\,a^{4/3}\,d} - \frac{2}{a\,d\,\left(\mathrm{e}^{2c+2dx}-1\right)}$$
$$+ \frac{b^{1/3}\ln\left(\frac{256\,b^3\left(19\,a^2\,b-24\,a\,b^2+6\,a^3-b^3+8\,a^3\,\mathrm{e}^{2c+2dx}+b^3\,\mathrm{e}^{2c+2dx}+70\,a\,b^2\,\mathrm{e}^{2c+2dx}+113\,a^2\,b\,\mathrm{e}^{2c+2dx}\right)}{a^4\,(a+b)^6} + \frac{b^{1/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1536\,b^3}{a^4\,(a+b)^6}\right.}{}\right.}{}$$
$$- \frac{b^{1/3}\ln\left(\frac{256\,b^3\left(19\,a^2\,b-24\,a\,b^2+6\,a^3-b^3+8\,a^3\,\mathrm{e}^{2c+2dx}+b^3\,\mathrm{e}^{2c+2dx}+70\,a\,b^2\,\mathrm{e}^{2c+2dx}+113\,a^2\,b\,\mathrm{e}^{2c+2dx}\right)}{a^4\,(a+b)^6} - \frac{b^{1/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1536\,b^3}{}\right.}{}\right.}{}$$

input
```
int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^3)),x)
```

output

```
(b^(1/3)*log(a^(1/3) - b^(1/3) + a^(1/3)*exp(2*c + 2*d*x) + b^(1/3)*exp(2*
c + 2*d*x)))/(3*a^(4/3)*d) - 2/(a*d*(exp(2*c + 2*d*x) - 1)) + (b^(1/3)*log
((256*b^3*(19*a^2*b - 24*a*b^2 + 6*a^3 - b^3 + 8*a^3*exp(2*c + 2*d*x) + b^
3*exp(2*c + 2*d*x) + 70*a*b^2*exp(2*c + 2*d*x) + 113*a^2*b*exp(2*c + 2*d*x
)))/(a^4*(a + b)^6) + (b^(1/3)*((3^(1/2)*1i)/2 - 1/2)*((1536*b^3*d*(8*a^2
- 8*b^2 + 15*a^2*exp(2*c + 2*d*x) + 15*b^2*exp(2*c + 2*d*x) + 66*a*b*exp(2
*c + 2*d*x)))/(a^2*(a + b)^6) + (768*b^(7/3)*d*((3^(1/2)*1i)/2 - 1/2)*(24*
a^2*b - 19*a*b^2 + a^3 - 6*b^3 + a^3*exp(2*c + 2*d*x) + 8*b^3*exp(2*c + 2*
d*x) + 113*a*b^2*exp(2*c + 2*d*x) + 70*a^2*b*exp(2*c + 2*d*x)))/(a^(7/3)*(
a + b)^6)))/(3*a^(4/3)*d))*((3^(1/2)*1i)/2 - 1/2))/(3*a^(4/3)*d) - (b^(1/3
)*log((256*b^3*(19*a^2*b - 24*a*b^2 + 6*a^3 - b^3 + 8*a^3*exp(2*c + 2*d*x)
 + b^3*exp(2*c + 2*d*x) + 70*a*b^2*exp(2*c + 2*d*x) + 113*a^2*b*exp(2*c +
2*d*x)))/(a^4*(a + b)^6) - (b^(1/3)*((3^(1/2)*1i)/2 + 1/2)*((1536*b^3*d*(8
*a^2 - 8*b^2 + 15*a^2*exp(2*c + 2*d*x) + 15*b^2*exp(2*c + 2*d*x) + 66*a*b*
exp(2*c + 2*d*x)))/(a^2*(a + b)^6) - (768*b^(7/3)*d*((3^(1/2)*1i)/2 + 1/2)
*(24*a^2*b - 19*a*b^2 + a^3 - 6*b^3 + a^3*exp(2*c + 2*d*x) + 8*b^3*exp(2*c
 + 2*d*x) + 113*a*b^2*exp(2*c + 2*d*x) + 70*a^2*b*exp(2*c + 2*d*x)))/(a^(7
/3)*(a + b)^6)))/(3*a^(4/3)*d))*((3^(1/2)*1i)/2 + 1/2))/(3*a^(4/3)*d)
```

3.79 $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

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3.79.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = -i\operatorname{Int}\left(\frac{i\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)},x\right)$$

output
```
-I*Unintegrable(I*csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x)
```

3.79.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 214 vs. 2(33) = 66.

Time = 0.92 (sec) , antiderivative size = 214, normalized size of antiderivative = 9.30

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx =$$
$$-\frac{16b\operatorname{RootSum}\left[a-b+3a\#1^2+3b\#1^2+3a\#1^4-3b\#1^4+a\#1^6+b\#1^6\&,\ \frac{c\#1+dx\#1+2\log\left(-\cosh\left(\frac{1}{2}(c+d\right.\right.}{a+b}$$

input
```
Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^3),x]
```

output
```
-1/24*(16*b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*b*#1^4 + a*
#1^6 + b*#1^6 & , (c*#1 + d*x*#1 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*
x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1)/(a + b + 2*a*#1^2
 - 2*b*#1^2 + a*#1^4 + b*#1^4) & ] + 3*(Csch[(c + d*x)/2]^2 - 4*Log[Cosh[(
c + d*x)/2]] + 4*Log[Sinh[(c + d*x)/2]] + Sech[(c + d*x)/2]^2))/(a*d)
```

3.79.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 26, 4151}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{i}{\sin(ic+idx)^3\,(a+ib\tan(ic+idx)^3)}dx$$
$$\downarrow 26$$
$$-i\int \frac{1}{\sin(ic+idx)^3\,(ib\tan(ic+idx)^3+a)}dx$$
$$\downarrow 4151$$
$$-i\int \frac{i\operatorname{csch}^3(c+dx)}{b\tanh^3(c+dx)+a}dx$$

input
```
Int[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^3),x]
```

output
```
$Aborted
```

3.79.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4151
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Unintegrable[(d*Sin[e + f*x])^m*(a +
 b*(c*Tan[e + f*x])^n)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x]
```

3.79.4 Maple [N/A] (verified)

Time = 1.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 5.91

method	result
derivativedivides	$\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a}-\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4-2\_R^2+1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_R a}\right)}{3a}-\frac{}{8a\,t}}{d}$
default	$\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a}-\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4-2\_R^2+1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_R a}\right)}{3a}-\frac{}{8a\,t}}{d}$
risch	$-\frac{e^{dx+c}\left(e^{2dx+2c}+1\right)}{da\left(e^{2dx+2c}-1\right)^2}+8\left(\sum_{\_R=\mathrm{RootOf}\left(191102976d^6\_Z^6a^{10}+1728a^4b^2d^2\_Z^2+a^2b^2-b^4\right)}\_R\ln\left(e^{dx+c}+\frac{15}{a}\right.\right.$

input
```
int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a-1/3*b/a*sum((_R^4-2*_R^2+1)/(_R^5*a+2*_R^
3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*
_Z^3*b+3*_Z^2*a+a))-1/8/a/tanh(1/2*d*x+1/2*c)^2-1/2/a*ln(tanh(1/2*d*x+1/2*
c)))
```

3.79.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 2.94 (sec) , antiderivative size = 6846, normalized size of antiderivative = 297.65

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

3.79.6 Sympy [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**3), x)
```

3.79.7 Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 188, normalized size of antiderivative = 8.17

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}^3(dx+c)}{b\tanh^3(dx+c)+a}\,dx$$

input
```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-8*b*integrate(e^(3*d*x + 3*c)/(a^2 - a*b + (a^2*e^(6*c) + a*b*e^(6*c))*e^
(6*d*x) + 3*(a^2*e^(4*c) - a*b*e^(4*c))*e^(4*d*x) + 3*(a^2*e^(2*c) + a*b*e
^(2*c))*e^(2*d*x)), x) - (e^(3*d*x + 3*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4
*c) - 2*a*d*e^(2*d*x + 2*c) + a*d) + 1/2*log((e^(d*x + c) + 1)*e^(-c))/(a*
d) - 1/2*log((e^(d*x + c) - 1)*e^(-c))/(a*d)
```

3.79.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

3.79.9 Mupad [B] (verification not implemented)

Time = 27.78 (sec) , antiderivative size = 3643, normalized size of antiderivative = 158.39

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^3)),x)
```

output

```
exp(c + d*x)/(a*d - a*d*exp(2*c + 2*d*x)) - (2*exp(c + d*x))/(a*d - 2*a*d*
exp(2*c + 2*d*x) + a*d*exp(4*c + 4*d*x)) + symsum(log((570425344*a^4*b^6*e
xp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z,
 k)) - 33554432*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4
, z, k)*a*b^10*d - 553648128*a^2*b^8*exp(d*x)*exp(root(729*a^10*d^6*z^6 +
27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) - 167772160*a^3*b^7*exp(d*x)*ex
p(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) - 167
77216*b^10*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b
^2 - b^4, z, k)) + 192937984*a^5*b^5*exp(d*x)*exp(root(729*a^10*d^6*z^6 +
27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) + 2617245696*root(729*a^10*d^6*
z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^3*a^5*b^8*d^3 - 150994944*
root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^3*a^6*b^
7*d^3 - 1384120320*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 -
b^4, z, k)^3*a^7*b^6*d^3 + 2415919104*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d
^2*z^2 + a^2*b^2 - b^4, z, k)^3*a^8*b^5*d^3 - 3498049536*root(729*a^10*d^6
*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^3*a^9*b^4*d^3 + 543581798
4*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^5*a^8*
b^7*d^5 + 679477248*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 -
 b^4, z, k)^5*a^9*b^6*d^5 - 70665633792*root(729*a^10*d^6*z^6 + 27*a^4*b^2
*d^2*z^2 + a^2*b^2 - b^4, z, k)^5*a^10*b^5*d^5 + 52319748096*root(729*a...
```

3.80 $\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

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3.80.1 Optimal result

Integrand size = 23, antiderivative size = 215

$$\begin{aligned}\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = &-\frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}d}+\frac{\coth(c+dx)}{ad}\\ &-\frac{\coth^3(c+dx)}{3ad}-\frac{b\log(\tanh(c+dx))}{a^2d}\\ &-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d}\\ &+\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6a^{4/3}d}\\ &+\frac{b\log\left(a+b\tanh^3(c+dx)\right)}{3a^2d}\end{aligned}$$

output

```
coth(d*x+c)/a/d-1/3*coth(d*x+c)^3/a/d-b*ln(tanh(d*x+c))/a^2/d-1/3*b^(1/3)*
ln(a^(1/3)+b^(1/3)*tanh(d*x+c))/a^(4/3)/d+1/6*b^(1/3)*ln(a^(2/3)-a^(1/3)*b
^(1/3)*tanh(d*x+c)+b^(2/3)*tanh(d*x+c)^2)/a^(4/3)/d+1/3*b*ln(a+b*tanh(d*x+
c)^3)/a^2/d-1/3*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*tanh(d*x+c))/a^(1/3)
*3^(1/2))/a^(4/3)/d*3^(1/2)
```

3.80.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.12 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.50

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

$$= \frac{-a\coth(c+dx)\left(-2+\operatorname{csch}^2(c+dx)\right)+3b(c+dx-\log(\sinh(c+dx)))+b\text{RootSum}\left[a-b+3a\#1+3\right.}{}$$

input
```
Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^3),x]
```

output
```
(-(a*Coth[c + d*x]*(-2 + Csch[c + d*x]^2)) + 3*b*(c + d*x - Log[Sinh[c + d
*x]]) + b*RootSum[a - b + 3*a*#1 + 3*b*#1 + 3*a*#1^2 - 3*b*#1^2 + a*#1^3 +
 b*#1^3 & , (-2*a*c + 2*b*c - 2*a*d*x + 2*b*d*x + a*Log[E^(2*(c + d*x)) -
#1] - b*Log[E^(2*(c + d*x)) - #1] - 8*a*c*#1 - 4*b*c*#1 - 8*a*d*x*#1 - 4*b
*d*x*#1 + 4*a*Log[E^(2*(c + d*x)) - #1]*#1 + 2*b*Log[E^(2*(c + d*x)) - #1]
*#1 + 2*a*c*#1^2 + 2*b*c*#1^2 + 2*a*d*x*#1^2 + 2*b*d*x*#1^2 - a*Log[E^(2*(
c + d*x)) - #1]*#1^2 - b*Log[E^(2*(c + d*x)) - #1]*#1^2)/(a - b + 2*a*#1 +
 2*b*#1 + a*#1^2 - b*#1^2) & ])/(3*a^2*d)
```

3.80.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4146, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sin(ic+idx)^4\left(a+ib\tan(ic+idx)^3\right)}dx$$

$$\downarrow 4146$$

$$\frac{\int \frac{\coth^4(c+dx)(1-\tanh^2(c+dx))}{b\tanh^3(c+dx)+a}\,d\tanh(c+dx)}{d}$$

$$\downarrow 2373$$

$$\frac{\int \left(\frac{\coth^4(c+dx)}{a}-\frac{\coth^2(c+dx)}{a}-\frac{b\coth(c+dx)}{a^2}+\frac{b\tanh(c+dx)(a+b\tanh(c+dx))}{a^2(b\tanh^3(c+dx)+a)}\right)d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{-\frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}}+\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6a^{4/3}}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}}+}{d}$$

input
```
Int[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^3),x]
```

output
```
(-((b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Tanh[c + d*x])/(Sqrt[3]*a^(1/3))])
/(Sqrt[3]*a^(4/3))) + Coth[c + d*x]/a - Coth[c + d*x]^3/(3*a) - (b*Log[Tan
h[c + d*x]])/a^2 - (b^(1/3)*Log[a^(1/3) + b^(1/3)*Tanh[c + d*x]])/(3*a^(4/
3)) + (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tanh[c + d*x] + b^(2/3)*Tanh[
c + d*x]^2])/(6*a^(4/3)) + (b*Log[a + b*Tanh[c + d*x]^3])/(3*a^2))/d
```

3.80.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.80.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.60 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^{5}{}_{a+4}\_R^{2}{}_{b+3}\_R_a\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{5}{}_{a+2}\_R^{3}{}_{a+4}\_R^{2}{}_{b+}\_R_a}\right)}{3a^2}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}}{d}$
default	$\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^{5}{}_{a+4}\_R^{2}{}_{b+3}\_R_a\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{5}{}_{a+2}\_R^{3}{}_{a+4}\_R^{2}{}_{b+}\_R_a}\right)}{3a^2}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}}{d}$
risch	$-\frac{4\left(3\,\mathrm{e}^{2dx+2c}-1\right)}{3da\left(\mathrm{e}^{2dx+2c}-1\right)^3}+16\left(\sum_{\_R=\mathrm{RootOf}\left(110592a^6d^3\_Z^3-6912a^4b\,d^2\_Z^2+144a^2b^2d\_Z+a^2b-b^3\right)}\_R\ln\left(\mathrm{e}^{2dx}\right.\right.$

input

```
int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output

```
1/d*(1/3/a^2*b*sum((_R^5*a+4*_R^2*b+3*_R*a)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a
)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a
))-1/8/a*(1/3*tanh(1/2*d*x+1/2*c)^3-3*tanh(1/2*d*x+1/2*c))-1/24/a/tanh(1/2
*d*x+1/2*c)^3+3/8/a/tanh(1/2*d*x+1/2*c)-1/a^2*b*ln(tanh(1/2*d*x+1/2*c)))
```

3.80.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.25 (sec) , antiderivative size = 3726, normalized size of antiderivative = 17.33

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
-1/12*(48*a*cosh(d*x + c)^2 + 2*(a^2*d*cosh(d*x + c)^6 + 6*a^2*d*cosh(d*x
+ c)*sinh(d*x + c)^5 + a^2*d*sinh(d*x + c)^6 - 3*a^2*d*cosh(d*x + c)^4 + 3
*a^2*d*cosh(d*x + c)^2 + 3*(5*a^2*d*cosh(d*x + c)^2 - a^2*d)*sinh(d*x + c)
^4 + 4*(5*a^2*d*cosh(d*x + c)^3 - 3*a^2*d*cosh(d*x + c))*sinh(d*x + c)^3 -
 a^2*d + 3*(5*a^2*d*cosh(d*x + c)^4 - 6*a^2*d*cosh(d*x + c)^2 + a^2*d)*sin
h(d*x + c)^2 + 6*(a^2*d*cosh(d*x + c)^5 - 2*a^2*d*cosh(d*x + c)^3 + a^2*d*
cosh(d*x + c))*sinh(d*x + c))*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) -
b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))*log(1/2*((1/
2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6
*d^3))^(1/3) - 2*b/(a^2*d))^2*a^4*d^2 - (a^3 - 2*a^2*b)*((1/2)^(1/3)*(I*sq
rt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) -
 2*b/(a^2*d))*d + (a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c
)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - 3*a*b + 2*b^2) + 96*
a*cosh(d*x + c)*sinh(d*x + c) + 48*a*sinh(d*x + c)^2 - (6*b*cosh(d*x + c)^
6 + 36*b*cosh(d*x + c)*sinh(d*x + c)^5 + 6*b*sinh(d*x + c)^6 - 18*b*cosh(d
*x + c)^4 + 18*(5*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^4 + 24*(5*b*cosh(d*
x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 18*b*cosh(d*x + c)^2 + 18*
(5*b*cosh(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + (a^2*d*c
osh(d*x + c)^6 + 6*a^2*d*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*d*sinh(d*x +
c)^6 - 3*a^2*d*cosh(d*x + c)^4 + 3*a^2*d*cosh(d*x + c)^2 + 3*(5*a^2*d*c...
```

3.80.6 Sympy [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**3), x)
```

3.80.7 Maxima [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)^4}{b\tanh\left(dx+c\right)^3+a}\,dx$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
2*a*b*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) + 3*
a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4
*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 - a^2*b) - (d*x + c)/((a
^3 - a^2*b)*d)) - 2*b^2*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^
(2*d*x + 2*c) + 3*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a
 - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 - a^2*
b) - (d*x + c)/((a^3 - a^2*b)*d)) + 2*b*integrate(e^(4*d*x + 4*c)/((a + b)
*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) +
 a - b), x)/a + 2*b^2*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) +
 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/a^2 -
8*b*integrate(e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*
x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/a - 4*b^2*integrate(e^(2
*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a +
b)*e^(2*d*x + 2*c) + a - b), x)/a^2 + 2/3*(3*b*d*x*e^(6*d*x + 6*c) - 9*b*d
*x*e^(4*d*x + 4*c) - 3*b*d*x + 3*(3*b*d*x*e^(2*c) - 2*a*e^(2*c))*e^(2*d*x)
 + 2*a)/(a^2*d*e^(6*d*x + 6*c) - 3*a^2*d*e^(4*d*x + 4*c) + 3*a^2*d*e^(2*d*
x + 2*c) - a^2*d) - b*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) - b*log((e^(d*
x + c) - 1)*e^(-c))/(a^2*d)
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.84

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$
$$= \frac{\frac{2\,b\log\left(\left|ae^{(6\,dx+6\,c)}+be^{(6\,dx+6\,c)}+3\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}+3\,ae^{(2\,dx+2\,c)}+3\,be^{(2\,dx+2\,c)}+a-b\right|\right)}{a^2} - \frac{6\,b\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{a^2} + \frac{11\,be^{(6\,dx+6}}{6\,d}$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
1/6*(2*b*log(abs(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) + 3*a*e^(4*d*x + 4*
c) - 3*b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c) + a -
 b))/a^2 - 6*b*log(abs(e^(2*d*x + 2*c) - 1))/a^2 + (11*b*e^(6*d*x + 6*c) -
 33*b*e^(4*d*x + 4*c) - 24*a*e^(2*d*x + 2*c) + 33*b*e^(2*d*x + 2*c) + 8*a
- 11*b)/(a^2*(e^(2*d*x + 2*c) - 1)^3))/d
```

3.80.9 Mupad [B] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 4563, normalized size of antiderivative = 21.22

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^3)),x)
```

output
```
8/(3*(a*d - 3*a*d*exp(2*c + 2*d*x) + 3*a*d*exp(4*c + 4*d*x) - a*d*exp(6*c
+ 6*d*x))) - 4/(a*d - 2*a*d*exp(2*c + 2*d*x) + a*d*exp(4*c + 4*d*x)) + sym
sum(log((1507328*a*b^9 + 1572864*b^10 - 5242880*a^2*b^8 - 7479296*a^3*b^7
+ 3948544*a^4*b^6 + 5963776*a^5*b^5 - 278528*a^6*b^4 + 8192*a^7*b^3 - 1572
864*b^10*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^
2*b - b^3, z, k))*exp(2*d*x) - 1769472*a*b^9*exp(2*root(27*a^6*d^3*z^3 - 2
7*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 4246732
8*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z,
 k)^2*a^4*b^8*d^2 + 21626880*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^
2*b^2*d*z + a^2*b - b^3, z, k)^2*a^5*b^7*d^2 - 70189056*root(27*a^6*d^3*z^
3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^6*b^6*d^2 +
18038784*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b -
b^3, z, k)^2*a^7*b^5*d^2 - 11993088*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2
 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^8*b^4*d^2 + 147456*root(27*a^6*d
^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^9*b^3*d
^2 - 98304*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b
- b^3, z, k)^2*a^10*b^2*d^2 - 42467328*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*
z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^3*a^6*b^7*d^3 - 12091392*root(27*
a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^3*a^7*
b^6*d^3 + 22708224*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d...
```

3.81 $\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

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3.81.1 Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{1}{8}(3a-b)x + \frac{(3a-b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)}{4d}$$

output
```
1/8*(3*a-b)*x+1/8*(3*a-b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*(a+b)*cosh(d*x+c)^
3*sinh(d*x+c)/d
```

3.81.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{-4bdx + 12a(c+dx) + 8a\sinh(2(c+dx)) + (a+b)\sinh(4(c+dx))}{32d}$$

input
```
Integrate[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-4*b*d*x + 12*a*(c + d*x) + 8*a*Sinh[2*(c + d*x)] + (a + b)*Sinh[4*(c + d
*x)])/(32*d)
```

3.81.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4158, 298, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a-b\tan(ic+idx)^2}{\sec(ic+idx)^4}dx$$

$$\downarrow 4158$$

$$\frac{\int \frac{b\tanh^2(c+dx)+a}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\downarrow 298$$

$$\frac{\frac{1}{4}(3a-b)\int \frac{1}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)+\frac{(a+b)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 215$$

$$\frac{\frac{1}{4}(3a-b)\left(\frac{1}{2}\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{(a+b)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 219$$

$$\frac{\frac{1}{4}(3a-b)\left(\frac{1}{2}\operatorname{arctanh}(\tanh(c+dx))+\frac{\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{(a+b)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

input
```
Int[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(((a + b)*Tanh[c + d*x])/(4*(1 - Tanh[c + d*x]^2)^2) + ((3*a - b)*(ArcTanh
[Tanh[c + d*x]]/2 + Tanh[c + d*x]/(2*(1 - Tanh[c + d*x]^2))))/4)/d
```

3.81.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.81.4 Maple [A] (verified)

Time = 4.87 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{a\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)}{d}$
default	$\frac{a\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)}{d}$
risch	$\frac{3ax}{8}-\frac{bx}{8}+\frac{e^{4dx+4c}a}{64d}+\frac{e^{4dx+4c}b}{64d}+\frac{a\,e^{2dx+2c}}{8d}-\frac{a\,e^{-2dx-2c}}{8d}-\frac{e^{-4dx-4c}a}{64d}-\frac{e^{-4dx-4c}b}{64d}$

input
```
int(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)+b*(
1/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1/8*c))
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{(a+b)\cosh(dx+c)\sinh(dx+c)^3+(3\,a-b)dx+\left((a+b)\cosh(dx+c)^3+4\,a\cosh(dx+c)\right)\sinh(dx+}{8\,d}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/8*((a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a - b)*d*x + ((a + b)*cosh
(d*x + c)^3 + 4*a*cosh(d*x + c))*sinh(d*x + c))/d
```

3.81.6 Sympy [F]

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2(c+dx)\right)\cosh^4(c+dx)\,dx$$

input
```
integrate(cosh(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*cosh(c + d*x)**4, x)
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.65

$$\begin{aligned}&\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx\\&=\frac{1}{64}a\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)\\&\quad-\frac{1}{64}b\left(\frac{8(dx+c)}{d}-\frac{e^{(4dx+4c)}}{d}+\frac{e^{(-4dx-4c)}}{d}\right)\end{aligned}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/64*a*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*c
)/d - e^(-4*d*x - 4*c)/d) - 1/64*b*(8*(d*x + c)/d - e^(4*d*x + 4*c)/d + e^
(-4*d*x - 4*c)/d)
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.65

$$\begin{aligned}&\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx\\&=\frac{8(dx+c)(3a-b)+ae^{(4dx+4c)}+be^{(4dx+4c)}+8ae^{(2dx+2c)}-\left(18ae^{(4dx+4c)}-6be^{(4dx+4c)}+8ae^{(2dx+2c)}-\right.}{64d}\end{aligned}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/64*(8*(d*x + c)*(3*a - b) + a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 8*a*
e^(2*d*x + 2*c) - (18*a*e^(4*d*x + 4*c) - 6*b*e^(4*d*x + 4*c) + 8*a*e^(2*d
*x + 2*c) + a + b)*e^(-4*d*x - 4*c))/d
```

3.81.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.17

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = x\left(\frac{3\,a}{8}-\frac{b}{8}\right)-\frac{\mathrm{e}^{-4\,c-4\,d\,x}\,(a+b)}{64\,d} + \frac{\mathrm{e}^{4\,c+4\,d\,x}\,(a+b)}{64\,d}-\frac{a\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,d}+\frac{a\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,d}$$

input
```
int(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2),x)
```

output
```
x*((3*a)/8 - b/8) - (exp(- 4*c - 4*d*x)*(a + b))/(64*d) + (exp(4*c + 4*d*x
)*(a + b))/(64*d) - (a*exp(- 2*c - 2*d*x))/(8*d) + (a*exp(2*c + 2*d*x))/(8
*d)
```

3.82 $\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

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3.82.1 Optimal result

Integrand size = 21, antiderivative size = 30

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{a\sinh(c+dx)}{d}+\frac{(a+b)\sinh^3(c+dx)}{3d}$$

output
```
a*sinh(d*x+c)/d+1/3*(a+b)*sinh(d*x+c)^3/d
```

3.82.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{a\sinh(c+dx)}{d}+\frac{a\sinh^3(c+dx)}{3d}+\frac{b\sinh^3(c+dx)}{3d}$$

input
```
Integrate[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a*Sinh[c + d*x])/d + (a*Sinh[c + d*x]^3)/(3*d) + (b*Sinh[c + d*x]^3)/(3*d
)
```

3.82.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a-b\tan(ic+idx)^2}{\sec(ic+idx)^3}dx$$

$$\downarrow 4159$$

$$\frac{\int \left((a+b)\sinh^2(c+dx)+a\right)d\sinh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{3}(a+b)\sinh^3(c+dx)+a\sinh(c+dx)}{d}$$

input
```
Int[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a*Sinh[c + d*x] + ((a + b)*Sinh[c + d*x]^3)/3)/d
```

3.82.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.82.4 Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

method	result	size
derivativedivides	$\frac{a\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+\frac{b\sinh(dx+c)^3}{3}}{d}$	37
default	$\frac{a\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+\frac{b\sinh(dx+c)^3}{3}}{d}$	37
risch	$\frac{e^{3dx+3c}a}{24d}+\frac{e^{3dx+3c}b}{24d}+\frac{3e^{dx+c}a}{8d}-\frac{e^{dx+c}b}{8d}-\frac{3e^{-dx-c}a}{8d}+\frac{e^{-dx-c}b}{8d}-\frac{e^{-3dx-3c}a}{24d}-\frac{e^{-3dx-3c}b}{24d}$	116

input
```
int(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c)+1/3*b*sinh(d*x+c)^3)
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.50

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{(a+b)\sinh(dx+c)^3+3\left((a+b)\cosh(dx+c)^2+3a-b\right)\sinh(dx+c)}{12d}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/12*((a + b)*sinh(d*x + c)^3 + 3*((a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh
(d*x + c))/d
```

3.82.6 Sympy [F]

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)\cosh^3\left(c+dx\right)dx$$

input
```
integrate(cosh(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*cosh(c + d*x)**3, x)
```

3.82.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(28) = 56$.

Time $= 0.20$ (sec) , antiderivative size $= 83$, normalized size of antiderivative $= 2.77$

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3}{24\,d} + \frac{1}{24}\,a\left(\frac{e^{(3\,dx+3\,c)}}{d} + \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} - \frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/24*b*(e^(d*x + c) - e^(-d*x - c))^3/d + 1/24*a*(e^(3*d*x + 3*c)/d + 9*e^
(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d)
```

3.82.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(28) = 56$.

Time $= 0.30$ (sec) , antiderivative size $= 84$, normalized size of antiderivative $= 2.80$

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{ae^{(3\,dx+3\,c)} + be^{(3\,dx+3\,c)} + 9\,ae^{(dx+c)} - 3\,be^{(dx+c)} - \left(9\,ae^{(2\,dx+2\,c)} - 3\,be^{(2\,dx+2\,c)} + a + b\right)e^{(-3\,dx-3\,c)}}{24\,d}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/24*(a*e^(3*d*x + 3*c) + b*e^(3*d*x + 3*c) + 9*a*e^(d*x + c) - 3*b*e^(d*x
 + c) - (9*a*e^(2*d*x + 2*c) - 3*b*e^(2*d*x + 2*c) + a + b)*e^(-3*d*x - 3*
c))/d
```

3.82.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.47

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{e^{3c+3dx}(a+b)}{24d} - \frac{e^{-3c-3dx}(a+b)}{24d} + \frac{e^{c+dx}(3a-b)}{8d} - \frac{e^{-c-dx}(3a-b)}{8d}$$

input
```
int(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2),x)
```

output
```
(exp(3*c + 3*d*x)*(a + b))/(24*d) - (exp(- 3*c - 3*d*x)*(a + b))/(24*d) +
(exp(c + d*x)*(3*a - b))/(8*d) - (exp(- c - d*x)*(3*a - b))/(8*d)
```

3.83 $\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

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3.83.1 Optimal result

Integrand size = 21, antiderivative size = 33

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{1}{2}(a-b)x + \frac{(a+b)\cosh(c+dx)\sinh(c+dx)}{2d}$$

output
```
1/2*(a-b)*x+1/2*(a+b)*cosh(d*x+c)*sinh(d*x+c)/d
```

3.83.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{2(a-b)(c+dx)+(a+b)\sinh(2(c+dx))}{4d}$$

input
```
Integrate[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(2*(a - b)*(c + d*x) + (a + b)*Sinh[2*(c + d*x)])/(4*d)
```

3.83.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4158, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a-b\tan(ic+idx)^2}{\sec(ic+idx)^2}dx$$

$$\downarrow 4158$$

$$\frac{\int \frac{b\tanh^2(c+dx)+a}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 298$$

$$\frac{\frac{1}{2}(a-b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{(a+b)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 219$$

$$\frac{\frac{1}{2}(a-b)\operatorname{arctanh}(\tanh(c+dx))+\frac{(a+b)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}}{d}$$

input
```
Int[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(((a - b)*ArcTanh[Tanh[c + d*x]])/2 + ((a + b)*Tanh[c + d*x])/(2*(1 - Tanh
[c + d*x]^2)))/d
```

3.83.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.83.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.64

method	result	size
derivativedivides	$\frac{b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	54
default	$\frac{b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	54
risch	$\frac{ax}{2}-\frac{bx}{2}+\frac{a\,\mathrm{e}^{2dx+2c}}{8d}+\frac{\mathrm{e}^{2dx+2c}b}{8d}-\frac{a\,\mathrm{e}^{-2dx-2c}}{8d}-\frac{\mathrm{e}^{-2dx-2c}b}{8d}$	70

input
```
int(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+a*(1/2*cosh(d*x+c)*sinh
(d*x+c)+1/2*d*x+1/2*c))
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(a-b)dx+(a+b)\cosh(dx+c)\sinh(dx+c)}{2\,d}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/2*((a - b)*d*x + (a + b)*cosh(d*x + c)*sinh(d*x + c))/d
```

3.83.6 Sympy [F]

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2(c+dx)\right)\cosh^2(c+dx)\,dx$$

input
```
integrate(cosh(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*cosh(c + d*x)**2, x)
```

3.83.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(29) = 58.

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.09

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{1}{8}a\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{8}b\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)$$

input
```
integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/8*a*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/8*b*(4*x - e^(2*d
*x + 2*c)/d + e^(-2*d*x - 2*c)/d)
```

3.83.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(29) = 58$.

Time $= 0.29$ (sec) , antiderivative size $= 78$, normalized size of antiderivative $= 2.36$

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{4\,(dx+c)(a-b)+ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-\left(2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)e^{(-2\,dx-2\,c)}}{8\,d}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/8*(4*(d*x + c)*(a - b) + a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - (2*a*e^
(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*e^(-2*d*x - 2*c))/d
```

3.83.9 Mupad [B] (verification not implemented)

Time $= 1.85$ (sec) , antiderivative size $= 27$, normalized size of antiderivative $= 0.82$

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = x\left(\frac{a}{2}-\frac{b}{2}\right)+\frac{\sinh(2\,c+2\,d\,x)\,\left(a+b\right)}{4\,d}$$

input
```
int(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2),x)
```

output
```
x*(a/2 - b/2) + (sinh(2*c + 2*d*x)*(a + b))/(4*d)
```

3.84 $\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

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3.84.2 Mathematica [A] (verified) . . . 697
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3.84.5 Fricas [B] (verification not implemented) . . . 700
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3.84.8 Giac [A] (verification not implemented) . . . 701
3.84.9 Mupad [B] (verification not implemented) . . . 701

3.84.1 Optimal result

Integrand size = 19, antiderivative size = 27

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{b\arctan(\sinh(c+dx))}{d}+\frac{(a+b)\sinh(c+dx)}{d}$$

output
```
-b*arctan(sinh(d*x+c))/d+(a+b)*sinh(d*x+c)/d
```

3.84.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{b\arctan(\sinh(c+dx))}{d}+\frac{a\cosh(dx)\sinh(c)}{d}+\frac{a\cosh(c)\sinh(dx)}{d}+\frac{b\sinh(c+dx)}{d}$$

input
```
Integrate[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-((b*ArcTan[Sinh[c + d*x]])/d) + (a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh
[d*x])/d + (b*Sinh[c + d*x])/d
```

3.84.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4159, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a-b\tan(ic+idx)^2}{\sec(ic+idx)}dx$$

$$\downarrow 4159$$

$$\frac{\int \frac{(a+b)\sinh^2(c+dx)+a}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{d}$$

$$\downarrow 299$$

$$\frac{(a+b)\sinh(c+dx)-b\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{d}$$

$$\downarrow 216$$

$$\frac{(a+b)\sinh(c+dx)-b\arctan(\sinh(c+dx))}{d}$$

input
```
Int[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-(b*ArcTan[Sinh[c + d*x]]) + (a + b)*Sinh[c + d*x])/d
```

3.84.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))  Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.84.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

method	result	size
derivativedivides	$\frac{a\sinh(dx+c)+b(\sinh(dx+c)-2\arctan(e^{dx+c}))}{d}$	32
default	$\frac{a\sinh(dx+c)+b(\sinh(dx+c)-2\arctan(e^{dx+c}))}{d}$	32
risch	$\frac{e^{dx+c}a}{2d}+\frac{e^{dx+c}b}{2d}-\frac{e^{-dx-c}a}{2d}-\frac{e^{-dx-c}b}{2d}+\frac{ib\ln(e^{dx+c}-i)}{d}-\frac{ib\ln(e^{dx+c}+i)}{d}$	90

input
```
int(cosh(d*x+c)*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*sinh(d*x+c)+b*(sinh(d*x+c)-2*arctan(exp(d*x+c))))
```

3.84.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(27) = 54$.

Time $= 0.27$ (sec) , antiderivative size $= 102$, normalized size of antiderivative $= 3.78$

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{(a+b)\cosh(dx+c)^2 + 2(a+b)\cosh(dx+c)\sinh(dx+c) + (a+b)\sinh(dx+c)^2 - 4(b\cosh(dx+c)}{2(d\cosh(dx+c)+d\sinh(dx+c))}$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a
+ b)*sinh(d*x + c)^2 - 4*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(cosh(d
*x + c) + sinh(d*x + c)) - a - b)/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

3.84.6 Sympy [F]

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2(c+dx)\right)\cosh(c+dx)\,dx$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*cosh(c + d*x), x)
```

3.84.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(27) = 54$.

Time $= 0.28$ (sec) , antiderivative size $= 55$, normalized size of antiderivative $= 2.04$

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{1}{2}b\left(\frac{4\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{e^{(dx+c)}}{d} - \frac{e^{(-dx-c)}}{d}\right) + \frac{a\sinh(dx+c)}{d}$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*b*(4*arctan(e^(-d*x - c))/d + e^(d*x + c)/d - e^(-d*x - c)/d) + a*sinh
(d*x + c)/d
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{4\,b\arctan\left(e^{(dx+c)}\right)-ae^{(dx+c)}-be^{(dx+c)}+(a+b)e^{(-dx-c)}}{2\,d}$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/2*(4*b*arctan(e^(d*x + c)) - a*e^(d*x + c) - b*e^(d*x + c) + (a + b)*e^
(-d*x - c))/d
```

3.84.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.44

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{\mathrm{e}^{c+d\,x}\,(a+b)}{2\,d}-\frac{2\,\mathrm{atan}\left(\frac{b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{d^2}} - \frac{\mathrm{e}^{-c-d\,x}\,(a+b)}{2\,d}$$

input
```
int(cosh(c + d*x)*(a + b*tanh(c + d*x)^2),x)
```

output
```
(exp(c + d*x)*(a + b))/(2*d) - (2*atan((b*exp(d*x)*exp(c)*(d^2)^(1/2))/(d*
(b^2)^(1/2)))*(b^2)^(1/2))/(d^2)^(1/2) - (exp(- c - d*x)*(a + b))/(2*d)
```

3.85 $\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

3.85.1 Optimal result 702
3.85.2 Mathematica [A] (verified) 702
3.85.3 Rubi [A] (verified) 703
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3.85.6 Sympy [F] 705
3.85.7 Maxima [B] (verification not implemented) 706
3.85.8 Giac [B] (verification not implemented) 706
3.85.9 Mupad [B] (verification not implemented) 707

3.85.1 Optimal result

Integrand size = 19, antiderivative size = 40

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(2a+b)\arctan(\sinh(c+dx))}{2d} - \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

output
```
1/2*(2*a+b)*arctan(sinh(d*x+c))/d-1/2*b*sech(d*x+c)*tanh(d*x+c)/d
```

3.85.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{a\arctan(\sinh(c+dx))}{d} + \frac{b\arctan(\sinh(c+dx))}{2d} - \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

input
```
Integrate[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a*ArcTan[Sinh[c + d*x]])/d + (b*ArcTan[Sinh[c + d*x]])/(2*d) - (b*Sech[c
+ d*x]*Tanh[c + d*x])/(2*d)
```

3.85.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4159, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \sec(ic+idx)\left(a-b\tan(ic+idx)^2\right)dx$$

$$\downarrow 4159$$

$$\frac{\int \frac{(a+b)\sinh^2(c+dx)+a}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)}{d}$$

$$\downarrow 298$$

$$\frac{\frac{1}{2}(2a+b)\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)-\frac{b\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}}{d}$$

$$\downarrow 216$$

$$\frac{\frac{1}{2}(2a+b)\arctan(\sinh(c+dx))-\frac{b\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}}{d}$$

input
```
Int[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(((2*a + b)*ArcTan[Sinh[c + d*x]])/2 - (b*Sinh[c + d*x])/(2*(1 + Sinh[c +
d*x]^2)))/d
```

3.85.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.85.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

method	result	size
derivativedivides	$\frac{2a\arctan\left(e^{dx+c}\right)+b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)}{d}$	56
default	$\frac{2a\arctan\left(e^{dx+c}\right)+b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)}{d}$	56
risch	$-\frac{b e^{dx+c}\left(e^{2dx+2c}-1\right)}{d\left(e^{2dx+2c}+1\right)^2}+\frac{i\ln\left(e^{dx+c}+i\right)a}{d}+\frac{ib\ln\left(e^{dx+c}+i\right)}{2d}-\frac{i\ln\left(e^{dx+c}-i\right)a}{d}-\frac{ib\ln\left(e^{dx+c}-i\right)}{2d}$	106

input
```
int(sech(d*x+c)*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*a*arctan(exp(d*x+c))+b*(-sinh(d*x+c)/cosh(d*x+c)^2+1/2*sech(d*x+c)*
tanh(d*x+c)+arctan(exp(d*x+c))))
```

3.85.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(36) = 72$.

Time $= 0.25$ (sec) , antiderivative size $= 323$, normalized size of antiderivative $= 8.08$

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx =$$

$$-\frac{b\cosh(dx+c)^3+3\,b\cosh(dx+c)\sinh(dx+c)^2+b\sinh(dx+c)^3-\left((2\,a+b)\cosh(dx+c)^4+4\,(2\,a$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^
3 - ((2*a + b)*cosh(d*x + c)^4 + 4*(2*a + b)*cosh(d*x + c)*sinh(d*x + c)^3
 + (2*a + b)*sinh(d*x + c)^4 + 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*(2*a + b
)*cosh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)^2 + 4*((2*a + b)*cosh(d*x + c)^
3 + (2*a + b)*cosh(d*x + c))*sinh(d*x + c) + 2*a + b)*arctan(cosh(d*x + c)
 + sinh(d*x + c)) - b*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 - b)*sinh(d*x +
 c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x +
 c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2
+ 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

3.85.6 Sympy [F]

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2(c+dx)\right)\operatorname{sech}(c+dx)\,dx$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x), x)
```

3.85.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(36) = 72$.

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.00

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= -b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)}\right) + \frac{a\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2
*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a*arctan(sinh(d*x + c))/d
```

3.85.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(36) = 72$.

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.10

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)(2\,a+b) - \frac{4\,b\left(e^{(dx+c)} - e^{(-dx-c)}\right)}{\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4}}{4\,d}$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/4*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(2*a + b) - 4
*b*(e^(d*x + c) - e^(-d*x - c))/((e^(d*x + c) - e^(-d*x - c))^2 + 4))/d
```

3.85.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 3.12

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{\operatorname{atan}\left(\frac{e^{dx}e^{c}\left(2a\sqrt{d^2}+b\sqrt{d^2}\right)}{d\sqrt{4a^2+4ab+b^2}}\right)\sqrt{4a^2+4ab+b^2}}{\sqrt{d^2}} - \frac{b\,e^{c+dx}}{d\left(e^{2c+2dx}+1\right)} + \frac{2b\,e^{c+dx}}{d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)/cosh(c + d*x),x)
```

output
```
(atan((exp(d*x)*exp(c)*(2*a*(d^2)^(1/2) + b*(d^2)^(1/2)))/(d*(4*a*b + 4*a^
2 + b^2)^(1/2)))*(4*a*b + 4*a^2 + b^2)^(1/2))/(d^2)^(1/2) - (b*exp(c + d*x
))/(d*(exp(2*c + 2*d*x) + 1)) + (2*b*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x)
+ exp(4*c + 4*d*x) + 1))
```

3.86 $\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

3.86.1 Optimal result 708
3.86.2 Mathematica [A] (verified) 708
3.86.3 Rubi [A] (verified) 709
3.86.4 Maple [A] (verified) 710
3.86.5 Fricas [B] (verification not implemented) 710
3.86.6 Sympy [F] 711
3.86.7 Maxima [A] (verification not implemented) 711
3.86.8 Giac [B] (verification not implemented) 711
3.86.9 Mupad [B] (verification not implemented) 712

3.86.1 Optimal result

Integrand size = 21, antiderivative size = 28

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}$$

output
```
a*tanh(d*x+c)/d+1/3*b*tanh(d*x+c)^3/d
```

3.86.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}$$

input
```
Integrate[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a*Tanh[c + d*x])/d + (b*Tanh[c + d*x]^3)/(3*d)
```

3.86.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4158, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \sec(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)dx$$

$$\downarrow 4158$$

$$\frac{\int \left(b\tanh^2(c+dx)+a\right)d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{a\tanh(c+dx)+\frac{1}{3}b\tanh^3(c+dx)}{d}$$

input
```
Int[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a*Tanh[c + d*x] + (b*Tanh[c + d*x]^3)/3)/d
```

3.86.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.86.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{b\tanh(dx+c)^3}{3}+a\tanh(dx+c)}{d}$	25
default	$\frac{\frac{b\tanh(dx+c)^3}{3}+a\tanh(dx+c)}{d}$	25
risch	$-\frac{2\left(3a\,\mathrm{e}^{4dx+4c}+3b\,\mathrm{e}^{4dx+4c}+6\,\mathrm{e}^{2dx+2c}a+3a+b\right)}{3d\left(\mathrm{e}^{2dx+2c}+1\right)^3}$	60

input
```
int(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*b*tanh(d*x+c)^3+a*tanh(d*x+c))
```

3.86.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(26) = 52.

Time = 0.26 (sec) , antiderivative size = 159, normalized size of antiderivative = 5.68

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx =$$

$$-\frac{4\left((3\,a+2\,b)\cosh(dx+c)^2+2\,b\cosh(dx+c)\sinh(dx+c)\right.}{3\left(d\cosh(dx+c)^4+4\,d\cosh(dx+c)\sinh(dx+c)^3+d\sinh(dx+c)^4+4\,d\cosh(dx+c)^2+2\left(3\,d\cos\right.\right.}$$

input
```
integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-4/3*((3*a + 2*b)*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + (3*a
 + 2*b)*sinh(d*x + c)^2 + 3*a)/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh
(d*x + c)^3 + d*sinh(d*x + c)^4 + 4*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x +
c)^2 + 2*d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh
(d*x + c) + 3*d)
```

3.86.6 Sympy [F]

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)\operatorname{sech}^2\left(c+dx\right)dx$$

input
```
integrate(sech(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x)**2, x)
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{b\tanh\left(dx+c\right)^3}{3\,d} + \frac{2\,a}{d(e^{(-2\,dx-2\,c)}+1)}$$

input
```
integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/3*b*tanh(d*x + c)^3/d + 2*a/(d*(e^(-2*d*x - 2*c) + 1))
```

3.86.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(26) = 52$.

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{2\left(3\,ae^{(4\,dx+4\,c)}+3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+3\,a+b\right)}{3\,d(e^{(2\,dx+2\,c)}+1)^3}$$

input
```
integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-2/3*(3*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) + 3*
a + b)/(d*(e^(2*d*x + 2*c) + 1)^3)
```

3.86.9 Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{2\left(3\,a+b+6\,a\,\mathrm{e}^{2\,c+2\,d\,x}+3\,a\,\mathrm{e}^{4\,c+4\,d\,x}+3\,b\,\mathrm{e}^{4\,c+4\,d\,x}\right)}{3\,d\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)^3}$$

input
```
int((a + b*tanh(c + d*x)^2)/cosh(c + d*x)^2,x)
```

output
```
-(2*(3*a + b + 6*a*exp(2*c + 2*d*x) + 3*a*exp(4*c + 4*d*x) + 3*b*exp(4*c +
 4*d*x)))/(3*d*(exp(2*c + 2*d*x) + 1)^3)
```

3.87 $\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

3.87.1 Optimal result 713
3.87.2 Mathematica [A] (verified) 713
3.87.3 Rubi [A] (verified) 714
3.87.4 Maple [A] (verified) 716
3.87.5 Fricas [B] (verification not implemented) 716
3.87.6 Sympy [F] 717
3.87.7 Maxima [B] (verification not implemented) 718
3.87.8 Giac [B] (verification not implemented) 718
3.87.9 Mupad [B] (verification not implemented) 719

3.87.1 Optimal result

Integrand size = 21, antiderivative size = 66

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(4a+b)\arctan(\sinh(c+dx))}{8d} + \frac{(4a+b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} - \frac{b\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d}$$

output

```
1/8*(4*a+b)*arctan(sinh(d*x+c))/d+1/8*(4*a+b)*sech(d*x+c)*tanh(d*x+c)/d-1/
4*b*sech(d*x+c)^3*tanh(d*x+c)/d
```

3.87.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.41

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{a\arctan(\sinh(c+dx))}{2d} + \frac{b\arctan(\sinh(c+dx))}{8d} + \frac{a\operatorname{sech}(c+dx)\tanh(c+dx)}{2d} + \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} - \frac{b\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d}$$

input
```
Integrate[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a*ArcTan[Sinh[c + d*x]])/(2*d) + (b*ArcTan[Sinh[c + d*x]])/(8*d) + (a*Sec
h[c + d*x]*Tanh[c + d*x])/(2*d) + (b*Sech[c + d*x]*Tanh[c + d*x])/(8*d) -
(b*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

3.87.3 Rubi [A] (verified)

Time $= 0.24$ (sec) , antiderivative size $= 71$, normalized size of antiderivative $= 1.08$, number of steps used $= 6$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used $= \{3042, 4159, 298, 215, 216\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \sec(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)dx$$

$$\downarrow 4159$$

$$\frac{\int \frac{(a+b)\sinh^2(c+dx)+a}{\left(\sinh^2(c+dx)+1\right)^3}d\sinh(c+dx)}{d}$$

$$\downarrow 298$$

$$\frac{\frac{1}{4}(4a+b)\int \frac{1}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)-\frac{b\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

$$\downarrow 215$$

$$\frac{\frac{1}{4}(4a+b)\left(\frac{1}{2}\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)+\frac{\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)-\frac{b\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

$$\downarrow 216$$

$$\frac{\frac{1}{4}(4a+b)\left(\frac{1}{2}\arctan(\sinh(c+dx))+\frac{\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)-\frac{b\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-1/4*(b*Sinh[c + d*x])/(1 + Sinh[c + d*x]^2)^2 + ((4*a + b)*(ArcTan[Sinh[
c + d*x]]/2 + Sinh[c + d*x]/(2*(1 + Sinh[c + d*x]^2))))/4)/d
```

3.87.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.87.4 Maple [A] (verified)

Time = 4.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{a\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b\left(-\frac{\sinh(dx+c)}{3\cosh(dx+c)^4}+\frac{\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)}{3}+\frac{\arctan\left(e^{dx+c}\right.}{4}\right.}{d}$
default	$\frac{a\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b\left(-\frac{\sinh(dx+c)}{3\cosh(dx+c)^4}+\frac{\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)}{3}+\frac{\arctan\left(e^{dx+c}\right.}{4}\right.}{d}$
risch	$\frac{e^{dx+c}\left(4a\,e^{6dx+6c}+b\,e^{6dx+6c}+4a\,e^{4dx+4c}-7b\,e^{4dx+4c}-4\,e^{2dx+2c}a+7b\,e^{2dx+2c}-4a-b\right)}{4d\left(e^{2dx+2c}+1\right)^4}+\frac{i\ln\left(e^{dx+c}+i\right)a}{2d}+\frac{ib\ln\left(e^{d}\right.}{8}$

input
```
int(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/2*sech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+b*(-1/3/cosh(d*x+c
)^4*sinh(d*x+c)+1/3*(1/4*sech(d*x+c)^3+3/8*sech(d*x+c))*tanh(d*x+c)+1/4*ar
ctan(exp(d*x+c))))
```

3.87.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. 2(60) = 120.

Time = 0.27 (sec) , antiderivative size = 1046, normalized size of antiderivative = 15.85

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output

```
1/4*((4*a + b)*cosh(d*x + c)^7 + 7*(4*a + b)*cosh(d*x + c)*sinh(d*x + c)^6
 + (4*a + b)*sinh(d*x + c)^7 + (4*a - 7*b)*cosh(d*x + c)^5 + (21*(4*a + b)
*cosh(d*x + c)^2 + 4*a - 7*b)*sinh(d*x + c)^5 + 5*(7*(4*a + b)*cosh(d*x +
c)^3 + (4*a - 7*b)*cosh(d*x + c))*sinh(d*x + c)^4 - (4*a - 7*b)*cosh(d*x +
 c)^3 + (35*(4*a + b)*cosh(d*x + c)^4 + 10*(4*a - 7*b)*cosh(d*x + c)^2 - 4
*a + 7*b)*sinh(d*x + c)^3 + (21*(4*a + b)*cosh(d*x + c)^5 + 10*(4*a - 7*b)
*cosh(d*x + c)^3 - 3*(4*a - 7*b)*cosh(d*x + c))*sinh(d*x + c)^2 + ((4*a +
b)*cosh(d*x + c)^8 + 8*(4*a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a + b)
*sinh(d*x + c)^8 + 4*(4*a + b)*cosh(d*x + c)^6 + 4*(7*(4*a + b)*cosh(d*x +
 c)^2 + 4*a + b)*sinh(d*x + c)^6 + 8*(7*(4*a + b)*cosh(d*x + c)^3 + 3*(4*a
 + b)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(4*a + b)*cosh(d*x + c)^4 + 2*(35
*(4*a + b)*cosh(d*x + c)^4 + 30*(4*a + b)*cosh(d*x + c)^2 + 12*a + 3*b)*si
nh(d*x + c)^4 + 8*(7*(4*a + b)*cosh(d*x + c)^5 + 10*(4*a + b)*cosh(d*x + c
)^3 + 3*(4*a + b)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(4*a + b)*cosh(d*x +
c)^2 + 4*(7*(4*a + b)*cosh(d*x + c)^6 + 15*(4*a + b)*cosh(d*x + c)^4 + 9*(
4*a + b)*cosh(d*x + c)^2 + 4*a + b)*sinh(d*x + c)^2 + 8*((4*a + b)*cosh(d*
x + c)^7 + 3*(4*a + b)*cosh(d*x + c)^5 + 3*(4*a + b)*cosh(d*x + c)^3 + (4*
a + b)*cosh(d*x + c))*sinh(d*x + c) + 4*a + b)*arctan(cosh(d*x + c) + sinh
(d*x + c)) - (4*a + b)*cosh(d*x + c) + (7*(4*a + b)*cosh(d*x + c)^6 + 5*(4
*a - 7*b)*cosh(d*x + c)^4 - 3*(4*a - 7*b)*cosh(d*x + c)^2 - 4*a - b)*si...
```

3.87.6 Sympy [F]

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)\operatorname{sech}^3\left(c+dx\right)dx$$

input

```
integrate(sech(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

output

```
Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x)**3, x)
```

3.87.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(60) = 120$.

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.74

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= -\frac{1}{4}b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - 7\,e^{(-3\,dx-3\,c)} + 7\,e^{(-5\,dx-5\,c)} - e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)} + 6\,e^{(-4\,dx-4\,c)} + 4\,e^{(-6\,dx-6\,c)} + e^{(-8\,dx-8\,c)} + 1)}\right)$$
$$- a\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)}\right)$$

input
```
integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/4*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c) + 7*e^
(-5*d*x - 5*c) - e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x -
4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a*(arctan(e^(-d*x -
c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d
*x - 4*c) + 1)))
```

3.87.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(60) = 120$.

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.32

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{\left(\pi + 2\arctan\left(\tfrac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)(4\,a+b) + \frac{4\left(4\,a\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + b\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 16\,a\left(e^{(dx+c)} - e^{(}\right.\right.}{\left(\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4\right)^2}}{16\,d}$$

input
```
integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/16*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(4*a + b) +
4*(4*a*(e^(d*x + c) - e^(-d*x - c))^3 + b*(e^(d*x + c) - e^(-d*x - c))^3 +
 16*a*(e^(d*x + c) - e^(-d*x - c)) - 4*b*(e^(d*x + c) - e^(-d*x - c)))/((e
^(d*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

3.87.9 Mupad [B] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 280, normalized size of antiderivative = 4.24

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{e^{dx}e^{c}\left(4a\sqrt{d^2}+b\sqrt{d^2}\right)}{d\sqrt{16a^2+8ab+b^2}}\right)\sqrt{16a^2+8ab+b^2}}{4\sqrt{d^2}}$$
$$- \frac{\frac{e^{5c+5dx}(a+b)}{d}+\frac{2e^{3c+3dx}(a-b)}{d}+\frac{e^{c+dx}(a+b)}{d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1} - \frac{e^{c+dx}(2a+3b)}{2d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}$$
$$+ \frac{e^{c+dx}(4a+b)}{4d\left(e^{2c+2dx}+1\right)} + \frac{2be^{c+dx}}{d\left(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)/cosh(c + d*x)^3,x)
```

output
```
(atan((exp(d*x)*exp(c)*(4*a*(d^2)^(1/2) + b*(d^2)^(1/2)))/(d*(8*a*b + 16*a
^2 + b^2)^(1/2)))*(8*a*b + 16*a^2 + b^2)^(1/2))/(4*(d^2)^(1/2)) - ((exp(5*
c + 5*d*x)*(a + b))/d + (2*exp(3*c + 3*d*x)*(a - b))/d + (exp(c + d*x)*(a
+ b))/d)/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + e
xp(8*c + 8*d*x) + 1) - (exp(c + d*x)*(2*a + 3*b))/(2*d*(2*exp(2*c + 2*d*x)
 + exp(4*c + 4*d*x) + 1)) + (exp(c + d*x)*(4*a + b))/(4*d*(exp(2*c + 2*d*x
) + 1)) + (2*b*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) +
 exp(6*c + 6*d*x) + 1))
```

3.88 $\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

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3.88.1 Optimal result

Integrand size = 21, antiderivative size = 48

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{a\tanh(c+dx)}{d} - \frac{(a-b)\tanh^3(c+dx)}{3d} - \frac{b\tanh^5(c+dx)}{5d}$$

output
```
a*tanh(d*x+c)/d-1/3*(a-b)*tanh(d*x+c)^3/d-1/5*b*tanh(d*x+c)^5/d
```

3.88.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.79

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{a\tanh(c+dx)}{d} + \frac{2b\tanh(c+dx)}{15d} + \frac{b\operatorname{sech}^2(c+dx)\tanh(c+dx)}{15d} - \frac{b\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d} - \frac{a\tanh^3(c+dx)}{3d}$$

input
```
Integrate[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a*Tanh[c + d*x])/d + (2*b*Tanh[c + d*x])/(15*d) + (b*Sech[c + d*x]^2*Tanh
[c + d*x])/(15*d) - (b*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d) - (a*Tanh[c +
d*x]^3)/(3*d)
```

3.88.3 Rubi [A] (verified)

Time $= 0.25$ (sec) , antiderivative size $= 43$, normalized size of antiderivative $= 0.90$, number of steps used $= 5$, number of rules used $= 4$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used $= \{3042, 4158, 290, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \sec(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)dx$$

$$\downarrow 4158$$

$$\frac{\int \left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)d\tanh(c+dx)}{d}$$

$$\downarrow 290$$

$$\frac{\int \left(-b\tanh^4(c+dx)-(a-b)\tanh^2(c+dx)+a\right)d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{3}(a-b)\tanh^3(c+dx)+a\tanh(c+dx)-\frac{1}{5}b\tanh^5(c+dx)}{d}$$

input
```
Int[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a*Tanh[c + d*x] - ((a - b)*Tanh[c + d*x]^3)/3 - (b*Tanh[c + d*x]^5)/5)/d
```

3.88.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.88.4 Maple [A] (verified)

Time = 8.92 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{\frac{\tanh(dx+c)^5 b}{5}+\frac{(a-b)\tanh(dx+c)^3}{3}-a\tanh(dx+c)}{d}$	42
default	$-\frac{\frac{\tanh(dx+c)^5 b}{5}+\frac{(a-b)\tanh(dx+c)^3}{3}-a\tanh(dx+c)}{d}$	42
risch	$-\frac{4\left(15a\,\mathrm{e}^{6dx+6c}+15b\,\mathrm{e}^{6dx+6c}+35a\,\mathrm{e}^{4dx+4c}-5b\,\mathrm{e}^{4dx+4c}+25\,\mathrm{e}^{2dx+2c}a+5b\,\mathrm{e}^{2dx+2c}+5a+b\right)}{15d\left(\mathrm{e}^{2dx+2c}+1\right)^5}$	96

input
```
int(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(1/5*tanh(d*x+c)^5*b+1/3*(a-b)*tanh(d*x+c)^3-a*tanh(d*x+c))
```

3.88.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(44) = 88$.

Time $= 0.26$ (sec) , antiderivative size $= 345$, normalized size of antiderivative $= 7.19$

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx =$$

$$-\frac{}{15\left(d\cosh\left(dx+c\right)^7+7\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^6+d\sinh\left(dx+c\right)^7+5\,d\cosh\left(dx+c\right)^5+\left(21\,d\,\mathrm{co}\right.}$$

input
```
integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-8/15*(2*(5*a + 4*b)*cosh(d*x + c)^3 + 6*(5*a + 4*b)*cosh(d*x + c)*sinh(d*
x + c)^2 + (5*a + 7*b)*sinh(d*x + c)^3 + 30*a*cosh(d*x + c) + (3*(5*a + 7*
b)*cosh(d*x + c)^2 + 5*a - 5*b)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*co
sh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 5*d*cosh(d*x + c)^5 + (2
1*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 5*d*
cosh(d*x + c))*sinh(d*x + c)^4 + 11*d*cosh(d*x + c)^3 + (35*d*cosh(d*x + c
)^4 + 50*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3 + (21*d*cosh(d*x + c)^5
+ 50*d*cosh(d*x + c)^3 + 33*d*cosh(d*x + c))*sinh(d*x + c)^2 + 15*d*cosh(d
*x + c) + (7*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 27*d*cosh(d*x + c)
^2 + 5*d)*sinh(d*x + c))
```

3.88.6 Sympy [F]

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)\operatorname{sech}^4\left(c+dx\right)dx$$

input
```
integrate(sech(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x)**4, x)
```

3.88.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(44) = 88$.

Time = 0.20 (sec) , antiderivative size = 371, normalized size of antiderivative = 7.73

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{4}{15}b\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)} - \frac{}{d(5\,e^{(-2\,dx-2\,c)}+}\right.$$
$$\left. + \frac{4}{3}a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)} + \frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)\right.$$

input
```
integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
4/15*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^
(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
 - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6
*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5
*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) +
10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*
x - 10*c) + 1))) + 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^
(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(
-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

3.88.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(44) = 88$.

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.98

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx =$$
$$-\frac{4\left(15\,ae^{(6\,dx+6\,c)}+15\,be^{(6\,dx+6\,c)}+35\,ae^{(4\,dx+4\,c)}-5\,be^{(4\,dx+4\,c)}+25\,ae^{(2\,dx+2\,c)}+5\,be^{(2\,dx+2\,c)}+5\,a+b\right)}{15\,d(e^{(2\,dx+2\,c)}+1)^5}$$

input
```
integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-4/15*(15*a*e^(6*d*x + 6*c) + 15*b*e^(6*d*x + 6*c) + 35*a*e^(4*d*x + 4*c)
- 5*b*e^(4*d*x + 4*c) + 25*a*e^(2*d*x + 2*c) + 5*b*e^(2*d*x + 2*c) + 5*a +
 b)/(d*(e^(2*d*x + 2*c) + 1)^5)
```

3.88.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 304, normalized size of antiderivative = 6.33

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= -\frac{\frac{8(a-b)}{15d}+\frac{4e^{2c+2dx}(a+b)}{5d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}$$
$$-\frac{\frac{8e^{2c+2dx}(a+b)}{5d}+\frac{8e^{6c+6dx}(a+b)}{5d}+\frac{16e^{4c+4dx}(a-b)}{5d}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1}$$
$$-\frac{\frac{2(a+b)}{5d}+\frac{6e^{4c+4dx}(a+b)}{5d}+\frac{8e^{2c+2dx}(a-b)}{5d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1}-\frac{2(a+b)}{5d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)/cosh(c + d*x)^4,x)
```

output
```
- ((8*(a - b))/(15*d) + (4*exp(2*c + 2*d*x)*(a + b))/(5*d))/(3*exp(2*c + 2
*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((8*exp(2*c + 2*d*x)*
(a + b))/(5*d) + (8*exp(6*c + 6*d*x)*(a + b))/(5*d) + (16*exp(4*c + 4*d*x)
*(a - b))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c +
6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*(a + b))/(5*d)
 + (6*exp(4*c + 4*d*x)*(a + b))/(5*d) + (8*exp(2*c + 2*d*x)*(a - b))/(5*d)
)/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c
+ 8*d*x) + 1) - (2*(a + b))/(5*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) +
1))
```

3.89 $\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

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3.89.1 Optimal result

Integrand size = 23, antiderivative size = 85

$$\begin{aligned}&\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx\\&=\frac{1}{8}\left(3a^2-2ab+3b^2\right)x+\frac{3(a^2-b^2)\cosh(c+dx)\sinh(c+dx)}{8d}\\&\quad+\frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)}{4d}\end{aligned}$$

output
```
1/8*(3*a^2-2*a*b+3*b^2)*x+3/8*(a^2-b^2)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*(a+b
)*cosh(d*x+c)^3*sinh(d*x+c)*(a+b*tanh(d*x+c)^2)/d
```

3.89.2 Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.74

$$\begin{aligned}&\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx\\&=\frac{4(3a^2-2ab+3b^2)(c+dx)+8(a^2-b^2)\sinh(2(c+dx))+(a+b)^2\sinh(4(c+dx))}{32d}\end{aligned}$$

input
```
Integrate[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(4*(3*a^2 - 2*a*b + 3*b^2)*(c + d*x) + 8*(a^2 - b^2)*Sinh[2*(c + d*x)] + (
a + b)^2*Sinh[4*(c + d*x)])/(32*d)
```

3.89.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.27, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4158, 315, 25, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\sec(ic+idx)^4}dx$$

$$\downarrow 4158$$

$$\frac{\int \frac{\left(b\tanh^2(c+dx)+a\right)^2}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\downarrow 315$$

$$\frac{\frac{(a+b)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)}{4\left(1-\tanh^2(c+dx)\right)^2} - \frac{1}{4}\int -\frac{(a-3b)b\tanh^2(c+dx)+a(3a-b)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 25$$

$$\frac{\frac{1}{4}\int \frac{(a-3b)b\tanh^2(c+dx)+a(3a-b)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx) + \frac{(a+b)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 298$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(3a^2-2ab+3b^2\right)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx) + \frac{3\left(a^2-b^2\right)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right) + \frac{(a+b)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 219$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(3a^2-2ab+3b^2\right)\operatorname{arctanh}(\tanh(c+dx)) + \frac{3\left(a^2-b^2\right)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right) + \frac{(a+b)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

input
```
Int[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(((a + b)*Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2))/(4*(1 - Tanh[c + d*x]^2)^
2) + (((3*a^2 - 2*a*b + 3*b^2)*ArcTanh[Tanh[c + d*x]])/2 + (3*(a^2 - b^2)*
Tanh[c + d*x])/(2*(1 - Tanh[c + d*x]^2)))/4)/d
```

3.89.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.89.4 Maple [A] (verified)

Time = 13.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{b^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)}{d}$
default	$\frac{b^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)}{d}$
risch	$\frac{3a^2x}{8}-\frac{abx}{4}+\frac{3b^2x}{8}+\frac{e^{4dx+4c}a^2}{64d}+\frac{e^{4dx+4c}ab}{32d}+\frac{e^{4dx+4c}b^2}{64d}+\frac{e^{2dx+2c}a^2}{8d}-\frac{e^{2dx+2c}b^2}{8d}-\frac{e^{-2dx-2c}a^2}{8d}+$

input
```
int(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+2
*a*b*(1/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1/
8*c)+a^2*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c))
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{(a^2+2\,ab+b^2)\cosh(dx+c)\sinh(dx+c)^3+(3\,a^2-2\,ab+3\,b^2)dx+\left((a^2+2\,ab+b^2)\cosh(dx+c)^3+\right.}{8\,d}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 - 2*a*b +
3*b^2)*d*x + ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 4*(a^2 - b^2)*cosh(d*x
 + c))*sinh(d*x + c))/d
```

3.89.6 Sympy [F]

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2(c+dx)\right)^2\cosh^4(c+dx)\,dx$$

input
```
integrate(cosh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*cosh(c + d*x)**4, x)
```

3.89.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(79) = 158$.

Time $= 0.20$ (sec) , antiderivative size $= 171$, normalized size of antiderivative $= 2.01$

$$\begin{aligned}
&\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx \\
&= \frac{1}{64}a^2\left(24x + \frac{e^{(4dx+4c)}}{d} + \frac{8e^{(2dx+2c)}}{d} - \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) \\
&\quad + \frac{1}{64}b^2\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) \\
&\quad - \frac{1}{32}ab\left(\frac{8(dx+c)}{d} - \frac{e^{(4dx+4c)}}{d} + \frac{e^{(-4dx-4c)}}{d}\right)
\end{aligned}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/64*a^2*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) + 1/64*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*
d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) - 1/32*a*b*(8*(d
*x + c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)
```

3.89.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(79) = 158$.

Time $= 0.36$ (sec) , antiderivative size $= 186$, normalized size of antiderivative $= 2.19$

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{a^2e^{(4\,dx+4\,c)} + 2\,abe^{(4\,dx+4\,c)} + b^2e^{(4\,dx+4\,c)} + 8\,a^2e^{(2\,dx+2\,c)} - 8\,b^2e^{(2\,dx+2\,c)} + 8\,(3\,a^2 - 2\,ab + 3\,b^2)(dx+c) -}{6}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/64*(a^2*e^(4*d*x + 4*c) + 2*a*b*e^(4*d*x + 4*c) + b^2*e^(4*d*x + 4*c) +
8*a^2*e^(2*d*x + 2*c) - 8*b^2*e^(2*d*x + 2*c) + 8*(3*a^2 - 2*a*b + 3*b^2)*
(d*x + c) - (18*a^2*e^(4*d*x + 4*c) - 12*a*b*e^(4*d*x + 4*c) + 18*b^2*e^(4
*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) - 8*b^2*e^(2*d*x + 2*c) + a^2 + 2*a*b
+ b^2)*e^(-4*d*x - 4*c))/d
```

3.89.9 Mupad [B] (verification not implemented)

Time $= 0.25$ (sec) , antiderivative size $= 102$, normalized size of antiderivative $= 1.20$

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = x\left(\frac{3\,a^2}{8} - \frac{a\,b}{4} + \frac{3\,b^2}{8}\right) - \frac{\mathrm{e}^{-2\,c-2\,d\,x}\,(a^2-b^2)}{8\,d} + \frac{\mathrm{e}^{2\,c+2\,d\,x}\,(a^2-b^2)}{8\,d} - \frac{\mathrm{e}^{-4\,c-4\,d\,x}\,(a+b)^2}{64\,d} + \frac{\mathrm{e}^{4\,c+4\,d\,x}\,(a+b)^2}{64\,d}$$

input
```
int(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
x*((3*a^2)/8 - (a*b)/4 + (3*b^2)/8) - (exp(- 2*c - 2*d*x)*(a^2 - b^2))/(8*
d) + (exp(2*c + 2*d*x)*(a^2 - b^2))/(8*d) - (exp(- 4*c - 4*d*x)*(a + b)^2)
/(64*d) + (exp(4*c + 4*d*x)*(a + b)^2)/(64*d)
```

3.90 $\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

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3.90.1 Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{b^2\arctan(\sinh(c+dx))}{d}+\frac{(a^2-b^2)\sinh(c+dx)}{d}+\frac{(a+b)^2\sinh^3(c+dx)}{3d}$$

output
```
b^2*arctan(sinh(d*x+c))/d+(a^2-b^2)*sinh(d*x+c)/d+1/3*(a+b)^2*sinh(d*x+c)^
3/d
```

3.90.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\sinh(c+dx)\left(\frac{3b^2\operatorname{arctanh}\left(\sqrt{-\sinh^2(c+dx)}\right)}{\sqrt{-\sinh^2(c+dx)}}+(a+b)\left(3(a-b)+(a+b)\sinh^2(c+dx)\right)\right)}{3d}$$

input
```
Integrate[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(Sinh[c + d*x]*((3*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/Sqrt[-Sinh[c + d*x
]^2] + (a + b)*(3*(a - b) + (a + b)*Sinh[c + d*x]^2)))/(3*d)
```

3.90.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4159, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\sec(ic+idx)^3}dx$$

$$\downarrow 4159$$

$$\frac{\int \frac{\left((a+b)\sinh^2(c+dx)+a\right)^2}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{d}$$

$$\downarrow 300$$

$$\frac{\int \left(a^2-b^2+(a+b)^2\sinh^2(c+dx)+\frac{b^2}{\sinh^2(c+dx)+1}\right)d\sinh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\left(a^2-b^2\right)\sinh(c+dx)+\frac{1}{3}(a+b)^2\sinh^3(c+dx)+b^2\arctan(\sinh(c+dx))}{d}$$

input
```
Int[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(b^2*ArcTan[Sinh[c + d*x]] + (a^2 - b^2)*Sinh[c + d*x] + ((a + b)^2*Sinh[c
 + d*x]^3)/3)/d
```

3.90.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.90.4 Maple [A] (verified)

Time = 5.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{a^2\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+\frac{2ab\sinh(dx+c)^3}{3}+b^2\left(\frac{\sinh(dx+c)^3}{3}-\sinh(dx+c)+2\arctan\left(e^{dx+c}\right)\right)}{d}$
default	$\frac{a^2\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+\frac{2ab\sinh(dx+c)^3}{3}+b^2\left(\frac{\sinh(dx+c)^3}{3}-\sinh(dx+c)+2\arctan\left(e^{dx+c}\right)\right)}{d}$
risch	$\frac{e^{3dx+3c}a^2}{24d}+\frac{e^{3dx+3c}ab}{12d}+\frac{e^{3dx+3c}b^2}{24d}+\frac{3e^{dx+c}a^2}{8d}-\frac{e^{dx+c}ab}{4d}-\frac{5e^{dx+c}b^2}{8d}-\frac{3e^{-dx-c}a^2}{8d}+\frac{e^{-dx-c}ab}{4d}+\frac{5}{}$

input
```
int(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c)+2/3*a*b*sinh(d*x+c)^3+b^2*(1/
3*sinh(d*x+c)^3-sinh(d*x+c)+2*arctan(exp(d*x+c))))
```

3.90.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(52) = 104$.

Time = 0.26 (sec) , antiderivative size = 519, normalized size of antiderivative = 9.61

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{(a^2+2\,ab+b^2)\cosh(dx+c)^6+6\,(a^2+2\,ab+b^2)\cosh(dx+c)\sinh(dx+c)^5+(a^2+2\,ab+b^2)\sinh(dx$$

input
```
integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/24*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + 3*(3*a^2 - 2
*a*b - 5*b^2)*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 +
 3*a^2 - 2*a*b - 5*b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)^3 + 3*(3*a^2 - 2*a*b - 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(3
*a^2 - 2*a*b - 5*b^2)*cosh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^4 + 6*(3*a^2 - 2*a*b - 5*b^2)*cosh(d*x + c)^2 - 3*a^2 + 2*a*b + 5*b^2
)*sinh(d*x + c)^2 - a^2 - 2*a*b - b^2 + 48*(b^2*cosh(d*x + c)^3 + 3*b^2*co
sh(d*x + c)^2*sinh(d*x + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*si
nh(d*x + c)^3)*arctan(cosh(d*x + c) + sinh(d*x + c)) + 6*((a^2 + 2*a*b + b
^2)*cosh(d*x + c)^5 + 2*(3*a^2 - 2*a*b - 5*b^2)*cosh(d*x + c)^3 - (3*a^2 -
 2*a*b - 5*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cos
h(d*x + c)^2*sinh(d*x + c) + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + d*sinh(d*
x + c)^3)
```

3.90.6 Sympy [F]

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2(c+dx)\right)^2\cosh^3(c+dx)\,dx$$

input
```
integrate(cosh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*cosh(c + d*x)**3, x)
```

3.90.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(52) = 104$.

Time = 0.28 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.98

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3}{12\,d}$$
$$-\frac{1}{24}b^2\left(\frac{\left(15\,e^{(-2\,dx-2\,c)}-1\right)e^{(3\,dx+3\,c)}}{d}-\frac{15\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{48\arctan\left(e^{(-dx-c)}\right)}{d}\right)$$
$$+\frac{1}{24}a^2\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/12*a*b*(e^(d*x + c) - e^(-d*x - c))^3/d - 1/24*b^2*((15*e^(-2*d*x - 2*c)
 - 1)*e^(3*d*x + 3*c)/d - (15*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + 48*arct
an(e^(-d*x - c))/d) + 1/24*a^2*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^
(-d*x - c)/d - e^(-3*d*x - 3*c)/d)
```

3.90.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(52) = 104$.

Time = 0.35 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.81

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{48\,b^2\arctan\left(e^{(dx+c)}\right)+a^2e^{(3\,dx+3\,c)}+2\,abe^{(3\,dx+3\,c)}+b^2e^{(3\,dx+3\,c)}+9\,a^2e^{(dx+c)}-6\,abe^{(dx+c)}-15\,b^2e^{(dx+c)}}{24\,d}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/24*(48*b^2*arctan(e^(d*x + c)) + a^2*e^(3*d*x + 3*c) + 2*a*b*e^(3*d*x +
3*c) + b^2*e^(3*d*x + 3*c) + 9*a^2*e^(d*x + c) - 6*a*b*e^(d*x + c) - 15*b^
2*e^(d*x + c) - (9*a^2*e^(2*d*x + 2*c) - 6*a*b*e^(2*d*x + 2*c) - 15*b^2*e^
(2*d*x + 2*c) + a^2 + 2*a*b + b^2)*e^(-3*d*x - 3*c))/d
```

3.90.9 Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.41

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{e^{3c+3dx}(a+b)^2}{24d} - \frac{e^{-3c-3dx}(a+b)^2}{24d} - \frac{e^{c+dx}(-3a^2+2ab+5b^2)}{8d} + \frac{2\operatorname{atan}\left(\frac{b^2 e^{dx} e^c \sqrt{d^2}}{d\sqrt{b^4}}\right)\sqrt{b^4}}{\sqrt{d^2}} + \frac{e^{-c-dx}(-3a^2+2ab+5b^2)}{8d}$$

input
```
int(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
(exp(3*c + 3*d*x)*(a + b)^2)/(24*d) - (exp(- 3*c - 3*d*x)*(a + b)^2)/(24*d
) - (exp(c + d*x)*(2*a*b - 3*a^2 + 5*b^2))/(8*d) + (2*atan((b^2*exp(d*x)*e
xp(c)*(d^2)^(1/2))/(d*(b^4)^(1/2)))*(b^4)^(1/2))/(d^2)^(1/2) + (exp(- c -
d*x)*(2*a*b - 3*a^2 + 5*b^2))/(8*d)
```

3.91 $\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

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3.91.1 Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{1}{2}(a-3b)(a+b)x + \frac{(a+b)^2\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b^2\tanh(c+dx)}{d}$$

output
```
1/2*(a-3*b)*(a+b)*x+1/2*(a+b)^2*cosh(d*x+c)*sinh(d*x+c)/d+b^2*tanh(d*x+c)/
d
```

3.91.2 Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{(a-3b)(a+b)(c+dx)}{2d} + \frac{(a+b)^2\sinh(2(c+dx))}{4d} + \frac{b^2\tanh(c+dx)}{d}$$

input
```
Integrate[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((a - 3*b)*(a + b)*(c + d*x))/(2*d) + ((a + b)^2*Sinh[2*(c + d*x)])/(4*d)
+ (b^2*Tanh[c + d*x])/d
```

3.91.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\sec(ic+idx)^2}dx$$

$$\downarrow 4158$$

$$\frac{\int \frac{\left(b\tanh^2(c+dx)+a\right)^2}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 300$$

$$\frac{\int \left(b^2+\frac{a^2-b^2+2b(a+b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^2}\right)d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{2}(a-3b)(a+b)\text{arctanh}(\tanh(c+dx))+\frac{(a+b)^2\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}+b^2\tanh(c+dx)}{d}$$

input
```
Int[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(((a - 3*b)*(a + b)*ArcTanh[Tanh[c + d*x]])/2 + b^2*Tanh[c + d*x] + ((a +
b)^2*Tanh[c + d*x])/(2*(1 - Tanh[c + d*x]^2)))/d
```

3.91.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.91.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(47) = 94$.

Time = 1.94 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.88

method	result
derivativedivides	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b^2\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)}{d}$
default	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b^2\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)}{d}$
risch	$\frac{a^2x}{2}-abx-\frac{3b^2x}{2}+\frac{e^{2dx+2c}a^2}{8d}+\frac{e^{2dx+2c}ab}{4d}+\frac{e^{2dx+2c}b^2}{8d}-\frac{e^{-2dx-2c}a^2}{8d}-\frac{e^{-2dx-2c}ab}{4d}-\frac{e^{-2dx-2c}b^2}{8d}-$

input
```
int(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+2*a*b*(1/2*cosh(d*x+c
)*sinh(d*x+c)-1/2*d*x-1/2*c)+b^2*(1/2*sinh(d*x+c)^3/cosh(d*x+c)-3/2*d*x-3/
2*c+3/2*tanh(d*x+c)))
```

3.91.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(47) = 94$.

Time $= 0.25$ (sec) , antiderivative size $= 105$, normalized size of antiderivative $= 2.06$

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{(a^2+2\,ab+b^2)\sinh(dx+c)^3+4\left((a^2-2\,ab-3\,b^2)dx-2\,b^2\right)\cosh(dx+c)+\left(3\,(a^2+2\,ab+b^2)\cosh(d\right.}{8\,d\cosh(dx+c)}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/8*((a^2 + 2*a*b + b^2)*sinh(d*x + c)^3 + 4*((a^2 - 2*a*b - 3*b^2)*d*x -
2*b^2)*cosh(d*x + c) + (3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*
b + 9*b^2)*sinh(d*x + c))/(d*cosh(d*x + c))
```

3.91.6 Sympy [F]

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2(c+dx)\right)^2\cosh^2(c+dx)\,dx$$

input
```
integrate(cosh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*cosh(c + d*x)**2, x)
```

3.91.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(47) = 94$.

Time $= 0.19$ (sec) , antiderivative size $= 140$, normalized size of antiderivative $= 2.75$

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{1}{8}a^2\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{4}ab\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)$$
$$-\frac{1}{8}b^2\left(\frac{12\,(dx+c)}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}-\frac{17\,e^{(-2\,dx-2\,c)}+1}{d(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)})}\right)$$

input
```
integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/8*a^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/4*a*b*(4*x - e^
(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/8*b^2*(12*(d*x + c)/d + e^(-2*d*
x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x -
4*c))))
```

3.91.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(47) = 94$.

Time $= 0.35$ (sec) , antiderivative size $= 167$, normalized size of antiderivative $= 3.27$

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{a^2e^{(2\,dx+2\,c)} + 2\,abe^{(2\,dx+2\,c)} + b^2e^{(2\,dx+2\,c)} + 4\,(a^2 - 2\,ab - 3\,b^2)(dx+c) - \frac{a^2e^{(4\,dx+4\,c)} - 2\,abe^{(4\,dx+4\,c)} - 3\,b^2e^{(4\,dx+4\,c}}{e^{(4\,dx+4\,c}}}{8\,d}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/8*(a^2*e^(2*d*x + 2*c) + 2*a*b*e^(2*d*x + 2*c) + b^2*e^(2*d*x + 2*c) + 4
*(a^2 - 2*a*b - 3*b^2)*(d*x + c) - (a^2*e^(4*d*x + 4*c) - 2*a*b*e^(4*d*x +
 4*c) - 3*b^2*e^(4*d*x + 4*c) + 2*a^2*e^(2*d*x + 2*c) + 14*b^2*e^(2*d*x +
2*c) + a^2 + 2*a*b + b^2)/(e^(4*d*x + 4*c) + e^(2*d*x + 2*c)))/d
```

3.91.9 Mupad [B] (verification not implemented)

Time $= 1.93$ (sec) , antiderivative size $= 77$, normalized size of antiderivative $= 1.51$

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\mathrm{e}^{2\,c+2\,dx}\,(a+b)^2}{8\,d} - \frac{2\,b^2}{d\,\left(\mathrm{e}^{2\,c+2\,dx}+1\right)} - \frac{\mathrm{e}^{-2\,c-2\,dx}\,(a+b)^2}{8\,d} - x\left(-\frac{a^2}{2} + a\,b + \frac{3\,b^2}{2}\right)$$

input
```
int(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
(exp(2*c + 2*d*x)*(a + b)^2)/(8*d) - (2*b^2)/(d*(exp(2*c + 2*d*x) + 1)) -
(exp(- 2*c - 2*d*x)*(a + b)^2)/(8*d) - x*(a*b - a^2/2 + (3*b^2)/2)
```

3.92 $\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

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3.92.1 Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{b(4a+3b)\arctan(\sinh(c+dx))}{2d} + \frac{(a+b)^2\sinh(c+dx)}{d} + \frac{b^2\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

output
```
-1/2*b*(4*a+3*b)*arctan(sinh(d*x+c))/d+(a+b)^2*sinh(d*x+c)/d+1/2*b^2*sech(
d*x+c)*tanh(d*x+c)/d
```

3.92.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{2(a+b)^2\sinh(c+dx)+b(-((4a+3b)\arctan(\sinh(c+dx)))+b\operatorname{sech}(c+dx)\tanh(c+dx))}{2d}$$

input
```
Integrate[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(2*(a + b)^2*Sinh[c + d*x] + b*(-((4*a + 3*b)*ArcTan[Sinh[c + d*x]]) + b*S
ech[c + d*x]*Tanh[c + d*x]))/(2*d)
```

3.92.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4159, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\sec(ic+idx)}dx$$

$$\downarrow 4159$$

$$\frac{\int \frac{\left((a+b)\sinh^2(c+dx)+a\right)^2}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)}{d}$$

$$\downarrow 300$$

$$\frac{\int \left((a+b)^2-\frac{2b(a+b)\sinh^2(c+dx)+b(2a+b)}{\left(\sinh^2(c+dx)+1\right)^2}\right)d\sinh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{2}b(4a+3b)\arctan(\sinh(c+dx))+(a+b)^2\sinh(c+dx)+\frac{b^2\sinh(c+dx)}{2(\sinh^2(c+dx)+1)}}{d}$$

input
```
Int[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-1/2*(b*(4*a + 3*b)*ArcTan[Sinh[c + d*x]]) + (a + b)^2*Sinh[c + d*x] + (b
^2*Sinh[c + d*x])/(2*(1 + Sinh[c + d*x]^2)))/d
```

3.92.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.92.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

method	result
derivativedivides	$\frac{a^2\sinh(dx+c)+2ab\left(\sinh(dx+c)-2\arctan\left(e^{dx+c}\right)\right)+b^2\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\ \mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan($
default	$\frac{a^2\sinh(dx+c)+2ab\left(\sinh(dx+c)-2\arctan\left(e^{dx+c}\right)\right)+b^2\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\ \mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan($
risch	$\frac{e^{dx+c}a^2}{2d}+\frac{e^{dx+c}ab}{d}+\frac{e^{dx+c}b^2}{2d}-\frac{e^{-dx-c}a^2}{2d}-\frac{e^{-dx-c}ab}{d}-\frac{e^{-dx-c}b^2}{2d}+\frac{b^2e^{dx+c}\left(e^{2dx+2c}-1\right)}{d\left(e^{2dx+2c}+1\right)^2}+\frac{2ib\ln\left(e^{dx+}\right.}{d}$

input
```
int(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*sinh(d*x+c)+2*a*b*(sinh(d*x+c)-2*arctan(exp(d*x+c)))+b^2*(sinh(d*
x+c)^3/cosh(d*x+c)^2+3*sinh(d*x+c)/cosh(d*x+c)^2-3/2*sech(d*x+c)*tanh(d*x+
c)-3*arctan(exp(d*x+c))))
```

3.92.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(56) = 112$.

Time = 0.28 (sec) , antiderivative size = 774, normalized size of antiderivative = 12.90

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{(a^2+2\,ab+b^2)\cosh(dx+c)^6+6\,(a^2+2\,ab+b^2)\cosh(dx+c)\sinh(dx+c)^5+(a^2+2\,ab+b^2)\sinh(dx$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/2*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + (a^2 + 2*a*b
+ 3*b^2)*cosh(d*x + c)^4 + (15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 +
 2*a*b + 3*b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3
 + (a^2 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (a^2 + 2*a*b + 3
*b^2)*cosh(d*x + c)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 +
 2*a*b + 3*b^2)*cosh(d*x + c)^2 - a^2 - 2*a*b - 3*b^2)*sinh(d*x + c)^2 - a
^2 - 2*a*b - b^2 - 2*((4*a*b + 3*b^2)*cosh(d*x + c)^5 + 5*(4*a*b + 3*b^2)*
cosh(d*x + c)*sinh(d*x + c)^4 + (4*a*b + 3*b^2)*sinh(d*x + c)^5 + 2*(4*a*b
 + 3*b^2)*cosh(d*x + c)^3 + 2*(5*(4*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*a*b +
 3*b^2)*sinh(d*x + c)^3 + 2*(5*(4*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(4*a*b
+ 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + (4*a*b + 3*b^2)*cosh(d*x + c) +
(5*(4*a*b + 3*b^2)*cosh(d*x + c)^4 + 6*(4*a*b + 3*b^2)*cosh(d*x + c)^2 + 4
*a*b + 3*b^2)*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(3*
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 2*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c
)^3 - (a^2 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)
^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 + 2*d*cosh(d*x
+ c)^3 + 2*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + 2*(5*d*cosh(d*x + c
)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*d*cosh(d*x
 + c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

3.92.6 Sympy [F]

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^2\cosh\left(c+dx\right)dx$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*cosh(c + d*x), x)
```

3.92.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(56) = 112$.

Time $= 0.28$ (sec) , antiderivative size $= 152$, normalized size of antiderivative $= 2.53$

$$\begin{aligned}
&\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx \\
&= \frac{1}{2}b^2\left(\frac{6\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)}}{d} + \frac{4\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} + 1}{d(e^{(-dx-c)} + 2\,e^{(-3\,dx-3\,c)} + e^{(-5\,dx-5\,c)})}\right) \\
&\quad + ab\left(\frac{4\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{e^{(dx+c)}}{d} - \frac{e^{(-dx-c)}}{d}\right) + \frac{a^2\sinh\left(dx+c\right)}{d}
\end{aligned}$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*b^2*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) -
 e^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x -
 5*c)))) + a*b*(4*arctan(e^(-d*x - c))/d + e^(d*x + c)/d - e^(-d*x - c)/d)
 + a^2*sinh(d*x + c)/d
```

3.92.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(56) = 112$.

Time $= 0.33$ (sec) , antiderivative size $= 160$, normalized size of antiderivative $= 2.67$

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{2a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)+4ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)-\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}\right.\right.\right.}{4d}$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/4*(2*a^2*(e^(d*x + c) - e^(-d*x - c)) + 4*a*b*(e^(d*x + c) - e^(-d*x - c
)) + 2*b^2*(e^(d*x + c) - e^(-d*x - c)) - (pi + 2*arctan(1/2*(e^(2*d*x + 2
*c) - 1)*e^(-d*x - c)))*(4*a*b + 3*b^2) + 4*b^2*(e^(d*x + c) - e^(-d*x - c
))/((e^(d*x + c) - e^(-d*x - c))^2 + 4))/d
```

3.92.9 Mupad [B] (verification not implemented)

Time $= 0.21$ (sec) , antiderivative size $= 182$, normalized size of antiderivative $= 3.03$

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{e^{c+dx}(a+b)^2}{2d} - \frac{e^{-c-dx}(a+b)^2}{2d}$$
$$- \frac{\operatorname{atan}\left(\frac{e^{dx}e^{c}\left(3b^2\sqrt{d^2}+4ab\sqrt{d^2}\right)}{d\sqrt{16a^2b^2+24ab^3+9b^4}}\right)\sqrt{16a^2b^2+24ab^3+9b^4}}{\sqrt{d^2}}$$
$$+ \frac{b^2e^{c+dx}}{d\left(e^{2c+2dx}+1\right)} - \frac{2b^2e^{c+dx}}{d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}$$

input
```
int(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
(exp(c + d*x)*(a + b)^2)/(2*d) - (exp(- c - d*x)*(a + b)^2)/(2*d) - (atan(
(exp(d*x)*exp(c)*(3*b^2*(d^2)^(1/2) + 4*a*b*(d^2)^(1/2)))/(d*(24*a*b^3 + 9
*b^4 + 16*a^2*b^2)^(1/2)))*(24*a*b^3 + 9*b^4 + 16*a^2*b^2)^(1/2))/(d^2)^(1
/2) + (b^2*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)) - (2*b^2*exp(c + d*x))
/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.93 $\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

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3.93.1 Optimal result

Integrand size = 21, antiderivative size = 91

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{(8a^2+8ab+3b^2)\arctan(\sinh(c+dx))}{8d} - \frac{3b(2a+b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} - \frac{b\operatorname{sech}^3(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{4d}$$

output

```
1/8*(8*a^2+8*a*b+3*b^2)*arctan(sinh(d*x+c))/d-3/8*b*(2*a+b)*sech(d*x+c)*ta
nh(d*x+c)/d-1/4*b*sech(d*x+c)^3*(a+(a+b)*sinh(d*x+c)^2)*tanh(d*x+c)/d
```

3.93.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.31 (sec) , antiderivative size = 427, normalized size of antiderivative = 4.69

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{\operatorname{csch}^3(c+dx)\left(128\,{}_5F_4\left(\frac{3}{2},2,2,2,2;1,1,1,\frac{9}{2};-\sinh^2(c+dx)\right)\sinh^6(c+dx)\left(a+a\sinh^2(c+dx)+b\sinh^2(c+dx)\right)\right.}{}$$

input
```
Integrate[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-1/6720*(Csch[c + d*x]^3*(128*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1,
1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6*(a + a*Sinh[c + d*x]^2 + b*Sinh
[c + d*x]^2)^2 + 128*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[
c + d*x]^2]*Sinh[c + d*x]^6*(5*b^2*Sinh[c + d*x]^4 + 2*a*b*Sinh[c + d*x]^2
*(6 + 5*Sinh[c + d*x]^2) + a^2*(7 + 12*Sinh[c + d*x]^2 + 5*Sinh[c + d*x]^4
)) + 35*(b^2*Sinh[c + d*x]^4*(1947 + 485*Sinh[c + d*x]^2) + 2*a*b*Sinh[c +
 d*x]^2*(2625 + 2554*Sinh[c + d*x]^2 + 485*Sinh[c + d*x]^4) + a^2*(3375 +
5907*Sinh[c + d*x]^2 + 3161*Sinh[c + d*x]^4 + 485*Sinh[c + d*x]^6)) - (105
*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(b^2*Sinh[c + d*x]^4*(649 + 378*Sinh[c +
d*x]^2 + 9*Sinh[c + d*x]^4) + 2*a*b*Sinh[c + d*x]^2*(875 + 1143*Sinh[c + d
*x]^2 + 389*Sinh[c + d*x]^4 + 9*Sinh[c + d*x]^6) + a^2*(1125 + 2344*Sinh[c
 + d*x]^2 + 1674*Sinh[c + d*x]^4 + 400*Sinh[c + d*x]^6 + 9*Sinh[c + d*x]^8
)))/Sqrt[-Sinh[c + d*x]^2]))/d
```

3.93.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4159, 315, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \sec(ic+idx)\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\downarrow 4159$$

$$\frac{\int \frac{\left((a+b)\sinh^2(c+dx)+a\right)^2}{\left(\sinh^2(c+dx)+1\right)^3} d\sinh(c+dx)}{d}$$

$$\downarrow 315$$

$$\frac{\frac{1}{4}\int \frac{(a+b)(4a+3b)\sinh^2(c+dx)+a(4a+b)}{\left(\sinh^2(c+dx)+1\right)^2} d\sinh(c+dx) - \frac{b\sinh(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)}{4\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

$$\downarrow 298$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8a^2+8ab+3b^2\right)\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)-\frac{3b(2a+b)\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)-\frac{b\sinh(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)}{4\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

$$\downarrow 216$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(8a^2+8ab+3b^2\right)\arctan(\sinh(c+dx))-\frac{3b(2a+b)\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)-\frac{b\sinh(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)}{4\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-1/4*(b*Sinh[c + d*x]*(a + (a + b)*Sinh[c + d*x]^2))/(1 + Sinh[c + d*x]^2
)^2 + (((8*a^2 + 8*a*b + 3*b^2)*ArcTan[Sinh[c + d*x]])/2 - (3*b*(2*a + b)*
Sinh[c + d*x])/(2*(1 + Sinh[c + d*x]^2)))/4)/d
```

3.93.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.93.4 Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.47

method	result
derivativedivides	$\frac{2a^2\arctan\left(e^{dx+c}\right)+2ab\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b^2\left(-\frac{\sinh(dx+c)^3}{\cosh(dx+c)^4}-\frac{\sinh(dx+c)}{\cosh(dx+c)^4}+\left(\text{se}\right.\right.}{d}$
default	$\frac{2a^2\arctan\left(e^{dx+c}\right)+2ab\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b^2\left(-\frac{\sinh(dx+c)^3}{\cosh(dx+c)^4}-\frac{\sinh(dx+c)}{\cosh(dx+c)^4}+\left(\text{se}\right.\right.}{d}$
risch	$-\frac{b\,e^{dx+c}\left(8a\,e^{6dx+6c}+5b\,e^{6dx+6c}+8a\,e^{4dx+4c}-3b\,e^{4dx+4c}-8\,e^{2dx+2c}a+3b\,e^{2dx+2c}-8a-5b\right)}{4d\left(e^{2dx+2c}+1\right)^4}+\frac{i\ln\left(e^{dx+c}+i\right)a^2}{d}+$

input
```
int(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*a^2*arctan(exp(d*x+c))+2*a*b*(-sinh(d*x+c)/cosh(d*x+c)^2+1/2*sech(d
*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+b^2*(-sinh(d*x+c)^3/cosh(d*x+c)^4-1/
cosh(d*x+c)^4*sinh(d*x+c)+(1/4*sech(d*x+c)^3+3/8*sech(d*x+c))*tanh(d*x+c)+
3/4*arctan(exp(d*x+c))))
```

3.93.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1373 vs. 2(85) = 170.

Time = 0.26 (sec) , antiderivative size = 1373, normalized size of antiderivative = 15.09

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/4*((8*a*b + 5*b^2)*cosh(d*x + c)^7 + 7*(8*a*b + 5*b^2)*cosh(d*x + c)*si
nh(d*x + c)^6 + (8*a*b + 5*b^2)*sinh(d*x + c)^7 + (8*a*b - 3*b^2)*cosh(d*x
 + c)^5 + (21*(8*a*b + 5*b^2)*cosh(d*x + c)^2 + 8*a*b - 3*b^2)*sinh(d*x +
c)^5 + 5*(7*(8*a*b + 5*b^2)*cosh(d*x + c)^3 + (8*a*b - 3*b^2)*cosh(d*x + c
))*sinh(d*x + c)^4 - (8*a*b - 3*b^2)*cosh(d*x + c)^3 + (35*(8*a*b + 5*b^2)
*cosh(d*x + c)^4 + 10*(8*a*b - 3*b^2)*cosh(d*x + c)^2 - 8*a*b + 3*b^2)*sin
h(d*x + c)^3 + (21*(8*a*b + 5*b^2)*cosh(d*x + c)^5 + 10*(8*a*b - 3*b^2)*co
sh(d*x + c)^3 - 3*(8*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - ((8*a^2
 + 8*a*b + 3*b^2)*cosh(d*x + c)^8 + 8*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c
)*sinh(d*x + c)^7 + (8*a^2 + 8*a*b + 3*b^2)*sinh(d*x + c)^8 + 4*(8*a^2 + 8
*a*b + 3*b^2)*cosh(d*x + c)^6 + 4*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)
^2 + 8*a^2 + 8*a*b + 3*b^2)*sinh(d*x + c)^6 + 8*(7*(8*a^2 + 8*a*b + 3*b^2)
*cosh(d*x + c)^3 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^
5 + 6*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(8*a^2 + 8*a*b + 3*b
^2)*cosh(d*x + c)^4 + 30*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 24*a^2
+ 24*a*b + 9*b^2)*sinh(d*x + c)^4 + 8*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x
+ c)^5 + 10*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(8*a^2 + 8*a*b + 3
*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x
+ c)^2 + 4*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^6 + 15*(8*a^2 + 8*a*b
+ 3*b^2)*cosh(d*x + c)^4 + 9*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + ...
```

3.93.6 Sympy [F]

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^2 \operatorname{sech}\left(c+dx\right)dx$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x), x)
```

3.93.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(85) = 170$.

Time $= 0.29$ (sec) , antiderivative size $= 199$, normalized size of antiderivative $= 2.19$

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$
$$-\frac{1}{4}b^2\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{5\,e^{(-dx-c)}-3\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}-5\,e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)$$
$$-2\,ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)+\frac{a^2\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*b^2*(3*arctan(e^(-d*x - c))/d + (5*e^(-d*x - c) - 3*e^(-3*d*x - 3*c)
+ 3*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-
4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 2*a*b*(arcta
n(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*
c) + e^(-4*d*x - 4*c) + 1))) + a^2*arctan(sinh(d*x + c))/d
```

3.93.8 Giac [A] (verification not implemented)

Time $= 0.32$ (sec) , antiderivative size $= 170$, normalized size of antiderivative $= 1.87$

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{\left(\pi+2\arctan\left(\tfrac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(8\,a^2+8\,ab+3\,b^2)-\frac{4\left(8\,ab(e^{(dx+c)}-e^{(-dx-c)})^3+5\,b^2(e^{(dx+c)}-e^{(-dx-c)})^3\right.}{\left((e^{(dx+c)}-e^{(}\right.}}{16\,d}$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/16*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(8*a^2 + 8*a
*b + 3*b^2) - 4*(8*a*b*(e^(d*x + c) - e^(-d*x - c))^3 + 5*b^2*(e^(d*x + c)
 - e^(-d*x - c))^3 + 32*a*b*(e^(d*x + c) - e^(-d*x - c)) + 12*b^2*(e^(d*x
+ c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

3.93.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.33

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$= \frac{\operatorname{atan}\left(\frac{e^{dx}e^{c}\left(8a^2\sqrt{d^2}+3b^2\sqrt{d^2}+8ab\sqrt{d^2}\right)}{d\sqrt{64a^4+128a^3b+112a^2b^2+48ab^3+9b^4}}\right)\sqrt{64a^4+128a^3b+112a^2b^2+48ab^3+9b^4}}{4\sqrt{d^2}}$$
$$- \frac{6b^2e^{c+dx}}{d\left(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1\right)}$$
$$+ \frac{4b^2e^{c+dx}}{d\left(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1\right)}$$
$$- \frac{e^{c+dx}\left(5b^2+8ab\right)}{4d\left(e^{2c+2dx}+1\right)} + \frac{e^{c+dx}\left(9b^2+8ab\right)}{2d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)^2/cosh(c + d*x),x)
```

output
```
(atan((exp(d*x)*exp(c)*(8*a^2*(d^2)^(1/2) + 3*b^2*(d^2)^(1/2) + 8*a*b*(d^2
)^(1/2)))/(d*(48*a*b^3 + 128*a^3*b + 64*a^4 + 9*b^4 + 112*a^2*b^2)^(1/2)))
*(48*a*b^3 + 128*a^3*b + 64*a^4 + 9*b^4 + 112*a^2*b^2)^(1/2))/(4*(d^2)^(1/
2)) - (6*b^2*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + e
xp(6*c + 6*d*x) + 1)) + (4*b^2*exp(c + d*x))/(d*(4*exp(2*c + 2*d*x) + 6*ex
p(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (exp(c + d*
x)*(8*a*b + 5*b^2))/(4*d*(exp(2*c + 2*d*x) + 1)) + (exp(c + d*x)*(8*a*b +
9*b^2))/(2*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.94 $\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

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3.94.1 Optimal result

Integrand size = 23, antiderivative size = 49

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{a^2\tanh(c+dx)}{d} + \frac{2ab\tanh^3(c+dx)}{3d} + \frac{b^2\tanh^5(c+dx)}{5d}$$

output

```
a^2*tanh(d*x+c)/d+2/3*a*b*tanh(d*x+c)^3/d+1/5*b^2*tanh(d*x+c)^5/d
```

3.94.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{a^2\tanh(c+dx)}{d} + \frac{2ab\tanh^3(c+dx)}{3d} + \frac{b^2\tanh^5(c+dx)}{5d}$$

input

```
Integrate[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output

```
(a^2*Tanh[c + d*x])/d + (2*a*b*Tanh[c + d*x]^3)/(3*d) + (b^2*Tanh[c + d*x]
^5)/(5*d)
```

3.94.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \sec(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\downarrow 4158$$

$$\frac{\int \left(b\tanh^2(c+dx)+a\right)^2 d\tanh(c+dx)}{d}$$

$$\downarrow 210$$

$$\frac{\int \left(b^2\tanh^4(c+dx)+2ab\tanh^2(c+dx)+a^2\right) d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{a^2\tanh(c+dx)+\frac{2}{3}ab\tanh^3(c+dx)+\frac{1}{5}b^2\tanh^5(c+dx)}{d}$$

input
```
Int[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(a^2*Tanh[c + d*x] + (2*a*b*Tanh[c + d*x]^3)/3 + (b^2*Tanh[c + d*x]^5)/5)/
d
```

3.94.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.94.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(45) = 90$.

Time = 6.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.57

method	result
derivativedivides	$\frac{a^2\tanh(dx+c)+2ab\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+b^2\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\frac{\sec}{}\right.}{}}{d}$
default	$\frac{a^2\tanh(dx+c)+2ab\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+b^2\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\frac{\sec}{}\right.}{}}{d}$
risch	$-\frac{2\left(15a^2e^{8dx+8c}+30ab\,e^{8dx+8c}+15b^2e^{8dx+8c}+60a^2e^{6dx+6c}+60ab\,e^{6dx+6c}+90a^2e^{4dx+4c}+40ab\,e^{4dx+4c}+30\,e^{4dx+4cb}\right.}{15d\left(e^{2dx+2c}+1\right)^5}$

input
```
int(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*tanh(d*x+c)+2*a*b*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3+1/3*se
ch(d*x+c)^2)*tanh(d*x+c))+b^2*(-1/2*sinh(d*x+c)^3/cosh(d*x+c)^5-3/8*sinh(d
*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d
*x+c)))
```

3.94.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(45) = 90$.

Time $= 0.27$ (sec) , antiderivative size $= 391$, normalized size of antiderivative $= 7.98$

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$

$$-\frac{4\left(\left(15\,a^2+20\,ab+9\,b^2\right)\cosh\left(dx+c\right)^4+8\left(5\,ab+3\,b^2\right)\cosh\left(dx+c\right)\sinh\left(dx+\right.\right.}{15\left(d\cosh\left(dx+c\right)^6+6\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+d\sinh\left(dx+c\right)^6+6\,d\cosh\left(dx+c\right)^4+3\left(5\,d\,\mathrm{co}\right.}$$

input
```
integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-4/15*((15*a^2 + 20*a*b + 9*b^2)*cosh(d*x + c)^4 + 8*(5*a*b + 3*b^2)*cosh(
d*x + c)*sinh(d*x + c)^3 + (15*a^2 + 20*a*b + 9*b^2)*sinh(d*x + c)^4 + 20*
(3*a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*(15*a^2 + 20*a*b + 9*b^2)*cosh(d*x
+ c)^2 + 30*a^2 + 20*a*b)*sinh(d*x + c)^2 + 45*a^2 + 20*a*b + 15*b^2 + 8*(
(5*a*b + 3*b^2)*cosh(d*x + c)^3 + 5*a*b*cosh(d*x + c))*sinh(d*x + c))/(d*c
osh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 + 6
*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^4 + 4*(5*
d*cosh(d*x + c)^3 + 4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 15*d*cosh(d*x + c
)^2 + 3*(5*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^2
 + 2*(3*d*cosh(d*x + c)^5 + 8*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(
d*x + c) + 10*d)
```

3.94.6 Sympy [F]

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^2\operatorname{sech}^2\left(c+dx\right)\,dx$$

input
```
integrate(sech(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x)**2, x)
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{b^2\tanh(dx+c)^5}{5\,d} + \frac{2\,ab\tanh(dx+c)^3}{3\,d} + \frac{2\,a^2}{d(e^{(-2\,dx-2\,c)}+1)}$$

input
```
integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/5*b^2*tanh(d*x + c)^5/d + 2/3*a*b*tanh(d*x + c)^3/d + 2*a^2/(d*(e^(-2*d*
x - 2*c) + 1))
```

3.94.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(45) = 90.

Time = 0.33 (sec) , antiderivative size = 169, normalized size of antiderivative = 3.45

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{2\left(15\,a^2e^{(8\,dx+8\,c)} + 30\,abe^{(8\,dx+8\,c)} + 15\,b^2e^{(8\,dx+8\,c)} + 60\,a^2e^{(6\,dx+6\,c)} + 60\,abe^{(6\,dx+6\,c)} + 90\,a^2e^{(4\,dx+4\,c)} + 4\right.}{15\,d(e^{(2\,dx+2\,c)}+1)}$$

input
```
integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-2/15*(15*a^2*e^(8*d*x + 8*c) + 30*a*b*e^(8*d*x + 8*c) + 15*b^2*e^(8*d*x +
 8*c) + 60*a^2*e^(6*d*x + 6*c) + 60*a*b*e^(6*d*x + 6*c) + 90*a^2*e^(4*d*x
+ 4*c) + 40*a*b*e^(4*d*x + 4*c) + 30*b^2*e^(4*d*x + 4*c) + 60*a^2*e^(2*d*x
 + 2*c) + 20*a*b*e^(2*d*x + 2*c) + 15*a^2 + 10*a*b + 3*b^2)/(d*(e^(2*d*x +
 2*c) + 1)^5)
```

3.94.9 Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 482, normalized size of antiderivative = 9.84

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= -\frac{\frac{2\left(a^2-b^2\right)}{5d}+\frac{2\,\mathrm{e}^{2c+2dx}(a+b)^2}{5d}}{2\,\mathrm{e}^{2c+2dx}+\mathrm{e}^{4c+4dx}+1}$$
$$-\frac{\frac{2\left(a^2-b^2\right)}{5d}+\frac{6\,\mathrm{e}^{4c+4dx}\left(a^2-b^2\right)}{5d}+\frac{2\,\mathrm{e}^{6c+6dx}(a+b)^2}{5d}+\frac{2\,\mathrm{e}^{2c+2dx}\left(3a^2-2ab+3b^2\right)}{5d}}{4\,\mathrm{e}^{2c+2dx}+6\,\mathrm{e}^{4c+4dx}+4\,\mathrm{e}^{6c+6dx}+\mathrm{e}^{8c+8dx}+1}$$
$$-\frac{\frac{2(a+b)^2}{5d}+\frac{8\,\mathrm{e}^{2c+2dx}\left(a^2-b^2\right)}{5d}+\frac{8\,\mathrm{e}^{6c+6dx}\left(a^2-b^2\right)}{5d}+\frac{2\,\mathrm{e}^{8c+8dx}(a+b)^2}{5d}+\frac{4\,\mathrm{e}^{4c+4dx}\left(3a^2-2ab+3b^2\right)}{5d}}{5\,\mathrm{e}^{2c+2dx}+10\,\mathrm{e}^{4c+4dx}+10\,\mathrm{e}^{6c+6dx}+5\,\mathrm{e}^{8c+8dx}+\mathrm{e}^{10c+10dx}+1}$$
$$-\frac{\frac{2\left(3a^2-2ab+3b^2\right)}{15d}+\frac{4\,\mathrm{e}^{2c+2dx}\left(a^2-b^2\right)}{5d}+\frac{2\,\mathrm{e}^{4c+4dx}(a+b)^2}{5d}}{3\,\mathrm{e}^{2c+2dx}+3\,\mathrm{e}^{4c+4dx}+\mathrm{e}^{6c+6dx}+1}-\frac{2(a+b)^2}{5d\left(\mathrm{e}^{2c+2dx}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)^2/cosh(c + d*x)^2,x)
```

output
```
- ((2*(a^2 - b^2))/(5*d) + (2*exp(2*c + 2*d*x)*(a + b)^2)/(5*d))/(2*exp(2*
c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*(a^2 - b^2))/(5*d) + (6*exp(4*c +
 4*d*x)*(a^2 - b^2))/(5*d) + (2*exp(6*c + 6*d*x)*(a + b)^2)/(5*d) + (2*exp
(2*c + 2*d*x)*(3*a^2 - 2*a*b + 3*b^2))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(
4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(a + b)^2)
/(5*d) + (8*exp(2*c + 2*d*x)*(a^2 - b^2))/(5*d) + (8*exp(6*c + 6*d*x)*(a^2
 - b^2))/(5*d) + (2*exp(8*c + 8*d*x)*(a + b)^2)/(5*d) + (4*exp(4*c + 4*d*x
)*(3*a^2 - 2*a*b + 3*b^2))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x
) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - (
(2*(3*a^2 - 2*a*b + 3*b^2))/(15*d) + (4*exp(2*c + 2*d*x)*(a^2 - b^2))/(5*d
) + (2*exp(4*c + 4*d*x)*(a + b)^2)/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c
+ 4*d*x) + exp(6*c + 6*d*x) + 1) - (2*(a + b)^2)/(5*d*(exp(2*c + 2*d*x) +
1))
```

3.95 $\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

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3.95.2 Mathematica [C] (warning: unable to verify) 762
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3.95.1 Optimal result

Integrand size = 23, antiderivative size = 125

$$\begin{aligned}
&\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx \\
&= \frac{(8a^2+4ab+b^2)\arctan(\sinh(c+dx))}{16d} + \frac{(8a^2+4ab+b^2)\operatorname{sech}(c+dx)\tanh(c+dx)}{16d} \\
&\quad - \frac{b(8a+3b)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{24d} \\
&\quad - \frac{b\operatorname{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{6d}
\end{aligned}$$

output

```
1/16*(8*a^2+4*a*b+b^2)*arctan(sinh(d*x+c))/d+1/16*(8*a^2+4*a*b+b^2)*sech(d
*x+c)*tanh(d*x+c)/d-1/24*b*(8*a+3*b)*sech(d*x+c)^3*tanh(d*x+c)/d-1/6*b*sec
h(d*x+c)^5*(a+(a+b)*sinh(d*x+c)^2)*tanh(d*x+c)/d
```

3.95.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 9.28 (sec) , antiderivative size = 792, normalized size of antiderivative = 6.34

$$\begin{aligned}
&\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx \\
&= \frac{a^2\sinh(c+dx)\left(-\frac{23555(a+b)}{a} - \frac{32970(a+b)^2}{a^2} - 14980\operatorname{csch}^2(c+dx) - \frac{91875(a+b)\operatorname{csch}^2(c+dx)}{a} - 65625\operatorname{csch}^4(c+\right.}{}
\end{aligned}$$

input
```
Integrate[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(a^2*Sinh[c + d*x]*((-23555*(a + b))/a - (32970*(a + b)^2)/a^2 - 14980*Csc
h[c + d*x]^2 - (91875*(a + b)*Csch[c + d*x]^2)/a - 65625*Csch[c + d*x]^4 -
 (8855*(a + b)^2*Sinh[c + d*x]^2)/a^2 - 620*HypergeometricPFQ[{3/2, 2, 2,
2}, {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^2 - 160*HypergeometricPFQ
[{3/2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^2 - 16
*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c + d*x]
^2]*Sinh[c + d*x]^2 - (968*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1
, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4)/a - (288*(a + b)*Hypergeometric
PFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4)/
a - (32*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2},
 -Sinh[c + d*x]^2]*Sinh[c + d*x]^4)/a - (380*(a + b)^2*HypergeometricPFQ[{
3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6)/a^2 - (128*
(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, -Sinh[c + d
*x]^2]*Sinh[c + d*x]^6)/a^2 - (16*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2,
2, 2, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6)/a^2 + (656
25*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/(-Sinh[c + d*x]^2)^(5/2) + (1680*ArcTa
nh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4)/(-Sinh[c + d*x]^2)^(5/2) - (36
855*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/(-Sinh[c + d*x]^2)^(3/2) - (91875*(a
+ b)*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/(a*(-Sinh[c + d*x]^2)^(3/2)) + (5418
0*(a + b)*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/(a*Sqrt[-Sinh[c + d*x]^2]) +...
```

3.95.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4159, 315, 298, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \sec(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\downarrow 4159$$

$$\frac{\int \frac{((a+b)\sinh^2(c+dx)+a)^2}{(\sinh^2(c+dx)+1)^4} d\sinh(c+dx)}{d}$$

$$\downarrow 315$$

$$\frac{\frac{1}{6}\int \frac{3(a+b)(2a+b)\sinh^2(c+dx)+a(6a+b)}{(\sinh^2(c+dx)+1)^3} d\sinh(c+dx) - \frac{b\sinh(c+dx)((a+b)\sinh^2(c+dx)+a)}{6(\sinh^2(c+dx)+1)^3}}{d}$$

$$\downarrow 298$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}(8a^2+4ab+b^2)\int \frac{1}{(\sinh^2(c+dx)+1)^2} d\sinh(c+dx) - \frac{b(8a+3b)\sinh(c+dx)}{4(\sinh^2(c+dx)+1)^2}\right) - \frac{b\sinh(c+dx)((a+b)\sinh^2(c+dx)+a)}{6(\sinh^2(c+dx)+1)^3}}{d}$$

$$\downarrow 215$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}(8a^2+4ab+b^2)\left(\frac{1}{2}\int \frac{1}{\sinh^2(c+dx)+1} d\sinh(c+dx) + \frac{\sinh(c+dx)}{2(\sinh^2(c+dx)+1)}\right) - \frac{b(8a+3b)\sinh(c+dx)}{4(\sinh^2(c+dx)+1)^2}\right) - \frac{b\sinh(c+dx)((a+b)}{6(\sinh^2(c+}}{d}$$

$$\downarrow 216$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}(8a^2+4ab+b^2)\left(\frac{1}{2}\arctan(\sinh(c+dx)) + \frac{\sinh(c+dx)}{2(\sinh^2(c+dx)+1)}\right) - \frac{b(8a+3b)\sinh(c+dx)}{4(\sinh^2(c+dx)+1)^2}\right) - \frac{b\sinh(c+dx)((a+b)\sinh^2(c+}{6(\sinh^2(c+dx)+1)^3}}{d}$$

input
```
Int[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-1/6*(b*Sinh[c + d*x]*(a + (a + b)*Sinh[c + d*x]^2))/(1 + Sinh[c + d*x]^2
)^3 + (-1/4*(b*(8*a + 3*b)*Sinh[c + d*x])/(1 + Sinh[c + d*x]^2)^2 + (3*(8*
a^2 + 4*a*b + b^2)*(ArcTan[Sinh[c + d*x]]/2 + Sinh[c + d*x]/(2*(1 + Sinh[c
 + d*x]^2))))/4)/6)/d
```

3.95.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.95.4 Maple [A] (verified)

Time = 14.47 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{a^2\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+2ab\left(-\frac{\sinh(dx+c)}{3\cosh(dx+c)^4}+\frac{\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)}{3}+\frac{\arctan\left(e^{d}\right.}{4}\right.}{d}$
default	$\frac{a^2\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+2ab\left(-\frac{\sinh(dx+c)}{3\cosh(dx+c)^4}+\frac{\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)}{3}+\frac{\arctan\left(e^{d}\right.}{4}\right.}{d}$
risch	$\frac{e^{dx+c}\left(24a^2e^{10dx+10c}+12ab\,e^{10dx+10c}+3b^2e^{10dx+10c}+72a^2e^{8dx+8c}-60ab\,e^{8dx+8c}-47b^2e^{8dx+8c}+48a^2e^{6dx+6c}-72ab\right.}{24d}$

input
```
int(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*sech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+2*a*b*(-1/3/cosh
(d*x+c)^4*sinh(d*x+c)+1/3*(1/4*sech(d*x+c)^3+3/8*sech(d*x+c))*tanh(d*x+c)+
1/4*arctan(exp(d*x+c)))+b^2*(-1/3*sinh(d*x+c)^3/cosh(d*x+c)^6-1/5*sinh(d*x
+c)/cosh(d*x+c)^6+1/5*(1/6*sech(d*x+c)^5+5/24*sech(d*x+c)^3+5/16*sech(d*x+
c))*tanh(d*x+c)+1/8*arctan(exp(d*x+c))))
```

3.95.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2824 vs. $2(117) = 234$.

Time $= 0.28$ (sec) , antiderivative size $= 2824$, normalized size of antiderivative $= 22.59$

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/24*(3*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^11 + 33*(8*a^2 + 4*a*b + b^2)*
cosh(d*x + c)*sinh(d*x + c)^10 + 3*(8*a^2 + 4*a*b + b^2)*sinh(d*x + c)^11
+ (72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^9 + (165*(8*a^2 + 4*a*b + b^2)*
cosh(d*x + c)^2 + 72*a^2 - 60*a*b - 47*b^2)*sinh(d*x + c)^9 + 9*(55*(8*a^2
 + 4*a*b + b^2)*cosh(d*x + c)^3 + (72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^8 + 6*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^7 + 6*(165*(
8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 6*(72*a^2 - 60*a*b - 47*b^2)*cosh(d
*x + c)^2 + 8*a^2 - 12*a*b + 13*b^2)*sinh(d*x + c)^7 + 42*(33*(8*a^2 + 4*a
*b + b^2)*cosh(d*x + c)^5 + 2*(72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^3 +
 (8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 - 6*(8*a^2 - 12*
a*b + 13*b^2)*cosh(d*x + c)^5 + 6*(231*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)
^6 + 21*(72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^4 + 21*(8*a^2 - 12*a*b +
13*b^2)*cosh(d*x + c)^2 - 8*a^2 + 12*a*b - 13*b^2)*sinh(d*x + c)^5 + 6*(16
5*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^7 + 21*(72*a^2 - 60*a*b - 47*b^2)*co
sh(d*x + c)^5 + 35*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^3 - 5*(8*a^2 -
12*a*b + 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 - (72*a^2 - 60*a*b - 47*b^
2)*cosh(d*x + c)^3 + (495*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^8 + 84*(72*a
^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^6 + 210*(8*a^2 - 12*a*b + 13*b^2)*cosh
(d*x + c)^4 - 60*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^2 - 72*a^2 + 60*a
*b + 47*b^2)*sinh(d*x + c)^3 + 3*(55*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c...
```

3.95.6 Sympy [F]

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^2 \operatorname{sech}^3\left(c+dx\right)\,dx$$

input
```
integrate(sech(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x)**3, x)
```

3.95.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(117) = 234$.

Time = 0.28 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.76

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$
$$-\frac{1}{24}b^2\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3e^{(-dx-c)}-47e^{(-3dx-3c)}+78e^{(-5dx-5c)}-78e^{(-7dx-7c)}+47e^{(-9dx-}}{d(6e^{(-2dx-2c)}+15e^{(-4dx-4c)}+20e^{(-6dx-6c)}+15e^{(-8dx-8c)}+6e^{(-10dx-}}\right.$$
$$-\frac{1}{2}ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-7e^{(-3dx-3c)}+7e^{(-5dx-5c)}-e^{(-7dx-7c)}}{d(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1)}\right)$$
$$\left.-a^2\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3dx-3c)}}{d(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1)}\right)\right.$$

input
```
integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/24*b^2*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) - 47*e^(-3*d*x - 3*c
) + 78*e^(-5*d*x - 5*c) - 78*e^(-7*d*x - 7*c) + 47*e^(-9*d*x - 9*c) - 3*e^
(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*
d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*
c) + 1))) - 1/2*a*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - 7*e^(-3*d*x
 - 3*c) + 7*e^(-5*d*x - 5*c) - e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6
*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a^2*(ar
ctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x -
 2*c) + e^(-4*d*x - 4*c) + 1)))
```

3.95.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(117) = 234$.

Time = 0.36 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.14

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)(8a^2+4ab+b^2)+\frac{4\left(24a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+12ab\left(e^{(dx+c)}-e^{(-dx-c)}\right.\right.}{}$$

input
```
integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/96*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(8*a^2 + 4
*a*b + b^2) + 4*(24*a^2*(e^(d*x + c) - e^(-d*x - c))^5 + 12*a*b*(e^(d*x +
c) - e^(-d*x - c))^5 + 3*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 192*a^2*(e^(
d*x + c) - e^(-d*x - c))^3 - 32*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 384*a
^2*(e^(d*x + c) - e^(-d*x - c)) - 192*a*b*(e^(d*x + c) - e^(-d*x - c)) - 4
8*b^2*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^3
)/d
```

3.95.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 572, normalized size of antiderivative = 4.58

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$=\frac{\operatorname{atan}\left(\frac{e^{dx}e^{c}\left(8a^2\sqrt{d^2}+b^2\sqrt{d^2}+4ab\sqrt{d^2}\right)}{d\sqrt{64a^4+64a^3b+32a^2b^2+8ab^3+b^4}}\right)\sqrt{64a^4+64a^3b+32a^2b^2+8ab^3+b^4}}{8\sqrt{d^2}}$$
$$-\frac{\frac{2e^{c+dx}(a+b)^2}{3d}+\frac{8e^{3c+3dx}(a^2-b^2)}{3d}+\frac{8e^{7c+7dx}(a^2-b^2)}{3d}+\frac{2e^{9c+9dx}(a+b)^2}{3d}+\frac{4e^{5c+5dx}(3a^2-2ab+3b^2)}{3d}}{6e^{2c+2dx}+15e^{4c+4dx}+20e^{6c+6dx}+15e^{8c+8dx}+6e^{10c+10dx}+e^{12c+12dx}+1}$$
$$-\frac{2e^{c+dx}(15b^2+4ab)}{3d\left(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1\right)}$$
$$+\frac{16b^2e^{c+dx}}{3d\left(5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1\right)}$$
$$+\frac{e^{c+dx}(8a^2+4ab+b^2)}{8d\left(e^{2c+2dx}+1\right)}-\frac{e^{c+dx}(16a^2+44ab+23b^2)}{12d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}$$
$$+\frac{e^{c+dx}(21b^2+20ab)}{3d\left(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)^2/cosh(c + d*x)^3,x)
```

output

```
(atan((exp(d*x)*exp(c)*(8*a^2*(d^2)^(1/2) + b^2*(d^2)^(1/2) + 4*a*b*(d^2)^
(1/2)))/(d*(8*a*b^3 + 64*a^3*b + 64*a^4 + b^4 + 32*a^2*b^2)^(1/2)))*(8*a*b
^3 + 64*a^3*b + 64*a^4 + b^4 + 32*a^2*b^2)^(1/2))/(8*(d^2)^(1/2)) - ((2*ex
p(c + d*x)*(a + b)^2)/(3*d) + (8*exp(3*c + 3*d*x)*(a^2 - b^2))/(3*d) + (8*
exp(7*c + 7*d*x)*(a^2 - b^2))/(3*d) + (2*exp(9*c + 9*d*x)*(a + b)^2)/(3*d)
 + (4*exp(5*c + 5*d*x)*(3*a^2 - 2*a*b + 3*b^2))/(3*d))/(6*exp(2*c + 2*d*x)
 + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp
(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1) - (2*exp(c + d*x)*(4*a*b + 15*b^
2))/(3*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + e
xp(8*c + 8*d*x) + 1)) + (16*b^2*exp(c + d*x))/(3*d*(5*exp(2*c + 2*d*x) + 1
0*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c +
 10*d*x) + 1)) + (exp(c + d*x)*(4*a*b + 8*a^2 + b^2))/(8*d*(exp(2*c + 2*d*
x) + 1)) - (exp(c + d*x)*(44*a*b + 16*a^2 + 23*b^2))/(12*d*(2*exp(2*c + 2*
d*x) + exp(4*c + 4*d*x) + 1)) + (exp(c + d*x)*(20*a*b + 21*b^2))/(3*d*(3*e
xp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1))
```

3.96 $\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

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3.96.1 Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{a^2\tanh(c+dx)}{d} - \frac{a(a-2b)\tanh^3(c+dx)}{3d} - \frac{(2a-b)b\tanh^5(c+dx)}{5d} - \frac{b^2\tanh^7(c+dx)}{7d}$$

output
```
a^2*tanh(d*x+c)/d-1/3*a*(a-2*b)*tanh(d*x+c)^3/d-1/5*(2*a-b)*b*tanh(d*x+c)^
5/d-1/7*b^2*tanh(d*x+c)^7/d
```

3.96.2 Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{(70a^2+28ab+6b^2+(35a^2+14ab+3b^2)\operatorname{sech}^2(c+dx)-6b(7a+4b)\operatorname{sech}^4(c+dx)+15b^2\operatorname{sech}^6(c+dx)}{105d}$$

input
```
Integrate[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((70*a^2 + 28*a*b + 6*b^2 + (35*a^2 + 14*a*b + 3*b^2)*Sech[c + d*x]^2 - 6*
b*(7*a + 4*b)*Sech[c + d*x]^4 + 15*b^2*Sech[c + d*x]^6)*Tanh[c + d*x])/(10
5*d)
```

3.96.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \sec(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\downarrow 4158$$

$$\frac{\int \left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2 d\tanh(c+dx)}{d}$$

$$\downarrow 290$$

$$\frac{\int \left(-b^2\tanh^6(c+dx)-(2a-b)b\tanh^4(c+dx)-a(a-2b)\tanh^2(c+dx)+a^2\right) d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{a^2\tanh(c+dx)-\frac{1}{5}b(2a-b)\tanh^5(c+dx)-\frac{1}{3}a(a-2b)\tanh^3(c+dx)-\frac{1}{7}b^2\tanh^7(c+dx)}{d}$$

input
```
Int[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(a^2*Tanh[c + d*x] - (a*(a - 2*b)*Tanh[c + d*x]^3)/3 - ((2*a - b)*b*Tanh[c
 + d*x]^5)/5 - (b^2*Tanh[c + d*x]^7)/7)/d
```

3.96.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.96.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(70) = 140$.

Time = 29.89 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.08

method	result
derivativedivides	$\frac{a^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+2ab\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{4}\right)+b^2\left(-\frac{\sinh(}{4\cosh(}\right.}{d}$
default	$\frac{a^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+2ab\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{4}\right)+b^2\left(-\frac{\sinh(}{4\cosh(}\right.}{d}$
risch	$-\frac{4\left(105a^2e^{10dx+10c}+210ab\,e^{10dx+10c}+105b^2e^{10dx+10c}+455a^2e^{8dx+8c}+350ab\,e^{8dx+8c}-105b^2e^{8dx+8c}+770a^2e^{6dx+6c}\right.}{}$

input
```
int(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+2*a*b*(-1/4*sinh(d*x+c)/cosh(
d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))+b^2*
(-1/4*sinh(d*x+c)^3/cosh(d*x+c)^7-1/8*sinh(d*x+c)/cosh(d*x+c)^7+1/8*(16/35
+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c)))
```

3.96.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(70) = 140$.

Time $= 0.25$ (sec) , antiderivative size $= 677$, normalized size of antiderivative $= 8.91$

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$

$$-\frac{8\left(2\left(35\right.\right.}{105\left(d\cosh\left(dx+c\right)^9+9\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^8+d\sinh\left(dx+c\right)^9+7\,d\cosh\left(dx+c\right)^7+\left(36\,d\,c\right.\right.}$$

input
```
integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-8/105*(2*(35*a^2 + 56*a*b + 27*b^2)*cosh(d*x + c)^5 + 10*(35*a^2 + 56*a*b
 + 27*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 + (35*a^2 + 98*a*b + 51*b^2)*sinh
(d*x + c)^5 + 14*(25*a^2 + 16*a*b - 3*b^2)*cosh(d*x + c)^3 + (10*(35*a^2 +
 98*a*b + 51*b^2)*cosh(d*x + c)^2 + 105*a^2 + 126*a*b - 63*b^2)*sinh(d*x +
 c)^3 + 2*(10*(35*a^2 + 56*a*b + 27*b^2)*cosh(d*x + c)^3 + 21*(25*a^2 + 16
*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 28*(25*a^2 + 4*a*b + 3*b^2)
*cosh(d*x + c) + (5*(35*a^2 + 98*a*b + 51*b^2)*cosh(d*x + c)^4 + 63*(5*a^2
 + 6*a*b - 3*b^2)*cosh(d*x + c)^2 + 70*a^2 + 28*a*b + 126*b^2)*sinh(d*x +
c))/(d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + d*sinh(d*x +
c)^9 + 7*d*cosh(d*x + c)^7 + (36*d*cosh(d*x + c)^2 + 7*d)*sinh(d*x + c)^7
+ 7*(12*d*cosh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)^6 + 22*d*cosh
(d*x + c)^5 + (126*d*cosh(d*x + c)^4 + 147*d*cosh(d*x + c)^2 + 20*d)*sinh(
d*x + c)^5 + (126*d*cosh(d*x + c)^5 + 245*d*cosh(d*x + c)^3 + 110*d*cosh(d
*x + c))*sinh(d*x + c)^4 + 42*d*cosh(d*x + c)^3 + (84*d*cosh(d*x + c)^6 +
245*d*cosh(d*x + c)^4 + 200*d*cosh(d*x + c)^2 + 28*d)*sinh(d*x + c)^3 + (3
6*d*cosh(d*x + c)^7 + 147*d*cosh(d*x + c)^5 + 220*d*cosh(d*x + c)^3 + 126*
d*cosh(d*x + c))*sinh(d*x + c)^2 + 56*d*cosh(d*x + c) + (9*d*cosh(d*x + c)
^8 + 49*d*cosh(d*x + c)^6 + 100*d*cosh(d*x + c)^4 + 84*d*cosh(d*x + c)^2 +
 14*d)*sinh(d*x + c))
```

3.96.6 Sympy [F]

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2(c+dx)\right)^2 \operatorname{sech}^4(c+dx)\,dx$$

input
```
integrate(sech(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x)**4, x)
```

3.96.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 928 vs. $2(70) = 140$.

Time $= 0.21$ (sec) , antiderivative size $= 928$, normalized size of antiderivative $= 12.21$

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
4/35*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
+ 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^
(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 14*e^(-4*d*x - 4*c)/(d*(7*e^
(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x
- 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c)
 + 1)) + 70*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
+ 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^
(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 35*e^(-8*d*x - 8*c)/(d*(7*e^
(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x
- 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c)
 + 1)) + 35*e^(-10*d*x - 10*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c
) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*
e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) +
 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-
10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 8/15*a
*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e
^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^(-4*
d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6
*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/
(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*...
```

3.96.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(70) = 140$.

Time $= 0.36$ (sec) , antiderivative size $= 238$, normalized size of antiderivative $= 3.13$

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$

$$-\frac{4\left(105\,a^2e^{(10\,dx+10\,c)}+210\,abe^{(10\,dx+10\,c)}+105\,b^2e^{(10\,dx+10\,c)}+455\,a^2e^{(8\,dx+8\,c)}+350\,abe^{(8\,dx+8\,c)}-105\,b^2\right.}{}$$

input
```
integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-4/105*(105*a^2*e^(10*d*x + 10*c) + 210*a*b*e^(10*d*x + 10*c) + 105*b^2*e^
(10*d*x + 10*c) + 455*a^2*e^(8*d*x + 8*c) + 350*a*b*e^(8*d*x + 8*c) - 105*
b^2*e^(8*d*x + 8*c) + 770*a^2*e^(6*d*x + 6*c) + 140*a*b*e^(6*d*x + 6*c) +
210*b^2*e^(6*d*x + 6*c) + 630*a^2*e^(4*d*x + 4*c) + 84*a*b*e^(4*d*x + 4*c)
 - 42*b^2*e^(4*d*x + 4*c) + 245*a^2*e^(2*d*x + 2*c) + 98*a*b*e^(2*d*x + 2*
c) + 21*b^2*e^(2*d*x + 2*c) + 35*a^2 + 14*a*b + 3*b^2)/(d*(e^(2*d*x + 2*c)
 + 1)^7)
```

3.96.9 Mupad [B] (verification not implemented)

Time $= 1.87$ (sec) , antiderivative size $= 732$, normalized size of antiderivative $= 9.63$

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$=-\frac{\frac{4\,(3\,a^2-2\,a\,b+3\,b^2)}{35\,d}+\frac{32\,e^{2\,c+2\,d\,x}\,(a^2-b^2)}{35\,d}+\frac{4\,e^{4\,c+4\,d\,x}\,(a+b)^2}{7\,d}}{4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1}-\frac{\frac{32\,(a^2-b^2)}{105\,d}+\frac{8\,e^{2\,c+2\,d\,x}\,(a+b)^2}{21\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1}$$

$$-\frac{\frac{32\,(a^2-b^2)}{105\,d}+\frac{64\,e^{4\,c+4\,d\,x}\,(a^2-b^2)}{35\,d}+\frac{16\,e^{6\,c+6\,d\,x}\,(a+b)^2}{21\,d}+\frac{16\,e^{2\,c+2\,d\,x}\,(3\,a^2-2\,a\,b+3\,b^2)}{35\,d}}{5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1}$$

$$-\frac{\frac{32\,e^{4\,c+4\,d\,x}\,(a^2-b^2)}{7\,d}+\frac{32\,e^{8\,c+8\,d\,x}\,(a^2-b^2)}{7\,d}+\frac{8\,e^{2\,c+2\,d\,x}\,(a+b)^2}{7\,d}+\frac{8\,e^{10\,c+10\,d\,x}\,(a+b)^2}{7\,d}+\frac{16\,e^{6\,c+6\,d\,x}\,(3\,a^2-2\,a\,b+3\,b^2)}{7\,d}}{7\,e^{2\,c+2\,d\,x}+21\,e^{4\,c+4\,d\,x}+35\,e^{6\,c+6\,d\,x}+35\,e^{8\,c+8\,d\,x}+21\,e^{10\,c+10\,d\,x}+7\,e^{12\,c+12\,d\,x}+e^{14\,c+14\,d\,x}+1}$$

$$-\frac{\frac{4\,(a+b)^2}{21\,d}+\frac{32\,e^{2\,c+2\,d\,x}\,(a^2-b^2)}{21\,d}+\frac{64\,e^{6\,c+6\,d\,x}\,(a^2-b^2)}{21\,d}+\frac{20\,e^{8\,c+8\,d\,x}\,(a+b)^2}{21\,d}+\frac{8\,e^{4\,c+4\,d\,x}\,(3\,a^2-2\,a\,b+3\,b^2)}{7\,d}}{6\,e^{2\,c+2\,d\,x}+15\,e^{4\,c+4\,d\,x}+20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}+6\,e^{10\,c+10\,d\,x}+e^{12\,c+12\,d\,x}+1}$$

$$-\frac{4\,(a+b)^2}{21\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)^2/cosh(c + d*x)^4,x)
```

output
```
- ((4*(3*a^2 - 2*a*b + 3*b^2))/(35*d) + (32*exp(2*c + 2*d*x)*(a^2 - b^2))/
(35*d) + (4*exp(4*c + 4*d*x)*(a + b)^2)/(7*d))/(4*exp(2*c + 2*d*x) + 6*exp
(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((32*(a^2 - b
^2))/(105*d) + (8*exp(2*c + 2*d*x)*(a + b)^2)/(21*d))/(3*exp(2*c + 2*d*x)
+ 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((32*(a^2 - b^2))/(105*d) +
 (64*exp(4*c + 4*d*x)*(a^2 - b^2))/(35*d) + (16*exp(6*c + 6*d*x)*(a + b)^2
)/(21*d) + (16*exp(2*c + 2*d*x)*(3*a^2 - 2*a*b + 3*b^2))/(35*d))/(5*exp(2*
c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x
) + exp(10*c + 10*d*x) + 1) - ((32*exp(4*c + 4*d*x)*(a^2 - b^2))/(7*d) + (
32*exp(8*c + 8*d*x)*(a^2 - b^2))/(7*d) + (8*exp(2*c + 2*d*x)*(a + b)^2)/(7
*d) + (8*exp(10*c + 10*d*x)*(a + b)^2)/(7*d) + (16*exp(6*c + 6*d*x)*(3*a^2
 - 2*a*b + 3*b^2))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*e
xp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c
 + 12*d*x) + exp(14*c + 14*d*x) + 1) - ((4*(a + b)^2)/(21*d) + (32*exp(2*c
 + 2*d*x)*(a^2 - b^2))/(21*d) + (64*exp(6*c + 6*d*x)*(a^2 - b^2))/(21*d) +
 (20*exp(8*c + 8*d*x)*(a + b)^2)/(21*d) + (8*exp(4*c + 4*d*x)*(3*a^2 - 2*a
*b + 3*b^2))/(7*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c
 + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x
) + 1) - (4*(a + b)^2)/(21*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.97 $\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

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3.97.1 Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{3}{8}(a+b)\left(a^2-2ab+5b^2\right)x + \frac{3(a-3b)(a+b)^2\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)^3\cosh^3(c+dx)\sinh(c+dx)}{4d} - \frac{b^3\tanh(c+dx)}{d}$$

output
```
3/8*(a+b)*(a^2-2*a*b+5*b^2)*x+3/8*(a-3*b)*(a+b)^2*cosh(d*x+c)*sinh(d*x+c)/
d+1/4*(a+b)^3*cosh(d*x+c)^3*sinh(d*x+c)/d-b^3*tanh(d*x+c)/d
```

3.97.2 Mathematica [A] (verified)

Time = 6.86 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.89

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{12(a^3-a^2b+3ab^2+5b^3)(c+dx)+8(a-2b)(a+b)^2\sinh(2(c+dx))+(a+b)^3\sinh(4(c+dx))-32b^3}{32d}$$

input
```
Integrate[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(12*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*(c + d*x) + 8*(a - 2*b)*(a + b)^2*Sinh
[2*(c + d*x)] + (a + b)^3*Sinh[4*(c + d*x)] - 32*b^3*Tanh[c + d*x])/(32*d)
```

3.97.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\sec(ic+idx)^4}dx$$

$$\downarrow 4158$$

$$\frac{\int \frac{\left(b\tanh^2(c+dx)+a\right)^3}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\downarrow 300$$

$$\frac{\int \left(\frac{3b^2(a+b)\tanh^4(c+dx)+3b(a^2-b^2)\tanh^2(c+dx)+a^3+b^3}{\left(1-\tanh^2(c+dx)\right)^3}-b^3\right)d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{3}{8}(a+b)\left(a^2-2ab+5b^2\right)\operatorname{arctanh}(\tanh(c+dx))+\frac{3(a-3b)(a+b)^2\tanh(c+dx)}{8\left(1-\tanh^2(c+dx)\right)}+\frac{(a+b)^3\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}+b^3(-\tanh(c+dx)}{d}$$

input
```
Int[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((3*(a + b)*(a^2 - 2*a*b + 5*b^2)*ArcTanh[Tanh[c + d*x]])/8 - b^3*Tanh[c +
 d*x] + ((a + b)^3*Tanh[c + d*x])/(4*(1 - Tanh[c + d*x]^2)^2) + (3*(a - 3*
b)*(a + b)^2*Tanh[c + d*x])/(8*(1 - Tanh[c + d*x]^2)))/d
```

3.97.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.97.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(85) = 170$.

Time = 55.73 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.02

method	result
derivativedivides	$\frac{a^3\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}$
default	$\frac{a^3\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}$
risch	$\frac{3a^3x}{8}-\frac{3ba^2x}{8}+\frac{9ab^2x}{8}+\frac{15b^3x}{8}+\frac{e^{4dx+4c}a^3}{64d}+\frac{3e^{4dx+4c}a^2b}{64d}+\frac{3e^{4dx+4c}ab^2}{64d}+\frac{e^{4dx+4c}b^3}{64d}+\frac{e^{2dx+2c}a^3}{8d}$

input
```
int(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)+3
*a^2*b*(1/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-
1/8*c)+3*a*b^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/
8*c)+b^3*(1/4*sinh(d*x+c)^5/cosh(d*x+c)-5/8*sinh(d*x+c)^3/cosh(d*x+c)+15/8
*d*x+15/8*c-15/8*tanh(d*x+c)))
```

3.97.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(85) = 170$.

Time $= 0.26$ (sec) , antiderivative size $= 227$, normalized size of antiderivative $= 2.49$

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{(a^3+3\,a^2b+3\,ab^2+b^3)\sinh(dx+c)^5+(9\,a^3+3\,a^2b-21\,ab^2-15\,b^3+10\,(a^3+3\,a^2b+3\,ab^2+b^3)\cosh$$

input
```
integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/64*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^5 + (9*a^3 + 3*a^2*b -
 21*a*b^2 - 15*b^3 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*s
inh(d*x + c)^3 + 8*(8*b^3 + 3*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*d*x)*cosh(d*
x + c) + (5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 8*a^3 - 24*a
*b^2 - 80*b^3 + 9*(3*a^3 + a^2*b - 7*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(
d*x + c))/(d*cosh(d*x + c))
```

3.97.6 Sympy [F]

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2(c+dx)\right)^3\cosh^4(c+dx)\,dx$$

input
```
integrate(cosh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*cosh(c + d*x)**4, x)
```

3.97.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(85) = 170$.

Time = 0.20 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.93

$$\begin{aligned}
&\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx \\
&= \frac{1}{64}a^3\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right) \\
&\quad+\frac{3}{64}ab^2\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right) \\
&\quad+\frac{1}{64}b^3\left(\frac{120(dx+c)}{d}+\frac{16e^{(-2dx-2c)}-e^{(-4dx-4c)}}{d}-\frac{15e^{(-2dx-2c)}+144e^{(-4dx-4c)}-1}{d\left(e^{(-4dx-4c)}+e^{(-6dx-6c)}\right)}\right) \\
&\quad-\frac{3}{64}a^2b\left(\frac{8(dx+c)}{d}-\frac{e^{(4dx+4c)}}{d}+\frac{e^{(-4dx-4c)}}{d}\right)
\end{aligned}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/64*a^3*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) + 3/64*a*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(
2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) + 1/64*b^3*(12
0*(d*x + c)/d + (16*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (15*e^(-2*d*x
 - 2*c) + 144*e^(-4*d*x - 4*c) - 1)/(d*(e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c
)))) - 3/64*a^2*b*(8*(d*x + c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)
```

3.97.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(85) = 170$.

Time = 0.54 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.10

$$\begin{aligned}
&\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx \\
&= \frac{a^3e^{(4dx+4c)}+3a^2be^{(4dx+4c)}+3ab^2e^{(4dx+4c)}+b^3e^{(4dx+4c)}+8a^3e^{(2dx+2c)}-24ab^2e^{(2dx+2c)}-16b^3e^{(2dx+2}}{}
\end{aligned}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/64*(a^3*e^(4*d*x + 4*c) + 3*a^2*b*e^(4*d*x + 4*c) + 3*a*b^2*e^(4*d*x + 4
*c) + b^3*e^(4*d*x + 4*c) + 8*a^3*e^(2*d*x + 2*c) - 24*a*b^2*e^(2*d*x + 2*
c) - 16*b^3*e^(2*d*x + 2*c) + 24*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*(d*x + c)
 + 128*b^3/(e^(2*d*x + 2*c) + 1) - (18*a^3*e^(4*d*x + 4*c) - 18*a^2*b*e^(4
*d*x + 4*c) + 54*a*b^2*e^(4*d*x + 4*c) + 90*b^3*e^(4*d*x + 4*c) + 8*a^3*e^
(2*d*x + 2*c) - 24*a*b^2*e^(2*d*x + 2*c) - 16*b^3*e^(2*d*x + 2*c) + a^3 +
3*a^2*b + 3*a*b^2 + b^3)*e^(-4*d*x - 4*c))/d
```

3.97.9 Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.46

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = x\left(\frac{3a^3}{8}-\frac{3a^2b}{8}+\frac{9ab^2}{8}+\frac{15b^3}{8}\right) + \frac{2b^3}{d\left(\mathrm{e}^{2c+2dx}+1\right)} - \frac{\mathrm{e}^{-4c-4dx}(a+b)^3}{64d} + \frac{\mathrm{e}^{4c+4dx}(a+b)^3}{64d} - \frac{\mathrm{e}^{-2c-2dx}(a+b)^2(a-2b)}{8d} + \frac{\mathrm{e}^{2c+2dx}(a+b)^2(a-2b)}{8d}$$

input
```
int(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
x*((9*a*b^2)/8 - (3*a^2*b)/8 + (3*a^3)/8 + (15*b^3)/8) + (2*b^3)/(d*(exp(2
*c + 2*d*x) + 1)) - (exp(- 4*c - 4*d*x)*(a + b)^3)/(64*d) + (exp(4*c + 4*d
*x)*(a + b)^3)/(64*d) - (exp(- 2*c - 2*d*x)*(a + b)^2*(a - 2*b))/(8*d) + (
exp(2*c + 2*d*x)*(a + b)^2*(a - 2*b))/(8*d)
```

3.98 $\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

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3.98.1 Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{b^2(6a+5b)\arctan(\sinh(c+dx))}{2d} + \frac{(a-2b)(a+b)^2\sinh(c+dx)}{d} + \frac{(a+b)^3\sinh^3(c+dx)}{3d} - \frac{b^3\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

output
```
1/2*b^2*(6*a+5*b)*arctan(sinh(d*x+c))/d+(a-2*b)*(a+b)^2*sinh(d*x+c)/d+1/3*
(a+b)^3*sinh(d*x+c)^3/d-1/2*b^3*sech(d*x+c)*tanh(d*x+c)/d
```

3.98.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.06 (sec) , antiderivative size = 494, normalized size of antiderivative = 5.68

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{\operatorname{csch}^5(c+dx)\left(-256\,{}_5F_4\left(\frac{3}{2},2,2,2,2;1,1,1,\frac{11}{2};-\sinh^2(c+dx)\right)\sinh^8(c+dx)\left(a+a\sinh^2(c+dx)+b\sinh^2(c+dx)\right)\right.}{}$$

input
```
Integrate[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(Csch[c + d*x]^5*(-256*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2
}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8*(a + a*Sinh[c + d*x]^2 + b*Sinh[c + d
*x]^2)^3 - (315*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(b^3*Sinh[c + d*x]^6*(2161
 + 1875*Sinh[c + d*x]^2 + 243*Sinh[c + d*x]^4 + Sinh[c + d*x]^6) + a^3*Cos
h[c + d*x]^6*(2401 + 1875*Sinh[c + d*x]^2 + 243*Sinh[c + d*x]^4 + Sinh[c +
 d*x]^6) + 3*a^2*b*(Sinh[c + d*x] + Sinh[c + d*x]^3)^2*(2401 + 1875*Sinh[c
 + d*x]^2 + 243*Sinh[c + d*x]^4 + Sinh[c + d*x]^6) + 3*a*b^2*Sinh[c + d*x]
^4*(2401 + 4180*Sinh[c + d*x]^2 + 2118*Sinh[c + d*x]^4 + 244*Sinh[c + d*x]
^6 + Sinh[c + d*x]^8)))/Sqrt[-Sinh[c + d*x]^2] + 21*(b^3*Sinh[c + d*x]^6*(
32415 + 17320*Sinh[c + d*x]^2 + 753*Sinh[c + d*x]^4) + 3*a*b^2*Sinh[c + d*
x]^4*(36015 + 50695*Sinh[c + d*x]^2 + 18073*Sinh[c + d*x]^4 + 753*Sinh[c +
 d*x]^6) + 3*a^2*b*Sinh[c + d*x]^2*(36015 + 88150*Sinh[c + d*x]^2 + 69728*
Sinh[c + d*x]^4 + 18826*Sinh[c + d*x]^6 + 753*Sinh[c + d*x]^8) + a^3*(3601
5 + 124165*Sinh[c + d*x]^2 + 157878*Sinh[c + d*x]^4 + 89514*Sinh[c + d*x]^
6 + 19579*Sinh[c + d*x]^8 + 753*Sinh[c + d*x]^10))))/(30240*d)
```

3.98.3 Rubi [A] (verified)

Time $= 0.31$ (sec) , antiderivative size $= 85$, normalized size of antiderivative $= 0.98$, number of steps used $= 5$, number of rules used $= 4$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used $= \{3042, 4159, 300, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\sec(ic+idx)^3}dx$$

$$\downarrow 4159$$

$$\frac{\int \frac{\left((a+b)\sinh^2(c+dx)+a\right)^3}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)}{d}$$

$$\downarrow 300$$

$$\frac{\int \left(\sinh^2(c+dx)(a+b)^3+(a-2b)(a+b)^2+\frac{3(a+b)\sinh^2(c+dx)b^2+(3a+2b)b^2}{\left(\sinh^2(c+dx)+1\right)^2}\right) d\sinh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{2}b^2(6a+5b)\arctan(\sinh(c+dx))+\frac{1}{3}(a+b)^3\sinh^3(c+dx)+(a-2b)(a+b)^2\sinh(c+dx)-\frac{b^3\sinh(c+dx)}{2(\sinh^2(c+dx)+1)}}{d}$$

input
```
Int[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((b^2*(6*a + 5*b)*ArcTan[Sinh[c + d*x]])/2 + (a - 2*b)*(a + b)^2*Sinh[c +
d*x] + ((a + b)^3*Sinh[c + d*x]^3)/3 - (b^3*Sinh[c + d*x])/(2*(1 + Sinh[c
+ d*x]^2)))/d
```

3.98.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.98.4 Maple [A] (verified)

Time = 22.77 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.78

method	result
derivativedivides	$\frac{a^3\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+a^2b\sinh(dx+c)^3+3ab^2\left(\frac{\sinh(dx+c)^3}{3}-\sinh(dx+c)+2\arctan\left(e^{dx+c}\right)\right)+b^3\left(\frac{\sinh(dx+c)}{3\cosh(dx+c)}\right.}{d}$
default	$\frac{a^3\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+a^2b\sinh(dx+c)^3+3ab^2\left(\frac{\sinh(dx+c)^3}{3}-\sinh(dx+c)+2\arctan\left(e^{dx+c}\right)\right)+b^3\left(\frac{\sinh(dx+c)}{3\cosh(dx+c)}\right.}{d}$
risch	$\frac{e^{3dx+3c}a^3}{24d}+\frac{e^{3dx+3c}a^2b}{8d}+\frac{e^{3dx+3c}ab^2}{8d}+\frac{e^{3dx+3c}b^3}{24d}+\frac{3e^{dx+c}a^3}{8d}-\frac{3e^{dx+c}a^2b}{8d}-\frac{15e^{dx+c}ab^2}{8d}-\frac{9b^3e^{dx+c}}{8d}$

input
```
int(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c)+a^2*b*sinh(d*x+c)^3+3*a*b^2*(
1/3*sinh(d*x+c)^3-sinh(d*x+c)+2*arctan(exp(d*x+c)))+b^3*(1/3*sinh(d*x+c)^5
/cosh(d*x+c)^2-5/3*sinh(d*x+c)^3/cosh(d*x+c)^2-5*sinh(d*x+c)/cosh(d*x+c)^2
+5/2*sech(d*x+c)*tanh(d*x+c)+5*arctan(exp(d*x+c))))
```

3.98.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1840 vs. 2(81) = 162.

Time = 0.28 (sec) , antiderivative size = 1840, normalized size of antiderivative = 21.15

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/24*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 10*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*sinh(d*x + c)^10 + (11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x
 + c)^8 + (11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3 + 45*(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*cosh(d*x + c)^3 + (11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*
x + c))*sinh(d*x + c)^7 + 2*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x
 + c)^6 + 2*(105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 5*a^3 -
 3*a^2*b - 21*a*b^2 - 25*b^3 + 14*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*c
osh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*co
sh(d*x + c)^5 + 14*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x + c)^3
+ 3*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 -
 2*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^4 + 2*(105*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 35*(11*a^3 - 3*a^2*b - 39*a*b^2
 - 25*b^3)*cosh(d*x + c)^4 - 5*a^3 + 3*a^2*b + 21*a*b^2 + 25*b^3 + 15*(5*a^
3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 7*(11*a^3 - 3*a^2*b - 39
*a*b^2 - 25*b^3)*cosh(d*x + c)^5 + 5*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)
*cosh(d*x + c)^3 - (5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c))*si
nh(d*x + c)^3 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 - (11*a^3 - 3*a^2*b - 39*...
```

3.98.6 Sympy [F]

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^3\cosh^3\left(c+dx\right)dx$$

input
```
integrate(cosh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*cosh(c + d*x)**3, x)
```

3.98.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(81) = 162$.

Time = 0.28 (sec) , antiderivative size = 284, normalized size of antiderivative = 3.26

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3}{8\,d}$$
$$-\frac{1}{8}\,ab^2\left(\frac{\left(15\,e^{(-2\,dx-2\,c)}-1\right)e^{(3\,dx+3\,c)}}{d}-\frac{15\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{48\,\arctan\left(e^{(-dx-c)}\right)}{d}\right)$$
$$+\frac{1}{24}\,b^3\left(\frac{27\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}-\frac{120\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{25\,e^{(-2\,dx-2\,c)}+77\,e^{(-4\,dx-4\,c)}+3\,e^{(-6\,dx-6\,c)}}{d\left(e^{(-3\,dx-3\,c)}+2\,e^{(-5\,dx-5\,c)}+e^{(-7\,dx-7\,c)}\right)}\right.$$
$$+\frac{1}{24}\,a^3\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/8*a^2*b*(e^(d*x + c) - e^(-d*x - c))^3/d - 1/8*a*b^2*((15*e^(-2*d*x - 2*
c) - 1)*e^(3*d*x + 3*c)/d - (15*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + 48*ar
ctan(e^(-d*x - c))/d) + 1/24*b^3*((27*e^(-d*x - c) - e^(-3*d*x - 3*c))/d -
 120*arctan(e^(-d*x - c))/d - (25*e^(-2*d*x - 2*c) + 77*e^(-4*d*x - 4*c) +
 3*e^(-6*d*x - 6*c) - 1)/(d*(e^(-3*d*x - 3*c) + 2*e^(-5*d*x - 5*c) + e^(-7
*d*x - 7*c)))) + 1/24*a^3*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x
 - c)/d - e^(-3*d*x - 3*c)/d)
```

3.98.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(81) = 162$.

Time = 0.47 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.03

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+3\,a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+3\,ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3-}{}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/24*(a^3*(e^(d*x + c) - e^(-d*x - c))^3 + 3*a^2*b*(e^(d*x + c) - e^(-d*x
- c))^3 + 3*a*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + b^3*(e^(d*x + c) - e^(-
d*x - c))^3 + 12*a^3*(e^(d*x + c) - e^(-d*x - c)) - 36*a*b^2*(e^(d*x + c)
- e^(-d*x - c)) - 24*b^3*(e^(d*x + c) - e^(-d*x - c)) - 24*b^3*(e^(d*x + c
) - e^(-d*x - c))/((e^(d*x + c) - e^(-d*x - c))^2 + 4) + 6*(pi + 2*arctan(
1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(6*a*b^2 + 5*b^3))/d
```

3.98.9 Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.67

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{e^{dx}e^{c}\left(5b^3\sqrt{d^2}+6ab^2\sqrt{d^2}\right)}{d\sqrt{36a^2b^4+60ab^5+25b^6}}\right)\sqrt{36a^2b^4+60ab^5+25b^6}}{\sqrt{d^2}} - \frac{e^{-3c-3dx}(a+b)^3}{24d}$$
$$+ \frac{e^{3c+3dx}(a+b)^3}{24d} + \frac{3e^{c+dx}(a+b)^2(a-3b)}{8d} - \frac{b^3e^{c+dx}}{d\left(e^{2c+2dx}+1\right)}$$
$$- \frac{3e^{-c-dx}(a+b)^2(a-3b)}{8d} + \frac{2b^3e^{c+dx}}{d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}$$

input
```
int(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
(atan((exp(d*x)*exp(c)*(5*b^3*(d^2)^(1/2) + 6*a*b^2*(d^2)^(1/2)))/(d*(60*a
*b^5 + 25*b^6 + 36*a^2*b^4)^(1/2)))*(60*a*b^5 + 25*b^6 + 36*a^2*b^4)^(1/2)
)/(d^2)^(1/2) - (exp(- 3*c - 3*d*x)*(a + b)^3)/(24*d) + (exp(3*c + 3*d*x)*
(a + b)^3)/(24*d) + (3*exp(c + d*x)*(a + b)^2*(a - 3*b))/(8*d) - (b^3*exp(
c + d*x))/(d*(exp(2*c + 2*d*x) + 1)) - (3*exp(- c - d*x)*(a + b)^2*(a - 3*
b))/(8*d) + (2*b^3*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x)
 + 1))
```

3.99 $\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

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3.99.1 Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{1}{2}(a-5b)(a+b)^2x + \frac{(a+b)^3\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b^2(3a+2b)\tanh(c+dx)}{d} + \frac{b^3\tanh^3(c+dx)}{3d}$$

output
```
1/2*(a-5*b)*(a+b)^2*x+1/2*(a+b)^3*cosh(d*x+c)*sinh(d*x+c)/d+b^2*(3*a+2*b)*
tanh(d*x+c)/d+1/3*b^3*tanh(d*x+c)^3/d
```

3.99.2 Mathematica [A] (verified)

Time = 2.56 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{6(a-5b)(a+b)^2(c+dx)+3(a+b)^3\sinh(2(c+dx))+4b^2\left(9a+7b-b\operatorname{sech}^2(c+dx)\right)\tanh(c+dx)}{12d}$$

input
```
Integrate[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(6*(a - 5*b)*(a + b)^2*(c + d*x) + 3*(a + b)^3*Sinh[2*(c + d*x)] + 4*b^2*(
9*a + 7*b - b*Sech[c + d*x]^2)*Tanh[c + d*x])/(12*d)
```

3.99.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\sec(ic+idx)^2}dx$$

$$\downarrow 4158$$

$$\frac{\int \frac{\left(b\tanh^2(c+dx)+a\right)^3}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 300$$

$$\frac{\int \left(\tanh^2(c+dx)b^3+(3a+2b)b^2+\frac{3b\tanh^2(c+dx)(a+b)^2+(a-2b)(a+b)^2}{\left(1-\tanh^2(c+dx)\right)^2}\right)d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{1}{2}(a-5b)(a+b)^2\operatorname{arctanh}(\tanh(c+dx))+b^2(3a+2b)\tanh(c+dx)+\frac{(a+b)^3\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}+\frac{1}{3}b^3\tanh^3(c+dx)}{d}$$

input
```
Int[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(((a - 5*b)*(a + b)^2*ArcTanh[Tanh[c + d*x]])/2 + b^2*(3*a + 2*b)*Tanh[c +
 d*x] + (b^3*Tanh[c + d*x]^3)/3 + ((a + b)^3*Tanh[c + d*x])/(2*(1 - Tanh[c
 + d*x]^2)))/d
```

3.99.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.99.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(72) = 144$.

Time = 6.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.90

method	result
derivativedivides	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3ab^2\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right.}{d}$
default	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3ab^2\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right.}{d}$
risch	$\frac{a^3x}{2}-\frac{3ba^2x}{2}-\frac{9ab^2x}{2}-\frac{5b^3x}{2}+\frac{e^{2dx+2c}a^3}{8d}+\frac{3e^{2dx+2c}a^2b}{8d}+\frac{3e^{2dx+2c}ab^2}{8d}+\frac{e^{2dx+2c}b^3}{8d}-\frac{e^{-2dx-2c}a^3}{8d}$

input
```
int(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*(1/2*cosh(d*x
+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+3*a*b^2*(1/2*sinh(d*x+c)^3/cosh(d*x+c)-3/2*
d*x-3/2*c+3/2*tanh(d*x+c))+b^3*(1/2*sinh(d*x+c)^5/cosh(d*x+c)^3-5/2*d*x-5/
2*c+5/2*tanh(d*x+c)+5/6*tanh(d*x+c)^3))
```

3.99.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(72) = 144$.

Time $= 0.26$ (sec) , antiderivative size $= 369$, normalized size of antiderivative $= 4.73$

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{3\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sinh\left(dx+c\right)^5-4\left(18\,ab^2+14\,b^3-3\left(a^3-3\,a^2b-9\,ab^2-5\,b^3\right)dx\right)\cosh\left(dx+\right.}{}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/24*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^5 - 4*(18*a*b^2 + 14
*b^3 - 3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c)^3 - 12*(18*a
*b^2 + 14*b^3 - 3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c)*sin
h(d*x + c)^2 + (9*a^3 + 27*a^2*b + 99*a*b^2 + 65*b^3 + 30*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 - 12*(18*a*b^2 + 14*b^3 -
 3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c) + 3*(5*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*a^3 + 6*a^2*b + 30*a*b^2 + 10*b^3
 + (9*a^3 + 27*a^2*b + 99*a*b^2 + 65*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/
(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c)
)
```

3.99.6 Sympy [F]

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^3\cosh^2\left(c+dx\right)dx$$

input
```
integrate(cosh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*cosh(c + d*x)**2, x)
```

3.99.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(72) = 144$.

Time = 0.21 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.28

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{1}{8}a^3\left(4x + \frac{e^{(2\,dx+2\,c)}}{d} - \frac{e^{(-2\,dx-2\,c)}}{d}\right) - \frac{3}{8}a^2b\left(4x - \frac{e^{(2\,dx+2\,c)}}{d} + \frac{e^{(-2\,dx-2\,c)}}{d}\right)$$
$$- \frac{1}{24}b^3\left(\frac{60\,(dx+c)}{d} + \frac{3\,e^{(-2\,dx-2\,c)}}{d} - \frac{121\,e^{(-2\,dx-2\,c)} + 201\,e^{(-4\,dx-4\,c)} + 147\,e^{(-6\,dx-6\,c)} + 3}{d(e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + 3\,e^{(-6\,dx-6\,c)} + e^{(-8\,dx-8\,c)})}\right)$$
$$- \frac{3}{8}ab^2\left(\frac{12\,(dx+c)}{d} + \frac{e^{(-2\,dx-2\,c)}}{d} - \frac{17\,e^{(-2\,dx-2\,c)} + 1}{d(e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)})}\right)$$

input
```
integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/8*a^3*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 3/8*a^2*b*(4*x -
e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/24*b^3*(60*(d*x + c)/d + 3*e^(
-2*d*x - 2*c)/d - (121*e^(-2*d*x - 2*c) + 201*e^(-4*d*x - 4*c) + 147*e^(-6
*d*x - 6*c) + 3)/(d*(e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 3*e^(-6*d*x -
 6*c) + e^(-8*d*x - 8*c)))) - 3/8*a*b^2*(12*(d*x + c)/d + e^(-2*d*x - 2*c)
/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c))))
```

3.99.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(72) = 144$.

Time = 0.47 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.40

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{3\,a^3e^{(2\,dx+2\,c)} + 9\,a^2be^{(2\,dx+2\,c)} + 9\,ab^2e^{(2\,dx+2\,c)} + 3\,b^3e^{(2\,dx+2\,c)} + 12\,(a^3 - 3\,a^2b - 9\,ab^2 - 5\,b^3)(dx+c) - 3}{}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/24*(3*a^3*e^(2*d*x + 2*c) + 9*a^2*b*e^(2*d*x + 2*c) + 9*a*b^2*e^(2*d*x +
 2*c) + 3*b^3*e^(2*d*x + 2*c) + 12*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*(d*x
+ c) - 3*(2*a^3*e^(2*d*x + 2*c) - 6*a^2*b*e^(2*d*x + 2*c) - 18*a*b^2*e^(2*
d*x + 2*c) - 10*b^3*e^(2*d*x + 2*c) + a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-2
*d*x - 2*c) - 16*(9*a*b^2*e^(4*d*x + 4*c) + 9*b^3*e^(4*d*x + 4*c) + 18*a*b
^2*e^(2*d*x + 2*c) + 12*b^3*e^(2*d*x + 2*c) + 9*a*b^2 + 7*b^3)/(e^(2*d*x +
 2*c) + 1)^3)/d
```

3.99.9 Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.12

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{e^{2c+2dx}(a+b)^3}{8d} - \frac{\frac{2(b^3+3ab^2)}{3d} + \frac{2e^{2c+2dx}(b^3+ab^2)}{d}}{2e^{2c+2dx}+e^{4c+4dx}+1} - \frac{2(b^3+ab^2)}{d(e^{2c+2dx}+1)} - \frac{e^{-2c-2dx}(a+b)^3}{8d}$$
$$- \frac{\frac{2(b^3+ab^2)}{d} + \frac{4e^{2c+2dx}(b^3+3ab^2)}{3d} + \frac{2e^{4c+4dx}(b^3+ab^2)}{d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1} + \frac{x(a+b)^2(a-5b)}{2}$$

input
```
int(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
(exp(2*c + 2*d*x)*(a + b)^3)/(8*d) - ((2*(3*a*b^2 + b^3))/(3*d) + (2*exp(2
*c + 2*d*x)*(a*b^2 + b^3))/d)/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)
- (2*(a*b^2 + b^3))/(d*(exp(2*c + 2*d*x) + 1)) - (exp(- 2*c - 2*d*x)*(a +
b)^3)/(8*d) - ((2*(a*b^2 + b^3))/d + (4*exp(2*c + 2*d*x)*(3*a*b^2 + b^3))/
(3*d) + (2*exp(4*c + 4*d*x)*(a*b^2 + b^3))/d)/(3*exp(2*c + 2*d*x) + 3*exp(
4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + (x*(a + b)^2*(a - 5*b))/2
```

3.100 $\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

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3.100.1 Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{3b(4(a+b)^2+(2a+b)^2)\arctan(\sinh(c+dx))}{8d} + \frac{(a+b)^3\sinh(c+dx)}{d} + \frac{3b^2(4a+3b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} - \frac{b^3\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d}$$

output
```
-3/8*b*(4*(a+b)^2+(2*a+b)^2)*arctan(sinh(d*x+c))/d+(a+b)^3*sinh(d*x+c)/d+3
/8*b^2*(4*a+3*b)*sech(d*x+c)*tanh(d*x+c)/d-1/4*b^3*sech(d*x+c)^3*tanh(d*x+
c)/d
```

3.100.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.90

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{-3b(8a^2+12ab+5b^2)\arctan(\sinh(c+dx))+8(a+b)^3\sinh(c+dx)+3b^2(4a+3b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d}$$

input
```
Integrate[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-3*b*(8*a^2 + 12*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]] + 8*(a + b)^3*Sinh[c
+ d*x] + 3*b^2*(4*a + 3*b)*Sech[c + d*x]*Tanh[c + d*x] - 2*b^3*Sech[c + d*
x]^3*Tanh[c + d*x])/(8*d)
```

3.100.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4159, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\sec(ic+idx)}dx$$

$$\downarrow 4159$$

$$\frac{\int \frac{\left((a+b)\sinh^2(c+dx)+a\right)^3}{\left(\sinh^2(c+dx)+1\right)^3}d\sinh(c+dx)}{d}$$

$$\downarrow 300$$

$$\frac{\int \left((a+b)^3-\frac{3b(a+b)^2\sinh^4(c+dx)+3b(a+b)(2a+b)\sinh^2(c+dx)+b(3a^2+3ba+b^2)}{\left(\sinh^2(c+dx)+1\right)^3}\right)d\sinh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{-\frac{3}{8}b\left(4(a+b)^2+(2a+b)^2\right)\arctan(\sinh(c+dx))+\frac{3b^2(4a+3b)\sinh(c+dx)}{8\left(\sinh^2(c+dx)+1\right)}+(a+b)^3\sinh(c+dx)-\frac{b^3\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

input
```
Int[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((-3*b*(4*(a + b)^2 + (2*a + b)^2)*ArcTan[Sinh[c + d*x]])/8 + (a + b)^3*Si
nh[c + d*x] - (b^3*Sinh[c + d*x])/(4*(1 + Sinh[c + d*x]^2)^2) + (3*b^2*(4*
a + 3*b)*Sinh[c + d*x])/(8*(1 + Sinh[c + d*x]^2)))/d
```

3.100.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.100.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(93) = 186$.

Time = 2.81 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.95

method	result
derivativedivides	$\frac{\sinh(dx+c)a^3+3a^2b(\sinh(dx+c)-2\arctan(e^{dx+c}))+3ab^2\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\ \mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\arcta}{}$
default	$\frac{\sinh(dx+c)a^3+3a^2b(\sinh(dx+c)-2\arctan(e^{dx+c}))+3ab^2\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\ \mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\arcta}{}$
risch	$\frac{e^{dx+c}a^3}{2d}+\frac{3e^{dx+c}a^2b}{2d}+\frac{3e^{dx+c}ab^2}{2d}+\frac{b^3e^{dx+c}}{2d}-\frac{e^{-dx-c}a^3}{2d}-\frac{3e^{-dx-c}a^2b}{2d}-\frac{3e^{-dx-c}ab^2}{2d}-\frac{e^{-dx-c}b^3}{2d}+$

input
```
int(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(sinh(d*x+c)*a^3+3*a^2*b*(sinh(d*x+c)-2*arctan(exp(d*x+c)))+3*a*b^2*(s
inh(d*x+c)^3/cosh(d*x+c)^2+3*sinh(d*x+c)/cosh(d*x+c)^2-3/2*sech(d*x+c)*tan
h(d*x+c)-3*arctan(exp(d*x+c)))+b^3*(sinh(d*x+c)^5/cosh(d*x+c)^4+5*sinh(d*x
+c)^3/cosh(d*x+c)^4+5/cosh(d*x+c)^4*sinh(d*x+c)-5*(1/4*sech(d*x+c)^3+3/8*s
ech(d*x+c))*tanh(d*x+c)-15/4*arctan(exp(d*x+c))))
```

3.100.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2411 vs. $2(93) = 186$.

Time $= 0.28$ (sec) , antiderivative size $= 2411$, normalized size of antiderivative $= 24.35$

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/4*(2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 20*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + 2*(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*sinh(d*x + c)^10 + 3*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(
d*x + c)^8 + 3*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3 + 30*(a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 24*(10*(a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*cosh(d*x + c)^3 + (2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(
d*x + c))*sinh(d*x + c)^7 + (4*a^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x
 + c)^6 + (420*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*a^3 + 1
2*a^2*b + 24*a*b^2 + 5*b^3 + 84*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^6 + 6*(84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d
*x + c)^5 + 28*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (4*a
^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - (4*a^3
+ 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + (420*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*cosh(d*x + c)^6 + 210*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cos
h(d*x + c)^4 - 4*a^3 - 12*a^2*b - 24*a*b^2 - 5*b^3 + 15*(4*a^3 + 12*a^2*b
+ 24*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(60*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 42*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^
3)*cosh(d*x + c)^5 + 5*(4*a^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)
^3 - (4*a^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^3
- 2*a^3 - 6*a^2*b - 6*a*b^2 - 2*b^3 - 3*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5...
```

3.100.6 Sympy [F]

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^3\cosh\left(c+dx\right)dx$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*cosh(c + d*x), x)
```

3.100.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(93) = 186.

Time = 0.29 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.98

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{1}{4}b^3\left(\frac{15\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{2\,e^{(-dx-c)}}{d} + \frac{17\,e^{(-2\,dx-2\,c)} + 13\,e^{(-4\,dx-4\,c)} + 7\,e^{(-6\,dx-6\,c)} - 7\,e^{(-8\,dx-8\,c)} +}{d(e^{(-dx-c)} + 4\,e^{(-3\,dx-3\,c)} + 6\,e^{(-5\,dx-5\,c)} + 4\,e^{(-7\,dx-7\,c)} + e^{(-9\,dx}\right.$$
$$+ \frac{3}{2}ab^2\left(\frac{6\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)}}{d} + \frac{4\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} + 1}{d(e^{(-dx-c)} + 2\,e^{(-3\,dx-3\,c)} + e^{(-5\,dx-5\,c)})}\right)$$
$$+ \frac{3}{2}a^2b\left(\frac{4\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{e^{(dx+c)}}{d} - \frac{e^{(-dx-c)}}{d}\right) + \frac{a^3\sinh\left(dx+c\right)}{d}$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*b^3*(15*arctan(e^(-d*x - c))/d - 2*e^(-d*x - c)/d + (17*e^(-2*d*x - 2*
c) + 13*e^(-4*d*x - 4*c) + 7*e^(-6*d*x - 6*c) - 7*e^(-8*d*x - 8*c) + 2)/(d
*(e^(-d*x - c) + 4*e^(-3*d*x - 3*c) + 6*e^(-5*d*x - 5*c) + 4*e^(-7*d*x - 7
*c) + e^(-9*d*x - 9*c)))) + 3/2*a*b^2*(6*arctan(e^(-d*x - c))/d - e^(-d*x
- c)/d + (4*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*
e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c)))) + 3/2*a^2*b*(4*arctan(e^(-d*x - c))
/d + e^(d*x + c)/d - e^(-d*x - c)/d) + a^3*sinh(d*x + c)/d
```

3.100.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(93) = 186$.

Time $= 0.41$ (sec) , antiderivative size $= 272$, normalized size of antiderivative $= 2.75$

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{8a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)+24a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)+24ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)+8b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{}$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/16*(8*a^3*(e^(d*x + c) - e^(-d*x - c)) + 24*a^2*b*(e^(d*x + c) - e^(-d*x
 - c)) + 24*a*b^2*(e^(d*x + c) - e^(-d*x - c)) + 8*b^3*(e^(d*x + c) - e^(-
d*x - c)) - 3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(8*a
^2*b + 12*a*b^2 + 5*b^3) + 4*(12*a*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 9*
b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 48*a*b^2*(e^(d*x + c) - e^(-d*x - c))
 + 28*b^3*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 +
4)^2)/d
```

3.100.9 Mupad [B] (verification not implemented)

Time $= 0.29$ (sec) , antiderivative size $= 355$, normalized size of antiderivative $= 3.59$

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{e^{c+dx}(a+b)^3}{2d} - \frac{e^{-c-dx}(a+b)^3}{2d}$$
$$- \frac{3\,\mathrm{atan}\left(\frac{e^{dx}e^{c}\left(5b^3\sqrt{d^2}+12ab^2\sqrt{d^2}+8a^2b\sqrt{d^2}\right)}{d\sqrt{64a^4b^2+192a^3b^3+224a^2b^4+120ab^5+25b^6}}\right)\sqrt{64a^4b^2+192a^3b^3+224a^2b^4+120ab^5+25b^6}}{4\sqrt{d^2}}$$
$$+ \frac{3e^{c+dx}(3b^3+4ab^2)}{4d\,(e^{2c+2dx}+1)} + \frac{6b^3e^{c+dx}}{d\,(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1)}$$
$$- \frac{e^{c+dx}(13b^3+12ab^2)}{2d\,(2e^{2c+2dx}+e^{4c+4dx}+1)} - \frac{4b^3e^{c+dx}}{d\,(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1)}$$

input
```
int(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^3,x)
```

output

```
(exp(c + d*x)*(a + b)^3)/(2*d) - (exp(- c - d*x)*(a + b)^3)/(2*d) - (3*ata
n((exp(d*x)*exp(c)*(5*b^3*(d^2)^(1/2) + 12*a*b^2*(d^2)^(1/2) + 8*a^2*b*(d^
2)^(1/2)))/(d*(120*a*b^5 + 25*b^6 + 224*a^2*b^4 + 192*a^3*b^3 + 64*a^4*b^2
)^(1/2)))*(120*a*b^5 + 25*b^6 + 224*a^2*b^4 + 192*a^3*b^3 + 64*a^4*b^2)^(1
/2))/(4*(d^2)^(1/2)) + (3*exp(c + d*x)*(4*a*b^2 + 3*b^3))/(4*d*(exp(2*c +
2*d*x) + 1)) + (6*b^3*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4
*d*x) + exp(6*c + 6*d*x) + 1)) - (exp(c + d*x)*(12*a*b^2 + 13*b^3))/(2*d*(
2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (4*b^3*exp(c + d*x))/(d*(4*e
xp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*
x) + 1))
```

3.101 $\int \mathbf{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

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3.101.1 Optimal result

Integrand size = 21, antiderivative size = 149

$$\begin{aligned}
&\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx \\
&= \frac{(2a+b)\left(8a^2+8ab+5b^2\right)\arctan(\sinh(c+dx))}{16d} \\
&\quad - \frac{b(44a^2+44ab+15b^2)\operatorname{sech}(c+dx)\tanh(c+dx)}{48d} \\
&\quad - \frac{5b(2a+b)\operatorname{sech}^3(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{24d} \\
&\quad - \frac{b\operatorname{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)^2\tanh(c+dx)}{6d}
\end{aligned}$$

output

```
1/16*(2*a+b)*(8*a^2+8*a*b+5*b^2)*arctan(sinh(d*x+c))/d-1/48*b*(44*a^2+44*a
*b+15*b^2)*sech(d*x+c)*tanh(d*x+c)/d-5/24*b*(2*a+b)*sech(d*x+c)^3*(a+(a+b)
*sinh(d*x+c)^2)*tanh(d*x+c)/d-1/6*b*sech(d*x+c)^5*(a+(a+b)*sinh(d*x+c)^2)^
2*tanh(d*x+c)/d
```

3.101.2 Mathematica [A] (verified)

Time = 11.48 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.80

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{6(16a^3+24a^2b+18ab^2+5b^3)\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-3b(24a^2+30ab+11b^2)\operatorname{sech}(c+dx)\tanh(c+$$
$$48d$$

input
```
Integrate[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(6*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*ArcTan[Tanh[(c + d*x)/2]] - 3*b*
(24*a^2 + 30*a*b + 11*b^2)*Sech[c + d*x]*Tanh[c + d*x] + 2*b^2*(18*a + 13*
b)*Sech[c + d*x]^3*Tanh[c + d*x] - 8*b^3*Sech[c + d*x]^5*Tanh[c + d*x])/(4
8*d)
```

3.101.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4159, 315, 401, 25, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$\downarrow 3042$$
$$\int \sec(ic+idx)\left(a-b\tan(ic+idx)^2\right)^3\,dx$$
$$\downarrow 4159$$
$$\frac{\int \frac{((a+b)\sinh^2(c+dx)+a)^3}{(\sinh^2(c+dx)+1)^4}\,d\sinh(c+dx)}{d}$$
$$\downarrow 315$$
$$\frac{\frac{1}{6}\int \frac{((a+b)\sinh^2(c+dx)+a)\left((a+b)(6a+5b)\sinh^2(c+dx)+a(6a+b)\right)}{(\sinh^2(c+dx)+1)^3}\,d\sinh(c+dx)-\frac{b\sinh(c+dx)((a+b)\sinh^2(c+dx)+a)^2}{6(\sinh^2(c+dx)+1)^3}}{d}$$

$$\downarrow 401$$

$$\frac{\frac{1}{6}\left(-\frac{1}{4}\int -\frac{(a+b)\left(24a^2+34ba+15b^2\right)\sinh^2(c+dx)+a\left(24a^2+14ba+5b^2\right)}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)-\frac{5b(2a+b)\sinh(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)}{4\left(\sinh^2(c+dx)+1\right)^2}\right)-}{d}$$

$$\downarrow 25$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\int \frac{(a+b)\left(24a^2+34ba+15b^2\right)\sinh^2(c+dx)+a\left(24a^2+14ba+5b^2\right)}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)-\frac{5b(2a+b)\sinh(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)}{4\left(\sinh^2(c+dx)+1\right)^2}\right)-\frac{bs}{}}{d}$$

$$\downarrow 298$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}(2a+b)\left(8a^2+8ab+5b^2\right)\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)-\frac{b\left(44a^2+44ab+15b^2\right)\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)-\frac{5b(2a+b)\sinh(c+dx)(}{4\left(\sinh^2(c}\right.}{d}$$

$$\downarrow 216$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}(2a+b)\left(8a^2+8ab+5b^2\right)\arctan(\sinh(c+dx))-\frac{b\left(44a^2+44ab+15b^2\right)\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)-\frac{5b(2a+b)\sinh(c+dx)\left((a+b)\sinh\right.}{4\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-1/6*(b*Sinh[c + d*x]*(a + (a + b)*Sinh[c + d*x]^2)^2)/(1 + Sinh[c + d*x]
^2)^3 + ((-5*b*(2*a + b)*Sinh[c + d*x]*(a + (a + b)*Sinh[c + d*x]^2))/(4*(
1 + Sinh[c + d*x]^2)^2) + ((3*(2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*ArcTan[Sin
h[c + d*x]])/2 - (b*(44*a^2 + 44*a*b + 15*b^2)*Sinh[c + d*x])/(2*(1 + Sinh
[c + d*x]^2)))/4)/6)/d
```

3.101.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.101.4 Maple [A] (verified)

Time = 6.98 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{2a^3\arctan\left(e^{dx+c}\right)+3a^2b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3ab^2\left(-\frac{\sinh(dx+c)^3}{\cosh(dx+c)^4}-\frac{\sinh(dx+c)}{\cosh(dx+c)^4}+$
default	$\frac{2a^3\arctan\left(e^{dx+c}\right)+3a^2b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3ab^2\left(-\frac{\sinh(dx+c)^3}{\cosh(dx+c)^4}-\frac{\sinh(dx+c)}{\cosh(dx+c)^4}+$
risch	$-\frac{be^{dx+c}\left(72a^2e^{10dx+10c}+90abe^{10dx+10c}+33b^2e^{10dx+10c}+216a^2e^{8dx+8c}+126abe^{8dx+8c}-5b^2e^{8dx+8c}+144a^2e^{6dx+6c}\right.}{}$

input
```
int(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*a^3*arctan(exp(d*x+c))+3*a^2*b*(-sinh(d*x+c)/cosh(d*x+c)^2+1/2*sech
(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+3*a*b^2*(-sinh(d*x+c)^3/cosh(d*x+c
)^4-1/cosh(d*x+c)^4*sinh(d*x+c)+(1/4*sech(d*x+c)^3+3/8*sech(d*x+c))*tanh(d
*x+c)+3/4*arctan(exp(d*x+c)))+b^3*(-sinh(d*x+c)^5/cosh(d*x+c)^6-5/3*sinh(d
*x+c)^3/cosh(d*x+c)^6-sinh(d*x+c)/cosh(d*x+c)^6+(1/6*sech(d*x+c)^5+5/24*se
ch(d*x+c)^3+5/16*sech(d*x+c))*tanh(d*x+c)+5/8*arctan(exp(d*x+c))))
```

3.101.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3465 vs. 2(141) = 282.

Time = 0.28 (sec) , antiderivative size = 3465, normalized size of antiderivative = 23.26

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/24*(3*(24*a^2*b + 30*a*b^2 + 11*b^3)*cosh(d*x + c)^11 + 33*(24*a^2*b +
30*a*b^2 + 11*b^3)*cosh(d*x + c)*sinh(d*x + c)^10 + 3*(24*a^2*b + 30*a*b^2
 + 11*b^3)*sinh(d*x + c)^11 + (216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c
)^9 + (216*a^2*b + 126*a*b^2 - 5*b^3 + 165*(24*a^2*b + 30*a*b^2 + 11*b^3)*
cosh(d*x + c)^2)*sinh(d*x + c)^9 + 9*(55*(24*a^2*b + 30*a*b^2 + 11*b^3)*co
sh(d*x + c)^3 + (216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x +
c)^8 + 18*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 18*(55*(24*a^2*b +
 30*a*b^2 + 11*b^3)*cosh(d*x + c)^4 + 8*a^2*b + 2*a*b^2 + 5*b^3 + 2*(216*a
^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 42*(33*(24*a^
2*b + 30*a*b^2 + 11*b^3)*cosh(d*x + c)^5 + 2*(216*a^2*b + 126*a*b^2 - 5*b^
3)*cosh(d*x + c)^3 + 3*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^6 - 18*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 18*(77*(24*a^2*
b + 30*a*b^2 + 11*b^3)*cosh(d*x + c)^6 + 7*(216*a^2*b + 126*a*b^2 - 5*b^3)
*cosh(d*x + c)^4 - 8*a^2*b - 2*a*b^2 - 5*b^3 + 21*(8*a^2*b + 2*a*b^2 + 5*b
^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 18*(55*(24*a^2*b + 30*a*b^2 + 11*b^
3)*cosh(d*x + c)^7 + 7*(216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c)^5 + 3
5*(8*a^2*b + 2*a*b^2 + 5*b^3)*cosh(d*x + c)^3 - 5*(8*a^2*b + 2*a*b^2 + 5*b
^3)*cosh(d*x + c))*sinh(d*x + c)^4 - (216*a^2*b + 126*a*b^2 - 5*b^3)*cosh(
d*x + c)^3 + (495*(24*a^2*b + 30*a*b^2 + 11*b^3)*cosh(d*x + c)^8 + 84*(216
*a^2*b + 126*a*b^2 - 5*b^3)*cosh(d*x + c)^6 + 630*(8*a^2*b + 2*a*b^2 + ...
```

3.101.6 Sympy [F]

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^3 \operatorname{sech}\left(c+dx\right)dx$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x), x)
```

3.101.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(141) = 282$.

Time = 0.28 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.43

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$
$$-\frac{1}{24}b^3\left(\frac{15\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{33\,e^{(-dx-c)}-5\,e^{(-3\,dx-3\,c)}+90\,e^{(-5\,dx-5\,c)}-90\,e^{(-7\,dx-7\,c)}+5\,e^{(-9\,dx-}\right.$$
$$-\frac{3}{4}ab^2\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{5\,e^{(-dx-c)}-3\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}-5\,e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)$$
$$-3\,a^2b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)+\frac{a^3\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/24*b^3*(15*arctan(e^(-d*x - c))/d + (33*e^(-d*x - c) - 5*e^(-3*d*x - 3*
c) + 90*e^(-5*d*x - 5*c) - 90*e^(-7*d*x - 7*c) + 5*e^(-9*d*x - 9*c) - 33*e
^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6
*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12
*c) + 1))) - 3/4*a*b^2*(3*arctan(e^(-d*x - c))/d + (5*e^(-d*x - c) - 3*e^(
-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x -
 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1)))
- 3*a^2*b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(
2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a^3*arctan(sinh(d*x + c))/d
```

3.101.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(141) = 282$.

Time = 0.40 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.08

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(16\,a^3+24\,a^2b+18\,ab^2+5\,b^3)-\frac{4\left(72\,a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+90\,ab\right.}{}$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/96*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(16*a^3 +
24*a^2*b + 18*a*b^2 + 5*b^3) - 4*(72*a^2*b*(e^(d*x + c) - e^(-d*x - c))^5
+ 90*a*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 33*b^3*(e^(d*x + c) - e^(-d*x
- c))^5 + 576*a^2*b*(e^(d*x + c) - e^(-d*x - c))^3 + 576*a*b^2*(e^(d*x + c
) - e^(-d*x - c))^3 + 160*b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 1152*a^2*b*
(e^(d*x + c) - e^(-d*x - c)) + 864*a*b^2*(e^(d*x + c) - e^(-d*x - c)) + 24
0*b^3*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^3
)/d
```

3.101.9 Mupad [B] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 535, normalized size of antiderivative = 3.59

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$=\frac{\operatorname{atan}\left(\frac{e^{dx}e^{c}\left(16a^3\sqrt{d^2}+5b^3\sqrt{d^2}+18ab^2\sqrt{d^2}+24a^2b\sqrt{d^2}\right)}{d\sqrt{256a^6+768a^5b+1152a^4b^2+1024a^3b^3+564a^2b^4+180ab^5+25b^6}}\right)\sqrt{256a^6+768a^5b+1152a^4b^2+1024a^3b^3+}}{8\sqrt{d^2}}$$

$$-\frac{e^{c+dx}\left(55b^3+54ab^2\right)}{3d\left(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1\right)}$$

$$-\frac{80b^3e^{c+dx}}{3d\left(5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1\right)}$$

$$+\frac{6e^{c+dx}\left(5b^3+2ab^2\right)}{d\left(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1\right)}$$

$$+\frac{32b^3e^{c+dx}}{3d\left(6e^{2c+2dx}+15e^{4c+4dx}+20e^{6c+6dx}+15e^{8c+8dx}+6e^{10c+10dx}+e^{12c+12dx}+1\right)}$$

$$-\frac{e^{c+dx}\left(24a^2b+30ab^2+11b^3\right)}{8d\left(e^{2c+2dx}+1\right)}+\frac{e^{c+dx}\left(72a^2b+162ab^2+85b^3\right)}{12d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)^3/cosh(c + d*x),x)
```

output

```
(atan((exp(d*x)*exp(c)*(16*a^3*(d^2)^(1/2) + 5*b^3*(d^2)^(1/2) + 18*a*b^2*
(d^2)^(1/2) + 24*a^2*b*(d^2)^(1/2)))/(d*(180*a*b^5 + 768*a^5*b + 256*a^6 +
 25*b^6 + 564*a^2*b^4 + 1024*a^3*b^3 + 1152*a^4*b^2)^(1/2)))*(180*a*b^5 +
768*a^5*b + 256*a^6 + 25*b^6 + 564*a^2*b^4 + 1024*a^3*b^3 + 1152*a^4*b^2)^
(1/2))/(8*(d^2)^(1/2)) - (exp(c + d*x)*(54*a*b^2 + 55*b^3))/(3*d*(3*exp(2*
c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (80*b^3*exp(c +
 d*x))/(3*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x
) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (6*exp(c + d*x)*(2*a*b
^2 + 5*b^3))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d
*x) + exp(8*c + 8*d*x) + 1)) + (32*b^3*exp(c + d*x))/(3*d*(6*exp(2*c + 2*d
*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*
exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (exp(c + d*x)*(30*a*b^2 +
24*a^2*b + 11*b^3))/(8*d*(exp(2*c + 2*d*x) + 1)) + (exp(c + d*x)*(162*a*b^
2 + 72*a^2*b + 85*b^3))/(12*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.102 $\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

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3.102.1 Optimal result

Integrand size = 23, antiderivative size = 67

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{a^3\tanh(c+dx)}{d} + \frac{a^2b\tanh^3(c+dx)}{d} + \frac{3ab^2\tanh^5(c+dx)}{5d} + \frac{b^3\tanh^7(c+dx)}{7d}$$

output
```
a^3*tanh(d*x+c)/d+a^2*b*tanh(d*x+c)^3/d+3/5*a*b^2*tanh(d*x+c)^5/d+1/7*b^3*
tanh(d*x+c)^7/d
```

3.102.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{a^3\tanh(c+dx)}{d} + \frac{a^2b\tanh^3(c+dx)}{d} + \frac{3ab^2\tanh^5(c+dx)}{5d} + \frac{b^3\tanh^7(c+dx)}{7d}$$

input
```
Integrate[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(a^3*Tanh[c + d*x])/d + (a^2*b*Tanh[c + d*x]^3)/d + (3*a*b^2*Tanh[c + d*x]
^5)/(5*d) + (b^3*Tanh[c + d*x]^7)/(7*d)
```

3.102.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\downarrow 3042$$

$$\int \sec(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^3\,dx$$

$$\downarrow 4158$$

$$\frac{\int \left(b\tanh^2(c+dx)+a\right)^3\,d\tanh(c+dx)}{d}$$

$$\downarrow 210$$

$$\frac{\int \left(b^3\tanh^6(c+dx)+3ab^2\tanh^4(c+dx)+3a^2b\tanh^2(c+dx)+a^3\right)\,d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{a^3\tanh(c+dx)+a^2b\tanh^3(c+dx)+\frac{3}{5}ab^2\tanh^5(c+dx)+\frac{1}{7}b^3\tanh^7(c+dx)}{d}$$

input
```
Int[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(a^3*Tanh[c + d*x] + a^2*b*Tanh[c + d*x]^3 + (3*a*b^2*Tanh[c + d*x]^5)/5 +
 (b^3*Tanh[c + d*x]^7)/7)/d
```

3.102.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.102.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(63) = 126$.

Time = 21.44 (sec) , antiderivative size = 227, normalized size of antiderivative = 3.39

method	result
derivativedivides	$\frac{a^3\tanh(dx+c)+3a^2b\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+3ab^2\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}-$
default	$\frac{a^3\tanh(dx+c)+3a^2b\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+3ab^2\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}-$
risch	$-\frac{2\left(105a^2b\,e^{12dx+12c}+21ab^2+105ab^2e^{12dx+12c}+35b^3e^{12dx+12c}+35a^3+105\,e^{4dx+4c}b^3+35a^2b+42\,e^{2dx+2c}ab^2+140a^$

input
```
int(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*tanh(d*x+c)+3*a^2*b*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3+1/3*
sech(d*x+c)^2)*tanh(d*x+c))+3*a*b^2*(-1/2*sinh(d*x+c)^3/cosh(d*x+c)^5-3/8*
sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*
tanh(d*x+c))+b^3*(-1/2*sinh(d*x+c)^5/cosh(d*x+c)^7-5/8*sinh(d*x+c)^3/cosh(
d*x+c)^7-5/16*sinh(d*x+c)/cosh(d*x+c)^7+5/16*(16/35+1/7*sech(d*x+c)^6+6/35
*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c)))
```

3.102.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 786 vs. $2(63) = 126$.

Time = 0.27 (sec) , antiderivative size = 786, normalized size of antiderivative = 11.73

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$

$$-\frac{4\left(\left(35\,a^3+70\,a^2b+63\,ab^2+20\,b^3\right)\cosh\left(dx+c\right)^6+6\left(35\,a^2b+42\,ab^2+15\,b^3\right)\cosh\left(dx+c\right)\sinh\left(dx+\right.\right.}{35\left(d\cosh\left(d\right.\right.}$$

input
```
integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-4/35*((35*a^3 + 70*a^2*b + 63*a*b^2 + 20*b^3)*cosh(d*x + c)^6 + 6*(35*a^2
*b + 42*a*b^2 + 15*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (35*a^3 + 70*a^2*b
 + 63*a*b^2 + 20*b^3)*sinh(d*x + c)^6 + 14*(15*a^3 + 20*a^2*b + 9*a*b^2)*c
osh(d*x + c)^4 + (210*a^3 + 280*a^2*b + 126*a*b^2 + 15*(35*a^3 + 70*a^2*b
+ 63*a*b^2 + 20*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(35*a^2*b + 4
2*a*b^2 + 15*b^3)*cosh(d*x + c)^3 + 28*(5*a^2*b + 3*a*b^2)*cosh(d*x + c))*
sinh(d*x + c)^3 + 350*a^3 + 280*a^2*b + 210*a*b^2 + 7*(75*a^3 + 70*a^2*b +
 39*a*b^2 + 20*b^3)*cosh(d*x + c)^2 + (15*(35*a^3 + 70*a^2*b + 63*a*b^2 +
20*b^3)*cosh(d*x + c)^4 + 525*a^3 + 490*a^2*b + 273*a*b^2 + 140*b^3 + 84*(
15*a^3 + 20*a^2*b + 9*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(35*a
^2*b + 42*a*b^2 + 15*b^3)*cosh(d*x + c)^5 + 56*(5*a^2*b + 3*a*b^2)*cosh(d*
x + c)^3 + 7*(25*a^2*b + 6*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d
*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 +
 8*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^6 + 4*(
14*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^5 + 28*d*cosh(d*x
+ c)^4 + 2*(35*d*cosh(d*x + c)^4 + 60*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x +
 c)^4 + 8*(7*d*cosh(d*x + c)^5 + 15*d*cosh(d*x + c)^3 + 7*d*cosh(d*x + c))
*sinh(d*x + c)^3 + 56*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 30*d*co
sh(d*x + c)^4 + 42*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c)^2 + 4*(2*d*cosh
(d*x + c)^7 + 9*d*cosh(d*x + c)^5 + 14*d*cosh(d*x + c)^3 + 7*d*cosh(d*x...
```

3.102.6 Sympy [F]

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2(c+dx)\right)^3 \operatorname{sech}^2(c+dx)\,dx$$

input
```
integrate(sech(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x)**2, x)
```

3.102.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{b^3\tanh(dx+c)^7}{7\,d} + \frac{3\,ab^2\tanh(dx+c)^5}{5\,d} + \frac{a^2b\tanh(dx+c)^3}{d} + \frac{2\,a^3}{d(e^{(-2\,dx-2\,c)}+1)}$$

input
```
integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/7*b^3*tanh(d*x + c)^7/d + 3/5*a*b^2*tanh(d*x + c)^5/d + a^2*b*tanh(d*x +
 c)^3/d + 2*a^3/(d*(e^(-2*d*x - 2*c) + 1))
```

3.102.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(63) = 126.

Time = 0.43 (sec) , antiderivative size = 347, normalized size of antiderivative = 5.18

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{2\left(35\,a^3e^{(12\,dx+12\,c)} + 105\,a^2be^{(12\,dx+12\,c)} + 105\,ab^2e^{(12\,dx+12\,c)} + 35\,b^3e^{(12\,dx+12\,c)} + 210\,a^3e^{(10\,dx+10\,c)} + 420\right.}{}$$

input
```
integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-2/35*(35*a^3*e^(12*d*x + 12*c) + 105*a^2*b*e^(12*d*x + 12*c) + 105*a*b^2*
e^(12*d*x + 12*c) + 35*b^3*e^(12*d*x + 12*c) + 210*a^3*e^(10*d*x + 10*c) +
 420*a^2*b*e^(10*d*x + 10*c) + 210*a*b^2*e^(10*d*x + 10*c) + 525*a^3*e^(8*
d*x + 8*c) + 665*a^2*b*e^(8*d*x + 8*c) + 315*a*b^2*e^(8*d*x + 8*c) + 175*b
^3*e^(8*d*x + 8*c) + 700*a^3*e^(6*d*x + 6*c) + 560*a^2*b*e^(6*d*x + 6*c) +
 420*a*b^2*e^(6*d*x + 6*c) + 525*a^3*e^(4*d*x + 4*c) + 315*a^2*b*e^(4*d*x
+ 4*c) + 231*a*b^2*e^(4*d*x + 4*c) + 105*b^3*e^(4*d*x + 4*c) + 210*a^3*e^(
2*d*x + 2*c) + 140*a^2*b*e^(2*d*x + 2*c) + 42*a*b^2*e^(2*d*x + 2*c) + 35*a
^3 + 35*a^2*b + 21*a*b^2 + 5*b^3)/(d*(e^(2*d*x + 2*c) + 1)^7)
```

3.102.9 Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 1050, normalized size of antiderivative = 15.67

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int((a + b*tanh(c + d*x)^2)^3/cosh(c + d*x)^2,x)
```

output
```
- ((2*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(35*d) + (2*exp(6*c + 6*d*x)*(a
 + b)^3)/(7*d) - (6*exp(2*c + 2*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(35*
d) + (6*exp(4*c + 4*d*x)*(a + b)^2*(a - b))/(7*d))/(4*exp(2*c + 2*d*x) + 6
*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(a +
b)^3)/(7*d) + (2*exp(12*c + 12*d*x)*(a + b)^3)/(7*d) - (6*exp(4*c + 4*d*x)
*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(7*d) - (6*exp(8*c + 8*d*x)*(a*b^2 + a^2
*b - 5*a^3 - 5*b^3))/(7*d) + (8*exp(6*c + 6*d*x)*(3*a*b^2 - 3*a^2*b + 5*a^
3 - 5*b^3))/(7*d) + (12*exp(2*c + 2*d*x)*(a + b)^2*(a - b))/(7*d) + (12*ex
p(10*c + 10*d*x)*(a + b)^2*(a - b))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*
c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*
d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1) - ((2*exp(4*c + 4*d*
x)*(a + b)^3)/(7*d) - (2*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(35*d) + (4*exp(
2*c + 2*d*x)*(a + b)^2*(a - b))/(7*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4
*d*x) + exp(6*c + 6*d*x) + 1) - ((2*(a + b)^2*(a - b))/(7*d) + (2*exp(2*c
+ 2*d*x)*(a + b)^3)/(7*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (
(2*exp(8*c + 8*d*x)*(a + b)^3)/(7*d) - (2*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))
/(35*d) - (12*exp(4*c + 4*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(35*d) + (
8*exp(2*c + 2*d*x)*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(35*d) + (8*exp(6*
c + 6*d*x)*(a + b)^2*(a - b))/(7*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*
d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + ...
```

3.103 $\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

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3.103.1 Optimal result

Integrand size = 23, antiderivative size = 198

$$\begin{aligned}
&\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx \\
&= \frac{(64a^3+48a^2b+24ab^2+5b^3)\arctan(\sinh(c+dx))}{128d} \\
&\quad + \frac{(64a^3+48a^2b+24ab^2+5b^3)\operatorname{sech}(c+dx)\tanh(c+dx)}{128d} \\
&\quad - \frac{b(72a^2+52ab+15b^2)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{192d} \\
&\quad - \frac{b(12a+5b)\operatorname{sech}^5(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)\tanh(c+dx)}{48d} \\
&\quad - \frac{b\operatorname{sech}^7(c+dx)\left(a+(a+b)\sinh^2(c+dx)\right)^2\tanh(c+dx)}{8d}
\end{aligned}$$

output

```
1/128*(64*a^3+48*a^2*b+24*a*b^2+5*b^3)*arctan(sinh(d*x+c))/d+1/128*(64*a^3
+48*a^2*b+24*a*b^2+5*b^3)*sech(d*x+c)*tanh(d*x+c)/d-1/192*b*(72*a^2+52*a*b
+15*b^2)*sech(d*x+c)^3*tanh(d*x+c)/d-1/48*b*(12*a+5*b)*sech(d*x+c)^5*(a+(a
+b)*sinh(d*x+c)^2)*tanh(d*x+c)/d-1/8*b*sech(d*x+c)^7*(a+(a+b)*sinh(d*x+c)^
2)^2*tanh(d*x+c)/d
```

3.103.2 Mathematica [A] (verified)

Time = 13.01 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.80

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{6(64a^3+48a^2b+24ab^2+5b^3)\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+3(64a^3+48a^2b+24ab^2+5b^3)\operatorname{sech}(c+dx)\,\mathrm{t}}{}$$

input
```
Integrate[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(6*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*ArcTan[Tanh[(c + d*x)/2]] + 3*(6
4*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*Sech[c + d*x]*Tanh[c + d*x] - 2*b*(14
4*a^2 + 168*a*b + 59*b^2)*Sech[c + d*x]^3*Tanh[c + d*x] + 8*b^2*(24*a + 17
*b)*Sech[c + d*x]^5*Tanh[c + d*x] - 48*b^3*Sech[c + d*x]^7*Tanh[c + d*x])/
(384*d)
```

3.103.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4159, 315, 401, 25, 298, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$\downarrow 3042$$
$$\int \sec(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^3\,dx$$
$$\downarrow 4159$$
$$\frac{\int \frac{\left((a+b)\sinh^2(c+dx)+a\right)^3}{\left(\sinh^2(c+dx)+1\right)^5}\,d\sinh(c+dx)}{d}$$
$$\downarrow 315$$
$$\frac{\frac{1}{8}\int \frac{\left((a+b)\sinh^2(c+dx)+a\right)\left((a+b)(8a+5b)\sinh^2(c+dx)+a(8a+b)\right)}{\left(\sinh^2(c+dx)+1\right)^4}\,d\sinh(c+dx)-\frac{b\sinh(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)^2}{8\left(\sinh^2(c+dx)+1\right)^4}}{d}$$

$$\downarrow 401$$

$$\frac{\frac{1}{8}\left(-\frac{1}{6}\int -\frac{3(a+b)\left(16a^2+14ba+5b^2\right)\sinh^2(c+dx)+a\left(48a^2+18ba+5b^2\right)}{\left(\sinh^2(c+dx)+1\right)^3}d\sinh(c+dx)-\frac{b(12a+5b)\sinh(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)}{6\left(\sinh^2(c+dx)+1\right)^3}\right)}{d}$$

$$\downarrow 25$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\int \frac{3(a+b)\left(16a^2+14ba+5b^2\right)\sinh^2(c+dx)+a\left(48a^2+18ba+5b^2\right)}{\left(\sinh^2(c+dx)+1\right)^3}d\sinh(c+dx)-\frac{b(12a+5b)\sinh(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)}{6\left(\sinh^2(c+dx)+1\right)^3}\right)-\frac{b}{}}{d}$$

$$\downarrow 298$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(64a^3+48a^2b+24ab^2+5b^3\right)\int \frac{1}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)-\frac{b\left(72a^2+52ab+15b^2\right)\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}\right)-\frac{b(12a+5b)\sin}{}\right.}{d}$$

$$\downarrow 215$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(64a^3+48a^2b+24ab^2+5b^3\right)\left(\frac{1}{2}\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)+\frac{\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)-\frac{b\left(72a^2+52ab+15b^2\right)\sinh(c}{4\left(\sinh^2(c+dx)+1\right)^2}\right.\right.}{d}$$

$$\downarrow 216$$

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(64a^3+48a^2b+24ab^2+5b^3\right)\left(\frac{1}{2}\arctan(\sinh(c+dx))+\frac{\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)-\frac{b\left(72a^2+52ab+15b^2\right)\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}\right)-\right.}{d}$$

input
```
Int[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-1/8*(b*Sinh[c + d*x]*(a + (a + b)*Sinh[c + d*x]^2)^2)/(1 + Sinh[c + d*x]
^2)^4 + (-1/6*(b*(12*a + 5*b)*Sinh[c + d*x]*(a + (a + b)*Sinh[c + d*x]^2))
/(1 + Sinh[c + d*x]^2)^3 + (-1/4*(b*(72*a^2 + 52*a*b + 15*b^2)*Sinh[c + d*
x])/(1 + Sinh[c + d*x]^2)^2 + (3*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*(A
rcTan[Sinh[c + d*x]]/2 + Sinh[c + d*x]/(2*(1 + Sinh[c + d*x]^2))))/4)/6)/8
)/d
```

3.103. $\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$

3.103.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.103.4 Maple [A] (verified)

Time = 43.36 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.47

method	result
derivativedivides	$a^3\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3a^2b\left(-\frac{\sinh(dx+c)}{3\cosh(dx+c)^4}+\frac{\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)}{3}+\frac{\arctan\left(e^{dx+c}\right)}{4}\right.$
default	$a^3\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3a^2b\left(-\frac{\sinh(dx+c)}{3\cosh(dx+c)^4}+\frac{\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)}{3}+\frac{\arctan\left(e^{dx+c}\right)}{4}\right.$
risch	$\frac{e^{dx+c}\left(-432a^2b\,e^{12dx+12c}-72a\,b^2-984a\,b^2e^{12dx+12c}-397b^3e^{12dx+12c}-192a^3-895\,e^{4dx+4c}b^3+72a\,b^2e^{14dx+14c}+192a\right.}{}$

input

```
int(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output

```
1/d*(a^3*(1/2*sech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+3*a^2*b*(-1/3/co
sh(d*x+c)^4*sinh(d*x+c)+1/3*(1/4*sech(d*x+c)^3+3/8*sech(d*x+c))*tanh(d*x+c
)+1/4*arctan(exp(d*x+c)))+3*a*b^2*(-1/3*sinh(d*x+c)^3/cosh(d*x+c)^6-1/5*si
nh(d*x+c)/cosh(d*x+c)^6+1/5*(1/6*sech(d*x+c)^5+5/24*sech(d*x+c)^3+5/16*sec
h(d*x+c))*tanh(d*x+c)+1/8*arctan(exp(d*x+c)))+b^3*(-1/3*sinh(d*x+c)^5/cosh
(d*x+c)^8-1/3*sinh(d*x+c)^3/cosh(d*x+c)^8-1/7*sinh(d*x+c)/cosh(d*x+c)^8+1/
7*(1/8*sech(d*x+c)^7+7/48*sech(d*x+c)^5+35/192*sech(d*x+c)^3+35/128*sech(d
*x+c))*tanh(d*x+c)+5/64*arctan(exp(d*x+c))))
```

3.103.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6114 vs. 2(188) = 376.

Time = 0.32 (sec) , antiderivative size = 6114, normalized size of antiderivative = 30.88

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.103.6 Sympy [F]

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2(c+dx)\right)^3 \operatorname{sech}^3(c+dx)\,dx$$

input
```
integrate(sech(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x)**3, x)
```

3.103.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 553 vs. 2(188) = 376.

Time = 0.30 (sec) , antiderivative size = 553, normalized size of antiderivative = 2.79

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$

$$-\frac{1}{192}b^3\left(\frac{15\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{15\,e^{(-dx-c)}-397\,e^{(-3\,dx-3\,c)}+895\,e^{(-5\,dx-5\,c)}-1765\,e^{(-7\,dx-7\,c)}+1765}{d(8\,e^{(-2\,dx-2\,c)}+28\,e^{(-4\,dx-4\,c)}+56\,e^{(-6\,dx-6\,c)}+70\,e^{(-8\,dx-8\,c)}+56\,e^{(}}\right.$$

$$-\frac{1}{8}ab^2\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3\,e^{(-dx-c)}-47\,e^{(-3\,dx-3\,c)}+78\,e^{(-5\,dx-5\,c)}-78\,e^{(-7\,dx-7\,c)}+47\,e^{(-9\,dx-}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,dx-}}\right.$$

$$-\frac{3}{4}a^2b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-7\,e^{(-3\,dx-3\,c)}+7\,e^{(-5\,dx-5\,c)}-e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)$$

$$-a^3\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$

input
```
integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/192*b^3*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) - 397*e^(-3*d*x -
 3*c) + 895*e^(-5*d*x - 5*c) - 1765*e^(-7*d*x - 7*c) + 1765*e^(-9*d*x - 9*
c) - 895*e^(-11*d*x - 11*c) + 397*e^(-13*d*x - 13*c) - 15*e^(-15*d*x - 15*
c))/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 7
0*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(
-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1))) - 1/8*a*b^2*(3*arctan(e^(-d*x
 - c))/d - (3*e^(-d*x - c) - 47*e^(-3*d*x - 3*c) + 78*e^(-5*d*x - 5*c) - 78
*e^(-7*d*x - 7*c) + 47*e^(-9*d*x - 9*c) - 3*e^(-11*d*x - 11*c))/(d*(6*e^(-
2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x -
8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) + 1))) - 3/4*a^2*b*(arcta
n(e^(-d*x - c))/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c) + 7*e^(-5*d*x - 5*c
) - e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-
6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a^3*(arctan(e^(-d*x - c))/d - (e^
(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) +
 1)))
```

3.103.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 485 vs. $2(188) = 376$.

Time $= 0.41$ (sec) , antiderivative size $= 485$, normalized size of antiderivative $= 2.45$

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{3\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)\left(64a^3+48a^2b+24ab^2+5b^3\right)+\frac{4\left(192a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^7+144a\right.}{}}{}$$

input
```
integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output

```
1/768*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(64*a^3 +
 48*a^2*b + 24*a*b^2 + 5*b^3) + 4*(192*a^3*(e^(d*x + c) - e^(-d*x - c))^7
+ 144*a^2*b*(e^(d*x + c) - e^(-d*x - c))^7 + 72*a*b^2*(e^(d*x + c) - e^(-d
*x - c))^7 + 15*b^3*(e^(d*x + c) - e^(-d*x - c))^7 + 2304*a^3*(e^(d*x + c)
 - e^(-d*x - c))^5 + 576*a^2*b*(e^(d*x + c) - e^(-d*x - c))^5 - 480*a*b^2*
(e^(d*x + c) - e^(-d*x - c))^5 - 292*b^3*(e^(d*x + c) - e^(-d*x - c))^5 +
9216*a^3*(e^(d*x + c) - e^(-d*x - c))^3 - 2304*a^2*b*(e^(d*x + c) - e^(-d*
x - c))^3 - 4224*a*b^2*(e^(d*x + c) - e^(-d*x - c))^3 - 880*b^3*(e^(d*x +
c) - e^(-d*x - c))^3 + 12288*a^3*(e^(d*x + c) - e^(-d*x - c)) - 9216*a^2*b
*(e^(d*x + c) - e^(-d*x - c)) - 4608*a*b^2*(e^(d*x + c) - e^(-d*x - c)) -
960*b^3*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)
^4)/d
```

3.103.9 Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 951, normalized size of antiderivative = 4.80

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$=\frac{\operatorname{atan}\left(\frac{e^{dx}e^{c}\left(64a^3\sqrt{d^2}+5b^3\sqrt{d^2}+24ab^2\sqrt{d^2}+48a^2b\sqrt{d^2}\right)}{d\sqrt{4096a^6+6144a^5b+5376a^4b^2+2944a^3b^3+1056a^2b^4+240ab^5+25b^6}}\right)\sqrt{4096a^6+6144a^5b+5376a^4b^2+2944a^3b^3+1056a^2b^4+240ab^5+25b^6}}{64\sqrt{d^2}}$$
$$-\frac{\frac{e^{c+dx}(a+b)^3}{2d}+\frac{e^{13c+13dx}(a+b)^3}{2d}-\frac{3e^{5c+5dx}\left(-5a^3+a^2b+ab^2-5b^3\right)}{2d}-\frac{3e^{9c+9dx}\left(-5a^3+a^2b+ab^2-5b^3\right)}{2d}+\frac{2e^{7c+7dx}\left(5a^3-3}{8e^{2c+2dx}+28e^{4c+4dx}+56e^{6c+6dx}+70e^{8c+8dx}+56e^{10c+10dx}+28e^{12c+12}}$$
$$+\frac{2e^{c+dx}\left(85b^3+48ab^2\right)}{3d\left(5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1\right)}$$
$$+\frac{16b^3e^{c+dx}}{d\left(7e^{2c+2dx}+21e^{4c+4dx}+35e^{6c+6dx}+35e^{8c+8dx}+21e^{10c+10dx}+7e^{12c+12dx}+e^{14c+14dx}+1\right)}$$
$$+\frac{e^{c+dx}\left(64a^3+48a^2b+24ab^2+5b^3\right)}{64d\left(e^{2c+2dx}+1\right)}$$
$$-\frac{4e^{c+dx}\left(35b^3+6ab^2\right)}{3d\left(6e^{2c+2dx}+15e^{4c+4dx}+20e^{6c+6dx}+15e^{8c+8dx}+6e^{10c+10dx}+e^{12c+12dx}+1\right)}$$
$$-\frac{e^{c+dx}\left(144a^3+576a^2b+600ab^2+203b^3\right)}{96d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}$$
$$+\frac{e^{c+dx}\left(288a^2b+600ab^2+305b^3\right)}{24d\left(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1\right)}$$
$$-\frac{e^{c+dx}\left(24a^2b+168ab^2+145b^3\right)}{4d\left(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)^3/cosh(c + d*x)^3,x)
```

output
```
(atan((exp(d*x)*exp(c)*(64*a^3*(d^2)^(1/2) + 5*b^3*(d^2)^(1/2) + 24*a*b^2*
(d^2)^(1/2) + 48*a^2*b*(d^2)^(1/2)))/(d*(240*a*b^5 + 6144*a^5*b + 4096*a^6
 + 25*b^6 + 1056*a^2*b^4 + 2944*a^3*b^3 + 5376*a^4*b^2)^(1/2)))*(240*a*b^5
 + 6144*a^5*b + 4096*a^6 + 25*b^6 + 1056*a^2*b^4 + 2944*a^3*b^3 + 5376*a^4
*b^2)^(1/2))/(64*(d^2)^(1/2)) - ((exp(c + d*x)*(a + b)^3)/(2*d) + (exp(13*
c + 13*d*x)*(a + b)^3)/(2*d) - (3*exp(5*c + 5*d*x)*(a*b^2 + a^2*b - 5*a^3
 - 5*b^3))/(2*d) - (3*exp(9*c + 9*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(2*
d) + (2*exp(7*c + 7*d*x)*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/d + (3*exp(3
*c + 3*d*x)*(a + b)^2*(a - b))/d + (3*exp(11*c + 11*d*x)*(a + b)^2*(a - b)
)/d)/(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*
exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(1
4*c + 14*d*x) + exp(16*c + 16*d*x) + 1) + (2*exp(c + d*x)*(48*a*b^2 + 85*b
^3))/(3*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x)
 + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (16*b^3*exp(c + d*x))/(d
*(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(
8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 1
4*d*x) + 1)) + (exp(c + d*x)*(24*a*b^2 + 48*a^2*b + 64*a^3 + 5*b^3))/(64*d
*(exp(2*c + 2*d*x) + 1)) - (4*exp(c + d*x)*(6*a*b^2 + 35*b^3))/(3*d*(6*exp
(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8
*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (exp(c + d*x)...
```

3.104 $\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

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3.104.1 Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{a^3\tanh(c+dx)}{d} - \frac{a^2(a-3b)\tanh^3(c+dx)}{3d} - \frac{3a(a-b)b\tanh^5(c+dx)}{5d} - \frac{(3a-b)b^2\tanh^7(c+dx)}{7d} - \frac{b^3\tanh^9(c+dx)}{9d}$$

output

```
a^3*tanh(d*x+c)/d-1/3*a^2*(a-3*b)*tanh(d*x+c)^3/d-3/5*a*(a-b)*b*tanh(d*x+c
)^5/d-1/7*(3*a-b)*b^2*tanh(d*x+c)^7/d-1/9*b^3*tanh(d*x+c)^9/d
```

3.104.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 218 vs. 2(102) = 204.

Time = 2.89 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.14

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{(5775a^3 - 1071a^2b + 621ab^2 - 725b^3 + 10(903a^3 - 63a^2b - 27ab^2 + 107b^3)\cosh(2(c+dx)) + 8(525a^3 +$$

input

```
Integrate[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((5775*a^3 - 1071*a^2*b + 621*a*b^2 - 725*b^3 + 10*(903*a^3 - 63*a^2*b - 2
7*a*b^2 + 107*b^3)*Cosh[2*(c + d*x)] + 8*(525*a^3 + 126*a^2*b - 81*a*b^2 -
 50*b^3)*Cosh[4*(c + d*x)] + 1050*a^3*Cosh[6*(c + d*x)] + 630*a^2*b*Cosh[6
*(c + d*x)] + 270*a*b^2*Cosh[6*(c + d*x)] + 50*b^3*Cosh[6*(c + d*x)] + 105
*a^3*Cosh[8*(c + d*x)] + 63*a^2*b*Cosh[8*(c + d*x)] + 27*a*b^2*Cosh[8*(c +
 d*x)] + 5*b^3*Cosh[8*(c + d*x)])*Sech[c + d*x]^8*Tanh[c + d*x])/(20160*d)
```

3.104.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\downarrow 3042$$

$$\int \sec(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)^3\,dx$$

$$\downarrow 4158$$

$$\frac{\int \left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3\,d\tanh(c+dx)}{d}$$

$$\downarrow 290$$

$$\frac{\int \left(-b^3\tanh^8(c+dx)-(3a-b)b^2\tanh^6(c+dx)-3a(a-b)b\tanh^4(c+dx)-a^2(a-3b)\tanh^2(c+dx)+a^3\right)d\,t}{d}$$

$$\downarrow 2009$$

$$\frac{a^3\tanh(c+dx)-\frac{1}{3}a^2(a-3b)\tanh^3(c+dx)-\frac{1}{7}b^2(3a-b)\tanh^7(c+dx)-\frac{3}{5}ab(a-b)\tanh^5(c+dx)-\frac{1}{9}b^3\tanh^{9}}{d}$$

input
```
Int[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(a^3*Tanh[c + d*x] - (a^2*(a - 3*b)*Tanh[c + d*x]^3)/3 - (3*a*(a - b)*b*Ta
nh[c + d*x]^5)/5 - ((3*a - b)*b^2*Tanh[c + d*x]^7)/7 - (b^3*Tanh[c + d*x]^
9)/9)/d
```

3.104.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.104.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(94) = 188$.

Time = 80.58 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.64

method	result
derivativedivides	$\frac{a^3\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+3a^2b\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{4}\right)+3ab^2\left(-\frac{s}{4}\right.}{}$
default	$\frac{a^3\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+3a^2b\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{4}\right)+3ab^2\left(-\frac{s}{4}\right.}{}$
risch	$-\frac{4\left(3465a^2b\,e^{12dx+12c}+27a\,b^2+945a\,b^2e^{12dx+12c}-525b^3e^{12dx+12c}+105a^3-135\,e^{4dx+4c}b^3+945a\,b^2e^{14dx+14c}+315a^3e\right.}{}$

input
```
int(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+3*a^2*b*(-1/4*sinh(d*x+c)/cos
h(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))+3*
a*b^2*(-1/4*sinh(d*x+c)^3/cosh(d*x+c)^7-1/8*sinh(d*x+c)/cosh(d*x+c)^7+1/8*
(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c
))+b^3*(-1/4*sinh(d*x+c)^5/cosh(d*x+c)^9-5/24*sinh(d*x+c)^3/cosh(d*x+c)^9-
5/64*sinh(d*x+c)/cosh(d*x+c)^9+5/64*(128/315+1/9*sech(d*x+c)^8+8/63*sech(d
*x+c)^6+16/105*sech(d*x+c)^4+64/315*sech(d*x+c)^2)*tanh(d*x+c)))
```

3.104.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. $2(94) = 188$.

Time $= 0.26$ (sec) , antiderivative size $= 1185$, normalized size of antiderivative $= 11.62$

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-8/315*(2*(105*a^3 + 252*a^2*b + 243*a*b^2 + 80*b^3)*cosh(d*x + c)^7 + 14*
(105*a^3 + 252*a^2*b + 243*a*b^2 + 80*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 +
 (105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*sinh(d*x + c)^7 + 6*(245*a^3
+ 336*a^2*b + 99*a*b^2 - 40*b^3)*cosh(d*x + c)^5 + 3*(175*a^3 + 483*a^2*b
+ 117*a*b^2 - 95*b^3 + 7*(105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(105*a^3 + 252*a^2*b + 243*a*b^2 + 80*
b^3)*cosh(d*x + c)^3 + 3*(245*a^3 + 336*a^2*b + 99*a*b^2 - 40*b^3)*cosh(d*
x + c))*sinh(d*x + c)^4 + 18*(245*a^3 + 168*a^2*b + 27*a*b^2 + 40*b^3)*cos
h(d*x + c)^3 + (35*(105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*cosh(d*x +
c)^4 + 945*a^3 + 1701*a^2*b + 459*a*b^2 + 855*b^3 + 30*(175*a^3 + 483*a^2*
b + 117*a*b^2 - 95*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 6*(7*(105*a^3 +
 252*a^2*b + 243*a*b^2 + 80*b^3)*cosh(d*x + c)^5 + 10*(245*a^3 + 336*a^2*b
 + 99*a*b^2 - 40*b^3)*cosh(d*x + c)^3 + 9*(245*a^3 + 168*a^2*b + 27*a*b^2
+ 40*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 210*(35*a^3 + 12*a^2*b + 9*a*b^
2)*cosh(d*x + c) + (7*(105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*cosh(d*x
 + c)^6 + 15*(175*a^3 + 483*a^2*b + 117*a*b^2 - 95*b^3)*cosh(d*x + c)^4 +
525*a^3 + 693*a^2*b + 567*a*b^2 - 945*b^3 + 27*(105*a^3 + 189*a^2*b + 51*a
*b^2 + 95*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^11 + 11*d*
cosh(d*x + c)*sinh(d*x + c)^10 + d*sinh(d*x + c)^11 + 9*d*cosh(d*x + c)^9
+ (55*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^9 + 3*(55*d*cosh(d*x + c)^...
```

3.104.6 Sympy [F]

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2(c+dx)\right)^3 \operatorname{sech}^4(c+dx)\,dx$$

input
```
integrate(sech(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x)**4, x)
```

3.104.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1847 vs. $2(94) = 188$.

Time $= 0.22$ (sec) , antiderivative size $= 1847$, normalized size of antiderivative $= 18.11$

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
4/63*b^3*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c)
+ 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84
*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-1
8*d*x - 18*c) + 1)) - 27*e^(-4*d*x - 4*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-
4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x
 - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x -
16*c) + e^(-18*d*x - 18*c) + 1)) + 189*e^(-6*d*x - 6*c)/(d*(9*e^(-2*d*x -
2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) +
126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9
*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) - 189*e^(-8*d*x - 8*c)/(d*(
9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8
*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d
*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 315*e^(-10*
d*x - 10*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x -
6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*
c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1
)) - 105*e^(-12*d*x - 12*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) +
 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*
e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18
*d*x - 18*c) + 1)) + 63*e^(-14*d*x - 14*c)/(d*(9*e^(-2*d*x - 2*c) + 36*...
```

3.104.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(94) = 188$.

Time = 0.44 (sec) , antiderivative size = 447, normalized size of antiderivative = 4.38

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$
$$-\frac{4\left(315\,a^3e^{(14\,dx+14\,c)}+945\,a^2be^{(14\,dx+14\,c)}+945\,ab^2e^{(14\,dx+14\,c)}+315\,b^3e^{(14\,dx+14\,c)}+1995\,a^3e^{(12\,dx+12\,c)}+\right.}{}$$

input
```
integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-4/315*(315*a^3*e^(14*d*x + 14*c) + 945*a^2*b*e^(14*d*x + 14*c) + 945*a*b^
2*e^(14*d*x + 14*c) + 315*b^3*e^(14*d*x + 14*c) + 1995*a^3*e^(12*d*x + 12*
c) + 3465*a^2*b*e^(12*d*x + 12*c) + 945*a*b^2*e^(12*d*x + 12*c) - 525*b^3*
e^(12*d*x + 12*c) + 5355*a^3*e^(10*d*x + 10*c) + 4725*a^2*b*e^(10*d*x + 10
*c) + 945*a*b^2*e^(10*d*x + 10*c) + 1575*b^3*e^(10*d*x + 10*c) + 7875*a^3*
e^(8*d*x + 8*c) + 3213*a^2*b*e^(8*d*x + 8*c) + 2457*a*b^2*e^(8*d*x + 8*c)
- 945*b^3*e^(8*d*x + 8*c) + 6825*a^3*e^(6*d*x + 6*c) + 1827*a^2*b*e^(6*d*x
 + 6*c) + 1323*a*b^2*e^(6*d*x + 6*c) + 945*b^3*e^(6*d*x + 6*c) + 3465*a^3*
e^(4*d*x + 4*c) + 1323*a^2*b*e^(4*d*x + 4*c) + 27*a*b^2*e^(4*d*x + 4*c) -
135*b^3*e^(4*d*x + 4*c) + 945*a^3*e^(2*d*x + 2*c) + 567*a^2*b*e^(2*d*x + 2
*c) + 243*a*b^2*e^(2*d*x + 2*c) + 45*b^3*e^(2*d*x + 2*c) + 105*a^3 + 63*a^
2*b + 27*a*b^2 + 5*b^3)/(d*(e^(2*d*x + 2*c) + 1)^9)
```

3.104.9 Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 1424, normalized size of antiderivative = 13.96

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int((a + b*tanh(c + d*x)^2)^3/cosh(c + d*x)^4,x)
```

output

```
- ((4*(a + b)^2*(a - b))/(21*d) + (2*exp(2*c + 2*d*x)*(a + b)^3)/(9*d))/(3
*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((5*exp(8
*c + 8*d*x)*(a + b)^3)/(9*d) - (a*b^2 + a^2*b - 5*a^3 - 5*b^3)/(21*d) - (1
0*exp(4*c + 4*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(21*d) + (16*exp(2*c +
 2*d*x)*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(63*d) + (40*exp(6*c + 6*d*x)
*(a + b)^2*(a - b))/(21*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20
*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c
+ 12*d*x) + 1) - ((4*(a + b)^2*(a - b))/(21*d) + (2*exp(10*c + 10*d*x)*(a
+ b)^3)/(3*d) - (2*exp(2*c + 2*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(7*d)
 - (20*exp(6*c + 6*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(21*d) + (16*exp(
4*c + 4*d*x)*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(21*d) + (20*exp(8*c + 8
*d*x)*(a + b)^2*(a - b))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x)
+ 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*ex
p(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1) - ((16*(3*a*b^2 - 3*a^2*b + 5*a
^3 - 5*b^3))/(315*d) + (4*exp(6*c + 6*d*x)*(a + b)^3)/(9*d) - (4*exp(2*c +
 2*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(21*d) + (8*exp(4*c + 4*d*x)*(a +
 b)^2*(a - b))/(7*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6
*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((8*exp(2*c +
 2*d*x)*(a + b)^3)/(9*d) + (8*exp(14*c + 14*d*x)*(a + b)^3)/(9*d) - (8*exp
(6*c + 6*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(3*d) - (8*exp(10*c + 10...
```

3.105 $\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.105.9 Mupad [B] (verification not implemented) . . . 842

3.105.1 Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(3a^2+10ab+15b^2)\,x}{8(a+b)^3} + \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^3 d} + \frac{(3a+7b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2 d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d}$$

output
```
1/8*(3*a^2+10*a*b+15*b^2)*x/(a+b)^3+1/8*(3*a+7*b)*cosh(d*x+c)*sinh(d*x+c)/
(a+b)^2/d+1/4*cosh(d*x+c)^3*sinh(d*x+c)/(a+b)/d+b^(5/2)*arctan(b^(1/2)*tan
h(d*x+c)/a^(1/2))/(a+b)^3/d/a^(1/2)
```

3.105.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.96

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(3a^2+10ab+15b^2)\,(c+dx)}{8(a+b)^3 d} + \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^3 d} + \frac{(a+2b)\sinh(2(c+dx))}{4(a+b)^2 d} + \frac{\sinh(4(c+dx))}{32(a+b)d}$$

input
```
Integrate[Cosh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
((3*a^2 + 10*a*b + 15*b^2)*(c + d*x))/(8*(a + b)^3*d) + (b^(5/2)*ArcTan[(S
qrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^3*d) + ((a + 2*b)*Sinh[2*
(c + d*x)])/(4*(a + b)^2*d) + Sinh[4*(c + d*x)]/(32*(a + b)*d)
```

3.105.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4158, 316, 402, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\downarrow 4158$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)^3\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{d}$$

$$\downarrow 316$$

$$\frac{\int \frac{3b\tanh^2(c+dx)+3a+4b}{\left(1-\tanh^2(c+dx)\right)^2\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{4(a+b)} + \frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 402$$

$$\frac{\frac{\int \frac{3a^2+7ba+8b^2+b(3a+7b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2(a+b)} + \frac{(3a+7b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}}{4(a+b)} + \frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 397$$

$$\frac{\frac{\left(3a^2+10ab+15b^2\right)\int \frac{1}{1-\tanh^2(c+dx)}\,d\tanh(c+dx)}{a+b}+\frac{8b^3\int \frac{1}{b\tanh^2(c+dx)+a}\,d\tanh(c+dx)}{a+b}}{2(a+b)}+\frac{(3a+7b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}}{4(a+b)}+\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 218$$

$$\frac{\frac{\left(3a^2+10ab+15b^2\right)\int \frac{1}{1-\tanh^2(c+dx)}\,d\tanh(c+dx)}{a+b}+\frac{8b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2(a+b)}+\frac{(3a+7b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}}{4(a+b)}+\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 219$$

$$\frac{\frac{\left(3a^2+10ab+15b^2\right)\operatorname{arctanh}(\tanh(c+dx))}{a+b}+\frac{8b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2(a+b)}+\frac{(3a+7b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}}{4(a+b)}+\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

input
```
Int[Cosh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(Tanh[c + d*x]/(4*(a + b)*(1 - Tanh[c + d*x]^2)^2) + (((8*b^(5/2)*ArcTan[(
Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)) + ((3*a^2 + 10*a*b + 15
*b^2)*ArcTanh[Tanh[c + d*x]])/(a + b))/(2*(a + b)) + ((3*a + 7*b)*Tanh[c +
 d*x])/(2*(a + b)*(1 - Tanh[c + d*x]^2)))/(4*(a + b)))/d
```

3.105.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.105.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(106) = 212.

Time = 11.20 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.28

method	result
risch	$\frac{3a^2x}{8(a+b)^3}+\frac{5axb}{4(a+b)^3}+\frac{15xb^2}{8(a+b)^3}+\frac{e^{4dx+4c}}{64d(a+b)}+\frac{e^{2dx+2c}a}{8(a+b)^2d}+\frac{e^{2dx+2c}b}{4(a+b)^2d}-\frac{e^{-2dx-2c}a}{8(a^2+2ab+b^2)d}-\frac{e^{-2dx-2c}b}{4(a^2+2ab+b^2)d}$
derivativedivides	$\frac{1}{2(2a+2b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{2}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{-7a-11b}{8(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-5a-9b}{8(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-3a^2$
default	$\frac{1}{2(2a+2b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{2}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{-7a-11b}{8(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-5a-9b}{8(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-3a^2$

input
```
int(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
3/8*a^2*x/(a+b)^3+5/4*a*x/(a+b)^3*b+15/8*x/(a+b)^3*b^2+1/64/d/(a+b)*exp(4*
d*x+4*c)+1/8/(a+b)^2/d*exp(2*d*x+2*c)*a+1/4/(a+b)^2/d*exp(2*d*x+2*c)*b-1/8
/(a^2+2*a*b+b^2)/d*exp(-2*d*x-2*c)*a-1/4/(a^2+2*a*b+b^2)/d*exp(-2*d*x-2*c)
*b-1/64/d/(a+b)*exp(-4*d*x-4*c)+1/2/a*(-a*b)^(1/2)*b^2/(a+b)^3/d*ln(exp(2*
d*x+2*c)+(2*(-a*b)^(1/2)+a-b)/(a+b))-1/2/a*(-a*b)^(1/2)*b^2/(a+b)^3/d*ln(e
xp(2*d*x+2*c)-(2*(-a*b)^(1/2)-a+b)/(a+b))
```

3.105.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 929 vs. 2(106) = 212.

Time = 0.30 (sec) , antiderivative size = 2180, normalized size of antiderivative = 18.17

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/64*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 8*(3*a^2 +
10*a*b + 15*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 + 3*a*b + 2*b^2)*cosh(d*x +
c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 + 6*a*b + 4*b^2)*s
inh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 6*(a^2 + 3*a*b
 + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)^4 + 4*(3*a^2 + 10*a*b + 15*b^2)*d*x + 60*(a^2 + 3*a*b + 2*b^2)*co
sh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5
+ 4*(3*a^2 + 10*a*b + 15*b^2)*d*x*cosh(d*x + c) + 20*(a^2 + 3*a*b + 2*b^2)
*cosh(d*x + c)^3)*sinh(d*x + c)^3 - 8*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^
2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 12*(3*a^2 + 10*a*b + 15*b^2
)*d*x*cosh(d*x + c)^2 + 30*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^4 - 2*a^2 -
 6*a*b - 4*b^2)*sinh(d*x + c)^2 + 32*(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x
 + c)^3*sinh(d*x + c) + 6*b^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^2*cosh
(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4)*sqrt(-b/a)*log(((a^2 + 2*
a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x
+ c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)
^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2
 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2
)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^...
```

3.105.6 Sympy [F]

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(cosh(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(cosh(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)
```

3.105.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(106) = 212$.

Time = 0.33 (sec) , antiderivative size = 514, normalized size of antiderivative = 4.28

$$\begin{aligned}\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = &-\frac{(ab-b^2)(dx+c)}{2\left(a^3+3a^2b+3ab^2+b^3\right)d}+\frac{\left(8be^{(-2dx-2c)}+a+b\right)e^{(4dx+4c)}}{64\left(a^2+2ab+b^2\right)d}\\&+\frac{b\log\left((a+b)e^{(4dx+4c)}+2(a-b)e^{(2dx+2c)}+a+b\right)}{4\left(a^2+2ab+b^2\right)d}\\&-\frac{b\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{4\left(a^2+2ab+b^2\right)d}\\&-\frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{4\left(a^2+2ab+b^2\right)\sqrt{ab}d}\\&-\frac{\left(a^2b-6ab^2+b^3\right)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{8\left(a^3+3a^2b+3ab^2+b^3\right)\sqrt{ab}d}\\&+\frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{4\left(a^2+2ab+b^2\right)\sqrt{ab}d}\\&-\frac{3b\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{8\sqrt{ab}(a+b)d}\\&-\frac{8be^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}}{64\left(a^2+2ab+b^2\right)d}\\&+\frac{3(dx+c)}{8(a+b)d}+\frac{e^{(2dx+2c)}}{8(a+b)d}-\frac{e^{(-2dx-2c)}}{8(a+b)d}\end{aligned}$$

input
```
integrate(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/2*(a*b - b^2)*(d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 1/64*(8*b
*e^(-2*d*x - 2*c) + a + b)*e^(4*d*x + 4*c)/((a^2 + 2*a*b + b^2)*d) + 1/4*b
*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^2 +
2*a*b + b^2)*d) - 1/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x
 - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/4*(a*b - b^2)*arctan(1/2*((a
+ b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d)
 - 1/8*(a^2*b - 6*a*b^2 + b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a -
b)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)*d) + 1/4*(a*b - b
^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b
 + b^2)*sqrt(a*b)*d) - 3/8*b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)
/sqrt(a*b))/(sqrt(a*b)*(a + b)*d) - 1/64*(8*b*e^(-2*d*x - 2*c) + (a + b)*e
^(-4*d*x - 4*c))/((a^2 + 2*a*b + b^2)*d) + 3/8*(d*x + c)/((a + b)*d) + 1/8
*e^(2*d*x + 2*c)/((a + b)*d) - 1/8*e^(-2*d*x - 2*c)/((a + b)*d)
```

3.105.8 Giac [F]

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\cosh\left(dx+c\right)^4}{b\tanh\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.105.9 Mupad [B] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 967, normalized size of antiderivative = 8.06

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x\left(3a^2+10ab+15b^2\right)}{8(a+b)^3} - \frac{e^{-4c-4dx}}{64d(a+b)} + \frac{e^{4c+4dx}}{64d(a+b)}$$
$$+ \frac{\operatorname{atan}\left(\left(e^{2c}e^{2dx}\left(\frac{4b^3}{d(a+b)^5\sqrt{b^5}\left(a^3+3a^2b+3ab^2+b^3\right)} + \frac{(a-b)\left(a^4d\sqrt{b^5}-b^4d\sqrt{b^5}-2ab^3d\sqrt{b^5}+2\right.}{b^3(a+b)^2\sqrt{ad^2(a+b)^6}\left(a^3+3a^2b+3ab^2+b^3\right)\sqrt{a^7d^2+6a^6bd^2+15a^5b^2d}\right.}\right.\right.}{}$$
$$- \frac{e^{-2c-2dx}(a+2b)}{8d(a+b)^2} + \frac{e^{2c+2dx}(a+2b)}{8d(a+b)^2}$$

input
```
int(cosh(c + d*x)^4/(a + b*tanh(c + d*x)^2),x)
```

output

```
(x*(10*a*b + 3*a^2 + 15*b^2))/(8*(a + b)^3) - exp(- 4*c - 4*d*x)/(64*d*(a
+ b)) + exp(4*c + 4*d*x)/(64*d*(a + b)) + (atan((exp(2*c)*exp(2*d*x)*((4*b
^3)/(d*(a + b)^5*(b^5)^(1/2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + ((a - b)*(
a^4*d*(b^5)^(1/2) - b^4*d*(b^5)^(1/2) - 2*a*b^3*d*(b^5)^(1/2) + 2*a^3*b*d*
(b^5)^(1/2)))/(b^3*(a + b)^2*(a*d^2*(a + b)^6)^(1/2)*(3*a*b^2 + 3*a^2*b +
a^3 + b^3)*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4
*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2))) + ((a - b)*(a^4*d*(b^5)^(1
/2) + b^4*d*(b^5)^(1/2) + 4*a*b^3*d*(b^5)^(1/2) + 4*a^3*b*d*(b^5)^(1/2) +
6*a^2*b^2*d*(b^5)^(1/2)))/(b^3*(a + b)^2*(a*d^2*(a + b)^6)^(1/2)*(3*a*b^2
+ 3*a^2*b + a^3 + b^3)*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2
+ 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2)))*((a^4*(a^7*d^2
 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d
^2 + 15*a^5*b^2*d^2)^(1/2))/2 + (b^4*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 +
6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2))/2
 + 2*a*b^3*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4
*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2) + 2*a^3*b*(a^7*d^2 + a*b^6*d
^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15*a^
5*b^2*d^2)^(1/2) + 3*a^2*b^2*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^
5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2)))*(b^5)^(1
/2))/(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^...
```

3.106 $\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.106.1 Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{b^2\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{5/2}d} + \frac{(a+2b)\sinh(c+dx)}{(a+b)^2 d} + \frac{\sinh^3(c+dx)}{3(a+b)d}$$

output
```
(a+2*b)*sinh(d*x+c)/(a+b)^2/d+1/3*sinh(d*x+c)^3/(a+b)/d+b^2*arctan(sinh(d*
x+c)*(a+b)^(1/2)/a^(1/2))/(a+b)^(5/2)/d/a^(1/2)
```

3.106.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{-\frac{12b^2\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{5/2}} + \frac{3(3a+7b)\sinh(c+dx)}{(a+b)^2} + \frac{\sinh(3(c+dx))}{a+b}}{12d}$$

input
```
Integrate[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
((-12*b^2*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5
/2)) + (3*(3*a + 7*b)*Sinh[c + d*x])/(a + b)^2 + Sinh[3*(c + d*x)]/(a + b)
)/(12*d)
```

3.106.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4159, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\downarrow 4159$$

$$\frac{\int \frac{\left(\sinh^2(c+dx)+1\right)^2}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{d}$$

$$\downarrow 300$$

$$\frac{\int \left(\frac{b^2}{(a+b)^2\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{\sinh^2(c+dx)}{a+b}+\frac{a+2b}{(a+b)^2}\right)d\sinh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{b^2\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{5/2}}+\frac{\sinh^3(c+dx)}{3(a+b)}+\frac{(a+2b)\sinh(c+dx)}{(a+b)^2}}{d}$$

input
```
Int[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
((b^2*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(5/2))
 + ((a + 2*b)*Sinh[c + d*x])/(a + b)^2 + Sinh[c + d*x]^3/(3*(a + b)))/d
```

3.106.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.106.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(70) = 140$.

Time = 3.97 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.88

method	result
risch	$\frac{e^{3dx+3c}}{24d(a+b)}+\frac{3e^{dx+c}a}{8(a+b)^2d}+\frac{7e^{dx+c}b}{8(a+b)^2d}-\frac{3e^{-dx-c}a}{8(a+b)^2d}-\frac{7e^{-dx-c}b}{8(a+b)^2d}-\frac{e^{-3dx-3c}}{24d(a+b)}-\frac{b^2\ln\left(e^{2dx+2c}-\frac{2ae^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}(a+b)^2d}$
derivativedivides	$-\frac{2}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3(2a+2b)}-\frac{1}{(2a+2b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{a+2b}{(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3(2a+2b)}+\frac{}{(2a+$
default	$-\frac{2}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3(2a+2b)}-\frac{1}{(2a+2b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{a+2b}{(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3(2a+2b)}+\frac{}{(2a+$

input
```
int(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/24/d/(a+b)*exp(3*d*x+3*c)+3/8/(a+b)^2/d*exp(d*x+c)*a+7/8/(a+b)^2/d*exp(d
*x+c)*b-3/8/(a+b)^2/d*exp(-d*x-c)*a-7/8/(a+b)^2/d*exp(-d*x-c)*b-1/24/d/(a+
b)*exp(-3*d*x-3*c)-1/2/(-a^2-a*b)^(1/2)*b^2/(a+b)^2/d*ln(exp(2*d*x+2*c)-2*
a/(-a^2-a*b)^(1/2)*exp(d*x+c)-1)+1/2/(-a^2-a*b)^(1/2)*b^2/(a+b)^2/d*ln(exp
(2*d*x+2*c)+2*a/(-a^2-a*b)^(1/2)*exp(d*x+c)-1)
```

3.106.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 860 vs. $2(70) = 140$.

Time $= 0.30$ (sec) , antiderivative size $= 1850$, normalized size of antiderivative $= 23.12$

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/24*((a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^6 + 6*(a^3 + 2*a^2*b + a*b^2)
*cosh(d*x + c)*sinh(d*x + c)^5 + (a^3 + 2*a^2*b + a*b^2)*sinh(d*x + c)^6 +
 3*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c)^4 + 3*(3*a^3 + 10*a^2*b + 7*
a*b^2 + 5*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*
(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^3 + 3*(3*a^3 + 10*a^2*b + 7*a*b^2)*c
osh(d*x + c))*sinh(d*x + c)^3 - a^3 - 2*a^2*b - a*b^2 - 3*(3*a^3 + 10*a^2*
b + 7*a*b^2)*cosh(d*x + c)^2 + 3*(5*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^
4 - 3*a^3 - 10*a^2*b - 7*a*b^2 + 6*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x +
 c)^2)*sinh(d*x + c)^2 - 12*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)^2*s
inh(d*x + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c)^3)*
sqrt(-a^2 - a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*si
nh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*
(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*
x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3
*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)
*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*
x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)
^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sin
h(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*
x + c) + a + b)) + 6*((a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^5 + 2*(3*a^...
```

3.106.6 Sympy [F]

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(cosh(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(cosh(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

3.106.7 Maxima [F]

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\cosh(dx+c)^3}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/24*((a*e^(6*c) + b*e^(6*c))*e^(6*d*x) + 3*(3*a*e^(4*c) + 7*b*e^(4*c))*e^
(4*d*x) - 3*(3*a*e^(2*c) + 7*b*e^(2*c))*e^(2*d*x) - a - b)*e^(-3*d*x)/(a^2
*d*e^(3*c) + 2*a*b*d*e^(3*c) + b^2*d*e^(3*c)) + 1/8*integrate(16*(b^2*e^(3
*d*x + 3*c) + b^2*e^(d*x + c))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + (a^3*e^(4*
c) + 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 2*(a^3*e
^(2*c) + a^2*b*e^(2*c) - a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

3.106.8 Giac [F]

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\cosh(dx+c)^3}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.106.9 Mupad [B] (verification not implemented)

Time = 3.44 (sec) , antiderivative size = 2194, normalized size of antiderivative = 27.42

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(cosh(c + d*x)^3/(a + b*tanh(c + d*x)^2),x)
```

output
```
exp(3*c + 3*d*x)/(24*d*(a + b)) - exp(- 3*c - 3*d*x)/(24*d*(a + b)) + ((b^
4)^(1/2)*(2*atan((exp(d*x)*exp(c)*((4*(10*a^2*d*(b^4)^(5/2) + 12*a^6*d*(b^
4)^(3/2) + 2*a*b^9*d*(b^4)^(1/2) + 10*a^3*b^3*d*(b^4)^(3/2) + 2*a^2*b^8*d*
(b^4)^(1/2) + 20*a^3*b^7*d*(b^4)^(1/2) + 40*a^4*b^6*d*(b^4)^(1/2) + 30*a^5
*b^5*d*(b^4)^(1/2) + 2*a^7*b^3*d*(b^4)^(1/2)))/(a*b^5*(a + b)^5*(a*d^2*(a
+ b)^5)^(1/2)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)*(5*a*b^4 + 5*a^4
*b + a^5 + b^5 + 10*a^2*b^3 + 10*a^3*b^2)*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d
^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2)) - (2*(b^9*(a^
6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*
b^2*d^2)^(1/2) + 4*a*b^8*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^
2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2) + 6*a^2*b^7*(a^6*d^2 + a*b^5*d^
2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2) +
 4*a^3*b^6*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3
*d^2 + 10*a^4*b^2*d^2)^(1/2) + a^4*b^5*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2
+ 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2)))/(a^2*b^3*d*(a +
 b)^7*(b^4)^(1/2)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)*(5*a*b^4 + 5
*a^4*b + a^5 + b^5 + 10*a^2*b^3 + 10*a^3*b^2)*(a^6*d^2 + a*b^5*d^2 + 5*a^5
*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2))) + (2*exp
(3*c)*exp(3*d*x)*(b^9*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 +
 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2) + 4*a*b^8*(a^6*d^2 + a*b^5*d^2 ...
```

3.107 $\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.107.1 Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(a+3b)x}{2(a+b)^2} + \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^2 d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d}$$

output
```
1/2*(a+3*b)*x/(a+b)^2+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d+b^(3/2)*arctan(b
^(1/2)*tanh(d*x+c)/a^(1/2))/(a+b)^2/d/a^(1/2)
```

3.107.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{2\sqrt{a}(a+3b)(c+dx) + 4b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) + \sqrt{a}(a+b)\sinh(2(c+dx))}{4\sqrt{a}(a+b)^2 d}$$

input
```
Integrate[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(2*Sqrt[a]*(a + 3*b)*(c + d*x) + 4*b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/
Sqrt[a]] + Sqrt[a]*(a + b)*Sinh[2*(c + d*x)])/(4*Sqrt[a]*(a + b)^2*d)
```

3.107.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4158, 316, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\downarrow 4158$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)^2\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{d}$$

$$\downarrow 316$$

$$\frac{\int \frac{b\tanh^2(c+dx)+a+2b}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2(a+b)} + \frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 397$$

$$\frac{\frac{2b^2\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b} + \frac{(a+3b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}}{2(a+b)} + \frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 218$$

$$\frac{\frac{(a+3b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b} + \frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2(a+b)} + \frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 219$$

$$\frac{\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)} + \frac{(a+3b)\operatorname{arctanh}(\tanh(c+dx))}{a+b}}{2(a+b)} + \frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}}{d}$$

input
```
Int[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(((2*b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)) +
((a + 3*b)*ArcTanh[Tanh[c + d*x]])/(a + b))/(2*(a + b)) + Tanh[c + d*x]/(2
*(a + b)*(1 - Tanh[c + d*x]^2)))/d
```

3.107.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.107.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(65) = 130.

Time = 1.24 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.06

method	result
risch	$\frac{ax}{2(a+b)^2}+\frac{3xb}{2(a+b)^2}+\frac{e^{2dx+2c}}{8d(a+b)}-\frac{e^{-2dx-2c}}{8d(a+b)}+\frac{\sqrt{-ab}\,b\ln\left(e^{2dx+2c}+\frac{2\sqrt{-ab}+a-b}{a+b}\right)}{2a(a+b)^2d}-\frac{\sqrt{-ab}\,b\ln\left(e^{2dx+2c}-\frac{2\sqrt{-a}}{a}\right.}{2a(a+b)^2d}$
derivativedivides	$-\frac{1}{(2a+2b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2}{(4a+4b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(a+3b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)^2}+\frac{1}{(2a+2b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{}{(4a+}$
default	$-\frac{1}{(2a+2b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2}{(4a+4b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(a+3b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)^2}+\frac{1}{(2a+2b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{}{(4a+}$

input
```
int(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x/(a+b)^2+3/2*x/(a+b)^2*b+1/8/d/(a+b)*exp(2*d*x+2*c)-1/8/d/(a+b)*exp
(-2*d*x-2*c)+1/2/a*(-a*b)^(1/2)*b/(a+b)^2/d*ln(exp(2*d*x+2*c)+(2*(-a*b)^(1
/2)+a-b)/(a+b))-1/2/a*(-a*b)^(1/2)*b/(a+b)^2/d*ln(exp(2*d*x+2*c)-(2*(-a*b)
^(1/2)-a+b)/(a+b))
```

3.107.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(65) = 130.

Time = 0.29 (sec) , antiderivative size = 948, normalized size of antiderivative = 12.31

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/8*(4*(a + 3*b)*d*x*cosh(d*x + c)^2 + (a + b)*cosh(d*x + c)^4 + 4*(a + b
)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(2*(a + 3*b)
*d*x + 3*(a + b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^2 +
 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2)*sqrt(-b/a)*log(((a^2
 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh
(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x
 + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x +
 c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2
- b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(
a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2
 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sin
h(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*
(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c
)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 4*(2*(a + 3*b)*d*x*
cosh(d*x + c) + (a + b)*cosh(d*x + c)^3)*sinh(d*x + c) - a - b)/((a^2 + 2*
a*b + b^2)*d*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(
d*x + c) + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^2), 1/8*(4*(a + 3*b)*d*x*co
sh(d*x + c)^2 + (a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x
 + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(2*(a + 3*b)*d*x + 3*(a + b)*cosh(d*
x + c)^2)*sinh(d*x + c)^2 + 8*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*si...
```

3.107.6 Sympy [F]

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(cosh(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(cosh(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

3.107.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(65) = 130$.

Time = 0.30 (sec) , antiderivative size = 316, normalized size of antiderivative = 4.10

$$\begin{aligned}\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = {} & \frac{b\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d} \\ & - \frac{b\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d} \\ & - \frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}d} \\ & + \frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}d} \\ & - \frac{b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}(a+b)d} \\ & + \frac{dx+c}{2\,(a+b)d} + \frac{e^{(2\,dx+2\,c)}}{8\,(a+b)d} - \frac{e^{(-2\,dx-2\,c)}}{8\,(a+b)d}\end{aligned}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a
^2 + 2*a*b + b^2)*d) - 1/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-
4*d*x - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/4*(a*b - b^2)*arctan(1/2
*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*
b)*d) + 1/4*(a*b - b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt
(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d) - 1/2*b*arctan(1/2*((a + b)*e^(-2
*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a + b)*d) + 1/2*(d*x + c)/((a
+ b)*d) + 1/8*e^(2*d*x + 2*c)/((a + b)*d) - 1/8*e^(-2*d*x - 2*c)/((a + b)*
d)
```

3.107.8 Giac [F]

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\cosh(dx+c)^2}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.107.9 Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 880, normalized size of antiderivative = 11.43

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{e^{2c+2dx}}{8d\,(a+b)} - \frac{e^{-2c-2dx}}{8d\,(a+b)} + \frac{x\,(a+3b)}{2\,(a+b)^2}$$
$$+ \frac{\operatorname{atan}\left(\left(e^{2c}e^{2dx}\left(\frac{2\left(2b^3\sqrt{a^5d^2+4a^4bd^2+6a^3b^2d^2+4a^2b^3d^2+ab^4d^2}+2ab^2\sqrt{a^5d^2+4a^4bd^2+6a^3b^2d^2+4a^2b^3d^2+ab^4d^2}\right)}{d(a+b)^5\sqrt{b^3}\,(a^3+3a^2b+3ab^2+b^3)\sqrt{a^5d^2+4a^4bd^2+6a^3b^2d^2+4a^2b^3d^2+ab^4d^2}} - \right.\right.\right.}{}$$

input
```
int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2),x)
```

output

```
exp(2*c + 2*d*x)/(8*d*(a + b)) - exp(- 2*c - 2*d*x)/(8*d*(a + b)) + (x*(a
+ 3*b))/(2*(a + b)^2) + (atan((exp(2*c)*exp(2*d*x)*((2*(2*b^3*(a^5*d^2 + a
*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) + 2*a*b^2*(a
^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)))/
(d*(a + b)^5*(b^3)^(1/2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(a^5*d^2 + a*b^4*
d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)) - ((a - b)*(2*a*
d*(b^3)^(3/2) + b*d*(b^3)^(3/2) - a^4*d*(b^3)^(1/2) - 2*a^3*b*d*(b^3)^(1/2
)))/(b^2*(a + b)^3*(a*d^2*(a + b)^4)^(1/2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)
*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)
)) + ((a - b)*(4*a*d*(b^3)^(3/2) + b*d*(b^3)^(3/2) + a^4*d*(b^3)^(1/2) + 4
*a^3*b*d*(b^3)^(1/2) + 6*a^2*b^2*d*(b^3)^(1/2)))/(b^2*(a + b)^3*(a*d^2*(a
+ b)^4)^(1/2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(a^5*d^2 + a*b^4*d^2 + 4*a^4
*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)))*((a^4*(a^5*d^2 + a*b^4*d^2
 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))/2 + (b^4*(a^5*d^2 +
 a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))/2 + 3*a^2
*b^2*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(
1/2) + 2*a*b^3*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*
b^2*d^2)^(1/2) + 2*a^3*b*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^
2 + 6*a^3*b^2*d^2)^(1/2)))*(b^3)^(1/2))/(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2
 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)
```

3.108 $\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.108.1 Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{b\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{3/2}d} + \frac{\sinh(c+dx)}{(a+b)d}$$

output
```
sinh(d*x+c)/(a+b)/d+b*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/(a+b)^(3/2)/
d/a^(1/2)
```

3.108.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{b\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{3/2}d} + \frac{\sinh(c+dx)}{(a+b)d}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(b*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(3/2)*d)
+ Sinh[c + d*x]/((a + b)*d)
```

3.108.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4159, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(ic+idx)\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\downarrow 4159$$

$$\frac{\int \frac{\sinh^2(c+dx)+1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{d}$$

$$\downarrow 299$$

$$\frac{\frac{b\int \frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{a+b}+\frac{\sinh(c+dx)}{a+b}}{d}$$

$$\downarrow 218$$

$$\frac{\frac{b\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{3/2}}+\frac{\sinh(c+dx)}{a+b}}{d}$$

input
```
Int[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
((b*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(3/2)) +
 Sinh[c + d*x]/(a + b))/d
```

3.108.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.108.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(45) = 90$.

Time $= 0.58$ (sec) , antiderivative size $= 149$, normalized size of antiderivative $= 2.81$

method	result
risch	$\frac{e^{dx+c}}{2d(a+b)}-\frac{e^{-dx-c}}{2d(a+b)}-\frac{b\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}\,(a+b)d}+\frac{b\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}\,(a+b)d}$
derivativedivides	$\frac{-\frac{2}{(2a+2b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2}{(2a+2b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2ba\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-b\right)}{2a\sqrt{(}}\right.}{a+b}}{d}$
default	$\frac{-\frac{2}{(2a+2b)\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2}{(2a+2b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2ba\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-b\right)}{2a\sqrt{(}}\right.}{a+b}}{d}$

input
```
int(cosh(d*x+c)/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d/(a+b)*exp(d*x+c)-1/2/d/(a+b)*exp(-d*x-c)-1/2/(-a^2-a*b)^(1/2)*b/(a+b
)/d*ln(exp(2*d*x+2*c)-2*a/(-a^2-a*b)^(1/2)*exp(d*x+c)-1)+1/2/(-a^2-a*b)^(1
/2)*b/(a+b)/d*ln(exp(2*d*x+2*c)+2*a/(-a^2-a*b)^(1/2)*exp(d*x+c)-1)
```

3.108.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(45)=90$.

Time $=0.29$ (sec) , antiderivative size $=766$, normalized size of antiderivative $=14.45$

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$=\left[\frac{(a^2+ab)\cosh(dx+c)^2+2(a^2+ab)\cosh(dx+c)\sinh(dx+c)+(a^2+ab)\sinh(dx+c)^2-\sqrt{-a^2-a}\right.$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x +
 c) + (a^2 + a*b)*sinh(d*x + c)^2 - sqrt(-a^2 - a*b)*(b*cosh(d*x + c) + b*
sinh(d*x + c))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh
(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3
*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x
+ c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*c
osh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*s
inh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x
+ c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4
 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(
d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x
+ c) + a + b)) - a^2 - a*b)/((a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c) + (a^
3 + 2*a^2*b + a*b^2)*d*sinh(d*x + c)), 1/2*((a^2 + a*b)*cosh(d*x + c)^2 +
2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 +
2*sqrt(a^2 + a*b)*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(1/2*((a + b)*
cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d
*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a - b
)*sinh(d*x + c))/sqrt(a^2 + a*b)) + 2*sqrt(a^2 + a*b)*(b*cosh(d*x + c) + b
*sinh(d*x + c))*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))
/a) - a^2 - a*b)/((a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c) + (a^3 + 2*a^...
```

3.108.6 Sympy [F]

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(cosh(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

3.108.7 Maxima [F]

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\cosh(dx+c)}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x)/(a*d*e^c + b*d*e^c) + 1/2*integrate(4*(
b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^2 + 2*a*b + b^2 + (a^2*e^(4*c) + 2*a
*b*e^(4*c) + b^2*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) - b^2*e^(2*c))*e^(2*d
*x)), x)
```

3.108.8 Giac [F]

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\cosh(dx+c)}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.108.9 Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.91

$$\begin{aligned}\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = {} & \frac{e^{c+dx}}{2d\,(a+b)} - \frac{e^{-c-dx}}{2d\,(a+b)} \\ & - \frac{b\ln\left(\sqrt{-a}\sqrt{a+b}+2ae^{c+dx}-\sqrt{-a}e^{2c+2dx}\sqrt{a+b}\right)}{2\sqrt{-a}d(a+b)^{3/2}} \\ & + \frac{b\ln\left(2ae^{c+dx}-\sqrt{-a}\sqrt{a+b}+\sqrt{-a}e^{2c+2dx}\sqrt{a+b}\right)}{2\sqrt{-a}d(a+b)^{3/2}}\end{aligned}$$

input
```
int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2),x)
```

output

```
exp(c + d*x)/(2*d*(a + b)) - exp(- c - d*x)/(2*d*(a + b)) - (b*log((-a)^(1
/2)*(a + b)^(1/2) + 2*a*exp(c + d*x) - (-a)^(1/2)*exp(2*c + 2*d*x)*(a + b)
^(1/2)))/(2*(-a)^(1/2)*d*(a + b)^(3/2)) + (b*log(2*a*exp(c + d*x) - (-a)^(
1/2)*(a + b)^(1/2) + (-a)^(1/2)*exp(2*c + 2*d*x)*(a + b)^(1/2)))/(2*(-a)^(
1/2)*d*(a + b)^(3/2))
```

3.109 $\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.109.1 Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}d}$$

output
```
arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/d/a^(1/2)/(a+b)^(1/2)
```

3.109.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}d}$$

input
```
Integrate[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]*d)
```

3.109.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4159, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sec(ic+idx)}{a-b\tan(ic+idx)^2}dx$$
$$\downarrow 4159$$
$$\frac{\int \frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{d}$$
$$\downarrow 218$$
$$\frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d\sqrt{a+b}}$$

input
```
Int[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]*d)
```

3.109.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.109.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(28) = 56$.

Time $= 0.90$ (sec) , antiderivative size $= 102$, normalized size of antiderivative $= 2.83$

method	result	size
risch	$-\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}\,d}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}\,d}$	102
derivativedivides	$\frac{2a\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{d}$	152
default	$\frac{2a\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{d}$	152

input

```
int(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output

```
-1/2/(-a^2-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)-2*a/(-a^2-a*b)^(1/2)*exp(d*x+c)-
1)+1/2/(-a^2-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)+2*a/(-a^2-a*b)^(1/2)*exp(d*x+c
)-1)
```

3.109.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(28) = 56.

Time = 0.27 (sec) , antiderivative size = 511, normalized size of antiderivative = 14.19

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$= \left[-\frac{\sqrt{-a^2-ab}\log\left(\frac{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)\sinh(dx+c)^4-2\,(3\,a+b)\cosh(dx+c)^2+2\left(3\,(a+b)\cosh(\right.}{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx+c)\sinh(dx+c)^3-}\right.$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a^2 - a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^
2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*c
osh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)
^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^
2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*c
osh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*
x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a -
b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*s
inh(d*x + c) + a + b))/((a^2 + a*b)*d), (sqrt(a^2 + a*b)*arctan(1/2*((a +
b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sin
h(d*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a
- b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2
 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/a))/((a^2 + a*b)*d)]
```

3.109.6 Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}\left(c+dx\right)}{a+b\tanh^2\left(c+dx\right)}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(sech(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

3.109.7 Maxima [F]

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}(dx+c)}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
integrate(sech(d*x + c)/(b*tanh(d*x + c)^2 + a), x)
```

3.109.8 Giac [F]

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}(dx+c)}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.109.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 4.08

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\operatorname{atan}\left(\frac{4a^2d^2e^{dx}e^c - e^{dx}e^c\sqrt{a^2d^2+bad^2}\sqrt{ad^2(a+b)} + e^{3c}e^{3dx}\sqrt{a^2d^2+bad^2}\sqrt{ad^2(a+b)}}{2ad\sqrt{ad^2(a+b)}}\right) + \operatorname{atan}\left(\frac{e^{dx}e^c\sqrt{ad^2(a+b)}}{2ad}\right)}{\sqrt{a^2d^2+bad^2}}$$

input
```
int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)),x)
```

output
```
(atan((4*a^2*d^2*exp(d*x)*exp(c) - exp(d*x)*exp(c)*(a^2*d^2 + a*b*d^2)^(1/
2)*(a*d^2*(a + b))^(1/2) + exp(3*c)*exp(3*d*x)*(a^2*d^2 + a*b*d^2)^(1/2)*(
a*d^2*(a + b))^(1/2))/(2*a*d*(a*d^2*(a + b))^(1/2))) + atan((exp(d*x)*exp(
c)*(a*d^2*(a + b))^(1/2))/(2*a*d)))/(a^2*d^2 + a*b*d^2)^(1/2)
```

3.110 $\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.110.1 Optimal result

Integrand size = 23, antiderivative size = 32

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

output
```
arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/d/a^(1/2)/b^(1/2)
```

3.110.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

input
```
Integrate[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

3.110.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4158, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sec(ic+idx)^2}{a-b\tan(ic+idx)^2}\,dx$$
$$\downarrow 4158$$
$$\frac{\int \frac{1}{b\tanh^2(c+dx)+a}\,d\tanh(c+dx)}{d}$$
$$\downarrow 218$$
$$\frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

input
```
Int[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

3.110.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.110.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(24) = 48$.

Time = 3.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.56

method	result	size
risch	$-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}-2ab}{(a+b)\sqrt{-ab}}\right)}{2\sqrt{-ab}\,d}+\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}+2ab}{(a+b)\sqrt{-ab}}\right)}{2\sqrt{-ab}\,d}$	114
derivativedivides	$\frac{2a\left(-\frac{\left(a-\sqrt{(a+b)b}+b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}+\frac{\left(-a-\sqrt{(a+b)b}-b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{d}$	160
default	$\frac{2a\left(-\frac{\left(a-\sqrt{(a+b)b}+b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}+\frac{\left(-a-\sqrt{(a+b)b}-b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{d}$	160

input

```
int(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output

```
-1/2/(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)+(a*(-a*b)^(1/2)-b*(-a*b)^(1/2)-2*a*b
)/(a+b)/(-a*b)^(1/2))+1/2/(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)+(a*(-a*b)^(1/2)
-b*(-a*b)^(1/2)+2*a*b)/(a+b)/(-a*b)^(1/2))
```

3.110.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(24) = 48$.

Time = 0.29 (sec) , antiderivative size = 455, normalized size of antiderivative = 14.22

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \left[-\frac{\sqrt{-ab}\log\left(\frac{(a^2+2\,ab+b^2)\cosh(dx+c)^4+4\,(a^2+2\,ab+b^2)\cosh(dx+c)\sinh(dx+c)^3+(a^2+2\,ab+b^2)\sinh(dx+c)^4+2\,(a^2-b^2)\cosh(dx+c)^{\cdots}}{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx+c)\sinh(dx\cdots}\right.}{\cdots}\right.$$

input
```
integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b
 + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^
4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)
^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^
2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)
*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*
x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*
x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)
^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*co
sh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b))/(a*b*d), sq
rt(a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh
(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b))/(a*b*d)]
```

3.110.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(sech(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

3.110.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}d}$$

input
```
integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*d)
```

3.110.8 Giac [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^2}{b\tanh\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.110.9 Mupad [B] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.53

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\operatorname{atan}\left(\frac{a\sqrt{a\,b\,d^2}-b\sqrt{a\,b\,d^2}+a\,e^{2\,c}\,e^{2\,d\,x}\sqrt{a\,b\,d^2}+b\,e^{2\,c}\,e^{2\,d\,x}\sqrt{a\,b\,d^2}}{2\,a\,b\,d}\right)}{\sqrt{a\,b\,d^2}}$$

input
```
int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)),x)
```

output
```
atan((a*(a*b*d^2)^(1/2) - b*(a*b*d^2)^(1/2) + a*exp(2*c)*exp(2*d*x)*(a*b*d
^2)^(1/2) + b*exp(2*c)*exp(2*d*x)*(a*b*d^2)^(1/2))/(2*a*b*d))/(a*b*d^2)^(1
/2)
```

3.111 $\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.111.1 Optimal result

Integrand size = 23, antiderivative size = 55

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\arctan(\sinh(c+dx))}{bd} + \frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}bd}$$

output
```
-arctan(sinh(d*x+c))/b/d+arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))*(a+b)^(1/
2)/b/d/a^(1/2)
```

3.111.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}} + 2\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd}$$

input
```
Integrate[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
-(((Sqrt[a + b]*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/Sqrt[a] + 2*A
rcTan[Tanh[(c + d*x)/2]])/(b*d))
```

3.111.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4159, 303, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(ic+idx)^3}{a-b\tan(ic+idx)^2}\,dx$$

$$\downarrow 4159$$

$$\frac{\int \frac{1}{(\sinh^2(c+dx)+1)\left((a+b)\sinh^2(c+dx)+a\right)}\,d\sinh(c+dx)}{d}$$

$$\downarrow 303$$

$$\frac{\frac{(a+b)\int \frac{1}{(a+b)\sinh^2(c+dx)+a}\,d\sinh(c+dx)}{b} - \frac{\int \frac{1}{\sinh^2(c+dx)+1}\,d\sinh(c+dx)}{b}}{d}$$

$$\downarrow 216$$

$$\frac{\frac{(a+b)\int \frac{1}{(a+b)\sinh^2(c+dx)+a}\,d\sinh(c+dx)}{b} - \frac{\arctan(\sinh(c+dx))}{b}}{d}$$

$$\downarrow 218$$

$$\frac{\frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ab}} - \frac{\arctan(\sinh(c+dx))}{b}}{d}$$

input
```
Int[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-(ArcTan[Sinh[c + d*x]]/b) + (Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Sinh[c + d*
x])/Sqrt[a]])/(Sqrt[a]*b))/d
```

3.111.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.111.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.71 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.62

method	result
risch	$\frac{i\ln\left(e^{dx+c}-i\right)}{db}-\frac{i\ln\left(e^{dx+c}+i\right)}{db}+\frac{\sqrt{-a(a+b)}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{-a(a+b)}\,e^{dx+c}}{a+b}-1\right)}{2adb}-\frac{\sqrt{-a(a+b)}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{}}{}\right.}{2adb}$
derivativedivides	$\frac{2a(a+b)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{b}-\frac{2\arctan\left(\tanh\left(\frac{d}{2}\right.\right.}{b}}{d}$
default	$\frac{2a(a+b)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{b}-\frac{2\arctan\left(\tanh\left(\frac{d}{2}\right.\right.}{b}}{d}$

input
```
int(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
I/d/b*ln(exp(d*x+c)-I)-I/d/b*ln(exp(d*x+c)+I)+1/2/a*(-a*(a+b))^(1/2)/d/b*l
n(exp(2*d*x+2*c)+2*(-a*(a+b))^(1/2)/(a+b)*exp(d*x+c)-1)-1/2/a*(-a*(a+b))^(
1/2)/d/b*ln(exp(2*d*x+2*c)-2*(-a*(a+b))^(1/2)/(a+b)*exp(d*x+c)-1)
```

3.111.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(47) = 94$.

Time $= 0.29$ (sec) , antiderivative size $= 540$, normalized size of antiderivative $= 9.82$

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$= \left[\frac{\sqrt{-\frac{a+b}{a}}\log\left(\frac{(a+b)\cosh(dx+c)^4+4(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)\sinh(dx+c)^4-2(3a+b)\cosh(dx+c)^2+2\left(3(a+b)\cosh(dx+c)^2\right.\right.}{(a+b)\cosh(dx+c)^4+4(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+\right.}\right.$$

input
```
integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-(a + b)/a)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^
2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*c
osh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x +
c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 - a*cosh(d*x
+ c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c))*sqrt(-(a + b)/a) + a + b)/
((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a +
b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c
)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d
*x + c))*sinh(d*x + c) + a + b)) - 4*arctan(cosh(d*x + c) + sinh(d*x + c))
)/(b*d), (sqrt((a + b)/a)*arctan(1/2*sqrt((a + b)/a)*(cosh(d*x + c) + sinh
(d*x + c))) + sqrt((a + b)/a)*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a +
 b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a - b)*co
sh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))*sqrt((a
 + b)/a)/(a + b)) - 2*arctan(cosh(d*x + c) + sinh(d*x + c)))/(b*d)]
```

3.111.6 Sympy [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(sech(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

3.111.7 Maxima [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}(dx+c)^3}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-2*arctan(e^(d*x + c))/(b*d) + 8*integrate(1/4*((a*e^(3*c) + b*e^(3*c))*e^
(3*d*x) + (a*e^c + b*e^c)*e^(d*x))/(a*b + b^2 + (a*b*e^(4*c) + b^2*e^(4*c)
)*e^(4*d*x) + 2*(a*b*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)), x)
```

3.111.8 Giac [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}(dx+c)^3}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.111.9 Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 449, normalized size of antiderivative = 8.16

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{\sqrt{a+b}\left(2\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\,\sqrt{a+b}\,\sqrt{a\,b^2\,d^2}}{2\,a\,b\,d}\right) - 2\,\mathrm{atan}\left(\frac{\left(e^{d\,x}\,e^{c}\left(\frac{64\left(2\,a\,b^2\,d\,\sqrt{a+b}-6\,a^2\,b\,d\,\sqrt{a+b}\right)}{a^3\,b^3\,d^2\,(a+b)^2\left(a^2+2\,a\,b+b^2\right)}+\frac{32\left(3\,a^2\,\sqrt{a\,b^2\,d^2}-b^2\,\sqrt{a\,b^2\,d^2}+2\,a\,b\,\sqrt{a\,b\cdots}\right.}{a^3\,b^2\,d\,(a+b)^{3/2}\left(a^2+2\,a\,b+b^2\right)\sqrt{a\,b^2\cdots}}\right.\right.}{2\,\sqrt{a\,b^2\cdots}}\right.}{}$$
$$- \frac{2\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\left(9\,a^2\,\sqrt{b^2\,d^2}+b^2\,\sqrt{b^2\,d^2}-6\,a\,b\,\sqrt{b^2\,d^2}\right)}{9\,d\,a^2\,b-6\,d\,a\,b^2+d\,b^3}\right)}{\sqrt{b^2\,d^2}}$$

input
```
int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)),x)
```

output
```
((a + b)^(1/2)*(2*atan((exp(d*x)*exp(c)*(a + b)^(1/2)*(a*b^2*d^2)^(1/2))/(
2*a*b*d)) - 2*atan(((exp(d*x)*exp(c)*((64*(2*a*b^2*d*(a + b)^(1/2) - 6*a^2
*b*d*(a + b)^(1/2)))/(a^3*b^3*d^2*(a + b)^2*(2*a*b + a^2 + b^2)) + (32*(3*
a^2*(a*b^2*d^2)^(1/2) - b^2*(a*b^2*d^2)^(1/2) + 2*a*b*(a*b^2*d^2)^(1/2)))/
(a^3*b^2*d*(a + b)^(3/2)*(2*a*b + a^2 + b^2)*(a*b^2*d^2)^(1/2))) - (32*exp
(3*c)*exp(3*d*x)*(3*a^2*(a*b^2*d^2)^(1/2) - b^2*(a*b^2*d^2)^(1/2) + 2*a*b*
(a*b^2*d^2)^(1/2)))/(a^3*b^2*d*(a + b)^(3/2)*(2*a*b + a^2 + b^2)*(a*b^2*d^
2)^(1/2)))*(a^4*b*(a + b)*(a*b^2*d^2)^(1/2) + a^2*b^3*(a + b)*(a*b^2*d^2)^
(1/2) + 2*a^3*b^2*(a + b)*(a*b^2*d^2)^(1/2)))/(192*a - 64*b))))/(2*(a*b^2*
d^2)^(1/2)) - (2*atan((exp(d*x)*exp(c)*(9*a^2*(b^2*d^2)^(1/2) + b^2*(b^2*d
^2)^(1/2) - 6*a*b*(b^2*d^2)^(1/2)))/(b^3*d - 6*a*b^2*d + 9*a^2*b*d)))/(b^2
*d^2)^(1/2)
```

3.112 $\int \frac{\text{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.112.1 Optimal result

Integrand size = 23, antiderivative size = 50

$$\int \frac{\text{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}d} - \frac{\tanh(c+dx)}{bd}$$

output
```
(a+b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/b^(3/2)/d/a^(1/2)-tanh(d*x+c)/b/
d
```

3.112.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}d} - \frac{\tanh(c+dx)}{bd}$$

input
```
Integrate[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
((a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d) - Ta
nh[c + d*x]/(b*d)
```

3.112.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(ic+idx)^4}{a-b\tan(ic+idx)^2}\,dx$$

$$\downarrow 4158$$

$$\frac{\int \frac{1-\tanh^2(c+dx)}{b\tanh^2(c+dx)+a}\,d\tanh(c+dx)}{d}$$

$$\downarrow 299$$

$$\frac{\frac{(a+b)\int \frac{1}{b\tanh^2(c+dx)+a}\,d\tanh(c+dx)}{b} - \frac{\tanh(c+dx)}{b}}{d}$$

$$\downarrow 218$$

$$\frac{\frac{(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}} - \frac{\tanh(c+dx)}{b}}{d}$$

input
```
Int[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(((a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)) - Tan
h[c + d*x]/b)/d
```

3.112.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.112.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(42) = 84$.

Time = 21.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.94

method	result
derivativedivides	$\frac{2a(a+b)\left(-\frac{\left(a-\sqrt{(a+b)b}+b\right)\arctanh\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}+\frac{\left(-a-\sqrt{(a+b)b}-b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{b}-\frac{2\,ta}{b\left(\tanh}}{d}$
default	$\frac{2a(a+b)\left(-\frac{\left(a-\sqrt{(a+b)b}+b\right)\arctanh\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}+\frac{\left(-a-\sqrt{(a+b)b}-b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{b}-\frac{2\,ta}{b\left(\tanh}}{d}$
risch	$\frac{2}{bd\left(e^{2dx+2c}+1\right)}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}-2ab}{(a+b)\sqrt{-ab}}\right)a}{2\sqrt{-ab}\,db}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}-2ab}{(a+b)\sqrt{-ab}}\right)}{2\sqrt{-ab}\,d}+\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}}{(a-}\right.}{2\sqrt{-ab}}$

input
```
int(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b*a*(a+b)*(-1/2*(a-((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*
b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)
-a-2*b)*a)^(1/2))+1/2*(-a-((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*
b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+
a+2*b)*a)^(1/2)))-2/b*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+1))
```

3.112.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(42) = 84$.

Time $= 0.28$ (sec) , antiderivative size $= 649$, normalized size of antiderivative $= 12.98$

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$= \left[-\frac{\left((a+b)\cosh(dx+c)^2+2(a+b)\cosh(dx+c)\sinh(dx+c)+(a+b)\sinh(dx+c)^2+a+b\right)\sqrt{-ab}\,l}{}\right.$$

input
```
integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/2*(((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) +
(a + b)*sinh(d*x + c)^2 + a + b)*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(
d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 +
2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 +
 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b +
 b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))
*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh
(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x
 + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^
4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh
(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x
 + c) + a + b)) - 4*a*b)/(a*b^2*d*cosh(d*x + c)^2 + 2*a*b^2*d*cosh(d*x + c
)*sinh(d*x + c) + a*b^2*d*sinh(d*x + c)^2 + a*b^2*d), (((a + b)*cosh(d*x +
 c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 +
a + b)*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x
+ c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)) + 2
*a*b)/(a*b^2*d*cosh(d*x + c)^2 + 2*a*b^2*d*cosh(d*x + c)*sinh(d*x + c) + a
*b^2*d*sinh(d*x + c)^2 + a*b^2*d)]
```

3.112.6 Sympy [F]

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(sech(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)
```

3.112.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{(a+b)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}bd} - \frac{2}{(be^{(-2\,dx-2\,c)}+b)d}$$

input
```
integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-(a + b)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*
b)*b*d) - 2/((b*e^(-2*d*x - 2*c) + b)*d)
```

3.112.8 Giac [F]

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^4}{b\tanh\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.112.9 Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 176, normalized size of antiderivative = 3.52

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{2}{b\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)} + \frac{\ln\left(-\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}}{b}-\frac{2\,\left(a\,d+b\,d+a\,d\,\mathrm{e}^{2\,c+2\,d\,x}-b\,d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{\sqrt{-a}\,b^{3/2}\,d}\right)\,(a+b)}{2\,\sqrt{-a}\,b^{3/2}\,d} - \frac{\ln\left(\frac{2\,\left(a\,d+b\,d+a\,d\,\mathrm{e}^{2\,c+2\,d\,x}-b\,d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{\sqrt{-a}\,b^{3/2}\,d}-\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}}{b}\right)\,(a+b)}{2\,\sqrt{-a}\,b^{3/2}\,d}$$

input
```
int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)),x)
```

output
```
2/(b*d*(exp(2*c + 2*d*x) + 1)) + (log(- (4*exp(2*c + 2*d*x))/b - (2*(a*d +
 b*d + a*d*exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/((-a)^(1/2)*b^(3/2)*d
))*(a + b))/(2*(-a)^(1/2)*b^(3/2)*d) - (log((2*(a*d + b*d + a*d*exp(2*c + 
2*d*x) - b*d*exp(2*c + 2*d*x)))/((-a)^(1/2)*b^(3/2)*d) - (4*exp(2*c + 2*d*
x))/b)*(a + b))/(2*(-a)^(1/2)*b^(3/2)*d)
```

3.113 $\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.113.1 Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{(2a+3b)\arctan(\sinh(c+dx))}{2b^2d} + \frac{(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^2d} - \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd}$$

output
```
-1/2*(2*a+3*b)*arctan(sinh(d*x+c))/b^2/d+(a+b)^(3/2)*arctan(sinh(d*x+c)*(a
+b)^(1/2)/a^(1/2))/b^2/d/a^(1/2)-1/2*sech(d*x+c)*tanh(d*x+c)/b/d
```

3.113.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{2(a+b)^{3/2}\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}} + 2(2a+3b)\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + b\operatorname{sech}(c+dx)\tanh(c+dx)}{2b^2d}$$

input
```
Integrate[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2),x]
```

output
```
-1/2*((2*(a + b)^(3/2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/Sqrt[a
] + 2*(2*a + 3*b)*ArcTan[Tanh[(c + d*x)/2]] + b*Sech[c + d*x]*Tanh[c + d*x
])/(b^2*d)
```

3.113.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4159, 316, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(ic+idx)^5}{a-b\tan(ic+idx)^2}dx$$

$$\downarrow 4159$$

$$\frac{\int \frac{1}{\left(\sinh^2(c+dx)+1\right)^2\left((a+b)\sinh^2(c+dx)+a\right)}d\sinh(c+dx)}{d}$$

$$\downarrow 316$$

$$\frac{\int \frac{-\left((a+b)\sinh^2(c+dx)\right)+a+2b}{\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)}d\sinh(c+dx)}{2b} - \frac{\sinh(c+dx)}{2b\left(\sinh^2(c+dx)+1\right)}}{d}$$

$$\downarrow 397$$

$$\frac{\frac{2(a+b)^2\int \frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{b} - \frac{(2a+3b)\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{b}}{2b} - \frac{\sinh(c+dx)}{2b\left(\sinh^2(c+dx)+1\right)}}{d}$$

$$\downarrow 216$$

$$\frac{\frac{2(a+b)^2\int \frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{b} - \frac{(2a+3b)\arctan(\sinh(c+dx))}{b}}{2b} - \frac{\sinh(c+dx)}{2b\left(\sinh^2(c+dx)+1\right)}}{d}$$

$$\downarrow 218$$

$$\frac{\frac{2(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{(2a+3b)\arctan(\sinh(c+dx))}{b}}{2b}-\frac{\sinh(c+dx)}{2b(\sinh^2(c+dx)+1)}}{d}$$

input
```
Int[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2),x]
```

output
```
((-(((2*a + 3*b)*ArcTan[Sinh[c + d*x]])/b) + (2*(a + b)^(3/2)*ArcTan[(Sqrt
[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b))/(2*b) - Sinh[c + d*x]/(2*b*(
1 + Sinh[c + d*x]^2)))/d
```

3.113.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.113.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(74) = 148.

Time = 50.33 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.74

method	result
derivativedivides	$\frac{2\left(-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{2}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{(2a+3b)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)^2}}{b^2}+\frac{2\left(a^2+2ab+b^2\right)a\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh}{\sqrt{\left(2\sqrt{(a-}\right.}\right.}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-\right.}}}{d}$
default	$\frac{2\left(-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{2}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{(2a+3b)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)^2}}{b^2}+\frac{2\left(a^2+2ab+b^2\right)a\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh}{\sqrt{\left(2\sqrt{(a-}\right.}\right.}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-\right.}}}{d}$
risch	$-\frac{e^{dx+c}\left(e^{2dx+2c}-1\right)}{db\left(e^{2dx+2c}+1\right)^2}+\frac{i\ln\left(e^{dx+c}-i\right)a}{db^2}+\frac{3i\ln\left(e^{dx+c}-i\right)}{2db}-\frac{i\ln\left(e^{dx+c}+i\right)a}{db^2}-\frac{3i\ln\left(e^{dx+c}+i\right)}{2db}+\frac{\sqrt{-a(a+b)}\ \ln}{}$

input

```
int(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output

```
1/d*(-2/b^2*((-1/2*tanh(1/2*d*x+1/2*c)^3*b+1/2*b*tanh(1/2*d*x+1/2*c))/(tan
h(1/2*d*x+1/2*c)^2+1)^2+1/2*(2*a+3*b)*arctan(tanh(1/2*d*x+1/2*c)))+2/b^2*(
a^2+2*a*b+b^2)*a*(1/2*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^
(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2
*b)*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2
)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)
*a)^(1/2))))
```

3.113.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 750 vs. $2(74) = 148$.

Time $= 0.29$ (sec) , antiderivative size $= 1584$, normalized size of antiderivative $= 18.42$

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/2*(2*b*cosh(d*x + c)^3 + 6*b*cosh(d*x + c)*sinh(d*x + c)^2 + 2*b*sinh(
d*x + c)^3 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x +
 c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)
*cosh(d*x + c)^2 + a + b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (
a + b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*sqrt(-(a + b)/a)*log(((a + b)
*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(
d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 -
3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x
 + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x +
 c)^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*si
nh(d*x + c))*sqrt(-(a + b)/a) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b
)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh
(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((
a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) +
2*((2*a + 3*b)*cosh(d*x + c)^4 + 4*(2*a + 3*b)*cosh(d*x + c)*sinh(d*x + c)
^3 + (2*a + 3*b)*sinh(d*x + c)^4 + 2*(2*a + 3*b)*cosh(d*x + c)^2 + 2*(3*(2
*a + 3*b)*cosh(d*x + c)^2 + 2*a + 3*b)*sinh(d*x + c)^2 + 4*((2*a + 3*b)*co
sh(d*x + c)^3 + (2*a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 2*a + 3*b)*arct
an(cosh(d*x + c) + sinh(d*x + c)) - 2*b*cosh(d*x + c) + 2*(3*b*cosh(d*x +
c)^2 - b)*sinh(d*x + c))/(b^2*d*cosh(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)...
```

3.113.6 Sympy [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)**5/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(sech(c + d*x)**5/(a + b*tanh(c + d*x)**2), x)
```

3.113.7 Maxima [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}(dx+c)^5}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-(e^(3*d*x + 3*c) - e^(d*x + c))/(b*d*e^(4*d*x + 4*c) + 2*b*d*e^(2*d*x + 2
*c) + b*d) - (2*a*e^c + 3*b*e^c)*arctan(e^(d*x + c))*e^(-c)/(b^2*d) + 32*i
ntegrate(1/16*((a^2*e^(3*c) + 2*a*b*e^(3*c) + b^2*e^(3*c))*e^(3*d*x) + (a^
2*e^c + 2*a*b*e^c + b^2*e^c)*e^(d*x))/(a*b^2 + b^3 + (a*b^2*e^(4*c) + b^3*
e^(4*c))*e^(4*d*x) + 2*(a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

3.113.8 Giac [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}(dx+c)^5}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.113.9 Mupad [B] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 1012, normalized size of antiderivative = 11.77

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$= \left(2\operatorname{atan}\left(\frac{\left(e^{dx}e^{c}\left(\frac{64\left(12a^2b^4d\sqrt{a^3+3a^2b+3ab^2+b^3}-2ab^5d\sqrt{a^3+3a^2b+3ab^2+b^3}+18a^3b^3d\sqrt{a^3+3a^2b+3ab^2+b^3}+6a^4b^2d\sqrt{a^3+3a^2b+3ab^2}\right.\right.\right.\right.}{a^3b^9d^2(a+b)^2\left(a^2+2ab+b^2\right)}\right.$$

$$-\frac{\operatorname{atan}\left(\frac{e^{dx}e^{c}\left(18a^7\sqrt{b^4d^2}+3b^7\sqrt{b^4d^2}+30a^2b^5\sqrt{b^4d^2}+342a^3b^4\sqrt{b^4d^2}+555a^4b^3\sqrt{b^4d^2}+396a^5b^2\sqrt{\right.}\right.}{b^8d\sqrt{4a^2+12ab+9b^2}-12ab^7d\sqrt{4a^2+12ab+9b^2}+18a^2b^6d\sqrt{4a^2+12ab+9b^2}+102a^3b^5d\sqrt{4a^2+12ab+9b^2}+117a^4b^4d\sqrt{4a^2}}\right.$$

$$\left.-\frac{e^{c+dx}}{bd\left(e^{2c+2dx}+1\right)}+\frac{2e^{c+dx}}{bd\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}\right.$$

input
```
int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)),x)
```

output
```
((2*atan(((exp(d*x)*exp(c)*((64*(12*a^2*b^4*d*(3*a*b^2 + 3*a^2*b + a^3 + b
^3)^(1/2) - 2*a*b^5*d*(3*a*b^2 + 3*a^2*b + a^3 + b^3)^(1/2) + 18*a^3*b^3*d
*(3*a*b^2 + 3*a^2*b + a^3 + b^3)^(1/2) + 6*a^4*b^2*d*(3*a*b^2 + 3*a^2*b +
a^3 + b^3)^(1/2)))/(a^3*b^9*d^2*(a + b)^2*(2*a*b + a^2 + b^2)) - (32*(3*a^
5*(a*b^4*d^2)^(1/2) - b^5*(a*b^4*d^2)^(1/2) + 4*a*b^4*(a*b^4*d^2)^(1/2) +
15*a^4*b*(a*b^4*d^2)^(1/2) + 20*a^2*b^3*(a*b^4*d^2)^(1/2) + 27*a^3*b^2*(a*
b^4*d^2)^(1/2)))/(a^3*b^7*d*((a + b)^3)^(1/2)*(2*a*b + a^2 + b^2)*(a*b^4*d
^2)^(1/2))) + (32*exp(3*c)*exp(3*d*x)*(3*a^5*(a*b^4*d^2)^(1/2) - b^5*(a*b^
4*d^2)^(1/2) + 4*a*b^4*(a*b^4*d^2)^(1/2) + 15*a^4*b*(a*b^4*d^2)^(1/2) + 20
*a^2*b^3*(a*b^4*d^2)^(1/2) + 27*a^3*b^2*(a*b^4*d^2)^(1/2)))/(a^3*b^7*d*((a
 + b)^3)^(1/2)*(2*a*b + a^2 + b^2)*(a*b^4*d^2)^(1/2)))*(a^2*b^7*(a*b^4*d^2
)^(1/2) + 2*a^3*b^6*(a*b^4*d^2)^(1/2) + a^4*b^5*(a*b^4*d^2)^(1/2)))/(384*a
*b^2 + 576*a^2*b + 192*a^3 - 64*b^3)) + 2*atan((exp(d*x)*exp(c)*(a + b)^2*
(a*b^4*d^2)^(1/2))/(2*a*b^2*d*((a + b)^3)^(1/2))))*(3*a*b^2 + 3*a^2*b + a^
3 + b^3)^(1/2))/(2*(a*b^4*d^2)^(1/2)) - (atan((exp(d*x)*exp(c)*(18*a^7*(b^
4*d^2)^(1/2) + 3*b^7*(b^4*d^2)^(1/2) + 30*a^2*b^5*(b^4*d^2)^(1/2) + 342*a^
3*b^4*(b^4*d^2)^(1/2) + 555*a^4*b^3*(b^4*d^2)^(1/2) + 396*a^5*b^2*(b^4*d^2
)^(1/2) - 34*a*b^6*(b^4*d^2)^(1/2) + 135*a^6*b*(b^4*d^2)^(1/2)))/(b^8*d*(1
2*a*b + 4*a^2 + 9*b^2)^(1/2) - 12*a*b^7*d*(12*a*b + 4*a^2 + 9*b^2)^(1/2) +
 18*a^2*b^6*d*(12*a*b + 4*a^2 + 9*b^2)^(1/2) + 102*a^3*b^5*d*(12*a*b + ...
```

3.114 $\int \frac{\text{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.114.1 Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \frac{\text{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(a+b)^2\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}d} - \frac{(a+2b)\tanh(c+dx)}{b^2d} + \frac{\tanh^3(c+dx)}{3bd}$$

output
```
(a+b)^2*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/b^(5/2)/d/a^(1/2)-(a+2*b)*tanh
(d*x+c)/b^2/d+1/3*tanh(d*x+c)^3/b/d
```

3.114.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \frac{\text{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(a+b)^2\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}d} - \frac{\left(3a+5b+b\text{sech}^2(c+dx)\right)\tanh(c+dx)}{3b^2d}$$

input
```
Integrate[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2),x]
```

output
```
((a + b)^2*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)*d) -
((3*a + 5*b + b*Sech[c + d*x]^2)*Tanh[c + d*x])/(3*b^2*d)
```

3.114.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(ic+idx)^6}{a-b\tan(ic+idx)^2}\,dx$$

$$\downarrow 4158$$

$$\frac{\int \frac{\left(1-\tanh^2(c+dx)\right)^2}{b\tanh^2(c+dx)+a}\,d\tanh(c+dx)}{d}$$

$$\downarrow 300$$

$$\frac{\int \left(\frac{\tanh^2(c+dx)}{b}-\frac{a+2b}{b^2}+\frac{a^2+2ba+b^2}{b^2\left(b\tanh^2(c+dx)+a\right)}\right)d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{(a+b)^2\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}-\frac{(a+2b)\tanh(c+dx)}{b^2}+\frac{\tanh^3(c+dx)}{3b}}{d}$$

input
```
Int[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(((a + b)^2*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)) - (
(a + 2*b)*Tanh[c + d*x])/b^2 + Tanh[c + d*x]^3/(3*b))/d
```

3.114.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.114.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(65) = 130$.

Time = 115.36 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.36

method	result
derivativedivides	$\frac{2a\left(a^2+2ab+b^2\right)\left(-\frac{\left(a-\sqrt{(a+b)b}+b\right)\arctanh\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}+\frac{\left(-a-\sqrt{(a+b)b}-b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{b^2}+$
default	$\frac{2a\left(a^2+2ab+b^2\right)\left(-\frac{\left(a-\sqrt{(a+b)b}+b\right)\arctanh\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}+\frac{\left(-a-\sqrt{(a+b)b}-b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{b^2}+$
risch	$\frac{2a\,e^{4dx+4c}+2b\,e^{4dx+4c}+4\,e^{2dx+2c}a+8b\,e^{2dx+2c}+2a+\frac{10b}{3}}{b^2d\left(e^{2dx+2c}+1\right)^3}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}-2ab}{(a+b)\sqrt{-ab}}\right)a^2}{2\sqrt{-ab}\,d\,b^2}-\frac{\ln\left(e^{2dx+2c}+\frac{a}{\sqrt}\right.}{\sqrt}$

input
```
int(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b^2*a*(a^2+2*a*b+b^2)*(-1/2*(a-((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)
/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a
+b)*b)^(1/2)-a-2*b)*a)^(1/2))+1/2*(-a-((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)
/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+
b)*b)^(1/2)+a+2*b)*a)^(1/2)))+2/b^2*((-a-2*b)*tanh(1/2*d*x+1/2*c)^5+(-2*a-
8/3*b)*tanh(1/2*d*x+1/2*c)^3+(-a-2*b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1
/2*c)^2+1)^3)
```

3.114.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 864 vs. $2(65) = 130$.

Time $= 0.30$ (sec) , antiderivative size $= 2032$, normalized size of antiderivative $= 27.09$

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/6*(12*(a^2*b + a*b^2)*cosh(d*x + c)^4 + 48*(a^2*b + a*b^2)*cosh(d*x + c
)*sinh(d*x + c)^3 + 12*(a^2*b + a*b^2)*sinh(d*x + c)^4 + 12*a^2*b + 20*a*b
^2 + 24*(a^2*b + 2*a*b^2)*cosh(d*x + c)^2 + 24*(a^2*b + 2*a*b^2 + 3*(a^2*b
 + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*((a^2 + 2*a*b + b^2)*cosh(d
*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2
*a*b + b^2)*sinh(d*x + c)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 3*(5
*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^4
+ 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*
x + c))*sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*(5*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 +
 a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 6*((a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^5 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*
a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log(((a^2 + 2*a*b + b^
2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(
3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 -
 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d
*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x +
 c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*
cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sin...
```

3.114.6 Sympy [F]

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)**6/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(sech(c + d*x)**6/(a + b*tanh(c + d*x)**2), x)
```

3.114.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(65) = 130.

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.87

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{2\left(6\,(a+2\,b)e^{(-2\,dx-2\,c)}+3\,(a+b)e^{(-4\,dx-4\,c)}+3\,a+5\,b\right)}{3\left(3\,b^2e^{(-2\,dx-2\,c)}+3\,b^2e^{(-4\,dx-4\,c)}+b^2e^{(-6\,dx-6\,c)}+b^2\right)d} - \frac{\left(a^2+2\,ab+b^2\right)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}b^2d}$$

input
```
integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-2/3*(6*(a + 2*b)*e^(-2*d*x - 2*c) + 3*(a + b)*e^(-4*d*x - 4*c) + 3*a + 5*
b)/((3*b^2*e^(-2*d*x - 2*c) + 3*b^2*e^(-4*d*x - 4*c) + b^2*e^(-6*d*x - 6*c
) + b^2)*d) - (a^2 + 2*a*b + b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a
 - b)/sqrt(a*b))/(sqrt(a*b)*b^2*d)
```

3.114.8 Giac [F]

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^6}{b\tanh\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
sage0*x
```

3.114.9 Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.36

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{4}{b\,d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)} - \frac{8}{3\,b\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)} + \frac{2\,(a+b)}{b^2\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)} + \frac{\ln\left(-\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(a+b)}{b^2}-\frac{2\,(a+b)\,\left(a+b+a\,\mathrm{e}^{2\,c+2\,d\,x}-b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{\sqrt{-a}\,b^{5/2}}\right)\,(a+b)^2}{2\,\sqrt{-a}\,b^{5/2}\,d} - \frac{\ln\left(\frac{2\,(a+b)\,\left(a+b+a\,\mathrm{e}^{2\,c+2\,d\,x}-b\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{\sqrt{-a}\,b^{5/2}}-\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(a+b)}{b^2}\right)\,(a+b)^2}{2\,\sqrt{-a}\,b^{5/2}\,d}$$

input
```
int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)),x)
```

output
```
4/(b*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - 8/(3*b*d*(3*exp(2*c
+ 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (2*(a + b))/(b^2*
d*(exp(2*c + 2*d*x) + 1)) + (log(- (4*exp(2*c + 2*d*x)*(a + b))/b^2 - (2*(
a + b)*(a + b + a*exp(2*c + 2*d*x) - b*exp(2*c + 2*d*x)))/((-a)^(1/2)*b^(5
/2)))*(a + b)^2)/(2*(-a)^(1/2)*b^(5/2)*d) - (log((2*(a + b)*(a + b + a*exp
(2*c + 2*d*x) - b*exp(2*c + 2*d*x)))/((-a)^(1/2)*b^(5/2)) - (4*exp(2*c + 2
*d*x)*(a + b))/b^2)*(a + b)^2)/(2*(-a)^(1/2)*b^(5/2)*d)
```

3.115 $\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

3.115.1 Optimal result . . . 901
3.115.2 Mathematica [A] (verified) . . . 901
3.115.3 Rubi [A] (verified) . . . 902
3.115.4 Maple [B] (verified) . . . 903
3.115.5 Fricas [B] (verification not implemented) . . . 905
3.115.6 Sympy [F] . . . 905
3.115.7 Maxima [F] . . . 905
3.115.8 Giac [F] . . . 906
3.115.9 Mupad [F(-1)] . . . 906

3.115.1 Optimal result

Integrand size = 23, antiderivative size = 128

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{b^2(6a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{7/2}d} + \frac{(a+3b)\sinh(c+dx)}{(a+b)^3 d} + \frac{\sinh^3(c+dx)}{3(a+b)^2 d} + \frac{b^3\sinh(c+dx)}{2a(a+b)^3 d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

output
```
1/2*b^2*(6*a+b)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(3/2)/(a+b)^(7/2
)/d+(a+3*b)*sinh(d*x+c)/(a+b)^3/d+1/3*sinh(d*x+c)^3/(a+b)^2/d+1/2*b^3*sinh
(d*x+c)/a/(a+b)^3/d/(a+(a+b)*sinh(d*x+c)^2)
```

3.115.2 Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.87

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{-\frac{6b^2(6a+b)\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}(a+b)^{7/2}} + \frac{3\left(3a+11b+\frac{4b^3}{a(a-b+(a+b)\cosh(2(c+dx)))}\right)\sinh(c+dx)}{(a+b)^3} + \frac{\sinh(3(c+dx))}{(a+b)^2}}{12d}$$

input
```
Integrate[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((-6*b^2*(6*a + b)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/(a^(3/2)*(
a + b)^(7/2)) + (3*(3*a + 11*b + (4*b^3)/(a*(a - b + (a + b)*Cosh[2*(c + d
*x)])))*Sinh[c + d*x])/(a + b)^3 + Sinh[3*(c + d*x)]/(a + b)^2)/(12*d)
```

3.115.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4159, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\downarrow 4159$$

$$\frac{\int \frac{\left(\sinh^2(c+dx)+1\right)^3}{\left((a+b)\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{d}$$

$$\downarrow 300$$

$$\frac{\int \left(\frac{\sinh^2(c+dx)}{(a+b)^2}+\frac{a+3b}{(a+b)^3}+\frac{3(a+b)\sinh^2(c+dx)b^2+(3a+b)b^2}{(a+b)^3\left((a+b)\sinh^2(c+dx)+a\right)^2}\right)d\sinh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{b^2(6a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{7/2}}+\frac{b^3\sinh(c+dx)}{2a(a+b)^3\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{\sinh^3(c+dx)}{3(a+b)^2}+\frac{(a+3b)\sinh(c+dx)}{(a+b)^3}}{d}$$

input
```
Int[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((b^2*(6*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a
 + b)^(7/2)) + ((a + 3*b)*Sinh[c + d*x])/(a + b)^3 + Sinh[c + d*x]^3/(3*(a
 + b)^2) + (b^3*Sinh[c + d*x])/(2*a*(a + b)^3*(a + (a + b)*Sinh[c + d*x]^2
)))/d
```

3.115.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.115.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(114) = 228$.

Time $= 26.64$ (sec) , antiderivative size $= 378$, normalized size of antiderivative $= 2.95$

method	result
derivativedivides	$-\frac{1}{3(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{a+3b}{(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{3(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{}{2(a+b$
default	$-\frac{1}{3(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{a+3b}{(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{3(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{}{2(a+b$
risch	$\frac{e^{3dx+3c}}{24d(a^2+2ab+b^2)}+\frac{3e^{dx+c}a}{8(a^2+2ab+b^2)(a+b)d}+\frac{11e^{dx+c}b}{8(a^2+2ab+b^2)(a+b)d}-\frac{3e^{-dx-c}a}{8(a^3+3a^2b+3ab^2+b^3)d}-\frac{11e^{-dx-}}{8(a^3+3a^2b+3a}$

input
```
int(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/(a+b)^2/(tanh(1/2*d*x+1/2*
c)-1)^2-(a+3*b)/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)-1/3/(a+b)^2/(1+tanh(1/2*d*
x+1/2*c))^3+1/2/(a+b)^2/(1+tanh(1/2*d*x+1/2*c))^2-(a+3*b)/(a+b)^3/(1+tanh(
1/2*d*x+1/2*c))+2/(a+b)^3*b^2*((-1/2*b/a*tanh(1/2*d*x+1/2*c)^3+1/2*b/a*tan
h(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tan
h(1/2*d*x+1/2*c)^2*b+a)+1/2*(6*a+b)*(1/2*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(
1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*
((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)
/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a
+b)*b)^(1/2)-a-2*b)*a)^(1/2)))))
```

3.115. $\int \frac{\cosh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$

3.115.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3642 vs. 2(114) = 228.

Time = 0.34 (sec) , antiderivative size = 6934, normalized size of antiderivative = 54.17

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.115.6 Sympy [F]

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(cosh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(cosh(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)
```

3.115.7 Maxima [F]

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\cosh(dx+c)^3}{\left(b\tanh(dx+c)^2+a\right)^2}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/24*(a^3 + 2*a^2*b + a*b^2 - (a^3*e^(10*c) + 2*a^2*b*e^(10*c) + a*b^2*e^
(10*c))*e^(10*d*x) - (11*a^3*e^(8*c) + 42*a^2*b*e^(8*c) + 31*a*b^2*e^(8*c)
)*e^(8*d*x) - 2*(5*a^3*e^(6*c) + 4*a^2*b*e^(6*c) - 49*a*b^2*e^(6*c) + 12*b
^3*e^(6*c))*e^(6*d*x) + 2*(5*a^3*e^(4*c) + 4*a^2*b*e^(4*c) - 49*a*b^2*e^(4
*c) + 12*b^3*e^(4*c))*e^(4*d*x) + (11*a^3*e^(2*c) + 42*a^2*b*e^(2*c) + 31*
a*b^2*e^(2*c))*e^(2*d*x))/((a^5*d*e^(7*c) + 4*a^4*b*d*e^(7*c) + 6*a^3*b^2*
d*e^(7*c) + 4*a^2*b^3*d*e^(7*c) + a*b^4*d*e^(7*c))*e^(7*d*x) + 2*(a^5*d*e^
(5*c) + 2*a^4*b*d*e^(5*c) - 2*a^2*b^3*d*e^(5*c) - a*b^4*d*e^(5*c))*e^(5*d*
x) + (a^5*d*e^(3*c) + 4*a^4*b*d*e^(3*c) + 6*a^3*b^2*d*e^(3*c) + 4*a^2*b^3*
d*e^(3*c) + a*b^4*d*e^(3*c))*e^(3*d*x)) + 1/8*integrate(8*((6*a*b^2*e^(3*c
) + b^3*e^(3*c))*e^(3*d*x) + (6*a*b^2*e^c + b^3*e^c)*e^(d*x))/(a^5 + 4*a^4
*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + (a^5*e^(4*c) + 4*a^4*b*e^(4*c) + 6*a^
3*b^2*e^(4*c) + 4*a^2*b^3*e^(4*c) + a*b^4*e^(4*c))*e^(4*d*x) + 2*(a^5*e^(2
*c) + 2*a^4*b*e^(2*c) - 2*a^2*b^3*e^(2*c) - a*b^4*e^(2*c))*e^(2*d*x)), x)
```

3.115.8 Giac [F]

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\cosh\left(dx+c\right)^3}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\cosh(c+d\,x)^3}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(cosh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
int(cosh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2, x)
```

3.116 $\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

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3.116.2 Mathematica [A] (verified) . . . 907
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3.116.1 Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{(a+5b)x}{2(a+b)^3} + \frac{b^{3/2}(5a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^3 d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{(a-b)b\tanh(c+dx)}{2a(a+b)^2 d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
1/2*(a+5*b)*x/(a+b)^3+1/2*b^(3/2)*(5*a+b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/
2))/a^(3/2)/(a+b)^3/d+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c)
^2)-1/2*(a-b)*b*tanh(d*x+c)/a/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

3.116.2 Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.79

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{2(a+5b)(c+dx) + \frac{2b^{3/2}(5a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + (a+b)\sinh(2(c+dx)) + \frac{2b^2(a+b)\sinh(2(c+dx))}{a(a-b+(a+b)\cosh(2(c+dx)))}}{4(a+b)^3 d}$$

input
```
Integrate[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(2*(a + 5*b)*(c + d*x) + (2*b^(3/2)*(5*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x
])/Sqrt[a]])/a^(3/2) + (a + b)*Sinh[2*(c + d*x)] + (2*b^2*(a + b)*Sinh[2*(
c + d*x)])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(4*(a + b)^3*d)
```

3.116.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4158, 316, 402, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\downarrow 4158$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)^2\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 316$$

$$\frac{\int \frac{3b\tanh^2(c+dx)+a+2b}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{2(a+b)} + \frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 402$$

$$\frac{-\frac{\int -\frac{2\left(a^2+4ba+b^2+(a-b)b\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)} - \frac{b(a-b)\tanh(c+dx)}{a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)} + \frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 27$$

$$\frac{\frac{\int \frac{a^2+4ba+b^2+(a-b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a(a+b)}-\frac{b(a-b)\tanh(c+dx)}{a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)}+\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}$$
$$d$$

$$\downarrow 397$$

$$\frac{\frac{b^2(5a+b)\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}+\frac{a(a+5b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}}{a(a+b)}-\frac{b(a-b)\tanh(c+dx)}{a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)}+\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2($$
$$d$$

$$\downarrow 218$$

$$\frac{\frac{a(a+5b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{b^{3/2}(5a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a(a+b)}-\frac{b(a-b)\tanh(c+dx)}{a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)}+\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+d$$
$$d$$

$$\downarrow 219$$

$$\frac{\frac{b^{3/2}(5a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}+\frac{a(a+5b)\operatorname{arctanh}(\tanh(c+dx))}{a+b}}{a(a+b)}-\frac{b(a-b)\tanh(c+dx)}{a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)}+\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}$$
$$d$$

input
```
Int[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(Tanh[c + d*x]/(2*(a + b)*(1 - Tanh[c + d*x]^2)*(a + b*Tanh[c + d*x]^2)) +
 (((b^(3/2)*(5*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a
 + b)) + (a*(a + 5*b)*ArcTanh[Tanh[c + d*x]])/(a + b))/(a*(a + b)) - ((a -
 b)*b*Tanh[c + d*x])/(a*(a + b)*(a + b*Tanh[c + d*x]^2)))/(2*(a + b)))/d
```

3.116.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.116.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(124) = 248$.

Time = 4.57 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.76

method	result
derivativedivides	$-\frac{1}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{1}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(a+5b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)^3}+\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{}{2(a+}$
default	$-\frac{1}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{1}{2(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(a+5b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)^3}+\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{}{2(a+}$
risch	$\frac{xa}{2(a+b)(a^2+2ab+b^2)}+\frac{5xb}{2(a+b)(a^2+2ab+b^2)}+\frac{\mathrm{e}^{2dx+2c}}{8d(a^2+2ab+b^2)}-\frac{\mathrm{e}^{-2dx-2c}}{8d(a^2+2ab+b^2)}-\frac{b^2\left(\mathrm{e}^{2dx+2c}\right.}{d(a+b)^3a\left(a\,\mathrm{e}^{4dx+4c}+b\,\mathrm{e}^{4d}\right.}$

input
```
int(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/(a+b)^2/(1+tanh(1/2*d*x+1/2*c))^2+1/2/(a+b)^2/(1+tanh(1/2*d*x+1/
2*c))+1/2*(a+5*b)/(a+b)^3*ln(1+tanh(1/2*d*x+1/2*c))+1/2/(a+b)^2/(tanh(1/2*
d*x+1/2*c)-1)^2+1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)+1/2/(a+b)^3*(-a-5*b)*l
n(tanh(1/2*d*x+1/2*c)-1)-2/(a+b)^3*b^2*((-1/2*(a+b)/a*tanh(1/2*d*x+1/2*c)^
3-1/2*(a+b)/a*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x
+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)+1/2*(5*a+b)*(1/2*(a+((a+b)*b)^(1/
2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(
1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(-a+((a+b)*b)^(1/2
)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(
1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))
```

3.116.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2001 vs. 2(124) = 248.

Time = 0.34 (sec) , antiderivative size = 4324, normalized size of antiderivative = 30.89

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/8*((a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*
cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(d*x + c)^8 +
2*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^6 + 2*(a^3
 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x + 14*(a^3 + 2*a^2*b + a*b^2)*co
sh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x +
c)^3 + 3*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c))*si
nh(d*x + c)^5 - 8*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x +
 c)^4 + 2*(35*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^4 - 4*a*b^2 + 4*b^3 +
4*(a^3 + 4*a^2*b - 5*a*b^2)*d*x + 15*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a
*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + 2*a^2*b + a*b^2)
*cosh(d*x + c)^5 + 5*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(
d*x + c)^3 - 4*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)
)*sinh(d*x + c)^3 - a^3 - 2*a^2*b - a*b^2 - 2*(a^3 + 3*a*b^2 + 4*b^3 - 2*(
a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^2 + 2*(14*(a^3 + 2*a^2*b + a*b
^2)*cosh(d*x + c)^6 + 15*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*c
osh(d*x + c)^4 - a^3 - 3*a*b^2 - 4*b^3 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x -
 24*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*
x + c)^2 + 2*((5*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^6 + 6*(5*a^2*b + 6*a
*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^2*b + 6*a*b^2 + b^3)*sinh
(d*x + c)^6 + 2*(5*a^2*b - 4*a*b^2 - b^3)*cosh(d*x + c)^4 + (10*a^2*b -...
```

3.116.6 Sympy [F]

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(cosh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(cosh(c + d*x)**2/(a + b*tanh(c + d*x)**2)**2, x)
```

3.116.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(124) = 248$.

Time = 0.38 (sec) , antiderivative size = 840, normalized size of antiderivative = 6.00

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{b\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)d}$$
$$-\frac{b\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)d}$$
$$-\frac{(3\,a^2b-6\,ab^2-b^3)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{8\,(a^4+3\,a^3b+3\,a^2b^2+ab^3)\sqrt{ab}d}$$
$$+\frac{(3\,a^2b-6\,ab^2-b^3)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{8\,(a^4+3\,a^3b+3\,a^2b^2+ab^3)\sqrt{ab}d}$$
$$-\frac{(3\,ab+b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{4\,(a^3+2\,a^2b+ab^2)\sqrt{ab}d}$$
$$+\frac{a^2b-b^3+(a^2b-6\,ab^2+b^3)e^{(2\,dx+2\,c)}}{4\,(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4+(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4)e^{(4\,dx+4\,c)}+2\,(a^5+2\,a^4b-2\,a$$
$$-\frac{a^2b-b^3+(a^2b-6\,ab^2+b^3)e^{(-2\,dx-2\,c)}}{4\,(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4+2\,(a^5+2\,a^4b-2\,a^2b^3-ab^4)e^{(-2\,dx-2\,c)}+(a^5+4\,a^4b+6\,a^3b^2+$$
$$+\frac{ab+b^2+(ab-b^2)e^{(-2\,dx-2\,c)}}{2\,(a^4+3\,a^3b+3\,a^2b^2+ab^3+2\,(a^4+a^3b-a^2b^2-ab^3)e^{(-2\,dx-2\,c)}+(a^4+3\,a^3b+3\,a^2b^2+ab^3)e^{(-4\,dx-4}$$
$$+\frac{dx+c}{2\,(a^2+2\,ab+b^2)d}+\frac{e^{(2\,dx+2\,c)}}{8\,(a^2+2\,ab+b^2)d}-\frac{e^{(-2\,dx-2\,c)}}{8\,(a^2+2\,ab+b^2)d}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/2*b*log(2*(a - b)*e^(-2*d*x - 2*c) +
(a + b)*e^(-4*d*x - 4*c) + a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/
8*(3*a^2*b - 6*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/s
qrt(a*b))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) + 1/8*(3*a^2*b
 - 6*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))
/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) - 1/4*(3*a*b + b^2)*arc
tan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^3 + 2*a^2*b + a*
b^2)*sqrt(a*b)*d) + 1/4*(a^2*b - b^3 + (a^2*b - 6*a*b^2 + b^3)*e^(2*d*x +
2*c))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + (a^5 + 4*a^4*b + 6
*a^3*b^2 + 4*a^2*b^3 + a*b^4)*e^(4*d*x + 4*c) + 2*(a^5 + 2*a^4*b - 2*a^2*b
^3 - a*b^4)*e^(2*d*x + 2*c))*d) - 1/4*(a^2*b - b^3 + (a^2*b - 6*a*b^2 + b^
3)*e^(-2*d*x - 2*c))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + 2*(
a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*e^(-2*d*x - 2*c) + (a^5 + 4*a^4*b + 6*a
^3*b^2 + 4*a^2*b^3 + a*b^4)*e^(-4*d*x - 4*c))*d) + 1/2*(a*b + b^2 + (a*b -
 b^2)*e^(-2*d*x - 2*c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3
*b - a^2*b^2 - a*b^3)*e^(-2*d*x - 2*c) + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^
3)*e^(-4*d*x - 4*c))*d) + 1/2*(d*x + c)/((a^2 + 2*a*b + b^2)*d) + 1/8*e^(2
*d*x + 2*c)/((a^2 + 2*a*b + b^2)*d) - 1/8*e^(-2*d*x - 2*c)/((a^2 + 2*a*b +
 b^2)*d)
```

3.116.8 Giac [F]

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\cosh\left(dx+c\right)^2}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\cosh(c+d\,x)^2}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2, x)
```

3.117 $\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

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3.117.1 Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{b(4a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{5/2}d} + \frac{\sinh(c+dx)}{(a+b)^2 d} + \frac{b^2\sinh(c+dx)}{2a(a+b)^2 d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

output
```
1/2*b*(4*a+b)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(3/2)/(a+b)^(5/2)/
d+sinh(d*x+c)/(a+b)^2/d+1/2*b^2*sinh(d*x+c)/a/(a+b)^2/d/(a+(a+b)*sinh(d*x+
c)^2)
```

3.117.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{b(4a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a+b)^{5/2}} + \frac{\sinh(c+dx)\left(2+\frac{b^2}{a\left(a+(a+b)\sinh^2(c+dx)\right)}\right)}{(a+b)^2}}{2d}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((b*(4*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a + b
)^(5/2)) + (Sinh[c + d*x]*(2 + b^2/(a*(a + (a + b)*Sinh[c + d*x]^2))))/(a
+ b)^2)/(2*d)
```

3.117.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4159, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(ic+idx)\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\downarrow 4159$$

$$\frac{\int \frac{\left(\sinh^2(c+dx)+1\right)^2}{\left((a+b)\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{d}$$

$$\downarrow 300$$

$$\frac{\int \left(\frac{2b(a+b)\sinh^2(c+dx)+b(2a+b)}{(a+b)^2\left((a+b)\sinh^2(c+dx)+a\right)^2}+\frac{1}{(a+b)^2}\right)d\sinh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{b(4a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{5/2}}+\frac{b^2\sinh(c+dx)}{2a(a+b)^2\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{\sinh(c+dx)}{(a+b)^2}}{d}$$

input
```
Int[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((b*(4*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a +
 b)^(5/2)) + Sinh[c + d*x]/(a + b)^2 + (b^2*Sinh[c + d*x])/(2*a*(a + b)^2*
(a + (a + b)*Sinh[c + d*x]^2)))/d
```

3.117.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.117.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(89) = 178$.

Time $= 1.82$ (sec) , antiderivative size $= 286$, normalized size of antiderivative $= 2.83$

method	result
derivativedivides	$-\frac{1}{(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{2b\left(\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a}+\frac{(4a+b)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{\sqrt{(}}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(}\right.}\right.}{(a+b)^2}\right.}{d}$
default	$-\frac{1}{(a+b)^2\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{2b\left(\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a}+\frac{(4a+b)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{\sqrt{(}}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(}\right.}\right.}{(a+b)^2}\right.}{d}$
risch	$\frac{e^{dx+c}}{2d(a^2+2ab+b^2)}-\frac{e^{-dx-c}}{2d(a^2+2ab+b^2)}+\frac{b^2e^{dx+c}\left(e^{2dx+2c}-1\right)}{d(a+b)^2a\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)}-\frac{b\ln\left(e^{2dx+}\right.}{\sqrt{-}}$

input
```
int(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/(a+b)^2/(1+tanh(1/2*d*x+1/2*c))+2*b/(a+b)^2*((-1/2*b/a*tanh(1/2*d*
x+1/2*c)^3+1/2*b/a*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/
2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)+1/2*(4*a+b)*(1/2*(((a+b)*b)^
(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*ta
nh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(((a+b)*b)^(1/2
)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(
1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))-1/(a+b)^2/(tanh(1/2*
d*x+1/2*c)-1))
```

3.117. $\int \frac{\cosh(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$

3.117.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1815 vs. 2(89) = 178.

Time = 0.35 (sec) , antiderivative size = 3502, normalized size of antiderivative = 34.67

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^6 + 12*(a^4 + 2*a^3*b + a^
2*b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(a^4 + 2*a^3*b + a^2*b^2)*sinh(d*
x + c)^6 + 2*(a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^4 + 2*(a^4
- 2*a^3*b - a^2*b^2 + 2*a*b^3 + 15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)
^2)*sinh(d*x + c)^4 - 2*a^4 - 4*a^3*b - 2*a^2*b^2 + 8*(5*(a^4 + 2*a^3*b +
a^2*b^2)*cosh(d*x + c)^3 + (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x +
c))*sinh(d*x + c)^3 - 2*(a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^
2 + 2*(15*(a^4 + 2*a^3*b + a^2*b^2)*cosh(d*x + c)^4 - a^4 + 2*a^3*b + a^2*
b^2 - 2*a*b^3 + 6*(a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^2 - ((4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^5 + 5*(4*a^2*b + 5
*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (4*a^2*b + 5*a*b^2 + b^3)*si
nh(d*x + c)^5 + 2*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^3 + 2*(4*a^2*b -
 3*a*b^2 - b^3 + 5*(4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^3 + 2*(5*(4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^3 + 3*(4*a^2*b - 3*a*b^
2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + (4*a^2*b + 5*a*b^2 + b^3)*cosh(d
*x + c) + (5*(4*a^2*b + 5*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*a^2*b + 5*a*b^2
 + b^3 + 6*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(
-a^2 - a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*
x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a
 + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x ...
```

3.117.6 Sympy [F]

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\cosh\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(cosh(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

3.117.7 Maxima [F]

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\cosh\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(a^2 + a*b - (a^2*e^(6*c) + a*b*e^(6*c))*e^(6*d*x) - (a^2*e^(4*c) - 3
*a*b*e^(4*c) + 2*b^2*e^(4*c))*e^(4*d*x) + (a^2*e^(2*c) - 3*a*b*e^(2*c) + 2
*b^2*e^(2*c))*e^(2*d*x))/((a^4*d*e^(5*c) + 3*a^3*b*d*e^(5*c) + 3*a^2*b^2*d
*e^(5*c) + a*b^3*d*e^(5*c))*e^(5*d*x) + 2*(a^4*d*e^(3*c) + a^3*b*d*e^(3*c)
 - a^2*b^2*d*e^(3*c) - a*b^3*d*e^(3*c))*e^(3*d*x) + (a^4*d*e^c + 3*a^3*b*d
*e^c + 3*a^2*b^2*d*e^c + a*b^3*d*e^c)*e^(d*x)) + 1/2*integrate(2*((4*a*b*e
^(3*c) + b^2*e^(3*c))*e^(3*d*x) + (4*a*b*e^c + b^2*e^c)*e^(d*x))/(a^4 + 3*
a^3*b + 3*a^2*b^2 + a*b^3 + (a^4*e^(4*c) + 3*a^3*b*e^(4*c) + 3*a^2*b^2*e^(
4*c) + a*b^3*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + a^3*b*e^(2*c) - a^2*b^2
*e^(2*c) - a*b^3*e^(2*c))*e^(2*d*x)), x)
```

3.117.8 Giac [F]

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\cosh\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\cosh(c+d\,x)}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2)^2, x)
```

3.118 $\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

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3.118.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{(2a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}d} + \frac{b\sinh(c+dx)}{2a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

output
```
1/2*(2*a+b)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(3/2)/(a+b)^(3/2)/d+
1/2*b*sinh(d*x+c)/a/(a+b)/d/(a+(a+b)*sinh(d*x+c)^2)
```

3.118.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{(2a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}} + \frac{b\sinh(c+dx)}{a(a+(a+b)\sinh^2(c+dx))}}{2(a+b)d}$$

input
```
Integrate[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(((2*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(a^(3/2)*Sqrt[a +
 b]) + (b*Sinh[c + d*x])/(a*(a + (a + b)*Sinh[c + d*x]^2)))/(2*(a + b)*d)
```

3.118.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4159, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(ic+idx)}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\downarrow 4159$$

$$\frac{\int \frac{\sinh^2(c+dx)+1}{\left((a+b)\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{d}$$

$$\downarrow 298$$

$$\frac{\frac{(2a+b)\int \frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{2a(a+b)}+\frac{b\sinh(c+dx)}{2a(a+b)\left((a+b)\sinh^2(c+dx)+a\right)}}{d}$$

$$\downarrow 218$$

$$\frac{\frac{(2a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}}+\frac{b\sinh(c+dx)}{2a(a+b)\left((a+b)\sinh^2(c+dx)+a\right)}}{d}$$

input
```
Int[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(((2*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a + b
)^(3/2)) + (b*Sinh[c + d*x])/(2*a*(a + b)*(a + (a + b)*Sinh[c + d*x]^2)))/
d
```

3.118.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.118.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(71) = 142$.

Time $= 4.08$ (sec) , antiderivative size $= 252$, normalized size of antiderivative $= 3.04$

method	result
derivativedivides	$\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{a(a+b)}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a(a+b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a}+\frac{(2a+b)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-\right.}{2a\sqrt{}}}{a+b}}{d}$
default	$\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{a(a+b)}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a(a+b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a}+\frac{(2a+b)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-\right.}{2a\sqrt{}}}{a+b}}{d}$
risch	$\frac{b\,e^{dx+c}\left(e^{2dx+2c}-1\right)}{da(a+b)\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)}-\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}\,(a+b)d}-\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{d}}{\sqrt{-a^2}}\right.}{4\sqrt{-a^2-ab}\,(a-}$

input
```
int(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-1/2*b/a/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/2*b/a/(a+b)*tanh(1/2*d*x+1/
2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/
2*c)^2*b+a)+(2*a+b)/(a+b)*(1/2*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*(
(a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^
(1/2)+a+2*b)*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b
)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/
2)-a-2*b)*a)^(1/2))))
```

3.118.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1029 vs. $2(71) = 142$.

Time $= 0.29$ (sec) , antiderivative size $= 2041$, normalized size of antiderivative $= 24.59$

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*(a^2*b + a*b^2)*cosh(d*x + c)^3 + 12*(a^2*b + a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^2 + 4*(a^2*b + a*b^2)*sinh(d*x + c)^3 - ((2*a^2 + 3*a*b + b
^2)*cosh(d*x + c)^4 + 4*(2*a^2 + 3*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^
3 + (2*a^2 + 3*a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - a*b - b^2)*cosh(d*x
 + c)^2 + 2*(3*(2*a^2 + 3*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 - a*b - b^2)*
sinh(d*x + c)^2 + 2*a^2 + 3*a*b + b^2 + 4*((2*a^2 + 3*a*b + b^2)*cosh(d*x
+ c)^3 + (2*a^2 - a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 - a*b
)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh
(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a
 + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*
sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)
- cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a
 + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*
cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 +
4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)
) - 4*(a^2*b + a*b^2)*cosh(d*x + c) - 4*(a^2*b + a*b^2 - 3*(a^2*b + a*b^2)
*cosh(d*x + c)^2)*sinh(d*x + c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*
cosh(d*x + c)^4 + 4*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*
sinh(d*x + c)^3 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)...
```

3.118.6 Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

3.118.7 Maxima [F]

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^3*d + 2*a^2*b*d + a*b^2*d + (a^3*d*
e^(4*c) + 2*a^2*b*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(a^3*d*e^(2*c
) - a*b^2*d*e^(2*c))*e^(2*d*x)) + 2*integrate(1/2*((2*a*e^(3*c) + b*e^(3*c
))*e^(3*d*x) + (2*a*e^c + b*e^c)*e^(d*x))/(a^3 + 2*a^2*b + a*b^2 + (a^3*e^
(4*c) + 2*a^2*b*e^(4*c) + a*b^2*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*c) - a*b^
2*e^(2*c))*e^(2*d*x)), x)
```

3.118.8 Giac [F]

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^2), x)
```

3.119 $\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

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3.119.1 Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d} + \frac{\tanh(c+dx)}{2ad\left(a+b\tanh^2(c+dx)\right)}$$

output
```
1/2*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/d/b^(1/2)+1/2*tanh(d*x+c)/
a/d/(a+b*tanh(d*x+c)^2)
```

3.119.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{\sqrt{a}\tanh(c+dx)}{a+b\tanh^2(c+dx)}}{2a^{3/2}d}$$

input
```
Integrate[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/Sqrt[b] + (Sqrt[a]*Tanh[c + d*x])
/(a + b*Tanh[c + d*x]^2))/(2*a^(3/2)*d)
```

3.119.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(ic+idx)^2}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\downarrow 4158$$

$$\frac{\int \frac{1}{\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 215$$

$$\frac{\frac{\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{2a}+\frac{\tanh(c+dx)}{2a\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 218$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{\tanh(c+dx)}{2a\left(a+b\tanh^2(c+dx)\right)}}{d}$$

input
```
Int[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Tanh[c + d*
x]/(2*a*(a + b*Tanh[c + d*x]^2)))/d
```

3.119.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.119.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(54) = 108.

Time = 14.33 (sec) , antiderivative size = 210, normalized size of antiderivative = 3.18

method	result
risch	$-\frac{e^{2dx+2c}a-b\,e^{2dx+2c}+a+b}{(a+b)da\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}-2ab}{(a+b)\sqrt{-ab}}\right)}{4\sqrt{-ab}\,da}+\frac{\ln\left(e^{2dx+2c}\right.}{}$
derivativedivides	$\frac{-\frac{2\left(-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a}-\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}+\frac{\left(-a+\sqrt{(a+b)b}\right.}{2a\sqrt{(}}}{d}$
default	$\frac{-\frac{2\left(-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a}-\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}+\frac{\left(-a+\sqrt{(a+b)b}\right.}{2a\sqrt{(}}}{d}$

input
```
int(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-(exp(2*d*x+2*c)*a-b*exp(2*d*x+2*c)+a+b)/(a+b)/d/a/(a*exp(4*d*x+4*c)+b*exp
(4*d*x+4*c)+2*exp(2*d*x+2*c)*a-2*b*exp(2*d*x+2*c)+a+b)-1/4/(-a*b)^(1/2)/d/
a*ln(exp(2*d*x+2*c)+(a*(-a*b)^(1/2)-b*(-a*b)^(1/2)-2*a*b)/(a+b)/(-a*b)^(1/
2))+1/4/(-a*b)^(1/2)/d/a*ln(exp(2*d*x+2*c)+(a*(-a*b)^(1/2)-b*(-a*b)^(1/2)+
2*a*b)/(a+b)/(-a*b)^(1/2))
```

3.119.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(54) = 108$.

Time $= 0.30$ (sec) , antiderivative size $= 1515$, normalized size of antiderivative $= 22.95$

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*a^2*b + 4*a*b^2 + 4*(a^2*b - a*b^2)*cosh(d*x + c)^2 + 8*(a^2*b -
a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(a^2*b - a*b^2)*sinh(d*x + c)^2 + (
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*
sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh
(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d
*x + c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (
a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log(((a^2 + 2*a*b + b^
2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(
3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 -
 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d
*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x +
 c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*
cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d
*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a -
 b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*
sinh(d*x + c) + a + b)))/((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4
+ 4*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b
 + 2*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^4*b - a^2*b^3)*d*cosh(d*x
 + c)^2 + 2*(3*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^4*b...
```

3.119.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)**2/(a + b*tanh(c + d*x)**2)**2, x)
```

3.119.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(54) = 108$.

Time = 0.33 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.89

$$\begin{aligned}&\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx \\ &= \frac{(a-b)e^{(-2\,dx-2\,c)}+a+b}{(a^3+2\,a^2b+ab^2+2\,(a^3-ab^2)e^{(-2\,dx-2\,c)}+(a^3+2\,a^2b+ab^2)e^{(-4\,dx-4\,c)})d} \\ &\quad - \frac{\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}ad}\end{aligned}$$

input
```
integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
((a - b)*e^(-2*d*x - 2*c) + a + b)/((a^3 + 2*a^2*b + a*b^2 + 2*(a^3 - a*b^
2)*e^(-2*d*x - 2*c) + (a^3 + 2*a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d) - 1/2*a
rctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a*d)
```

3.119.8 Giac [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^2}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^2\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2), x)
```

3.120 $\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

3.120.1 Optimal result . . . 934
3.120.2 Mathematica [A] (verified) . . . 934
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3.120.1 Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}\,d} + \frac{\sinh(c+dx)}{2ad\left(a+(a+b)\sinh^2(c+dx)\right)}$$

output
```
1/2*sinh(d*x+c)/a/d/(a+(a+b)*sinh(d*x+c)^2)+1/2*arctan(sinh(d*x+c)*(a+b)^(
1/2)/a^(1/2))/a^(3/2)/d/(a+b)^(1/2)
```

3.120.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}} + \frac{\sqrt{a}\sinh(c+dx)}{a+(a+b)\sinh^2(c+dx)}}{2a^{3/2}d}$$

input
```
Integrate[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/Sqrt[a + b] + (Sqrt[a]*Sinh[c
 + d*x])/(a + (a + b)*Sinh[c + d*x]^2))/(2*a^(3/2)*d)
```

3.120.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4159, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(ic+idx)^3}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\downarrow 4159$$

$$\frac{\int \frac{1}{\left((a+b)\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{d}$$

$$\downarrow 215$$

$$\frac{\frac{\int \frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{2a}+\frac{\sinh(c+dx)}{2a\left((a+b)\sinh^2(c+dx)+a\right)}}{d}$$

$$\downarrow 218$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}}+\frac{\sinh(c+dx)}{2a\left((a+b)\sinh^2(c+dx)+a\right)}}{d}$$

input
```
Int[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a + b]) + Sin
h[c + d*x]/(2*a*(a + (a + b)*Sinh[c + d*x]^2)))/d
```

3.120.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.120.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(60) = 120$.

Time = 35.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.54

method	result
risch	$\frac{e^{dx+c}\left(e^{2dx+2c}-1\right)}{da\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)}-\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{4\sqrt{-a^2-ab}\,da}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}-\right.}{4\sqrt{-a^2-ab}\,da}$
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{a}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a}-\frac{\left(\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}+\frac{\left(\sqrt{(a+b)b}+b\right)\operatorname{arct}}{2a\sqrt{(a+b)b}}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{a}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a}-\frac{\left(\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}+\frac{\left(\sqrt{(a+b)b}+b\right)\operatorname{arct}}{2a\sqrt{(a+b)b}}$

input
```
int(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
exp(d*x+c)*(exp(2*d*x+2*c)-1)/d/a/(a*exp(4*d*x+4*c)+b*exp(4*d*x+4*c)+2*exp
(2*d*x+2*c)*a-2*b*exp(2*d*x+2*c)+a+b)-1/4/(-a^2-a*b)^(1/2)/d/a*ln(exp(2*d*
x+2*c)-2*a/(-a^2-a*b)^(1/2)*exp(d*x+c)-1)+1/4/(-a^2-a*b)^(1/2)/d/a*ln(exp(
2*d*x+2*c)+2*a/(-a^2-a*b)^(1/2)*exp(d*x+c)-1)
```

3.120.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(60) = 120$.

Time $= 0.29$ (sec) , antiderivative size $= 1555$, normalized size of antiderivative $= 21.60$

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*(a^2 + a*b)*cosh(d*x + c)^3 + 12*(a^2 + a*b)*cosh(d*x + c)*sinh(d*
x + c)^2 + 4*(a^2 + a*b)*sinh(d*x + c)^3 - ((a + b)*cosh(d*x + c)^4 + 4*(a
 + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*
cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 +
4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)
*sqrt(-a^2 - a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*s
inh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2
*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d
*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 +
3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1
)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d
*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c
)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*si
nh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d
*x + c) + a + b)) - 4*(a^2 + a*b)*cosh(d*x + c) + 4*(3*(a^2 + a*b)*cosh(d*
x + c)^2 - a^2 - a*b)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x
 + c)^4 + 4*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a
^4 + 2*a^3*b + a^2*b^2)*d*sinh(d*x + c)^4 + 2*(a^4 - a^2*b^2)*d*cosh(d*x +
 c)^2 + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 - a^2*b^2)
*d)*sinh(d*x + c)^2 + (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a^3*b +...
```

3.120.6 Sympy [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)
```

3.120.7 Maxima [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}(dx+c)^3}{\left(b\tanh(dx+c)^2+a\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
((a*e^(3*c) + b*e^(3*c))*e^(3*d*x) - (a*e^c + b*e^c)*e^(d*x))/(a^3*d + 2*a
^2*b*d + a*b^2*d + (a^3*d*e^(4*c) + 2*a^2*b*d*e^(4*c) + a*b^2*d*e^(4*c))*e
^(4*d*x) + 2*(a^3*d*e^(2*c) - a*b^2*d*e^(2*c))*e^(2*d*x)) + 8*integrate(1/
8*(e^(3*d*x + 3*c) + e^(d*x + c))/(a^2 + a*b + (a^2*e^(4*c) + a*b*e^(4*c))
*e^(4*d*x) + 2*(a^2*e^(2*c) - a*b*e^(2*c))*e^(2*d*x)), x)
```

3.120.8 Giac [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}(dx+c)^3}{\left(b\tanh(dx+c)^2+a\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{1}{\cosh(c+dx)^3\left(b\tanh(c+dx)^2+a\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2), x)
```

3.121 $\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

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3.121.1 Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{(a-b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d} + \frac{(a+b)\tanh(c+dx)}{2abd\left(a+b\tanh^2(c+dx)\right)}$$

output
```
-1/2*(a-b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/b^(3/2)/d+1/2*(a+b)
*tanh(d*x+c)/a/b/d/(a+b*tanh(d*x+c)^2)
```

3.121.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{(-a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) + \frac{\sqrt{a}\sqrt{b}(a+b)\sinh(2(c+dx))}{a-b+(a+b)\cosh(2(c+dx))}}{2a^{3/2}b^{3/2}d}$$

input
```
Integrate[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((-a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] + (Sqrt[a]*Sqrt[b]*(a +
b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(2*a^(3/2)*b^(3
/2)*d)
```

3.121.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(ic+idx)^4}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\downarrow 4158$$

$$\frac{\int \frac{1-\tanh^2(c+dx)}{\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 298$$

$$\frac{\frac{(a+b)\tanh(c+dx)}{2ab\left(a+b\tanh^2(c+dx)\right)} - \frac{(a-b)\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{2ab}}{d}$$

$$\downarrow 218$$

$$\frac{\frac{(a+b)\tanh(c+dx)}{2ab\left(a+b\tanh^2(c+dx)\right)} - \frac{(a-b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}}{d}$$

input
```
Int[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-1/2*((a - b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*b^(3/2))
+ ((a + b)*Tanh[c + d*x])/(2*a*b*(a + b*Tanh[c + d*x]^2)))/d
```

3.121.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.121.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(65) = 130$.

Time $= 73.20$ (sec) , antiderivative size $= 259$, normalized size of antiderivative $= 3.36$

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2ab}-\frac{(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ab}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a}-\frac{(a-b)\left(-\frac{\left(a-\sqrt{(a+b)b}+b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}+\frac{(-a}{b}\right.}{d}$
default	$\frac{-\frac{2\left(-\frac{(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2ab}-\frac{(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ab}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a}-\frac{(a-b)\left(-\frac{\left(a-\sqrt{(a+b)b}+b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}+\frac{(-a}{b}\right.}{d}$
risch	$-\frac{e^{2dx+2c}a-b\,e^{2dx+2c}+a+b}{dab\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}+2ab}{(a+b)\sqrt{-ab}}\right)}{4\sqrt{-ab}\,db}+\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{}}{4\sqrt{}}\right.}{}$

input
```
int(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*(a+b)/a/b*tanh(1/2*d*x+1/2*c)^3-1/2*(a+b)/a/b*tanh(1/2*d*x+1
/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1
/2*c)^2*b+a)-(a-b)/b*(-1/2*(a-((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a
+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(
1/2)-a-2*b)*a)^(1/2))+1/2*(-a-((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a
+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1
/2)+a+2*b)*a)^(1/2))))
```

3.121.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(65) = 130$.

Time $= 0.28$ (sec) , antiderivative size $= 1443$, normalized size of antiderivative $= 18.74$

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*a^2*b + 4*a*b^2 + 4*(a^2*b - a*b^2)*cosh(d*x + c)^2 + 8*(a^2*b -
a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(a^2*b - a*b^2)*sinh(d*x + c)^2 - (
(a^2 - b^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3
+ (a^2 - b^2)*sinh(d*x + c)^4 + 2*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2 + 2*
(3*(a^2 - b^2)*cosh(d*x + c)^2 + a^2 - 2*a*b + b^2)*sinh(d*x + c)^2 + a^2
- b^2 + 4*((a^2 - b^2)*cosh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*cosh(d*x + c)
)*sinh(d*x + c))*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(
a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sin
h(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*co
sh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 +
 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) -
 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a +
 b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a +
 b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*co
sh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*
((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)))
/((a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 + a^2*b^3)*d*cosh(d*x
 + c)*sinh(d*x + c)^3 + (a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^3*b^2
 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2
 + (a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^2 + (a^3*b^2 + a^2*b^3)*d + 4*(...
```

3.121.6 Sympy [F]

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)**4/(a + b*tanh(c + d*x)**2)**2, x)
```

3.121.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.65

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{(a-b)e^{(-2\,dx-2\,c)}+a+b}{(a^2b+ab^2+2\,(a^2b-ab^2)e^{(-2\,dx-2\,c)}+(a^2b+ab^2)e^{(-4\,dx-4\,c)})d} + \frac{(a-b)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}abd}$$

input
```
integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
((a - b)*e^(-2*d*x - 2*c) + a + b)/((a^2*b + a*b^2 + 2*(a^2*b - a*b^2)*e^(
-2*d*x - 2*c) + (a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d) + 1/2*(a - b)*arctan(
1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a*b*d)
```

3.121.8 Giac [F]

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^4}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^4\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2), x)
```

3.122 $\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

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3.122.1 Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\arctan(\sinh(c+dx))}{b^2 d} - \frac{(2a-b)\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2 d} + \frac{(a+b)\sinh(c+dx)}{2abd\left(a+(a+b)\sinh^2(c+dx)\right)}$$

output
```
arctan(sinh(d*x+c))/b^2/d+1/2*(a+b)*sinh(d*x+c)/a/b/d/(a+(a+b)*sinh(d*x+c)
^2)-1/2*(2*a-b)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))*(a+b)^(1/2)/a^(3/2
)/b^2/d
```

3.122.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.99

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{(a-b)\left((2a^2+ab-b^2)\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)+4a^{3/2}\sqrt{a+b}\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)+(a+b)\left((2a$$
$$2a^{3/2}b^2\sqrt{a+b}$$

input
```
Integrate[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((a - b)*((2*a^2 + a*b - b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]]
+ 4*a^(3/2)*Sqrt[a + b]*ArcTan[Tanh[(c + d*x)/2]]) + (a + b)*((2*a^2 + a*b
 - b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]] + 4*a^(3/2)*Sqrt[a + b
]*ArcTan[Tanh[(c + d*x)/2]])*Cosh[2*(c + d*x)] + 2*Sqrt[a]*b*(a + b)^(3/2)
*Sinh[c + d*x])/(2*a^(3/2)*b^2*Sqrt[a + b]*d*(a - b + (a + b)*Cosh[2*(c +
d*x)]))
```

3.122.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4159, 316, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(ic+idx)^5}{\left(a-b\tan(ic+idx)^2\right)^2}\,dx$$

$$\downarrow 4159$$

$$\frac{\int \frac{1}{\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)^2}\,d\sinh(c+dx)}{d}$$

$$\downarrow 316$$

$$\frac{\frac{(a+b)\sinh(c+dx)}{2ab\left((a+b)\sinh^2(c+dx)+a\right)} - \frac{\int \frac{-\left((a+b)\sinh^2(c+dx)\right)+a-b}{\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)}\,d\sinh(c+dx)}{2ab}}{d}$$

$$\downarrow 397$$

$$\frac{\frac{(a+b)\sinh(c+dx)}{2ab\left((a+b)\sinh^2(c+dx)+a\right)} - \frac{\frac{(2a-b)(a+b)\int \frac{1}{(a+b)\sinh^2(c+dx)+a}\,d\sinh(c+dx)}{b} - \frac{2a\int \frac{1}{\sinh^2(c+dx)+1}\,d\sinh(c+dx)}{b}}{2ab}}{d}$$

$$\downarrow 216$$

$$\frac{\frac{(a+b)\sinh(c+dx)}{2ab((a+b)\sinh^2(c+dx)+a)}-\frac{\frac{(2a-b)(a+b)\int\frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{b}-\frac{2a\arctan(\sinh(c+dx))}{b}}{2ab}}{d}$$

$$\downarrow 218$$

$$\frac{\frac{(a+b)\sinh(c+dx)}{2ab((a+b)\sinh^2(c+dx)+a)}-\frac{\frac{(2a-b)\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ab}}-\frac{2a\arctan(\sinh(c+dx))}{b}}{2ab}}{d}$$

input
```
Int[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-1/2*((-2*a*ArcTan[Sinh[c + d*x]])/b + ((2*a - b)*Sqrt[a + b]*ArcTan[(Sqr
t[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b))/(a*b) + ((a + b)*Sinh[c + d
*x])/(2*a*b*(a + (a + b)*Sinh[c + d*x]^2)))/d
```

3.122.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.122.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(90) = 180$.

Time = 136.48 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.68

method	result
derivativedivides	$\frac{-\frac{2\left(\frac{\frac{(a+b)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}-\frac{(a+b)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a}+\frac{\left(2a^2+ab-b^2\right)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\cdots\right)}{2}\cdots\right)}{b^2}\cdots}{d}$
default	$\frac{-\frac{2\left(\frac{\frac{(a+b)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}-\frac{(a+b)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a}+\frac{\left(2a^2+ab-b^2\right)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\cdots\right)}{2}\cdots\right)}{b^2}\cdots}{d}$
risch	$\frac{e^{dx+c}(a+b)\left(e^{2dx+2c}-1\right)}{dab\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)}+\frac{i\ln\left(e^{dx+c}+i\right)}{d\,b^2}-\frac{i\ln\left(e^{dx+c}-i\right)}{d\,b^2}+\frac{\sqrt{-a(a+b)}\,\ln\left(e^{2dx}\cdots\right.}{\cdots}$

input
```
int(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/b^2*((1/2*(a+b)*b/a*tanh(1/2*d*x+1/2*c)^3-1/2*(a+b)*b/a*tanh(1/2*d
*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d
*x+1/2*c)^2*b+a)+1/2*(2*a^2+a*b-b^2)*(1/2*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^
(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2
*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2
)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((
a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))+2/b^2*arctan(tanh(1/2*d*x+1/2*c)))
```

3.122.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1078 vs. $2(90) = 180$.

Time $= 0.35$ (sec) , antiderivative size $= 2140$, normalized size of antiderivative $= 20.98$

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*(a*b + b^2)*cosh(d*x + c)^3 + 12*(a*b + b^2)*cosh(d*x + c)*sinh(d*
x + c)^2 + 4*(a*b + b^2)*sinh(d*x + c)^3 - ((2*a^2 + a*b - b^2)*cosh(d*x +
 c)^4 + 4*(2*a^2 + a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + a*b
 - b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(
2*a^2 + a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sinh(d*x + c)^2
+ 2*a^2 + a*b - b^2 + 4*((2*a^2 + a*b - b^2)*cosh(d*x + c)^3 + (2*a^2 - 3*
a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-(a + b)/a)*log(((a + b)*cos
h(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x
+ c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a
- b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x + c
))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^
2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d
*x + c))*sqrt(-(a + b)/a) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*co
sh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x
 + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a +
b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 8*((
a^2 + a*b)*cosh(d*x + c)^4 + 4*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a^2 + a*b)*sinh(d*x + c)^4 + 2*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(a^2 +
 a*b)*cosh(d*x + c)^2 + a^2 - a*b)*sinh(d*x + c)^2 + a^2 + a*b + 4*((a^2 +
 a*b)*cosh(d*x + c)^3 + (a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))*arcta...
```

3.122.6 Sympy [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)**5/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)**5/(a + b*tanh(c + d*x)**2)**2, x)
```

3.122.7 Maxima [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}(dx+c)^5}{\left(b\tanh(dx+c)^2+a\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
((a*e^(3*c) + b*e^(3*c))*e^(3*d*x) - (a*e^c + b*e^c)*e^(d*x))/(a^2*b*d + a
*b^2*d + (a^2*b*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(a^2*b*d*e^(2*c
) - a*b^2*d*e^(2*c))*e^(2*d*x)) + 2*arctan(e^(d*x + c))/(b^2*d) - 32*integ
rate(1/32*((2*a^2*e^(3*c) + a*b*e^(3*c) - b^2*e^(3*c))*e^(3*d*x) + (2*a^2*
e^c + a*b*e^c - b^2*e^c)*e^(d*x))/(a^2*b^2 + a*b^3 + (a^2*b^2*e^(4*c) + a*
b^3*e^(4*c))*e^(4*d*x) + 2*(a^2*b^2*e^(2*c) - a*b^3*e^(2*c))*e^(2*d*x)), x
)
```

3.122.8 Giac [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}(dx+c)^5}{\left(b\tanh(dx+c)^2+a\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^5\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)^2), x)
```

3.123 $\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

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3.123.1 Optimal result

Integrand size = 23, antiderivative size = 97

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{(3a-b)(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d} + \frac{\tanh(c+dx)}{b^2d} + \frac{(a+b)^2\tanh(c+dx)}{2ab^2d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
-1/2*(3*a-b)*(a+b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/b^(5/2)/d+t
anh(d*x+c)/b^2/d+1/2*(a+b)^2*tanh(d*x+c)/a/b^2/d/(a+b*tanh(d*x+c)^2)
```

3.123.2 Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.05

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{-\frac{(3a-b)(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\sqrt{b}(a+b)^2\sinh(2(c+dx))}{a(a-b+(a+b)\cosh(2(c+dx)))} + 2\sqrt{b}\tanh(c+dx)}{2b^{5/2}d}$$

input
```
Integrate[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-(((3*a - b)*(a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2)) +
(Sqrt[b]*(a + b)^2*Sinh[2*(c + d*x)])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)
])) + 2*Sqrt[b]*Tanh[c + d*x])/(2*b^(5/2)*d)
```

3.123.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(ic+idx)^6}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\downarrow 4158$$

$$\frac{\int \frac{\left(1-\tanh^2(c+dx)\right)^2}{\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 300$$

$$\frac{\int \left(\frac{1}{b^2}-\frac{a^2-b^2+2b(a+b)\tanh^2(c+dx)}{b^2\left(b\tanh^2(c+dx)+a\right)^2}\right)d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{-\frac{(3a-b)(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{(a+b)^2\tanh(c+dx)}{2ab^2\left(a+b\tanh^2(c+dx)\right)}+\frac{\tanh(c+dx)}{b^2}}{d}$$

input
```
Int[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-1/2*((3*a - b)*(a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)
*b^(5/2)) + Tanh[c + d*x]/b^2 + ((a + b)^2*Tanh[c + d*x])/(2*a*b^2*(a + b*
Tanh[c + d*x]^2)))/d
```

3.123.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.123.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(85) = 170$.

Time = 258.30 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.15

method	result
derivativedivides	$\frac{2\left(\frac{\left(a^2+2ab+b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}+\frac{\left(a^2+2ab+b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a}+\left(3a^2+2ab-b^2\right)\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2\right.}}\right.}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right.}{b^2}$ $\frac{}{d}$
default	$\frac{2\left(\frac{\left(a^2+2ab+b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}+\frac{\left(a^2+2ab+b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a}+\left(3a^2+2ab-b^2\right)\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2\right.}}\right.}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right.}{b^2}$ $\frac{}{d}$
risch	$-\frac{3a^2e^{4dx+4c}+2ab\,e^{4dx+4c}-e^{4dx+4c}b^2+6a^2e^{2dx+2c}-2ab\,e^{2dx+2c}+3a^2+4ab+b^2}{a\,b^2d\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)\left(e^{2dx+2c}+1\right)}-\frac{3a\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}+2}{(a+b)\sqrt{-ab}}\right.}{4\sqrt{-ab}\,d\,b^2}$

input
```
int(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b^2*((1/2*(a^2+2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^3+1/2*(a^2+2*a*b+b^
2)/a*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2
*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)+1/2*(3*a^2+2*a*b-b^2)*(1/2*(a+((a+b)*b)^(1
/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh
(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(-a+((a+b)*b)^(1/
2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh
(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))+2/b^2*tanh(1/2*d*x+
1/2*c)/(tanh(1/2*d*x+1/2*c)^2+1))
```

3.123.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1282 vs. $2(85) = 170$.

Time = 0.31 (sec) , antiderivative size = 2869, normalized size of antiderivative = 29.58

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*(3*a^3*b + 2*a^2*b^2 - a*b^3)*cosh(d*x + c)^4 + 16*(3*a^3*b + 2*a
^2*b^2 - a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(3*a^3*b + 2*a^2*b^2 - a
*b^3)*sinh(d*x + c)^4 + 12*a^3*b + 16*a^2*b^2 + 4*a*b^3 + 8*(3*a^3*b - a^2
*b^2)*cosh(d*x + c)^2 + 8*(3*a^3*b - a^2*b^2 + 3*(3*a^3*b + 2*a^2*b^2 - a*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^3 + 5*a^2*b + a*b^2 - b^3)*c
osh(d*x + c)^6 + 6*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x
+ c)^5 + (3*a^3 + 5*a^2*b + a*b^2 - b^3)*sinh(d*x + c)^6 + (9*a^3 + 3*a^2*
b - 5*a*b^2 + b^3)*cosh(d*x + c)^4 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3 + 15
*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(
3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^3 + (9*a^3 + 3*a^2*b - 5*a*b^
2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a^3 + 5*a^2*b + a*b^2 - b^3 +
(9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^2 + (15*(3*a^3 + 5*a^2*b +
 a*b^2 - b^3)*cosh(d*x + c)^4 + 9*a^3 + 3*a^2*b - 5*a*b^2 + b^3 + 6*(9*a^3
 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(3*a^3
 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^5 + 2*(9*a^3 + 3*a^2*b - 5*a*b^2 +
 b^3)*cosh(d*x + c)^3 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c))*s
inh(d*x + c))*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2
 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d
*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 +...
```

3.123.6 Sympy [F]

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)**6/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)**6/(a + b*tanh(c + d*x)**2)**2, x)
```

3.123.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(85) = 170$.

Time = 0.39 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.15

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{3a^2+4ab+b^2+2(3a^2-ab)e^{(-2dx-2c)}+(3a^2+2ab-b^2)e^{(-4dx-4c)}}{(a^2b^2+ab^3+(3a^2b^2-ab^3)e^{(-2dx-2c)}+(3a^2b^2-ab^3)e^{(-4dx-4c)}+(a^2b^2+ab^3)e^{(-6dx-6c)})d}$$
$$+ \frac{(3a^2+2ab-b^2)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}ab^2d}$$

input
```
integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
(3*a^2 + 4*a*b + b^2 + 2*(3*a^2 - a*b)*e^(-2*d*x - 2*c) + (3*a^2 + 2*a*b -
 b^2)*e^(-4*d*x - 4*c))/((a^2*b^2 + a*b^3 + (3*a^2*b^2 - a*b^3)*e^(-2*d*x
- 2*c) + (3*a^2*b^2 - a*b^3)*e^(-4*d*x - 4*c) + (a^2*b^2 + a*b^3)*e^(-6*d*
x - 6*c))*d) + 1/2*(3*a^2 + 2*a*b - b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2
*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a*b^2*d)
```

3.123.8 Giac [F]

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^6}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{1}{\cosh(c+dx)^6\left(b\tanh(c+dx)^2+a\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)^2), x)
```

3.124 $\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

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3.124.1 Optimal result

Integrand size = 23, antiderivative size = 155

$$\begin{aligned}\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx &= \frac{(4a+5b)\arctan(\sinh(c+dx))}{2b^3d}\\ &\quad - \frac{(4a-b)(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d}\\ &\quad + \frac{(a+b)(2a+b)\sinh(c+dx)}{2ab^2d\left(a+(a+b)\sinh^2(c+dx)\right)}\\ &\quad - \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd\left(a+(a+b)\sinh^2(c+dx)\right)}\end{aligned}$$

output

```
1/2*(4*a+5*b)*arctan(sinh(d*x+c))/b^3/d-1/2*(4*a-b)*(a+b)^(3/2)*arctan(sin
h(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(3/2)/b^3/d+1/2*(a+b)*(2*a+b)*sinh(d*x+c)/
a/b^2/d/(a+(a+b)*sinh(d*x+c)^2)-1/2*sech(d*x+c)*tanh(d*x+c)/b/d/(a+(a+b)*s
inh(d*x+c)^2)
```

3.124.2 Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.71

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{2\sqrt{a}b(a+b)^{5/2}\sinh(c+dx)+(a-b)\left((4a-b)(a+b)^2\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)+2a^{3/2}\sqrt{a+b}(4a+5b)\text{ar}\right.}{}$$

input
```
Integrate[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(2*Sqrt[a]*b*(a + b)^(5/2)*Sinh[c + d*x] + (a - b)*((4*a - b)*(a + b)^2*Ar
cTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]] + 2*a^(3/2)*Sqrt[a + b]*(4*a + 5
*b)*ArcTan[Tanh[(c + d*x)/2]] + a^(3/2)*b*Sqrt[a + b]*Sech[c + d*x]*Tanh[c
 + d*x]) + (a + b)*Cosh[2*(c + d*x)]*((4*a - b)*(a + b)^2*ArcTan[(Sqrt[a]*
Csch[c + d*x])/Sqrt[a + b]] + 2*a^(3/2)*Sqrt[a + b]*(4*a + 5*b)*ArcTan[Tan
h[(c + d*x)/2]] + a^(3/2)*b*Sqrt[a + b]*Sech[c + d*x]*Tanh[c + d*x]))/(2*a
^(3/2)*b^3*Sqrt[a + b]*d*(a - b + (a + b)*Cosh[2*(c + d*x)]))
```

3.124.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4159, 316, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sec(ic+idx)^7}{\left(a-b\tan(ic+idx)^2\right)^2}\,dx$$
$$\downarrow 4159$$
$$\frac{\int \frac{1}{\left(\sinh^2(c+dx)+1\right)^2\left((a+b)\sinh^2(c+dx)+a\right)^2}\,d\sinh(c+dx)}{d}$$
$$\downarrow 316$$

$$\frac{\int \frac{-3(a+b)\sinh^2(c+dx)+a+2b}{\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)^2}\,d\sinh(c+dx)}{2b} - \frac{\sinh(c+dx)}{2b\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)}$$

$$\Big\downarrow 402$$

$$\frac{\frac{(a+b)(2a+b)\sinh(c+dx)}{ab\left((a+b)\sinh^2(c+dx)+a\right)} - \frac{\int \frac{2\left(2a^2+2ba-b^2-(a+b)(2a+b)\sinh^2(c+dx)\right)}{\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)}\,d\sinh(c+dx)}{2ab}}{2b} - \frac{\sinh(c+dx)}{2b\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)}$$

$$\Big\downarrow 27$$

$$\frac{\frac{(a+b)(2a+b)\sinh(c+dx)}{ab\left((a+b)\sinh^2(c+dx)+a\right)} - \frac{\int \frac{2a^2+2ba-b^2-(a+b)(2a+b)\sinh^2(c+dx)}{\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)}\,d\sinh(c+dx)}{ab}}{2b} - \frac{\sinh(c+dx)}{2b\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)}$$

$$\Big\downarrow 397$$

$$\frac{\frac{(a+b)(2a+b)\sinh(c+dx)}{ab\left((a+b)\sinh^2(c+dx)+a\right)} - \frac{(4a-b)(a+b)^2\int \frac{1}{(a+b)\sinh^2(c+dx)+a}\,d\sinh(c+dx)}{b} - \frac{a(4a+5b)\int \frac{1}{\sinh^2(c+dx)+1}\,d\sinh(c+dx)}{b}}{ab}}{2b} - \frac{\sinh(c+dx)}{2b\left(\sinh^2(c+dx)+1\right)\left((a+b)\sin\right.}$$

$$\Big\downarrow 216$$

$$\frac{\frac{(a+b)(2a+b)\sinh(c+dx)}{ab\left((a+b)\sinh^2(c+dx)+a\right)} - \frac{(4a-b)(a+b)^2\int \frac{1}{(a+b)\sinh^2(c+dx)+a}\,d\sinh(c+dx)}{b} - \frac{a(4a+5b)\arctan(\sinh(c+dx))}{b}}{ab}}{2b} - \frac{\sinh(c+dx)}{2b\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)\right.}$$

$$\Big\downarrow 218$$

$$\frac{\frac{(a+b)(2a+b)\sinh(c+dx)}{ab\left((a+b)\sinh^2(c+dx)+a\right)} - \frac{(4a-b)(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ab}} - \frac{a(4a+5b)\arctan(\sinh(c+dx))}{b}}{ab}}{2b} - \frac{\sinh(c+dx)}{2b\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)}$$

input
```
Int[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2)^2,x]
```

3.124. $\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

output
```
(-1/2*Sinh[c + d*x]/(b*(1 + Sinh[c + d*x]^2)*(a + (a + b)*Sinh[c + d*x]^2)
) + (-((-((a*(4*a + 5*b)*ArcTan[Sinh[c + d*x]])/b) + ((4*a - b)*(a + b)^(3
/2)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b))/(a*b)) + ((a
 + b)*(2*a + b)*Sinh[c + d*x])/(a*b*(a + (a + b)*Sinh[c + d*x]^2)))/(2*b))
/d
```

3.124.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.124.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(139) = 278$.

Time $= 0.30$ (sec) , antiderivative size $= 351$, normalized size of antiderivative $= 2.26$

$$\frac{\frac{2\left(-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 b}{2}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)^2}+(5b+4a)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3}-\frac{2\left(\frac{\frac{b\left(a^2+2ab+b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}-\frac{b\left(a^2+2ab+b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a}+\frac{\left(4a^3+7a\right.}{}\right.}{d}$$

input
```
int(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
1/d*(2/b^3*((-1/2*tanh(1/2*d*x+1/2*c)^3*b+1/2*b*tanh(1/2*d*x+1/2*c))/(tanh
(1/2*d*x+1/2*c)^2+1)^2+1/2*(5*b+4*a)*arctan(tanh(1/2*d*x+1/2*c)))-2/b^3*((
1/2*b*(a^2+2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^3-1/2*b*(a^2+2*a*b+b^2)/a*tanh
(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh
(1/2*d*x+1/2*c)^2*b+a)+1/2*(4*a^3+7*a^2*b+2*a*b^2-b^3)*(1/2*(((a+b)*b)^(1/
2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(
1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-b
)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2
*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)))))
```

3.124.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3490 vs. $2(139) = 278$.

Time $= 0.37$ (sec) , antiderivative size $= 6396$, normalized size of antiderivative $= 41.26$

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.124.6 Sympy [F]

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)**7/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)**7/(a + b*tanh(c + d*x)**2)**2, x)
```

3.124.7 Maxima [F]

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^7}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
((2*a^2*e^(7*c) + 3*a*b*e^(7*c) + b^2*e^(7*c))*e^(7*d*x) + (2*a^2*e^(5*c)
- a*b*e^(5*c) + b^2*e^(5*c))*e^(5*d*x) - (2*a^2*e^(3*c) - a*b*e^(3*c) + b^
2*e^(3*c))*e^(3*d*x) - (2*a^2*e^c + 3*a*b*e^c + b^2*e^c)*e^(d*x))/(4*a^2*b
^2*d*e^(6*d*x + 6*c) + 4*a^2*b^2*d*e^(2*d*x + 2*c) + a^2*b^2*d + a*b^3*d +
 (a^2*b^2*d*e^(8*c) + a*b^3*d*e^(8*c))*e^(8*d*x) + 2*(3*a^2*b^2*d*e^(4*c)
- a*b^3*d*e^(4*c))*e^(4*d*x)) + (4*a*e^c + 5*b*e^c)*arctan(e^(d*x + c))*e^
(-c)/(b^3*d) - 128*integrate(1/128*((4*a^3*e^(3*c) + 7*a^2*b*e^(3*c) + 2*a
*b^2*e^(3*c) - b^3*e^(3*c))*e^(3*d*x) + (4*a^3*e^c + 7*a^2*b*e^c + 2*a*b^2
*e^c - b^3*e^c)*e^(d*x))/(a^2*b^3 + a*b^4 + (a^2*b^3*e^(4*c) + a*b^4*e^(4*
c))*e^(4*d*x) + 2*(a^2*b^3*e^(2*c) - a*b^4*e^(2*c))*e^(2*d*x)), x)
```

3.124.8 Giac [F]

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^7}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
sage0*x
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^7\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)^7*(a + b*tanh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)^7*(a + b*tanh(c + d*x)^2)^2), x)
```

3.125 $\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

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3.125.1 Optimal result

Integrand size = 23, antiderivative size = 198

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{(a+7b)x}{2(a+b)^4} + \frac{b^{3/2}(35a^2+14ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^4 d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(2a-b)b\tanh(c+dx)}{4a(a+b)^2 d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a-3b)b(4a+b)\tanh(c+dx)}{8a^2(a+b)^3 d\left(a+b\tanh^2(c+dx)\right)}$$

output

```
1/2*(a+7*b)*x/(a+b)^4+1/8*b^(3/2)*(35*a^2+14*a*b+3*b^2)*arctan(b^(1/2)*tan
h(d*x+c)/a^(1/2))/a^(5/2)/(a+b)^4/d+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d/(a
+b*tanh(d*x+c)^2)^2-1/4*(2*a-b)*b*tanh(d*x+c)/a/(a+b)^2/d/(a+b*tanh(d*x+c)
^2)^2-1/8*(a-3*b)*b*(4*a+b)*tanh(d*x+c)/a^2/(a+b)^3/d/(a+b*tanh(d*x+c)^2)
```

3.125.2 Mathematica [A] (verified)

Time = 2.41 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.83

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{4(a+7b)(c+dx) + \frac{b^{3/2}(35a^2+14ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}} + 2(a+b)\sinh(2(c+dx)) + \frac{4b^3(a+b)\sinh(2(c+dx))}{a(a-b+(a+b)\cosh(2(c+dx)}}{8(a+b)^4 d}$$

input
```
Integrate[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(4*(a + 7*b)*(c + d*x) + (b^(3/2)*(35*a^2 + 14*a*b + 3*b^2)*ArcTan[(Sqrt[b
]*Tanh[c + d*x])/Sqrt[a]])/a^(5/2) + 2*(a + b)*Sinh[2*(c + d*x)] + (4*b^3*
(a + b)*Sinh[2*(c + d*x)])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])^2) + (b^
2*(a + b)*(13*a + 3*b)*Sinh[2*(c + d*x)])/(a^2*(a - b + (a + b)*Cosh[2*(c
+ d*x)])))/(8*(a + b)^4*d)
```

3.125.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4158, 316, 402, 27, 402, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\sec(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$
$$\downarrow 4158$$
$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)^2\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh(c+dx)}{d}$$
$$\downarrow 316$$

$$\frac{\int \frac{5b\tanh^2(c+dx)+a+2b}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh(c+dx)}{2(a+b)} + \frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}$$
$$d$$
$$\downarrow 402$$
$$\frac{-\frac{\int -\frac{2\left(2a^2+8ba+3b^2+3(2a-b)b\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}\,d\tanh(c+dx)}{4a(a+b)} - \frac{b(2a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{2(a+b)} + \frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}$$
$$d$$
$$\downarrow 27$$
$$\frac{\frac{\int \frac{2a^2+8ba+3b^2+3(2a-b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}\,d\tanh(c+dx)}{2a(a+b)} - \frac{b(2a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{2(a+b)} + \frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}$$
$$d$$
$$\downarrow 402$$
$$\frac{-\frac{\int -\frac{4a^3+24ba^2+11b^2a+3b^3+(a-3b)b(4a+b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}\,d\tanh(c+dx)}{2a(a+b)} - \frac{b(a-3b)(4a+b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)} - \frac{b(2a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{2(a+b)} + \frac{\text{ta}}{2(a+b)\left(1-\tanh^2(c\right.}$$
$$d$$
$$\downarrow 25$$
$$\frac{\frac{\int \frac{4a^3+24ba^2+11b^2a+3b^3+(a-3b)b(4a+b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}\,d\tanh(c+dx)}{2a(a+b)} - \frac{b(a-3b)(4a+b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)} - \frac{b(2a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{2(a+b)} + \frac{\tanh}{2(a+b)\left(1-\tanh^2(c+d\right.}$$
$$d$$
$$\downarrow 397$$
$$\frac{\frac{b^2\left(35a^2+14ab+3b^2\right)\int \frac{1}{b\tanh^2(c+dx)+a}\,d\tanh(c+dx)}{a+b} + \frac{4a^2(a+7b)\int \frac{1}{1-\tanh^2(c+dx)}\,d\tanh(c+dx)}{a+b}}{2a(a+b)} - \frac{b(a-3b)(4a+b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)} - \frac{b(2a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}$$
$$2(a+b)$$
$$d$$
$$\downarrow 218$$

3.125. $\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

$$\frac{\frac{4a^2(a+7b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{b^{3/2}\left(35a^2+14ab+3b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2a(a+b)}-\frac{b(a-3b)(4a+b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{b(2a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+$$
$$\frac{2(a+b)}{d}$$

$$\downarrow 219$$

$$\frac{\frac{b^{3/2}\left(35a^2+14ab+3b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}+\frac{4a^2(a+7b)\operatorname{arctanh}(\tanh(c+dx))}{a+b}}{2a(a+b)}-\frac{b(a-3b)(4a+b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{b(2a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+\frac{}{2(a+b}$$
$$d$$

input
```
Int[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(Tanh[c + d*x]/(2*(a + b)*(1 - Tanh[c + d*x]^2)*(a + b*Tanh[c + d*x]^2)^2)
 + (-1/2*((2*a - b)*b*Tanh[c + d*x])/(a*(a + b)*(a + b*Tanh[c + d*x]^2)^2)
 + (((b^(3/2)*(35*a^2 + 14*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqr
t[a]])/(Sqrt[a]*(a + b)) + (4*a^2*(a + 7*b)*ArcTanh[Tanh[c + d*x]])/(a + b
))/(2*a*(a + b)) - ((a - 3*b)*b*(4*a + b)*Tanh[c + d*x])/(2*a*(a + b)*(a +
 b*Tanh[c + d*x]^2)))/(2*a*(a + b)))/(2*(a + b)))/d
```

3.125.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.125.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(180) = 360$.

Time = 27.89 (sec) , antiderivative size = 501, normalized size of antiderivative = 2.53

method	result
derivativedivides	$-\frac{1}{2(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{1}{2(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(a+7b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)^4}+\frac{1}{2(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{}{2(a+}$
default	$-\frac{1}{2(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{1}{2(a+b)^3\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{(a+7b)\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)^4}+\frac{1}{2(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{}{2(a+}$
risch	$\frac{xa}{2(a+b)(a^3+3a^2b+3ab^2+b^3)}+\frac{7xb}{2(a+b)(a^3+3a^2b+3ab^2+b^3)}+\frac{e^{2dx+2c}}{8(a^3+3a^2b+3ab^2+b^3)d}-\frac{e^{-2dx-2c}}{8(a^3+3a^2b+3ab^2+b^3)d}$

input
```
int(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/(a+b)^3/(1+tanh(1/2*d*x+1/2*c))^2+1/2/(a+b)^3/(1+tanh(1/2*d*x+1/
2*c))+1/2*(a+7*b)/(a+b)^4*ln(1+tanh(1/2*d*x+1/2*c))+1/2/(a+b)^3/(tanh(1/2*
d*x+1/2*c)-1)^2+1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)+1/2/(a+b)^4*(-a-7*b)*l
n(tanh(1/2*d*x+1/2*c)-1)-2*b^2/(a+b)^4*((-1/8*(13*a^2+18*a*b+5*b^2)/a*tanh
(1/2*d*x+1/2*c)^7-1/8*(39*a^3+98*a^2*b+71*a*b^2+12*b^3)/a^2*tanh(1/2*d*x+1
/2*c)^5-1/8*(39*a^3+98*a^2*b+71*a*b^2+12*b^3)/a^2*tanh(1/2*d*x+1/2*c)^3-1/
8*(13*a^2+18*a*b+5*b^2)/a*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*
tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2+1/8/a*(35*a^2+14*a*
b+3*b^2)*(1/2*(a+((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+
a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)
^(1/2))-1/2*(-a+((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a
-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)
^(1/2)))))
```

3.125.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6780 vs. $2(180) = 360$.

Time $= 0.46$ (sec) , antiderivative size $= 13887$, normalized size of antiderivative $= 70.14$

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.125.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.125.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1806 vs. 2(180) = 360.

Time = 0.57 (sec) , antiderivative size = 1806, normalized size of antiderivative = 9.12

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
3/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a
^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 3/4*b*log(2*(a - b)*e^(-2*d
*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2
 + 4*a*b^3 + b^4)*d) - 3/32*(5*a^3*b - 15*a^2*b^2 - 5*a*b^3 - b^4)*arctan(1
/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^6 + 4*a^5*b + 6*a^4*b^
2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) + 3/32*(5*a^3*b - 15*a^2*b^2 - 5*a*b
^3 - b^4)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^6 +
 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) - 1/16*(15*a^2*b
 + 10*a*b^2 + 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b
))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)*d) + 1/16*(9*a^4*b + 4
*a^3*b^2 - 22*a^2*b^3 - 20*a*b^4 - 3*b^5 + 3*(3*a^4*b - 22*a^3*b^2 - 20*a^
2*b^3 + 6*a*b^4 + b^5)*e^(6*d*x + 6*c) + (27*a^4*b - 156*a^3*b^2 + 110*a^2
*b^3 - 36*a*b^4 - 9*b^5)*e^(4*d*x + 4*c) + (27*a^4*b - 86*a^3*b^2 - 84*a^2
*b^3 + 38*a*b^4 + 9*b^5)*e^(2*d*x + 2*c))/((a^8 + 6*a^7*b + 15*a^6*b^2 + 2
0*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6 + (a^8 + 6*a^7*b + 15*a^6*b^2
 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*e^(8*d*x + 8*c) + 4*(a^8
 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*e^(6*d*x + 6*c)
+ 2*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5
+ 3*a^2*b^6)*e^(4*d*x + 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 -
4*a^3*b^5 - a^2*b^6)*e^(2*d*x + 2*c))*d) - 1/16*(9*a^4*b + 4*a^3*b^2 - ...
```

3.125.8 Giac [F]

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\cosh\left(dx+c\right)^2}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\cosh(c+d\,x)^2}{\left(b\tanh(c+d\,x)^2+a\right)^3}\,dx$$

input
```
int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3, x)
```

3.126 $\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

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3.126.1 Optimal result

Integrand size = 21, antiderivative size = 154

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{3b(8a^2+4ab+b^2)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{7/2}d} + \frac{\sinh(c+dx)}{(a+b)^3 d} + \frac{b^3\sinh(c+dx)}{4a(a+b)^3 d\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{3b^2(4a+b)\sinh(c+dx)}{8a^2(a+b)^3 d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

output
```
3/8*b*(8*a^2+4*a*b+b^2)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(5/2)/(a
+b)^(7/2)/d+sinh(d*x+c)/(a+b)^3/d+1/4*b^3*sinh(d*x+c)/a/(a+b)^3/d/(a+(a+b)
*sinh(d*x+c)^2)^2+3/8*b^2*(4*a+b)*sinh(d*x+c)/a^2/(a+b)^3/d/(a+(a+b)*sinh(
d*x+c)^2)
```

3.126.2 Mathematica [A] (verified)

Time = 2.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.88

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\frac{3b(8a^2+4ab+b^2)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a+b)^{7/2}} + \frac{\sinh(c+dx)\left(8+\frac{3b^3}{a^2\left(a+(a+b)\sinh^2(c+dx)\right)}+\frac{2b^2\left(6a+b+6(a+b)\sinh^2(c+dx)\right)}{a\left(a+(a+b)\sinh^2(c+dx)\right)^2}\right)}{(a+b)^3}}{8d}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((3*b*(8*a^2 + 4*a*b + b^2)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(
a^(5/2)*(a + b)^(7/2)) + (Sinh[c + d*x]*(8 + (3*b^3)/(a^2*(a + (a + b)*Sin
h[c + d*x]^2)) + (2*b^2*(6*a + b + 6*(a + b)*Sinh[c + d*x]^2))/(a*(a + (a
+ b)*Sinh[c + d*x]^2)^2)))/(a + b)^3)/(8*d)
```

3.126.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4159, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sec(ic+idx)\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\downarrow 4159$$

$$\frac{\int \frac{\left(\sinh^2(c+dx)+1\right)^3}{\left((a+b)\sinh^2(c+dx)+a\right)^3}d\sinh(c+dx)}{d}$$

$$\downarrow 300$$

$$\frac{\int \left(\frac{3b(a+b)^2\sinh^4(c+dx)+3b(a+b)(2a+b)\sinh^2(c+dx)+b\left(3a^2+3ba+b^2\right)}{(a+b)^3\left((a+b)\sinh^2(c+dx)+a\right)^3}+\frac{1}{(a+b)^3}\right)d\sinh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\frac{3b^2(4a+b)\sinh(c+dx)}{8a^2(a+b)^3\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{3b\left(8a^2+4ab+b^2\right)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{7/2}}+\frac{b^3\sinh(c+dx)}{4a(a+b)^3\left((a+b)\sinh^2(c+dx)+a\right)^2}+\frac{\sinh(c+dx)}{(a+b)^3}}{d}$$

input
```
Int[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((3*b*(8*a^2 + 4*a*b + b^2)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(
8*a^(5/2)*(a + b)^(7/2)) + Sinh[c + d*x]/(a + b)^3 + (b^3*Sinh[c + d*x])/(
4*a*(a + b)^3*(a + (a + b)*Sinh[c + d*x]^2)^2) + (3*b^2*(4*a + b)*Sinh[c +
 d*x])/(8*a^2*(a + b)^3*(a + (a + b)*Sinh[c + d*x]^2)))/d
```

3.126.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.126.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(140) = 280$.

Time $= 7.20$ (sec) , antiderivative size $= 375$, normalized size of antiderivative $= 2.44$

method	result
derivativedivides	$2b\left(\frac{-\frac{b(12a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8a}-\frac{3\left(4a^2+15ab+4b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^2}+\frac{3\left(4a^2+15ab+4b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8a^2}+\frac{b(12a+5b)\tanh\left(\frac{dx}{2}+\right.}{8a}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}\right)}{(a+b$
default	$2b\left(\frac{-\frac{b(12a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8a}-\frac{3\left(4a^2+15ab+4b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^2}+\frac{3\left(4a^2+15ab+4b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8a^2}+\frac{b(12a+5b)\tanh\left(\frac{dx}{2}+\right.}{8a}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}\right)}{(a+b$
risch	$\frac{e^{dx+c}}{2(a^3+3a^2b+3ab^2+b^3)d}-\frac{e^{-dx-c}}{2(a^3+3a^2b+3ab^2+b^3)d}+\frac{b^2e^{dx+c}(12a^2e^{6dx+6c}+15abe^{6dx+6c}+3b^2e^{6dx+6c}+12a^2e^{4dx+}}{4(a^3+3a^2b+3ab^2+b^3)(a$

input
```
int(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*b/(a+b)^3*((-1/8*b*(12*a+5*b)/a*tanh(1/2*d*x+1/2*c)^7-3/8*(4*a^2+15
*a*b+4*b^2)/a^2*b*tanh(1/2*d*x+1/2*c)^5+3/8*(4*a^2+15*a*b+4*b^2)/a^2*b*tan
h(1/2*d*x+1/2*c)^3+1/8*b*(12*a+5*b)/a*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1
/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2+3/8/a*(
8*a^2+4*a*b+b^2)*(1/2*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^
(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2
*b)*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2
)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)
*a)^(1/2))))-1/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)-1/(a+b)^3/(1+tanh(1/2*d*x+1
/2*c)))
```

3.126. $\int \frac{\cosh(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

3.126.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6260 vs. 2(140) = 280.

Time = 0.43 (sec) , antiderivative size = 11392, normalized size of antiderivative = 73.97

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.126.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.126.7 Maxima [F]

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\cosh\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*a^4 + 4*a^3*b + 2*a^2*b^2 - 2*(a^4*e^(10*c) + 2*a^3*b*e^(10*c) + a
^2*b^2*e^(10*c))*e^(10*d*x) - (6*a^4*e^(8*c) - 4*a^3*b*e^(8*c) + 2*a^2*b^2
*e^(8*c) + 15*a*b^3*e^(8*c) + 3*b^4*e^(8*c))*e^(8*d*x) - (4*a^4*e^(6*c) -
8*a^3*b*e^(6*c) + 32*a^2*b^2*e^(6*c) - 25*a*b^3*e^(6*c) - 9*b^4*e^(6*c))*e
^(6*d*x) + (4*a^4*e^(4*c) - 8*a^3*b*e^(4*c) + 32*a^2*b^2*e^(4*c) - 25*a*b^
3*e^(4*c) - 9*b^4*e^(4*c))*e^(4*d*x) + (6*a^4*e^(2*c) - 4*a^3*b*e^(2*c) +
2*a^2*b^2*e^(2*c) + 15*a*b^3*e^(2*c) + 3*b^4*e^(2*c))*e^(2*d*x))/((a^7*d*e
^(9*c) + 5*a^6*b*d*e^(9*c) + 10*a^5*b^2*d*e^(9*c) + 10*a^4*b^3*d*e^(9*c) +
 5*a^3*b^4*d*e^(9*c) + a^2*b^5*d*e^(9*c))*e^(9*d*x) + 4*(a^7*d*e^(7*c) + 3
*a^6*b*d*e^(7*c) + 2*a^5*b^2*d*e^(7*c) - 2*a^4*b^3*d*e^(7*c) - 3*a^3*b^4*d
*e^(7*c) - a^2*b^5*d*e^(7*c))*e^(7*d*x) + 2*(3*a^7*d*e^(5*c) + 7*a^6*b*d*e
^(5*c) + 6*a^5*b^2*d*e^(5*c) + 6*a^4*b^3*d*e^(5*c) + 7*a^3*b^4*d*e^(5*c) +
 3*a^2*b^5*d*e^(5*c))*e^(5*d*x) + 4*(a^7*d*e^(3*c) + 3*a^6*b*d*e^(3*c) + 2
*a^5*b^2*d*e^(3*c) - 2*a^4*b^3*d*e^(3*c) - 3*a^3*b^4*d*e^(3*c) - a^2*b^5*d
*e^(3*c))*e^(3*d*x) + (a^7*d*e^c + 5*a^6*b*d*e^c + 10*a^5*b^2*d*e^c + 10*a
^4*b^3*d*e^c + 5*a^3*b^4*d*e^c + a^2*b^5*d*e^c)*e^(d*x)) + 1/2*integrate(3
/2*((8*a^2*b*e^(3*c) + 4*a*b^2*e^(3*c) + b^3*e^(3*c))*e^(3*d*x) + (8*a^2*b
*e^c + 4*a*b^2*e^c + b^3*e^c)*e^(d*x))/(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*
b^3 + a^2*b^4 + (a^6*e^(4*c) + 4*a^5*b*e^(4*c) + 6*a^4*b^2*e^(4*c) + 4*a^3
*b^3*e^(4*c) + a^2*b^4*e^(4*c))*e^(4*d*x) + 2*(a^6*e^(2*c) + 2*a^5*b*e^...
```

3.126.8 Giac [F]

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\cosh\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\cosh(c+d\,x)}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2)^3, x)
```

3.127 $\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

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3.127.1 Optimal result

Integrand size = 21, antiderivative size = 144

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{(8a^2+8ab+3b^2)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{5/2}d} + \frac{b\cosh^2(c+dx)\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{3b(2a+b)\sinh(c+dx)}{8a^2(a+b)^2d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

output
```
1/8*(8*a^2+8*a*b+3*b^2)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(5/2)/(a
+b)^(5/2)/d+1/4*b*cosh(d*x+c)^2*sinh(d*x+c)/a/(a+b)/d/(a+(a+b)*sinh(d*x+c)
^2)^2+3/8*b*(2*a+b)*sinh(d*x+c)/a^2/(a+b)^2/d/(a+(a+b)*sinh(d*x+c)^2)
```

3.127.2 Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.93

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{-\frac{(8a^2+8ab+3b^2)\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + \frac{2\sqrt{ab}(8a^2-ab-3b^2+(8a^2+11ab+3b^2)\cosh(2(c+dx)))\sinh(c+dx)}{(a+b)^2(a-b+(a+b)\cosh(2(c+dx)))^2}}{8a^{5/2}d}$$

input
```
Integrate[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-(((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/(
a + b)^(5/2)) + (2*Sqrt[a]*b*(8*a^2 - a*b - 3*b^2 + (8*a^2 + 11*a*b + 3*b^
2)*Cosh[2*(c + d*x)])*Sinh[c + d*x])/((a + b)^2*(a - b + (a + b)*Cosh[2*(c
 + d*x)])^2))/(8*a^(5/2)*d)
```

3.127.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4159, 315, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(ic+idx)}{\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$

$$\downarrow 4159$$

$$\frac{\int \frac{\left(\sinh^2(c+dx)+1\right)^2}{\left((a+b)\sinh^2(c+dx)+a\right)^3}\,d\sinh(c+dx)}{d}$$

$$\downarrow 315$$

$$\frac{\frac{\int \frac{(4a+b)\sinh^2(c+dx)+4a+3b}{\left((a+b)\sinh^2(c+dx)+a\right)^2}\,d\sinh(c+dx)}{4a(a+b)} + \frac{b\sinh(c+dx)\left(\sinh^2(c+dx)+1\right)}{4a(a+b)\left((a+b)\sinh^2(c+dx)+a\right)^2}}{d}$$

$$\downarrow 298$$

$$\frac{\frac{\left(8a^2+8ab+3b^2\right)\int \frac{1}{(a+b)\sinh^2(c+dx)+a}\,d\sinh(c+dx)}{2a(a+b)} + \frac{3b(2a+b)\sinh(c+dx)}{2a(a+b)\left((a+b)\sinh^2(c+dx)+a\right)}}{4a(a+b)} + \frac{b\sinh(c+dx)\left(\sinh^2(c+dx)+1\right)}{4a(a+b)\left((a+b)\sinh^2(c+dx)+a\right)^2}}{d}$$

$$\downarrow 218$$

$$\frac{\frac{\left(8a^2+8ab+3b^2\right)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}}+\frac{3b(2a+b)\sinh(c+dx)}{2a(a+b)\left((a+b)\sinh^2(c+dx)+a\right)}}{4a(a+b)}+\frac{b\sinh(c+dx)\left(\sinh^2(c+dx)+1\right)}{4a(a+b)\left((a+b)\sinh^2(c+dx)+a\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((b*Sinh[c + d*x]*(1 + Sinh[c + d*x]^2))/(4*a*(a + b)*(a + (a + b)*Sinh[c
+ d*x]^2)^2) + (((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x]
)/Sqrt[a]])/(2*a^(3/2)*(a + b)^(3/2)) + (3*b*(2*a + b)*Sinh[c + d*x])/(2*a
*(a + b)*(a + (a + b)*Sinh[c + d*x]^2)))/(4*a*(a + b)))/d
```

3.127.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.127.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(130) = 260$.

Time = 19.44 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.74

method	result
derivativedivides	$\frac{-\frac{b(8a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4a\left(a^2+2ab+b^2\right)}-\frac{\left(8a^2+29ab+12b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4a^2\left(a^2+2ab+b^2\right)}+\frac{\left(8a^2+29ab+12b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4a^2\left(a^2+2ab+b^2\right)}+\frac{b(8a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a\left(a^2+2ab+b^2\right)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}+\cdots}{d}$
default	$\frac{-\frac{b(8a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4a\left(a^2+2ab+b^2\right)}-\frac{\left(8a^2+29ab+12b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4a^2\left(a^2+2ab+b^2\right)}+\frac{\left(8a^2+29ab+12b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4a^2\left(a^2+2ab+b^2\right)}+\frac{b(8a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a\left(a^2+2ab+b^2\right)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}+\cdots}{d}$
risch	$\frac{\left(8a^2e^{6dx+6c}+11abe^{6dx+6c}+3b^2e^{6dx+6c}+8a^2e^{4dx+4c}-13abe^{4dx+4c}-9e^{4dx+4c}b^2-8a^2e^{2dx+2c}+13abe^{2dx+2c}+9e^{2dx+\cdots}\right.}{4(a^2+2ab+b^2)\left(ae^{4dx+4c}+be^{4dx+4c}+2e^{2dx+2c}a-2be^{2dx+2c}+a+b\right)^2da^2}$

input

```
int(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output

```
1/d*(2*(-1/8*b*(8*a+5*b)/a/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-1/8*(8*a^
2+29*a*b+12*b^2)/a^2*b/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5+1/8*(8*a^2+29
*a*b+12*b^2)/a^2*b/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+1/8*b*(8*a+5*b)/a
/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*
d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2+1/4*(8*a^2+8*a*b+3*b^2)/a/(a
^2+2*a*b+b^2)*(1/2*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/
2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)
*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a
-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)
^(1/2))))
```

3.127.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4377 vs. 2(130) = 260.

Time = 0.37 (sec) , antiderivative size = 7909, normalized size of antiderivative = 54.92

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.127.6 Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(sech(c + d*x)/(a + b*tanh(c + d*x)**2)**3, x)
```

3.127.7 Maxima [F]

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*((8*a^2*b*e^(7*c) + 11*a*b^2*e^(7*c) + 3*b^3*e^(7*c))*e^(7*d*x) + (8*a
^2*b*e^(5*c) - 13*a*b^2*e^(5*c) - 9*b^3*e^(5*c))*e^(5*d*x) - (8*a^2*b*e^(3
*c) - 13*a*b^2*e^(3*c) - 9*b^3*e^(3*c))*e^(3*d*x) - (8*a^2*b*e^c + 11*a*b^
2*e^c + 3*b^3*e^c)*e^(d*x))/(a^6*d + 4*a^5*b*d + 6*a^4*b^2*d + 4*a^3*b^3*d
 + a^2*b^4*d + (a^6*d*e^(8*c) + 4*a^5*b*d*e^(8*c) + 6*a^4*b^2*d*e^(8*c) +
4*a^3*b^3*d*e^(8*c) + a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(a^6*d*e^(6*c) + 2*
a^5*b*d*e^(6*c) - 2*a^3*b^3*d*e^(6*c) - a^2*b^4*d*e^(6*c))*e^(6*d*x) + 2*(
3*a^6*d*e^(4*c) + 4*a^5*b*d*e^(4*c) + 2*a^4*b^2*d*e^(4*c) + 4*a^3*b^3*d*e^
(4*c) + 3*a^2*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^6*d*e^(2*c) + 2*a^5*b*d*e^(2
*c) - 2*a^3*b^3*d*e^(2*c) - a^2*b^4*d*e^(2*c))*e^(2*d*x)) + 2*integrate(1/
8*((8*a^2*e^(3*c) + 8*a*b*e^(3*c) + 3*b^2*e^(3*c))*e^(3*d*x) + (8*a^2*e^c
+ 8*a*b*e^c + 3*b^2*e^c)*e^(d*x))/(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + (
a^5*e^(4*c) + 3*a^4*b*e^(4*c) + 3*a^3*b^2*e^(4*c) + a^2*b^3*e^(4*c))*e^(4*
d*x) + 2*(a^5*e^(2*c) + a^4*b*e^(2*c) - a^3*b^2*e^(2*c) - a^2*b^3*e^(2*c))
*e^(2*d*x)), x)
```

3.127.8 Giac [F]

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^3), x)
```

3.128 $\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

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3.128.1 Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{3\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d} + \frac{\tanh(c+dx)}{4ad\left(a+b\tanh^2(c+dx)\right)^2} + \frac{3\tanh(c+dx)}{8a^2d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
3/8*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/d/b^(1/2)+1/4*tanh(d*x+c)/
a/d/(a+b*tanh(d*x+c)^2)^2+3/8*tanh(d*x+c)/a^2/d/(a+b*tanh(d*x+c)^2)
```

3.128.2 Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\frac{3\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}} + \frac{\tanh(c+dx)\left(5a+3b\tanh^2(c+dx)\right)}{a^2\left(a+b\tanh^2(c+dx)\right)^2}}{8d}$$

input
```
Integrate[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((3*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[b]) + (Tanh[c +
 d*x]*(5*a + 3*b*Tanh[c + d*x]^2))/(a^2*(a + b*Tanh[c + d*x]^2)^2))/(8*d)
```

3.128.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4158, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(ic+idx)^2}{\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$

$$\downarrow 4158$$

$$\frac{\int \frac{1}{\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh(c+dx)}{d}$$

$$\downarrow 215$$

$$\frac{\frac{3\int \frac{1}{\left(b\tanh^2(c+dx)+a\right)^2}\,d\tanh(c+dx)}{4a}+\frac{\tanh(c+dx)}{4a\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 215$$

$$\frac{3\left(\frac{\int \frac{1}{b\tanh^2(c+dx)+a}\,d\tanh(c+dx)}{2a}+\frac{\tanh(c+dx)}{2a\left(a+b\tanh^2(c+dx)\right)}\right)}{4a}+\frac{\tanh(c+dx)}{4a\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 218$$

$$\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{\tanh(c+dx)}{2a\left(a+b\tanh^2(c+dx)\right)}\right)}{4a}+\frac{\tanh(c+dx)}{4a\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(Tanh[c + d*x]/(4*a*(a + b*Tanh[c + d*x]^2)^2) + (3*(ArcTan[(Sqrt[b]*Tanh[
c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Tanh[c + d*x]/(2*a*(a + b*Tanh[c
+ d*x]^2))))/(4*a))/d
```

3.128.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.128.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(82) = 164$.

Time = 63.99 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.96

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8a}-\frac{3(5a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^2}-\frac{3(5a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8a^2}-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}-3\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(-$ $\frac{}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(}\right.}}$ $\Big)$ $\Big)$ $\Big/$ d
default	$\frac{-\frac{2\left(-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8a}-\frac{3(5a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^2}-\frac{3(5a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8a^2}-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}-3\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(-$ $\frac{}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(}\right.}}$ $\Big)$ $\Big)$ $\Big/$ d
risch	$-\frac{5a^3e^{6dx+6c}-a^2be^{6dx+6c}-9ab^2e^{6dx+6c}-3e^{6dx+6c}b^3+15a^3e^{4dx+4c}-a^2be^{4dx+4c}+9ab^2e^{4dx+4c}+9e^{4dx+4c}b^3+15a^3}{4(a^2+2ab+b^2)(ae^{4dx+4c}+be^{4dx+4c}+2e^{2dx+2c}a-2be^{2}}$

input
```
int(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-5/8/a*tanh(1/2*d*x+1/2*c)^7-3/8*(5*a+4*b)/a^2*tanh(1/2*d*x+1/2*c
)^5-3/8*(5*a+4*b)/a^2*tanh(1/2*d*x+1/2*c)^3-5/8/a*tanh(1/2*d*x+1/2*c))/(ta
nh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+
a)^2-3/4/a*(1/2*(a+((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2
)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*
a)^(1/2))-1/2*(-a+((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)
-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*
a)^(1/2))))
```

3.128.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2768 vs. $2(82) = 164$.

Time $= 0.34$ (sec) , antiderivative size $= 5840$, normalized size of antiderivative $= 60.83$

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.128.6 Sympy [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(sech(c + d*x)**2/(a + b*tanh(c + d*x)**2)**3, x)
```

3.128.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(82) = 164$.

Time = 0.40 (sec) , antiderivative size = 366, normalized size of antiderivative = 3.81

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{5\,a^3+13\,a^2b+11\,ab^2+3\,b^3+(15\,a^3+13\,a^2b-11\,ab^2-9\,b^3)e^{(-2\,dx}}{4\,(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4+4\,(a^6+2\,a^5b-2\,a^3b^3-a^2b^4)e^{(-2\,dx-2\,c)}+2\,(3\,a^6+4\,a^5b+2\,a^4b^2}$$
$$-\frac{3\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{8\sqrt{ab}a^2d}$$

input
```
integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*(5*a^3 + 13*a^2*b + 11*a*b^2 + 3*b^3 + (15*a^3 + 13*a^2*b - 11*a*b^2 -
 9*b^3)*e^(-2*d*x - 2*c) + (15*a^3 - a^2*b + 9*a*b^2 + 9*b^3)*e^(-4*d*x -
4*c) + (5*a^3 - a^2*b - 9*a*b^2 - 3*b^3)*e^(-6*d*x - 6*c))/((a^6 + 4*a^5*b
 + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4 + 4*(a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^
4)*e^(-2*d*x - 2*c) + 2*(3*a^6 + 4*a^5*b + 2*a^4*b^2 + 4*a^3*b^3 + 3*a^2*b
^4)*e^(-4*d*x - 4*c) + 4*(a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*e^(-6*d*x -
 6*c) + (a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*e^(-8*d*x - 8*c)
)*d) - 3/8*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(
a*b)*a^2*d)
```

3.128.8 Giac [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^2}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^2\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3), x)
```

3.129 $\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

3.129.1 Optimal result . . . 996
3.129.2 Mathematica [A] (verified) . . . 996
3.129.3 Rubi [A] (verified) . . . 997
3.129.4 Maple [B] (verified) . . . 999
3.129.5 Fricas [B] (verification not implemented) . . . 1000
3.129.6 Sympy [F] . . . 1000
3.129.7 Maxima [F] . . . 1000
3.129.8 Giac [F] . . . 1001
3.129.9 Mupad [F(-1)] . . . 1001

3.129.1 Optimal result

Integrand size = 23, antiderivative size = 129

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{(4a+3b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{3/2}d} + \frac{b\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{(4a+3b)\sinh(c+dx)}{8a^2(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

output
```
1/8*(4*a+3*b)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(5/2)/(a+b)^(3/2)/
d+1/4*b*sinh(d*x+c)/a/(a+b)/d/(a+(a+b)*sinh(d*x+c)^2)^2+1/8*(4*a+3*b)*sinh
(d*x+c)/a^2/(a+b)/d/(a+(a+b)*sinh(d*x+c)^2)
```

3.129.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{-\frac{8\sinh(c+dx)}{\left(a+(a+b)\sinh^2(c+dx)\right)^2} + (4a+3b)\left(\frac{3\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a+b}} + \frac{5a\sinh(c+dx)+3(a+b)\sinh^3(c+dx)}{a^2\left(a+(a+b)\sinh^2(c+dx)\right)^2}\right)}{24(a+b)d}$$

input
```
Integrate[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((-8*Sinh[c + d*x])/(a + (a + b)*Sinh[c + d*x]^2)^2 + (4*a + 3*b)*((3*ArcT
an[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[a + b]) + (5*a*Sinh
[c + d*x] + 3*(a + b)*Sinh[c + d*x]^3)/(a^2*(a + (a + b)*Sinh[c + d*x]^2)^
2)))/(24*(a + b)*d)
```

3.129.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4159, 298, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(ic+idx)^3}{\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$

$$\downarrow 4159$$

$$\frac{\int \frac{\sinh^2(c+dx)+1}{\left((a+b)\sinh^2(c+dx)+a\right)^3}\,d\sinh(c+dx)}{d}$$

$$\downarrow 298$$

$$\frac{\frac{1}{4}\left(\frac{1}{a+b}+\frac{3}{a}\right)\int \frac{1}{\left((a+b)\sinh^2(c+dx)+a\right)^2}\,d\sinh(c+dx)+\frac{b\sinh(c+dx)}{4a(a+b)\left((a+b)\sinh^2(c+dx)+a\right)^2}}{d}$$

$$\downarrow 215$$

$$\frac{\frac{1}{4}\left(\frac{1}{a+b}+\frac{3}{a}\right)\left(\frac{\int \frac{1}{(a+b)\sinh^2(c+dx)+a}\,d\sinh(c+dx)}{2a}+\frac{\sinh(c+dx)}{2a\left((a+b)\sinh^2(c+dx)+a\right)}\right)+\frac{b\sinh(c+dx)}{4a(a+b)\left((a+b)\sinh^2(c+dx)+a\right)^2}}{d}$$

$$\downarrow 218$$

$$\frac{\frac{1}{4}\left(\frac{1}{a+b}+\frac{3}{a}\right)\left(\frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}}+\frac{\sinh(c+dx)}{2a\left((a+b)\sinh^2(c+dx)+a\right)}\right)+\frac{b\sinh(c+dx)}{4a(a+b)\left((a+b)\sinh^2(c+dx)+a\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((b*Sinh[c + d*x])/(4*a*(a + b)*(a + (a + b)*Sinh[c + d*x]^2)^2) + ((3/a +
 (a + b)^(-1))*(ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqr
t[a + b]) + Sinh[c + d*x]/(2*a*(a + (a + b)*Sinh[c + d*x]^2))))/4)/d
```

3.129.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.129.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(115) = 230$.

Time = 103.41 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.65

method	result
derivativedivides	$\dfrac{-\frac{(5b+4a)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4a(a+b)}-\frac{\left(4a^2+13ab+12b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4a^2(a+b)}+\frac{\left(4a^2+13ab+12b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4a^2(a+b)}+\frac{(5b+4a)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a(a+b)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}+\frac{(4a+}{d}$
default	$\dfrac{-\frac{(5b+4a)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4a(a+b)}-\frac{\left(4a^2+13ab+12b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4a^2(a+b)}+\frac{\left(4a^2+13ab+12b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4a^2(a+b)}+\frac{(5b+4a)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a(a+b)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}+\frac{(4a+}{d}$
risch	$\dfrac{\left(4a^2e^{6dx+6c}+7abe^{6dx+6c}+3b^2e^{6dx+6c}+4a^2e^{4dx+4c}-abe^{4dx+4c}-9e^{4dx+4c}b^2-4a^2e^{2dx+2c}+abe^{2dx+2c}+9e^{2dx+2c}b^2-\right.}{4(a+b)\left(ae^{4dx+4c}+be^{4dx+4c}+2e^{2dx+2c}a-2be^{2dx+2c}+a+b\right)^2da^2}$

input
```
int(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-1/8*(5*b+4*a)/a/(a+b)*tanh(1/2*d*x+1/2*c)^7-1/8*(4*a^2+13*a*b+12*
b^2)/a^2/(a+b)*tanh(1/2*d*x+1/2*c)^5+1/8*(4*a^2+13*a*b+12*b^2)/a^2/(a+b)*t
anh(1/2*d*x+1/2*c)^3+1/8*(5*b+4*a)/a/(a+b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*
d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2+1/
4/a*(4*a+3*b)/(a+b)*(1/2*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*
b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+
a+2*b)*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(
1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2
*b)*a)^(1/2))))
```

3.129.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3627 vs. 2(115) = 230.

Time = 0.36 (sec) , antiderivative size = 6614, normalized size of antiderivative = 51.27

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.129.6 Sympy [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}^3\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(sech(c + d*x)**3/(a + b*tanh(c + d*x)**2)**3, x)
```

3.129.7 Maxima [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^3}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*((4*a^2*e^(7*c) + 7*a*b*e^(7*c) + 3*b^2*e^(7*c))*e^(7*d*x) + (4*a^2*e^
(5*c) - a*b*e^(5*c) - 9*b^2*e^(5*c))*e^(5*d*x) - (4*a^2*e^(3*c) - a*b*e^(3
*c) - 9*b^2*e^(3*c))*e^(3*d*x) - (4*a^2*e^c + 7*a*b*e^c + 3*b^2*e^c)*e^(d*
x))/(a^5*d + 3*a^4*b*d + 3*a^3*b^2*d + a^2*b^3*d + (a^5*d*e^(8*c) + 3*a^4*
b*d*e^(8*c) + 3*a^3*b^2*d*e^(8*c) + a^2*b^3*d*e^(8*c))*e^(8*d*x) + 4*(a^5*
d*e^(6*c) + a^4*b*d*e^(6*c) - a^3*b^2*d*e^(6*c) - a^2*b^3*d*e^(6*c))*e^(6*
d*x) + 2*(3*a^5*d*e^(4*c) + a^4*b*d*e^(4*c) + a^3*b^2*d*e^(4*c) + 3*a^2*b^
3*d*e^(4*c))*e^(4*d*x) + 4*(a^5*d*e^(2*c) + a^4*b*d*e^(2*c) - a^3*b^2*d*e^
(2*c) - a^2*b^3*d*e^(2*c))*e^(2*d*x)) + 8*integrate(1/32*((4*a*e^(3*c) + 3
*b*e^(3*c))*e^(3*d*x) + (4*a*e^c + 3*b*e^c)*e^(d*x))/(a^4 + 2*a^3*b + a^2*
b^2 + (a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c))*e^(4*d*x) + 2*(a^4
*e^(2*c) - a^2*b^2*e^(2*c))*e^(2*d*x)), x)
```

3.129.8 Giac [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^3}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^3\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3), x)
```

3.130 $\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

3.130.1 Optimal result . . . 1002
3.130.2 Mathematica [A] (verified) . . . 1002
3.130.3 Rubi [A] (verified) . . . 1003
3.130.4 Maple [B] (verified) . . . 1005
3.130.5 Fricas [B] (verification not implemented) . . . 1006
3.130.6 Sympy [F] . . . 1006
3.130.7 Maxima [B] (verification not implemented) . . . 1006
3.130.8 Giac [F] . . . 1007
3.130.9 Mupad [F(-1)] . . . 1007

3.130.1 Optimal result

Integrand size = 23, antiderivative size = 115

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{(a-3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d} + \frac{(a+b)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a-3b)\tanh(c+dx)}{8a^2bd\left(a+b\tanh^2(c+dx)\right)}$$

output
```
-1/8*(a-3*b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/b^(3/2)/d+1/4*(a+
b)*tanh(d*x+c)/a/b/d/(a+b*tanh(d*x+c)^2)^2-1/8*(a-3*b)*tanh(d*x+c)/a^2/b/d
/(a+b*tanh(d*x+c)^2)
```

3.130.2 Mathematica [A] (verified)

Time = 1.96 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\frac{(-a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}} + \frac{\sqrt{a}\left(a^2+6ab-3b^2+\left(a^2+4ab+3b^2\right)\cosh(2(c+dx))\right)\sinh(2(c+dx))}{b\left(a-b+(a+b)\cosh(2(c+dx))\right)^2}}{8a^{5/2}d}$$

input
```
Integrate[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(((-a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/b^(3/2) + (Sqrt[a]*(
a^2 + 6*a*b - 3*b^2 + (a^2 + 4*a*b + 3*b^2)*Cosh[2*(c + d*x)])*Sinh[2*(c +
 d*x)])/(b*(a - b + (a + b)*Cosh[2*(c + d*x)])^2))/(8*a^(5/2)*d)
```

3.130.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4158, 298, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(ic+idx)^4}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\downarrow 4158$$

$$\frac{\int \frac{1-\tanh^2(c+dx)}{\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\downarrow 298$$

$$\frac{\frac{(a+b)\tanh(c+dx)}{4ab\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a-3b)\int \frac{1}{\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4ab}}{d}$$

$$\downarrow 215$$

$$\frac{\frac{(a+b)\tanh(c+dx)}{4ab\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a-3b)\left(\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{2a} + \frac{\tanh(c+dx)}{2a\left(a+b\tanh^2(c+dx)\right)}\right)}{4ab}}{d}$$

$$\downarrow 218$$

$$\frac{\frac{(a+b)\tanh(c+dx)}{4ab\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a-3b)\left(\frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{\tanh(c+dx)}{2a\left(a+b\tanh^2(c+dx)\right)}\right)}{4ab}}{d}$$

input
```
Int[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(((a + b)*Tanh[c + d*x])/(4*a*b*(a + b*Tanh[c + d*x]^2)^2) - ((a - 3*b)*(A
rcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Tanh[c + d*x]
/(2*a*(a + b*Tanh[c + d*x]^2))))/(4*a*b))/d
```

3.130.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.130.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(101) = 202$.

Time = 185.48 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.90

method	result
derivativedivides	$\frac{2\left(-\frac{(a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8ab}-\frac{\left(3a^2+11ab+12b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^2b}-\frac{\left(3a^2+11ab+12b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8a^2b}-\frac{(a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ab}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}-\cdots}{d}$
default	$\frac{2\left(-\frac{(a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8ab}-\frac{\left(3a^2+11ab+12b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^2b}-\frac{\left(3a^2+11ab+12b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8a^2b}-\frac{(a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ab}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}-\cdots}{d}$
risch	$-\frac{a^3e^{6dx+6c}-a^2be^{6dx+6c}-5ab^2e^{6dx+6c}-3e^{6dx+6c}b^3+3a^3e^{4dx+4c}+7a^2be^{4dx+4c}-3ab^2e^{4dx+4c}+9e^{4dx+4c}b^3+3a^3e^{2}}{4(a+b)\left(ae^{4dx+4c}+be^{4dx+4c}+2e^{2dx+2c}a-2be^{2dx+2c}+\right.}$

input
```
int(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/8*(a+5*b)/a/b*tanh(1/2*d*x+1/2*c)^7-1/8*(3*a^2+11*a*b+12*b^2)/
a^2/b*tanh(1/2*d*x+1/2*c)^5-1/8*(3*a^2+11*a*b+12*b^2)/a^2/b*tanh(1/2*d*x+1
/2*c)^3-1/8*(a+5*b)/a/b*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*ta
nh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2-1/4/a*(a-3*b)/b*(-1/2
*(a-((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/
2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))+1/2*
(-a-((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/
2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))))
```

3.130.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2677 vs. 2(101) = 202.

Time = 0.34 (sec) , antiderivative size = 5659, normalized size of antiderivative = 49.21

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.130.6 Sympy [F]

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(sech(c + d*x)**4/(a + b*tanh(c + d*x)**2)**3, x)
```

3.130.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(101) = 202.

Time = 0.46 (sec) , antiderivative size = 360, normalized size of antiderivative = 3.13

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{a^3+5\,a^2b+7\,ab^2+3\,b^3+(3\,a^3+13\,a^2b+ab^2-9\,b^3)e^{(-2\,dx-2\,c)}+(3\,a^3+7}{4\,(a^5b+3\,a^4b^2+3\,a^3b^3+a^2b^4+4\,(a^5b+a^4b^2-a^3b^3-a^2b^4)e^{(-2\,dx-2\,c)}+2\,(3\,a^5b+a^4b^2+a^3b^3+3\,a^2b^4)}$$
$$+\frac{(a-3\,b)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{8\sqrt{ab}a^2bd}$$

input
```
integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*(a^3 + 5*a^2*b + 7*a*b^2 + 3*b^3 + (3*a^3 + 13*a^2*b + a*b^2 - 9*b^3)*
e^(-2*d*x - 2*c) + (3*a^3 + 7*a^2*b - 3*a*b^2 + 9*b^3)*e^(-4*d*x - 4*c) +
(a^3 - a^2*b - 5*a*b^2 - 3*b^3)*e^(-6*d*x - 6*c))/((a^5*b + 3*a^4*b^2 + 3*
a^3*b^3 + a^2*b^4 + 4*(a^5*b + a^4*b^2 - a^3*b^3 - a^2*b^4)*e^(-2*d*x - 2*
c) + 2*(3*a^5*b + a^4*b^2 + a^3*b^3 + 3*a^2*b^4)*e^(-4*d*x - 4*c) + 4*(a^5
*b + a^4*b^2 - a^3*b^3 - a^2*b^4)*e^(-6*d*x - 6*c) + (a^5*b + 3*a^4*b^2 +
3*a^3*b^3 + a^2*b^4)*e^(-8*d*x - 8*c))*d) + 1/8*(a - 3*b)*arctan(1/2*((a +
 b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*b*d)
```

3.130.8 Giac [F]

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^4}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^4\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3), x)
```

3.131 $\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

3.131.1 Optimal result 1008
3.131.2 Mathematica [A] (verified) 1008
3.131.3 Rubi [A] (verified) 1009
3.131.4 Maple [B] (verified) 1010
3.131.5 Fricas [B] (verification not implemented) 1011
3.131.6 Sympy [F] 1011
3.131.7 Maxima [F] 1012
3.131.8 Giac [F] 1012
3.131.9 Mupad [F(-1)] 1012

3.131.1 Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{3\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{a+b}d} + \frac{\sinh(c+dx)}{4ad\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{3\sinh(c+dx)}{8a^2d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

output
```
1/4*sinh(d*x+c)/a/d/(a+(a+b)*sinh(d*x+c)^2)^2+3/8*sinh(d*x+c)/a^2/d/(a+(a+
b)*sinh(d*x+c)^2)+3/8*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(5/2)/d/(a
+b)^(1/2)
```

3.131.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\frac{3\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}} + \frac{\sqrt{a}\sinh(c+dx)\left(5a+3(a+b)\sinh^2(c+dx)\right)}{\left(a+(a+b)\sinh^2(c+dx)\right)^2}}{8a^{5/2}d}$$

input
```
Integrate[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((3*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/Sqrt[a + b] + (Sqrt[a]*Si
nh[c + d*x]*(5*a + 3*(a + b)*Sinh[c + d*x]^2))/(a + (a + b)*Sinh[c + d*x]^
2)^2)/(8*a^(5/2)*d)
```

3.131.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4159, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(ic+idx)^5}{\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$

$$\downarrow 4159$$

$$\frac{\int \frac{1}{\left((a+b)\sinh^2(c+dx)+a\right)^3}\,d\sinh(c+dx)}{d}$$

$$\downarrow 215$$

$$\frac{3\int \frac{1}{\left((a+b)\sinh^2(c+dx)+a\right)^2}\,d\sinh(c+dx)}{4a} + \frac{\sinh(c+dx)}{4a\left((a+b)\sinh^2(c+dx)+a\right)^2}}{d}$$

$$\downarrow 215$$

$$\frac{3\left(\frac{\int \frac{1}{(a+b)\sinh^2(c+dx)+a}\,d\sinh(c+dx)}{2a} + \frac{\sinh(c+dx)}{2a\left((a+b)\sinh^2(c+dx)+a\right)}\right)}{4a} + \frac{\sinh(c+dx)}{4a\left((a+b)\sinh^2(c+dx)+a\right)^2}}{d}$$

$$\downarrow 218$$

$$\frac{3\left(\frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}} + \frac{\sinh(c+dx)}{2a\left((a+b)\sinh^2(c+dx)+a\right)}\right)}{4a} + \frac{\sinh(c+dx)}{4a\left((a+b)\sinh^2(c+dx)+a\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(Sinh[c + d*x]/(4*a*(a + (a + b)*Sinh[c + d*x]^2)^2) + (3*(ArcTan[(Sqrt[a
+ b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a + b]) + Sinh[c + d*x]/(2*a*
(a + (a + b)*Sinh[c + d*x]^2))))/(4*a))/d
```

3.131.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.131.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(90) = 180$.

Time $= 0.22$ (sec) , antiderivative size $= 276$, normalized size of antiderivative $= 2.65$

$$\frac{-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4a}+\frac{3(a-4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4a^2}-\frac{3(a-4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4a^2}+\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 b+a\right)^2}+\frac{\frac{3\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{8a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{3\left(\sqrt{(a+b)b}-b\right)}{8a\sqrt{(a+}}}{a}}{d}$$

input
```
int(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
1/d*(2*(-5/8/a*tanh(1/2*d*x+1/2*c)^7+3/8*(a-4*b)/a^2*tanh(1/2*d*x+1/2*c)^5
-3/8*(a-4*b)/a^2*tanh(1/2*d*x+1/2*c)^3+5/8/a*tanh(1/2*d*x+1/2*c))/(tanh(1/
2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2+
3/4/a*(1/2*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b
)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2
))-1/2*(((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)
^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)))
)
```

3.131.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2712 vs. $2(90) = 180$.

Time $= 0.32$ (sec) , antiderivative size $= 5077$, normalized size of antiderivative $= 48.82$

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.131.6 Sympy [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}^5\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)**5/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(sech(c + d*x)**5/(a + b*tanh(c + d*x)**2)**3, x)
```

3.131.7 Maxima [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^5}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*(3*(a*e^(7*c) + b*e^(7*c))*e^(7*d*x) + (11*a*e^(5*c) - 9*b*e^(5*c))*e^
(5*d*x) - (11*a*e^(3*c) - 9*b*e^(3*c))*e^(3*d*x) - 3*(a*e^c + b*e^c)*e^(d*
x))/(a^4*d + 2*a^3*b*d + a^2*b^2*d + (a^4*d*e^(8*c) + 2*a^3*b*d*e^(8*c) + 
a^2*b^2*d*e^(8*c))*e^(8*d*x) + 4*(a^4*d*e^(6*c) - a^2*b^2*d*e^(6*c))*e^(6*
d*x) + 2*(3*a^4*d*e^(4*c) - 2*a^3*b*d*e^(4*c) + 3*a^2*b^2*d*e^(4*c))*e^(4*
d*x) + 4*(a^4*d*e^(2*c) - a^2*b^2*d*e^(2*c))*e^(2*d*x)) + 32*integrate(3/1
28*(e^(3*d*x + 3*c) + e^(d*x + c))/(a^3 + a^2*b + (a^3*e^(4*c) + a^2*b*e^(
4*c))*e^(4*d*x) + 2*(a^3*e^(2*c) - a^2*b*e^(2*c))*e^(2*d*x)), x)
```

3.131.8 Giac [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^5}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^5\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)^3), x)
```

3.132 $\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

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3.132.2 Mathematica [A] (verified) . . . 1013
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3.132.1 Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{(3a^2-2ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d} + \frac{(a+b)\operatorname{sech}^2(c+dx)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2} + \frac{3\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\tanh(c+dx)}{8d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
1/8*(3*a^2-2*a*b+3*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/b^(5/2
)/d+1/4*(a+b)*sech(d*x+c)^2*tanh(d*x+c)/a/b/d/(a+b*tanh(d*x+c)^2)^2+3/8*(1
/a^2-1/b^2)*tanh(d*x+c)/d/(a+b*tanh(d*x+c)^2)
```

3.132.2 Mathematica [A] (verified)

Time = 1.96 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{(3a^2-2ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) - \frac{\sqrt{a}\sqrt{b}(a+b)\left(3a^2-10ab+3b^2+3(a^2-b^2)\cosh(2(c+dx))\right)\sinh(2(c+dx))}{(a-b+(a+b)\cosh(2(c+dx)))^2}}{8a^{5/2}b^{5/2}d}$$

input
```
Integrate[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((3*a^2 - 2*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] - (Sqrt[a
]*Sqrt[b]*(a + b)*(3*a^2 - 10*a*b + 3*b^2 + 3*(a^2 - b^2)*Cosh[2*(c + d*x)
])*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)])^2)/(8*a^(5/2)*b^
(5/2)*d)
```

3.132.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4158, 315, 25, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(ic+idx)^6}{\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$

$$\downarrow 4158$$

$$\frac{\int \frac{\left(1-\tanh^2(c+dx)\right)^2}{\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh(c+dx)}{d}$$

$$\downarrow 315$$

$$\frac{\int -\frac{-\left((3a-b)\tanh^2(c+dx)\right)+a-3b}{\left(b\tanh^2(c+dx)+a\right)^2}\,d\tanh(c+dx)}{4ab} + \frac{(a+b)\tanh(c+dx)\left(1-\tanh^2(c+dx)\right)}{4ab\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{(a+b)\tanh(c+dx)\left(1-\tanh^2(c+dx)\right)}{4ab\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\int \frac{-\left((3a-b)\tanh^2(c+dx)\right)+a-3b}{\left(b\tanh^2(c+dx)+a\right)^2}\,d\tanh(c+dx)}{4ab}}{d}$$

$$\downarrow 298$$

$$\frac{\frac{(a+b)\tanh(c+dx)\left(1-\tanh^2(c+dx)\right)}{4ab\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{1}{2}\left(-\frac{3a}{b}-\frac{3b}{a}+2\right)\int\frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)+\frac{3\left(\frac{a}{b}-\frac{b}{a}\right)\tanh(c+dx)}{2\left(a+b\tanh^2(c+dx)\right)}}{4ab}}{d}$$

$$\downarrow 218$$

$$\frac{\frac{(a+b)\tanh(c+dx)\left(1-\tanh^2(c+dx)\right)}{4ab\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\left(-\frac{3a}{b}-\frac{3b}{a}+2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{3\left(\frac{a}{b}-\frac{b}{a}\right)\tanh(c+dx)}{2\left(a+b\tanh^2(c+dx)\right)}}{4ab}}{d}$$

input
```
Int[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(((a + b)*Tanh[c + d*x]*(1 - Tanh[c + d*x]^2))/(4*a*b*(a + b*Tanh[c + d*x]
^2)^2) - (((2 - (3*a)/b - (3*b)/a)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]
)/(2*Sqrt[a]*Sqrt[b]) + (3*(a/b - b/a)*Tanh[c + d*x])/(2*(a + b*Tanh[c + d
*x]^2)))/(4*a*b))/d
```

3.132.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.132.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(117) = 234$.

Time = 0.19 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.87

$$-\frac{2\left(\frac{\left(3a^2-2ab-5b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8a\,b^2}+\frac{\left(9a^3+14a^2b-7a\,b^2-12b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^2b^2}+\frac{\left(9a^3+14a^2b-7a\,b^2-12b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8a^2b^2}+\frac{\left(3a^2-2ab-5b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a\,b^2}\right.}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}$$

$$d$$

input
```
int(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
1/d*(-2*(1/8*(3*a^2-2*a*b-5*b^2)/a/b^2*tanh(1/2*d*x+1/2*c)^7+1/8*(9*a^3+14
*a^2*b-7*a*b^2-12*b^3)/a^2/b^2*tanh(1/2*d*x+1/2*c)^5+1/8*(9*a^3+14*a^2*b-7
*a*b^2-12*b^3)/a^2/b^2*tanh(1/2*d*x+1/2*c)^3+1/8*(3*a^2-2*a*b-5*b^2)/a/b^2
*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4
*tanh(1/2*d*x+1/2*c)^2*b+a)^2-1/4/a*(3*a^2-2*a*b+3*b^2)/b^2*(1/2*(a+((a+b)
*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(
a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(-a+((a+b)*
b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(
a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))
```

3.132.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2464 vs. 2(117) = 234.

Time = 0.34 (sec) , antiderivative size = 5233, normalized size of antiderivative = 39.95

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.132.6 Sympy [F]

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)**6/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(sech(c + d*x)**6/(a + b*tanh(c + d*x)**2)**3, x)
```

3.132.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(117) = 234.

Time = 0.49 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.53

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx =$$
$$-\frac{3a^3+3a^2b-3ab^2-3b^3+(9a^3-13a^2b-13ab^2+9b^3)e^{(-2dx-2c)}+3(3a^3-5a^2b+5ab^2-3b^3}{4(a^4b^2+2a^3b^3+a^2b^4+4(a^4b^2-a^2b^4)e^{(-2dx-2c)}+2(3a^4b^2-2a^3b^3+3a^2b^4)e^{(-4dx-4c)}+4(a^4b^2-}$$
$$-\frac{(3a^2-2ab+3b^2)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{8\sqrt{ab}a^2b^2d}$$

input
```
integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*(3*a^3 + 3*a^2*b - 3*a*b^2 - 3*b^3 + (9*a^3 - 13*a^2*b - 13*a*b^2 + 9
*b^3)*e^(-2*d*x - 2*c) + 3*(3*a^3 - 5*a^2*b + 5*a*b^2 - 3*b^3)*e^(-4*d*x -
 4*c) + (3*a^3 + a^2*b + a*b^2 + 3*b^3)*e^(-6*d*x - 6*c))/((a^4*b^2 + 2*a^
3*b^3 + a^2*b^4 + 4*(a^4*b^2 - a^2*b^4)*e^(-2*d*x - 2*c) + 2*(3*a^4*b^2 -
2*a^3*b^3 + 3*a^2*b^4)*e^(-4*d*x - 4*c) + 4*(a^4*b^2 - a^2*b^4)*e^(-6*d*x
- 6*c) + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*e^(-8*d*x - 8*c))*d) - 1/8*(3*a^2
 - 2*a*b + 3*b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))
/(sqrt(a*b)*a^2*b^2*d)
```

3.132.8 Giac [F]

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^6}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^6\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)^3), x)
```

3.133 $\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

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3.133.2 Mathematica [C] (verified) . . . 1020
3.133.3 Rubi [A] (verified) . . . 1020
3.133.4 Maple [B] (verified) . . . 1023
3.133.5 Fricas [B] (verification not implemented) . . . 1023
3.133.6 Sympy [F(-1)] . . . 1024
3.133.7 Maxima [F] . . . 1024
3.133.8 Giac [F] . . . 1025
3.133.9 Mupad [F(-1)] . . . 1025

3.133.1 Optimal result

Integrand size = 23, antiderivative size = 156

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\arctan(\sinh(c+dx))}{b^3 d} + \frac{\sqrt{a+b}(8a^2-4ab+3b^2)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3 d} + \frac{(a+b)\sinh(c+dx)}{4abd\left(a+(a+b)\sinh^2(c+dx)\right)^2} - \frac{(4a-3b)(a+b)\sinh(c+dx)}{8a^2b^2d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

output

```
-arctan(sinh(d*x+c))/b^3/d+1/4*(a+b)*sinh(d*x+c)/a/b/d/(a+(a+b)*sinh(d*x+c
)^2)^2-1/8*(4*a-3*b)*(a+b)*sinh(d*x+c)/a^2/b^2/d/(a+(a+b)*sinh(d*x+c)^2)+1
/8*(8*a^2-4*a*b+3*b^2)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))*(a+b)^(1/2)
/a^(5/2)/b^3/d
```

3.133.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.69 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.03

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx =$$
$$-\frac{\frac{2\sqrt{a+b}\left(8a^2-4ab+3b^2\right)\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}}+\frac{2\left(8a^3+4a^2b-ab^2+3b^3\right)\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}}+64\arctan\left(\tanh\left(\frac{1}{2}(c+d\right.\right.}$$

input
```
Integrate[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-1/32*((2*Sqrt[a + b]*(8*a^2 - 4*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x
])/Sqrt[a + b]])/a^(5/2) + (2*(8*a^3 + 4*a^2*b - a*b^2 + 3*b^3)*ArcTan[(Sq
rt[a]*Csch[c + d*x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]) + 64*ArcTan[Tanh[
(c + d*x)/2]] + (I*Sqrt[a + b]*(8*a^2 - 4*a*b + 3*b^2)*Log[a - b + (a + b)
*Cosh[2*(c + d*x)]])/a^(5/2) - (I*(8*a^3 + 4*a^2*b - a*b^2 + 3*b^3)*Log[a
- b + (a + b)*Cosh[2*(c + d*x)]])/(a^(5/2)*Sqrt[a + b]) - (32*b^2*(a + b)*
Sinh[c + d*x])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])^2) + (8*b*(4*a^2 + a
*b - 3*b^2)*Sinh[c + d*x])/(a^2*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(b^3
*d)
```

3.133.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4159, 316, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sec(ic+idx)^7}{\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$
$$\downarrow 4159$$

$$\frac{\int \frac{1}{\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)^3}\,d\sinh(c+dx)}{d}$$

$$\downarrow 316$$

$$\frac{\frac{(a+b)\sinh(c+dx)}{4ab\left((a+b)\sinh^2(c+dx)+a\right)^2}-\frac{\int \frac{-3(a+b)\sinh^2(c+dx)+a-3b}{\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)^2}\,d\sinh(c+dx)}{4ab}}{d}$$

$$\downarrow 402$$

$$\frac{\frac{(a+b)\sinh(c+dx)}{4ab\left((a+b)\sinh^2(c+dx)+a\right)^2}-\frac{\frac{(4a-3b)(a+b)\sinh(c+dx)}{2ab\left((a+b)\sinh^2(c+dx)+a\right)}-\frac{\int \frac{4a^2-ba+3b^2-(4a-3b)(a+b)\sinh^2(c+dx)}{\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)}\,d\sinh(c+dx)}{2ab}}{4ab}}{d}$$

$$\downarrow 397$$

$$\frac{\frac{(a+b)\sinh(c+dx)}{4ab\left((a+b)\sinh^2(c+dx)+a\right)^2}-\frac{\frac{(4a-3b)(a+b)\sinh(c+dx)}{2ab\left((a+b)\sinh^2(c+dx)+a\right)}-\frac{\frac{(a+b)\left(8a^2-4ab+3b^2\right)\int \frac{1}{(a+b)\sinh^2(c+dx)+a}\,d\sinh(c+dx)}{b}-\frac{8a^2\int \frac{1}{\sinh^2(c+dx)+1}\,d\sinh(c+d}{b}}{2ab}}{4ab}}{d}$$

$$\downarrow 216$$

$$\frac{\frac{(a+b)\sinh(c+dx)}{4ab\left((a+b)\sinh^2(c+dx)+a\right)^2}-\frac{\frac{(4a-3b)(a+b)\sinh(c+dx)}{2ab\left((a+b)\sinh^2(c+dx)+a\right)}-\frac{\frac{(a+b)\left(8a^2-4ab+3b^2\right)\int \frac{1}{(a+b)\sinh^2(c+dx)+a}\,d\sinh(c+dx)}{b}-\frac{8a^2\arctan(\sinh(c+dx))}{b}}{2ab}}{4ab}}{d}$$

$$\downarrow 218$$

$$\frac{\frac{(a+b)\sinh(c+dx)}{4ab\left((a+b)\sinh^2(c+dx)+a\right)^2}-\frac{\frac{(4a-3b)(a+b)\sinh(c+dx)}{2ab\left((a+b)\sinh^2(c+dx)+a\right)}-\frac{\frac{\sqrt{a+b}\left(8a^2-4ab+3b^2\right)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{8a^2\arctan(\sinh(c+dx))}{b}}{2ab}}{4ab}}{d}$$

input
```
Int[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(((a + b)*Sinh[c + d*x])/(4*a*b*(a + (a + b)*Sinh[c + d*x]^2)^2) - (-1/2*(
(-8*a^2*ArcTan[Sinh[c + d*x]])/b + (Sqrt[a + b]*(8*a^2 - 4*a*b + 3*b^2)*Ar
cTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b))/(a*b) + ((4*a - 3*
b)*(a + b)*Sinh[c + d*x])/(2*a*b*(a + (a + b)*Sinh[c + d*x]^2)))/(4*a*b))/
d
```

3.133. $\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

3.133.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.133.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(142) = 284$.

Time = 0.21 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.49

$$\frac{2\left(\frac{b\left(4a^2-ab-5b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8a}+\frac{\left(4a^3+23a^2b+7ab^2-12b^3\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^2}-\frac{\left(4a^3+23a^2b+7ab^2-12b^3\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8a^2}-\frac{b\left(4a^2-ab-5b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a\right)^2}$$

$$\frac{}{b^3}$$

$$a$$

input
```
int(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
1/d*(2/b^3*((1/8*b*(4*a^2-a*b-5*b^2)/a*tanh(1/2*d*x+1/2*c)^7+1/8*(4*a^3+23
*a^2*b+7*a*b^2-12*b^3)/a^2*b*tanh(1/2*d*x+1/2*c)^5-1/8*(4*a^3+23*a^2*b+7*a
*b^2-12*b^3)/a^2*b*tanh(1/2*d*x+1/2*c)^3-1/8*b*(4*a^2-a*b-5*b^2)/a*tanh(1/
2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/
2*d*x+1/2*c)^2*b+a)^2+1/8/a*(8*a^3+4*a^2*b-a*b^2+3*b^3)*(1/2*(((a+b)*b)^(1
/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh
(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-
b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/
2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))-2/b^3*arctan(tanh(1/2*
d*x+1/2*c)))
```

3.133.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4457 vs. $2(142) = 284$.

Time = 0.38 (sec) , antiderivative size = 8070, normalized size of antiderivative = 51.73

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.133.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)**7/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.133.7 Maxima [F]

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^7}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*((4*a^3*e^(7*c) + 5*a^2*b*e^(7*c) - 2*a*b^2*e^(7*c) - 3*b^3*e^(7*c))*
e^(7*d*x) + (4*a^3*e^(5*c) - 19*a^2*b*e^(5*c) - 14*a*b^2*e^(5*c) + 9*b^3*e
^(5*c))*e^(5*d*x) - (4*a^3*e^(3*c) - 19*a^2*b*e^(3*c) - 14*a*b^2*e^(3*c) +
 9*b^3*e^(3*c))*e^(3*d*x) - (4*a^3*e^c + 5*a^2*b*e^c - 2*a*b^2*e^c - 3*b^3
*e^c)*e^(d*x))/(a^4*b^2*d + 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^(8*c) +
 2*a^3*b^3*d*e^(8*c) + a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(a^4*b^2*d*e^(6*c)
 - a^2*b^4*d*e^(6*c))*e^(6*d*x) + 2*(3*a^4*b^2*d*e^(4*c) - 2*a^3*b^3*d*e^(
4*c) + 3*a^2*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^4*b^2*d*e^(2*c) - a^2*b^4*d*e
^(2*c))*e^(2*d*x)) - 2*arctan(e^(d*x + c))/(b^3*d) + 128*integrate(1/512*(
(8*a^3*e^(3*c) + 4*a^2*b*e^(3*c) - a*b^2*e^(3*c) + 3*b^3*e^(3*c))*e^(3*d*x
) + (8*a^3*e^c + 4*a^2*b*e^c - a*b^2*e^c + 3*b^3*e^c)*e^(d*x))/(a^3*b^3 +
a^2*b^4 + (a^3*b^3*e^(4*c) + a^2*b^4*e^(4*c))*e^(4*d*x) + 2*(a^3*b^3*e^(2*
c) - a^2*b^4*e^(2*c))*e^(2*d*x)), x)
```

3.133.8 Giac [F]

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^7}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^7\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)^7*(a + b*tanh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)^7*(a + b*tanh(c + d*x)^2)^3), x)
```

3.134 $\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

3.134.1 Optimal result 1026
3.134.2 Mathematica [A] (verified) 1026
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3.134.9 Mupad [B] (verification not implemented) 1031

3.134.1 Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = (a+b)x - \frac{(a+b)\tanh(c+dx)}{d} - \frac{(a+b)\tanh^3(c+dx)}{3d} - \frac{b\tanh^5(c+dx)}{5d}$$

output
```
(a+b)*x-(a+b)*tanh(d*x+c)/d-1/3*(a+b)*tanh(d*x+c)^3/d-1/5*b*tanh(d*x+c)^5/
d
```

3.134.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.80

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{a\operatorname{arctanh}(\tanh(c+dx))}{d} + \frac{b\operatorname{arctanh}(\tanh(c+dx))}{d} - \frac{a\tanh(c+dx)}{d} - \frac{b\tanh(c+dx)}{d} - \frac{a\tanh^3(c+dx)}{3d} - \frac{b\tanh^3(c+dx)}{3d} - \frac{b\tanh^5(c+dx)}{5d}$$

input
```
Integrate[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a*ArcTanh[Tanh[c + d*x]])/d + (b*ArcTanh[Tanh[c + d*x]])/d - (a*Tanh[c +
d*x])/d - (b*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3*d) - (b*Tanh[c + d*
x]^3)/(3*d) - (b*Tanh[c + d*x]^5)/(5*d)
```

3.134.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4114, 3042, 3954, 25, 3042, 25, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \tan(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)dx$$

$$\downarrow 4114$$

$$(a+b)\int \tanh^4(c+dx)dx - \frac{b\tanh^5(c+dx)}{5d}$$

$$\downarrow 3042$$

$$-\frac{b\tanh^5(c+dx)}{5d} + (a+b)\int \tan(ic+idx)^4 dx$$

$$\downarrow 3954$$

$$(a+b)\left(-\int -\tanh^2(c+dx)dx - \frac{\tanh^3(c+dx)}{3d}\right) - \frac{b\tanh^5(c+dx)}{5d}$$

$$\downarrow 25$$

$$(a+b)\left(\int \tanh^2(c+dx)dx - \frac{\tanh^3(c+dx)}{3d}\right) - \frac{b\tanh^5(c+dx)}{5d}$$

$$\downarrow 3042$$

$$-\frac{b\tanh^5(c+dx)}{5d} + (a+b)\left(-\frac{\tanh^3(c+dx)}{3d} + \int -\tan(ic+idx)^2 dx\right)$$

$$\downarrow 25$$

$$-\frac{b\tanh^5(c+dx)}{5d} + (a+b)\left(-\frac{\tanh^3(c+dx)}{3d} - \int \tan(ic+idx)^2 dx\right)$$

$$\downarrow 3954$$

$$(a+b)\left(\int 1dx - \frac{\tanh^3(c+dx)}{3d} - \frac{\tanh(c+dx)}{d}\right) - \frac{b\tanh^5(c+dx)}{5d}$$

$$\downarrow 24$$

$$(a+b)\left(-\frac{\tanh^3(c+dx)}{3d} - \frac{\tanh(c+dx)}{d} + x\right) - \frac{b\tanh^5(c+dx)}{5d}$$

input
```
Int[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-1/5*(b*Tanh[c + d*x]^5)/d + (a + b)*(x - Tanh[c + d*x]/d - Tanh[c + d*x]^
3/(3*d))
```

3.134.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4114
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Simp[(A - C)   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

3.134.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

method	result
parallelrisch	$\frac{-3\tanh(dx+c)^5 b-5\tanh(dx+c)^3 a-5b\tanh(dx+c)^3+15adx+15dxb-15a\tanh(dx+c)-15b\tanh(dx+c)}{15d}$
derivativedivides	$\frac{-\frac{\tanh(dx+c)^5 b}{5}-\frac{\tanh(dx+c)^3 a}{3}-\frac{b\tanh(dx+c)^3}{3}-a\tanh(dx+c)-b\tanh(dx+c)-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(a+b)\ln(\tanh(dx+}{d}$
default	$\frac{-\frac{\tanh(dx+c)^5 b}{5}-\frac{\tanh(dx+c)^3 a}{3}-\frac{b\tanh(dx+c)^3}{3}-a\tanh(dx+c)-b\tanh(dx+c)-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(a+b)\ln(\tanh(dx+}{d}$
parts	$\frac{b\left(-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{a\left(-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)\right.}{}$
risch	$ax+bx+\frac{4a\,e^{8dx+8c}+6b\,e^{8dx+8c}+12a\,e^{6dx+6c}+12b\,e^{6dx+6c}+\frac{44a\,e^{4dx+4c}}{3}+\frac{56b\,e^{4dx+4c}}{3}+\frac{28\,e^{2dx+2c}a}{3}+\frac{28b\,e^{2dx+2}}{3}}{d\left(e^{2dx+2c}+1\right)^5}$

input
```
int(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(-3*tanh(d*x+c)^5*b-5*tanh(d*x+c)^3*a-5*b*tanh(d*x+c)^3+15*a*d*x+15*d
*x*b-15*a*tanh(d*x+c)-15*b*tanh(d*x+c))/d
```

3.134.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(50) = 100.

Time = 0.27 (sec) , antiderivative size = 339, normalized size of antiderivative = 6.28

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{(15\,(a+b)dx+20\,a+23\,b)\cosh(dx+c)^5+5\,(15\,(a+b)dx+20\,a+23\,b)\cosh(dx+c)\sinh(dx+c)^4-}{}$$

input
```
integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/15*((15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c)^5 + 5*(15*(a + b)*d*x +
 20*a + 23*b)*cosh(d*x + c)*sinh(d*x + c)^4 - (20*a + 23*b)*sinh(d*x + c)^
5 + 5*(15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c)^3 - 5*(2*(20*a + 23*b)*
cosh(d*x + c)^2 + 8*a + 5*b)*sinh(d*x + c)^3 + 5*(2*(15*(a + b)*d*x + 20*a
 + 23*b)*cosh(d*x + c)^3 + 3*(15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c))
*sinh(d*x + c)^2 + 10*(15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c) - 5*((2
0*a + 23*b)*cosh(d*x + c)^4 + 3*(8*a + 5*b)*cosh(d*x + c)^2 + 4*a + 10*b)*
sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*
d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x +
 c)^2 + 10*d*cosh(d*x + c))
```

3.134.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.52

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \begin{cases} ax - \frac{a\tanh^3(c+dx)}{3d} - \frac{a\tanh(c+dx)}{d} + bx - \frac{b\tanh^5(c+dx)}{5d} - \frac{b\tanh^3(c+dx)}{3d} - \frac{b\tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a+b\tanh^2(c)\right)\tanh^4(c) & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

output
```
Piecewise((a*x - a*tanh(c + d*x)**3/(3*d) - a*tanh(c + d*x)/d + b*x - b*ta
nh(c + d*x)**5/(5*d) - b*tanh(c + d*x)**3/(3*d) - b*tanh(c + d*x)/d, Ne(d,
 0)), (x*(a + b*tanh(c)**2)*tanh(c)**4, True))
```

3.134.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(50) = 100$.

Time = 0.21 (sec) , antiderivative size = 199, normalized size of antiderivative = 3.69

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{1}{15}b\left(15x + \frac{15c}{d} - \frac{2\left(70e^{(-2dx-2c)} + 140e^{(-4dx-4c)} + 90e^{(-6dx-6c)} + 45e^{(-8dx-8c)} + 23\right)}{d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)}\right)$$
$$+ \frac{1}{3}a\left(3x + \frac{3c}{d} - \frac{4\left(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + 2\right)}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)}\right)$$

input
```
integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/15*b*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90
*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
 10*c) + 1))) + 1/3*a*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x -
 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c)
+ 1)))
```

3.134.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(50) = 100$.

Time $= 0.32$ (sec) , antiderivative size $= 134$, normalized size of antiderivative $= 2.48$

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{15\,(dx+c)(a+b) + \frac{2\,(30\,ae^{(8\,dx+8\,c)}+45\,be^{(8\,dx+8\,c)}+90\,ae^{(6\,dx+6\,c)}+90\,be^{(6\,dx+6\,c)}+110\,ae^{(4\,dx+4\,c)}+140\,be^{(4\,dx+4\,c)}+70\,ae^{(2\,dx+2\,c)}}{(e^{(2\,dx+2\,c)}+1)^5}}{15\,d}$$

input
```
integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/15*(15*(d*x + c)*(a + b) + 2*(30*a*e^(8*d*x + 8*c) + 45*b*e^(8*d*x + 8*c
) + 90*a*e^(6*d*x + 6*c) + 90*b*e^(6*d*x + 6*c) + 110*a*e^(4*d*x + 4*c) +
140*b*e^(4*d*x + 4*c) + 70*a*e^(2*d*x + 2*c) + 70*b*e^(2*d*x + 2*c) + 20*a
 + 23*b)/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.134.9 Mupad [B] (verification not implemented)

Time $= 1.86$ (sec) , antiderivative size $= 50$, normalized size of antiderivative $= 0.93$

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = x\,(a+b) - \frac{\tanh(c+d\,x)^3\,(a+b)}{3\,d} - \frac{b\tanh(c+d\,x)^5}{5\,d} - \frac{\tanh(c+d\,x)\,(a+b)}{d}$$

input
```
int(tanh(c + d*x)^4*(a + b*tanh(c + d*x)^2),x)
```

output
```
x*(a + b) - (tanh(c + d*x)^3*(a + b))/(3*d) - (b*tanh(c + d*x)^5)/(5*d) -
(tanh(c + d*x)*(a + b))/d
```

3.135 $\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

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3.135.2 Mathematica [A] (verified) 1033
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3.135.1 Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(a+b)\log(\cosh(c+dx))}{d} - \frac{(a+b)\tanh^2(c+dx)}{2d} - \frac{b\tanh^4(c+dx)}{4d}$$

output
```
(a+b)*ln(cosh(d*x+c))/d-1/2*(a+b)*tanh(d*x+c)^2/d-1/4*b*tanh(d*x+c)^4/d
```

3.135.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{-4(a+b)\log(\cosh(c+dx)) + 2(a+b)\tanh^2(c+dx) + b\tanh^4(c+dx)}{4d}$$

input
```
Integrate[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-1/4*(-4*(a + b)*Log[Cosh[c + d*x]] + 2*(a + b)*Tanh[c + d*x]^2 + b*Tanh[c
 + d*x]^4)/d
```

3.135.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 26, 4114, 26, 3042, 26, 3954, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int i\tan(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)dx$$

$$\downarrow 26$$

$$i\int \tan(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)dx$$

$$\downarrow 4114$$

$$i\left((a+b)\int -i\tanh^3(c+dx)dx+\frac{ib\tanh^4(c+dx)}{4d}\right)$$

$$\downarrow 26$$

$$i\left(\frac{ib\tanh^4(c+dx)}{4d}-i(a+b)\int \tanh^3(c+dx)dx\right)$$

$$\downarrow 3042$$

$$i\left(\frac{ib\tanh^4(c+dx)}{4d}-i(a+b)\int i\tan(ic+idx)^3dx\right)$$

$$\downarrow 26$$

$$i\left((a+b)\int \tan(ic+idx)^3dx+\frac{ib\tanh^4(c+dx)}{4d}\right)$$

$$\downarrow 3954$$

$$i\left((a+b)\left(\frac{i\tanh^2(c+dx)}{2d}-\int i\tanh(c+dx)dx\right)+\frac{ib\tanh^4(c+dx)}{4d}\right)$$

$$\downarrow 26$$

$$i\left((a+b)\left(\frac{i\tanh^2(c+dx)}{2d}-i\int \tanh(c+dx)dx\right)+\frac{ib\tanh^4(c+dx)}{4d}\right)$$

$$\downarrow 3042$$

$$i\left((a+b)\left(\frac{i\tanh^2(c+dx)}{2d}-i\int -i\tan(ic+idx)dx\right)+\frac{ib\tanh^4(c+dx)}{4d}\right)$$

$$\downarrow 26$$

$$i\left((a+b)\left(\frac{i\tanh^2(c+dx)}{2d}-\int \tan(ic+idx)dx\right)+\frac{ib\tanh^4(c+dx)}{4d}\right)$$

$$\downarrow 3956$$

$$i\left((a+b)\left(\frac{i\tanh^2(c+dx)}{2d}-\frac{i\log(\cosh(c+dx))}{d}\right)+\frac{ib\tanh^4(c+dx)}{4d}\right)$$

input
```
Int[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
I*(((I/4)*b*Tanh[c + d*x]^4)/d + (a + b)*(((-I)*Log[Cosh[c + d*x]])/d + ((
I/2)*Tanh[c + d*x]^2)/d))
```

3.135.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4114
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Simp[(A - C)   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

3.135.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.45

method	result
derivativedivides	$\frac{-\frac{\tanh(dx+c)^4 b}{4}-\frac{\tanh(dx+c)^2 a}{2}-\frac{b\tanh(dx+c)^2}{2}-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(-a-b)\ln(\tanh(dx+c)+1)}{2}}{d}$
default	$\frac{-\frac{\tanh(dx+c)^4 b}{4}-\frac{\tanh(dx+c)^2 a}{2}-\frac{b\tanh(dx+c)^2}{2}-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(-a-b)\ln(\tanh(dx+c)+1)}{2}}{d}$
parallelrisch	$-\frac{\tanh(dx+c)^4 b+4adx+4dxb+2\tanh(dx+c)^2 a+2b\tanh(dx+c)^2+4\ln(1-\tanh(dx+c))a+4\ln(1-\tanh(dx+c))b}{4d}$
parts	$\frac{a\left(-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b\left(-\frac{\tanh(dx+c)^4}{4}-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh}{d}$
risch	$-ax-bx-\frac{2ac}{d}-\frac{2bc}{d}+\frac{2\,\mathrm{e}^{2dx+2c}\left(a\,\mathrm{e}^{4dx+4c}+2b\,\mathrm{e}^{4dx+4c}+2\,\mathrm{e}^{2dx+2c}a+2b\,\mathrm{e}^{2dx+2c}+a+2b\right)}{d\left(\mathrm{e}^{2dx+2c}+1\right)^4}+\frac{\ln\left(\mathrm{e}^{2dx+2c}+1\right)}{d}$

input
```
int(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4*tanh(d*x+c)^4*b-1/2*tanh(d*x+c)^2*a-1/2*b*tanh(d*x+c)^2-1/2*(a+b
)*ln(tanh(d*x+c)-1)+1/2*(-a-b)*ln(tanh(d*x+c)+1))
```

3.135.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1205 vs. $2(45) = 90$.

Time = 0.27 (sec) , antiderivative size = 1205, normalized size of antiderivative = 24.59

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-((a + b)*d*x*cosh(d*x + c)^8 + 8*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^
7 + (a + b)*d*x*sinh(d*x + c)^8 + 2*(2*(a + b)*d*x - a - 2*b)*cosh(d*x + c
)^6 + 2*(14*(a + b)*d*x*cosh(d*x + c)^2 + 2*(a + b)*d*x - a - 2*b)*sinh(d*
x + c)^6 + 4*(14*(a + b)*d*x*cosh(d*x + c)^3 + 3*(2*(a + b)*d*x - a - 2*b)
*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*(a + b)*d*x - 2*a - 2*b)*cosh(d*x +
 c)^4 + 2*(35*(a + b)*d*x*cosh(d*x + c)^4 + 3*(a + b)*d*x + 15*(2*(a + b)*
d*x - a - 2*b)*cosh(d*x + c)^2 - 2*a - 2*b)*sinh(d*x + c)^4 + 8*(7*(a + b)
*d*x*cosh(d*x + c)^5 + 5*(2*(a + b)*d*x - a - 2*b)*cosh(d*x + c)^3 + (3*(a
 + b)*d*x - 2*a - 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 + (a + b)*d*x + 2*(2
*(a + b)*d*x - a - 2*b)*cosh(d*x + c)^2 + 2*(14*(a + b)*d*x*cosh(d*x + c)^
6 + 15*(2*(a + b)*d*x - a - 2*b)*cosh(d*x + c)^4 + 2*(a + b)*d*x + 6*(3*(a
 + b)*d*x - 2*a - 2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 - ((a +
b)*cosh(d*x + c)^8 + 8*(a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a + b)*sin
h(d*x + c)^8 + 4*(a + b)*cosh(d*x + c)^6 + 4*(7*(a + b)*cosh(d*x + c)^2 +
a + b)*sinh(d*x + c)^6 + 8*(7*(a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x
 + c))*sinh(d*x + c)^5 + 6*(a + b)*cosh(d*x + c)^4 + 2*(35*(a + b)*cosh(d*
x + c)^4 + 30*(a + b)*cosh(d*x + c)^2 + 3*a + 3*b)*sinh(d*x + c)^4 + 8*(7*
(a + b)*cosh(d*x + c)^5 + 10*(a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 4*(a + b)*cosh(d*x + c)^2 + 4*(7*(a + b)*cosh(d*x
+ c)^6 + 15*(a + b)*cosh(d*x + c)^4 + 9*(a + b)*cosh(d*x + c)^2 + a + b...
```

3.135.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(42) = 84$.

Time $= 0.12$ (sec) , antiderivative size $= 88$, normalized size of antiderivative $= 1.80$

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$= \begin{cases} ax - \frac{a\log(\tanh(c+dx)+1)}{d} - \frac{a\tanh^2(c+dx)}{2d} + bx - \frac{b\log(\tanh(c+dx)+1)}{d} - \frac{b\tanh^4(c+dx)}{4d} - \frac{b\tanh^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x\left(a+b\tanh^2(c)\right)\tanh^3(c) & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

output
```
Piecewise((a*x - a*log(tanh(c + d*x) + 1)/d - a*tanh(c + d*x)**2/(2*d) + b
*x - b*log(tanh(c + d*x) + 1)/d - b*tanh(c + d*x)**4/(4*d) - b*tanh(c + d*
x)**2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)*tanh(c)**3, True))
```

3.135.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(45) = 90$.

Time = 0.29 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.43

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)$$
$$+ a\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$

input
```
integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
b*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d*x
 - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) +
4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + a*(x + c/d + log(e^(-2*d*x
- 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*
c) + 1)))
```

3.135.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(45) = 90$.

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.88

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx =$$
$$-\frac{(dx+c)(a+b)-(a+b)\log\left(e^{(2\,dx+2\,c)}+1\right)-\frac{2\left((a+2\,b)e^{(6\,dx+6\,c)}+2\,(a+b)e^{(4\,dx+4\,c)}+(a+2\,b)e^{(2\,dx+2\,c)}\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^4}}{d}$$

input
```
integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-((d*x + c)*(a + b) - (a + b)*log(e^(2*d*x + 2*c) + 1) - 2*((a + 2*b)*e^(6
*d*x + 6*c) + 2*(a + b)*e^(4*d*x + 4*c) + (a + 2*b)*e^(2*d*x + 2*c))/(e^(2
*d*x + 2*c) + 1)^4)/d
```

3.135.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = x\,(a+b) - \frac{\tanh(c+d\,x)^2\,(a+b)}{2\,d} - \frac{b\tanh(c+d\,x)^4}{4\,d} - \frac{\ln\left(\tanh(c+d\,x)+1\right)\,(a+b)}{d}$$

input
```
int(tanh(c + d*x)^3*(a + b*tanh(c + d*x)^2),x)
```

output
```
x*(a + b) - (tanh(c + d*x)^2*(a + b))/(2*d) - (b*tanh(c + d*x)^4)/(4*d) -
(log(tanh(c + d*x) + 1)*(a + b))/d
```

3.136 $\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

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3.136.2 Mathematica [A] (verified) 1040
3.136.3 Rubi [A] (verified) 1041
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3.136.8 Giac [B] (verification not implemented) 1045
3.136.9 Mupad [B] (verification not implemented) 1045

3.136.1 Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = (a+b)x - \frac{(a+b)\tanh(c+dx)}{d} - \frac{b\tanh^3(c+dx)}{3d}$$

output
```
(a+b)*x-(a+b)*tanh(d*x+c)/d-1/3*b*tanh(d*x+c)^3/d
```

3.136.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.81

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{a\operatorname{arctanh}(\tanh(c+dx))}{d} + \frac{b\operatorname{arctanh}(\tanh(c+dx))}{d} - \frac{a\tanh(c+dx)}{d} - \frac{b\tanh(c+dx)}{d} - \frac{b\tanh^3(c+dx)}{3d}$$

input
```
Integrate[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a*ArcTanh[Tanh[c + d*x]])/d + (b*ArcTanh[Tanh[c + d*x]])/d - (a*Tanh[c +
d*x])/d - (b*Tanh[c + d*x])/d - (b*Tanh[c + d*x]^3)/(3*d)
```

3.136.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 25, 4114, 25, 3042, 25, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int -\tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)dx$$

$$\downarrow 25$$

$$-\int \tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)dx$$

$$\downarrow 4114$$

$$-(a+b)\int -\tanh^2(c+dx)dx - \frac{b\tanh^3(c+dx)}{3d}$$

$$\downarrow 25$$

$$(a+b)\int \tanh^2(c+dx)dx - \frac{b\tanh^3(c+dx)}{3d}$$

$$\downarrow 3042$$

$$-\frac{b\tanh^3(c+dx)}{3d} + (a+b)\int -\tan(ic+idx)^2 dx$$

$$\downarrow 25$$

$$-\frac{b\tanh^3(c+dx)}{3d} - (a+b)\int \tan(ic+idx)^2 dx$$

$$\downarrow 3954$$

$$-(a+b)\left(\frac{\tanh(c+dx)}{d} - \int 1dx\right) - \frac{b\tanh^3(c+dx)}{3d}$$

$$\downarrow 24$$

$$-(a+b)\left(\frac{\tanh(c+dx)}{d} - x\right) - \frac{b\tanh^3(c+dx)}{3d}$$

input
```
Int[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-1/3*(b*Tanh[c + d*x]^3)/d - (a + b)*(-x + Tanh[c + d*x]/d)
```

3.136.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4114
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Simp[(A - C)   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

3.136.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

method	result
parallelrisch	$-\frac{b\tanh(dx+c)^3-3adx-3dxb+3a\tanh(dx+c)+3b\tanh(dx+c)}{3d}$
derivativedivides	$\frac{-\frac{b\tanh(dx+c)^3}{3}-a\tanh(dx+c)-b\tanh(dx+c)-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(a+b)\ln(\tanh(dx+c)+1)}{2}}{d}$
default	$\frac{-\frac{b\tanh(dx+c)^3}{3}-a\tanh(dx+c)-b\tanh(dx+c)-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(a+b)\ln(\tanh(dx+c)+1)}{2}}{d}$
risch	$ax+bx+\frac{2a\,e^{4dx+4c}+4b\,e^{4dx+4c}+4\,e^{2dx+2c}a+4b\,e^{2dx+2c}+2a+\frac{8b}{3}}{d\left(e^{2dx+2c}+1\right)^3}$
parts	$\frac{a\left(-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b\left(-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh($

input
```
int(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*tanh(d*x+c)^3-3*a*d*x-3*d*x*b+3*a*tanh(d*x+c)+3*b*tanh(d*x+c))/d
```

3.136.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(34) = 68$.

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 4.44

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{(3\,(a+b)dx+3\,a+4\,b)\cosh(dx+c)^3+3\,(3\,(a+b)dx+3\,a+4\,b)\cosh(dx+c)\sinh(dx+c)^2-(3\,a+}{3\left(d\cosh(dx+c)^3+3\,d\cosh(dx-\right.}$$

input
```
integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/3*((3*(a + b)*d*x + 3*a + 4*b)*cosh(d*x + c)^3 + 3*(3*(a + b)*d*x + 3*a
+ 4*b)*cosh(d*x + c)*sinh(d*x + c)^2 - (3*a + 4*b)*sinh(d*x + c)^3 + 3*(3*
(a + b)*d*x + 3*a + 4*b)*cosh(d*x + c) - 3*((3*a + 4*b)*cosh(d*x + c)^2 +
a)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 +
 3*d*cosh(d*x + c))
```

3.136.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.50

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \begin{cases} ax - \frac{a\tanh(c+dx)}{d} + bx - \frac{b\tanh^3(c+dx)}{3d} - \frac{b\tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a+b\tanh^2(c)\right)\tanh^2(c) & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

output
```
Piecewise((a*x - a*tanh(c + d*x)/d + b*x - b*tanh(c + d*x)**3/(3*d) - b*ta
nh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)*tanh(c)**2, True))
```

3.136.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(34) = 68.

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.92

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{1}{3}b\left(3x + \frac{3c}{d} - \frac{4\left(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + 2\right)}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)}\right)$$
$$+ a\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2dx-2c)} + 1)}\right)$$

input
```
integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/3*b*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3
*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + a*(x +
c/d - 2/(d*(e^(-2*d*x - 2*c) + 1)))
```

3.136.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(34) = 68$.

Time $= 0.31$ (sec) , antiderivative size $= 86$, normalized size of antiderivative $= 2.39$

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{3\,(dx+c)(a+b)+\frac{2\left(3\,ae^{(4\,dx+4\,c)}+6\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+6\,be^{(2\,dx+2\,c)}+3\,a+4\,b\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

input
```
integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*(a + b) + 2*(3*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c) +
6*a*e^(2*d*x + 2*c) + 6*b*e^(2*d*x + 2*c) + 3*a + 4*b)/(e^(2*d*x + 2*c) +
1)^3)/d
```

3.136.9 Mupad [B] (verification not implemented)

Time $= 1.80$ (sec) , antiderivative size $= 34$, normalized size of antiderivative $= 0.94$

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = x\,(a+b)-\frac{b\tanh(c+d\,x)^3}{3\,d}-\frac{\tanh(c+d\,x)\,(a+b)}{d}$$

input
```
int(tanh(c + d*x)^2*(a + b*tanh(c + d*x)^2),x)
```

output
```
x*(a + b) - (b*tanh(c + d*x)^3)/(3*d) - (tanh(c + d*x)*(a + b))/d
```

3.137 $\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

3.137.1 Optimal result 1046
3.137.2 Mathematica [A] (verified) 1046
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3.137.1 Optimal result

Integrand size = 19, antiderivative size = 31

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(a+b)\log(\cosh(c+dx))}{d} - \frac{b\tanh^2(c+dx)}{2d}$$

output
```
(a+b)*ln(cosh(d*x+c))/d-1/2*b*tanh(d*x+c)^2/d
```

3.137.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{a\log(\cosh(c+dx))}{d} + \frac{b\log(\cosh(c+dx))}{d} - \frac{b\tanh^2(c+dx)}{2d}$$

input
```
Integrate[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a*Log[Cosh[c + d*x]])/d + (b*Log[Cosh[c + d*x]])/d - (b*Tanh[c + d*x]^2)/
(2*d)
```

3.137.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 26, 4114, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int -i\tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)\,dx$$

$$\downarrow 26$$

$$-i\int \tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)\,dx$$

$$\downarrow 4114$$

$$-i\left((a+b)\int i\tanh(c+dx)dx-\frac{ib\tanh^2(c+dx)}{2d}\right)$$

$$\downarrow 26$$

$$-i\left(i(a+b)\int \tanh(c+dx)dx-\frac{ib\tanh^2(c+dx)}{2d}\right)$$

$$\downarrow 3042$$

$$-i\left(i(a+b)\int -i\tan(ic+idx)dx-\frac{ib\tanh^2(c+dx)}{2d}\right)$$

$$\downarrow 26$$

$$-i\left((a+b)\int \tan(ic+idx)dx-\frac{ib\tanh^2(c+dx)}{2d}\right)$$

$$\downarrow 3956$$

$$-i\left(\frac{i(a+b)\log(\cosh(c+dx))}{d}-\frac{ib\tanh^2(c+dx)}{2d}\right)$$

input
```
Int[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-I)*((I*(a + b)*Log[Cosh[c + d*x]])/d - ((I/2)*b*Tanh[c + d*x]^2)/d)
```

3.137.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4114
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Simp[(A - C)   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

3.137.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

method	result	size
derivativedivides	$\frac{-\frac{b\tanh(dx+c)^2}{2}-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(-a-b)\ln(\tanh(dx+c)+1)}{2}}{d}$	49
default	$\frac{-\frac{b\tanh(dx+c)^2}{2}-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(-a-b)\ln(\tanh(dx+c)+1)}{2}}{d}$	49
parts	$\frac{a\ln(\cosh(dx+c))}{d}+\frac{b\left(-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}$	52
parallelrisch	$-\frac{2adx+2dxb+b\tanh(dx+c)^2+2\ln(1-\tanh(dx+c))a+2\ln(1-\tanh(dx+c))b}{2d}$	55
risch	$-ax-bx-\frac{2ac}{d}-\frac{2bc}{d}+\frac{2b\,e^{2dx+2c}}{d\left(e^{2dx+2c}+1\right)^2}+\frac{\ln\left(e^{2dx+2c}+1\right)a}{d}+\frac{\ln\left(e^{2dx+2c}+1\right)b}{d}$	86

input
```
int(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*b*tanh(d*x+c)^2-1/2*(a+b)*ln(tanh(d*x+c)-1)+1/2*(-a-b)*ln(tanh(d
*x+c)+1))
```

3.137.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(29) = 58$.

Time $= 0.27$ (sec) , antiderivative size $= 399$, normalized size of antiderivative $= 12.87$

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx =$$

$$-\frac{(a+b)dx\cosh(dx+c)^4+4(a+b)dx\cosh(dx+c)\sinh(dx+c)^3+(a+b)dx\sinh(dx+c)^4+(a+b)}$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-((a + b)*d*x*cosh(d*x + c)^4 + 4*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^
3 + (a + b)*d*x*sinh(d*x + c)^4 + (a + b)*d*x + 2*((a + b)*d*x - b)*cosh(d
*x + c)^2 + 2*(3*(a + b)*d*x*cosh(d*x + c)^2 + (a + b)*d*x - b)*sinh(d*x +
 c)^2 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3
 + (a + b)*sinh(d*x + c)^4 + 2*(a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh
(d*x + c)^2 + a + b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + b
)*cosh(d*x + c))*sinh(d*x + c) + a + b)*log(2*cosh(d*x + c)/(cosh(d*x + c)
 - sinh(d*x + c))) + 4*((a + b)*d*x*cosh(d*x + c)^3 + ((a + b)*d*x - b)*co
sh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*
x + c)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^
2 + d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x
+ c) + d)
```

3.137.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(26) = 52$.

Time $= 0.09$ (sec) , antiderivative size $= 60$, normalized size of antiderivative $= 1.94$

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$= \begin{cases} ax - \frac{a\log(\tanh(c+dx)+1)}{d} + bx - \frac{b\log(\tanh(c+dx)+1)}{d} - \frac{b\tanh^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x\left(a+b\tanh^2(c)\right)\tanh(c) & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

output
```
Piecewise((a*x - a*log(tanh(c + d*x) + 1)/d + b*x - b*log(tanh(c + d*x) +
1)/d - b*tanh(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)*tanh(c),
 True))
```

3.137.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(29) = 58$.

Time $= 0.28$ (sec) , antiderivative size $= 76$, normalized size of antiderivative $= 2.45$

$$\begin{aligned}&\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx\\&= b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)\\&\quad+\frac{a\log\left(\cosh\left(dx+c\right)\right)}{d}\end{aligned}$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
b*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*
d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a*log(cosh(d*x + c))/d
```

3.137.8 Giac [A] (verification not implemented)

Time $= 0.29$ (sec) , antiderivative size $= 57$, normalized size of antiderivative $= 1.84$

$$\begin{aligned}&\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx\\&= -\frac{(dx+c)(a+b)-(a+b)\log\left(e^{(2\,dx+2\,c)}+1\right)-\frac{2\,be^{(2\,dx+2\,c)}}{\left(e^{(2\,dx+2\,c)}+1\right)^2}}{d}\end{aligned}$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-((d*x + c)*(a + b) - (a + b)*log(e^(2*d*x + 2*c) + 1) - 2*b*e^(2*d*x + 2*
c)/(e^(2*d*x + 2*c) + 1)^2)/d
```

3.137.9 Mupad [B] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = x\,(a+b) - \frac{b\tanh(c+d\,x)^2}{2\,d} - \frac{\ln\left(\tanh(c+d\,x)+1\right)\,(a+b)}{d}$$

input
```
int(tanh(c + d*x)*(a + b*tanh(c + d*x)^2),x)
```

output
```
x*(a + b) - (b*tanh(c + d*x)^2)/(2*d) - (log(tanh(c + d*x) + 1)*(a + b))/d
```

3.138 $\int \left(a + b\tanh^2(c+dx)\right)\, dx$

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3.138.1 Optimal result

Integrand size = 12, antiderivative size = 19

$$\int \left(a + b\tanh^2(c+dx)\right)\, dx = ax + bx - \frac{b\tanh(c+dx)}{d}$$

output
```
a*x+b*x-b*tanh(d*x+c)/d
```

3.138.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \left(a + b\tanh^2(c+dx)\right)\, dx = ax + \frac{b\text{arctanh}(\tanh(c+dx))}{d} - \frac{b\tanh(c+dx)}{d}$$

input
```
Integrate[a + b*Tanh[c + d*x]^2,x]
```

output
```
a*x + (b*ArcTanh[Tanh[c + d*x]])/d - (b*Tanh[c + d*x])/d
```

3.138.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\tanh^2(c + dx))\, dx$$

$$\downarrow \text{2009}$$

$$ax - \frac{b\tanh(c + dx)}{d} + bx$$

input
```
Int[a + b*Tanh[c + d*x]^2,x]
```

output
```
a*x + b*x - (b*Tanh[c + d*x])/d
```

3.138.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.138.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

method	result	size
parallelrisch	$-\frac{b(-dx+\tanh(dx+c))}{d} + ax$	22
risch	$ax + bx + \frac{2b}{d(e^{2dx+2c}+1)}$	27
default	$ax + \frac{b\left(-\tanh(dx+c) - \frac{\ln(\tanh(dx+c)-1)}{2} + \frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}$	41
parts	$ax + \frac{b\left(-\tanh(dx+c) - \frac{\ln(\tanh(dx+c)-1)}{2} + \frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}$	41
derivativedivides	$\frac{-b\tanh(dx+c) + \frac{(-a-b)\ln(\tanh(dx+c)-1)}{2} - \frac{(-a-b)\ln(\tanh(dx+c)+1)}{2}}{d}$	51

input
```
int(a+b*tanh(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-b*(-d*x+tanh(d*x+c))/d+a*x
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \left(a + b\tanh^2(c+dx)\right)\,dx = \frac{((a+b)dx+b)\cosh(dx+c) - b\sinh(dx+c)}{d\cosh(dx+c)}$$

input
```
integrate(a+b*tanh(d*x+c)^2,x, algorithm="fricas")
```

output
```
(((a + b)*d*x + b)*cosh(d*x + c) - b*sinh(d*x + c))/(d*cosh(d*x + c))
```

3.138.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \left(a + b\tanh^2(c+dx)\right)\,dx = ax + b\left(\begin{cases} x - \frac{\tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x\tanh^2(c) & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*tanh(d*x+c)**2,x)
```

output
```
a*x + b*Piecewise((x - tanh(c + d*x)/d, Ne(d, 0)), (x*tanh(c)**2, True))
```

3.138.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \left(a + b\tanh^2(c+dx)\right)\,dx = b\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2\,dx-2\,c)}+1)}\right) + ax$$

input
```
integrate(a+b*tanh(d*x+c)^2,x, algorithm="maxima")
```

output
```
b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a*x
```

3.138.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \left(a + b \tanh^2(c + dx)\right)\, dx = ax + \frac{\left(dx + c + \frac{2}{e^{(2\,dx + 2\,c)} + 1}\right) b}{d}$$

input
```
integrate(a+b*tanh(d*x+c)^2,x, algorithm="giac")
```

output
```
a*x + (d*x + c + 2/(e^(2*d*x + 2*c) + 1))*b/d
```

3.138.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \left(a + b \tanh^2(c + dx)\right)\, dx = x\,(a + b) - \frac{b \tanh(c + d\,x)}{d}$$

input
```
int(a + b*tanh(c + d*x)^2,x)
```

output
```
x*(a + b) - (b*tanh(c + d*x))/d
```

3.139 $\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

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3.139.1 Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{b\log(\cosh(c+dx))}{d}+\frac{a\log(\sinh(c+dx))}{d}$$

output
```
b*ln(cosh(d*x+c))/d+a*ln(sinh(d*x+c))/d
```

3.139.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{b\log(\cosh(c+dx))}{d} + \frac{a(\log(\cosh(c+dx))+\log(\tanh(c+dx)))}{d}$$

input
```
Integrate[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(b*Log[Cosh[c + d*x]])/d + (a*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]]))/d
```

3.139.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 26, 4108, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{i\left(a-b\tan(ic+idx)^2\right)}{\tan(ic+idx)}dx$$

$$\downarrow 26$$

$$i\int \frac{a-b\tan(ic+idx)^2}{\tan(ic+idx)}dx$$

$$\downarrow 4108$$

$$i\left(a\int -i\coth(c+dx)dx - b\int i\tanh(c+dx)dx\right)$$

$$\downarrow 26$$

$$i\left(-ia\int \coth(c+dx)dx - ib\int \tanh(c+dx)dx\right)$$

$$\downarrow 3042$$

$$i\left(-ia\int -i\tan\left(ic+idx+\frac{\pi}{2}\right)dx - ib\int -i\tan(ic+idx)dx\right)$$

$$\downarrow 26$$

$$i\left(-a\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx - b\int \tan(ic+idx)dx\right)$$

$$\downarrow 3956$$

$$i\left(-\frac{ia\log(-i\sinh(c+dx))}{d} - \frac{ib\log(\cosh(c+dx))}{d}\right)$$

input
```
Int[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
I*(((-I)*b*Log[Cosh[c + d*x]])/d - (I*a*Log[(-I)*Sinh[c + d*x]])/d)
```

3.139.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4108
```
Int[((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/tan[(e_.) + (f_.)*(x_)], x_Sym
bol] :> Simp[A   Int[1/Tan[e + f*x], x], x] + Simp[C   Int[Tan[e + f*x], x]
, x] /; FreeQ[{e, f, A, C}, x] && NeQ[A, C]
```

3.139.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{a\ln(\sinh(dx+c))+b\ln(\cosh(dx+c))}{d}$	24
default	$\frac{a\ln(\sinh(dx+c))+b\ln(\cosh(dx+c))}{d}$	24
parallelrisch	$\frac{(-a-b)\ln(1-\tanh(dx+c))+a\ln(\tanh(dx+c))-(a+b)xd}{d}$	41
risch	$-ax-bx-\frac{2bc}{d}-\frac{2ac}{d}+\frac{\ln(e^{2dx+2c}+1)b}{d}+\frac{a\ln(e^{2dx+2c}-1)}{d}$	58

input
```
int(coth(d*x+c)*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*ln(sinh(d*x+c))+b*ln(cosh(d*x+c)))
```

3.139.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(25) = 50$.

Time $= 0.26$ (sec) , antiderivative size $= 70$, normalized size of antiderivative $= 2.80$

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{(a+b)dx - b\log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right) - a\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{d}$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-((a + b)*d*x - b*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - a
*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/d
```

3.139.6 Sympy [F]

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2(c+dx)\right)\coth(c+dx)\,dx$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*coth(c + d*x), x)
```

3.139.7 Maxima [A] (verification not implemented)

Time $= 0.20$ (sec) , antiderivative size $= 35$, normalized size of antiderivative $= 1.40$

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{b\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d} + \frac{a\log\left(\sinh(dx+c)\right)}{d}$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
b*log(e^(d*x + c) + e^(-d*x - c))/d + a*log(sinh(d*x + c))/d
```

3.139.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{(dx+c)(a+b)-b\log\left(e^{(2\,dx+2\,c)}+1\right)-a\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{d}$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-((d*x + c)*(a + b) - b*log(e^(2*d*x + 2*c) + 1) - a*log(abs(e^(2*d*x + 2*
c) - 1)))/d
```

3.139.9 Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 228, normalized size of antiderivative = 9.12

$$\begin{aligned}\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= \frac{a\,\ln\left(8\,a\,b-4\,a^2-4\,b^2+4\,a^2\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}+4\,b^2\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}-8\,a\,b\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}\right)}{2\,d}\\ &\quad -b\,x-\frac{\mathrm{atan}\left(\frac{a\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{-d^2}}{d\,\sqrt{a^2-2\,a\,b+b^2}}-\frac{b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{-d^2}}{d\,\sqrt{a^2-2\,a\,b+b^2}}\right)\,\sqrt{a^2-2\,a\,b+b^2}}{\sqrt{-d^2}}-a\,x\\ &\quad +\frac{b\,\ln\left(8\,a\,b-4\,a^2-4\,b^2+4\,a^2\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}+4\,b^2\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}-8\,a\,b\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}\right)}{2\,d}\end{aligned}$$

input
```
int(coth(c + d*x)*(a + b*tanh(c + d*x)^2),x)
```

output
```
(a*log(8*a*b - 4*a^2 - 4*b^2 + 4*a^2*exp(4*c)*exp(4*d*x) + 4*b^2*exp(4*c)*
exp(4*d*x) - 8*a*b*exp(4*c)*exp(4*d*x)))/(2*d) - b*x - (atan((a*exp(2*c)*e
xp(2*d*x)*(-d^2)^(1/2))/(d*(a^2 - 2*a*b + b^2)^(1/2)) - (b*exp(2*c)*exp(2*
d*x)*(-d^2)^(1/2))/(d*(a^2 - 2*a*b + b^2)^(1/2)))*(a^2 - 2*a*b + b^2)^(1/2
))/(-d^2)^(1/2) - a*x + (b*log(8*a*b - 4*a^2 - 4*b^2 + 4*a^2*exp(4*c)*exp(
4*d*x) + 4*b^2*exp(4*c)*exp(4*d*x) - 8*a*b*exp(4*c)*exp(4*d*x)))/(2*d)
```

3.140 $\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

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3.140.8 Giac [A] (verification not implemented) 1065
3.140.9 Mupad [B] (verification not implemented) 1065

3.140.1 Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = (a+b)x - \frac{a\coth(c+dx)}{d}$$

output
```
(a+b)*x-a*coth(d*x+c)/d
```

3.140.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\begin{aligned}&\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx \\ &= bx - \frac{a\coth(c+dx)\,\text{Hypergeometric2F1}\left(-\tfrac{1}{2},1,\tfrac{1}{2},\tanh^2(c+dx)\right)}{d}\end{aligned}$$

input
```
Integrate[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
b*x - (a*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2])/d
```

3.140.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 25, 4112, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int -\frac{a-b\tan(ic+idx)^2}{\tan(ic+idx)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{a-b\tan(ic+idx)^2}{\tan(ic+idx)^2}dx$$

$$\downarrow 4112$$

$$-\int(-a-b)dx-\frac{a\coth(c+dx)}{d}$$

$$\downarrow 24$$

$$x(a+b)-\frac{a\coth(c+dx)}{d}$$

input
```
Int[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a + b)*x - (a*Coth[c + d*x])/d
```

3.140.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4112

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[
e + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[
a^2 + b^2, 0]
```

3.140.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

method	result	size
risch	$ax+bx-\frac{2a}{d(\mathrm{e}^{2dx+2c}-1)}$	27
parallelrisch	$\frac{-a+\tanh(dx+c)dx(a+b)}{d\tanh(dx+c)}$	29
derivativedivides	$\frac{-\frac{a}{\tanh(dx+c)}+\left(-\frac{b}{2}-\frac{a}{2}\right)\ln(\tanh(dx+c)-1)+\left(\frac{b}{2}+\frac{a}{2}\right)\ln(\tanh(dx+c)+1)}{d}$	51
default	$\frac{-\frac{a}{\tanh(dx+c)}+\left(-\frac{b}{2}-\frac{a}{2}\right)\ln(\tanh(dx+c)-1)+\left(\frac{b}{2}+\frac{a}{2}\right)\ln(\tanh(dx+c)+1)}{d}$	51

input

```
int(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output

```
a*x+b*x-2*a/d/(exp(2*d*x+2*c)-1)
```

3.140.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(18) = 36$.

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{a\cosh(dx+c)-((a+b)dx+a)\sinh(dx+c)}{d\sinh(dx+c)}$$

input

```
integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output

```
-(a*cosh(d*x + c) - ((a + b)*d*x + a)*sinh(d*x + c))/(d*sinh(d*x + c))
```

3.140.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(14) = 28$.

Time $= 4.27$ (sec) , antiderivative size $= 90$, normalized size of antiderivative $= 5.00$

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= a\left(\begin{cases} x\coth^2(c) & \text{for } d=0 \\ -\frac{\log\left(-e^{-dx}\right)\coth^2\left(dx+\log\left(-e^{-dx}\right)\right)}{d} & \text{for } c=\log\left(-e^{-dx}\right) \\ x\coth^2\left(dx+\log\left(e^{-dx}\right)\right) & \text{for } c=\log\left(e^{-dx}\right) \\ x-\frac{1}{d\tanh(c+dx)} & \text{otherwise} \end{cases}\right)$$
$$+ b\left(\begin{cases} x & \text{for } |x|<1 \\ G_{2,2}^{1,1}\left(\begin{matrix}1 & 2\\ 1 & 0\end{matrix}\middle| x\right)+G_{2,2}^{0,2}\left(\begin{matrix}2,1 & \\ & 1,0\end{matrix}\middle| x\right) & \text{otherwise} \end{cases}\right)$$

input
```
integrate(coth(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

output
```
a*Piecewise((x*coth(c)**2, Eq(d, 0)), (-log(-exp(-d*x))*coth(d*x + log(-ex
p(-d*x)))**2/d, Eq(c, log(-exp(-d*x)))), (x*coth(d*x + log(exp(-d*x)))**2,
 Eq(c, log(exp(-d*x)))), (x - 1/(d*tanh(c + d*x)), True)) + b*Piecewise((x
, Abs(x) < 1), (meijerg(((1,), (2,)), ((1,), (0,)), x) + meijerg(((2, 1),
()), ((), (1, 0)), x), True))
```

3.140.7 Maxima [A] (verification not implemented)

Time $= 0.20$ (sec) , antiderivative size $= 31$, normalized size of antiderivative $= 1.72$

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = a\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2\,dx-2\,c)}-1)}\right)+bx$$

input
```
integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
a*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + b*x
```

3.140.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(dx+c)(a+b)-\frac{2a}{e^{(2dx+2c)}-1}}{d}$$

input
```
integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
((d*x + c)*(a + b) - 2*a/(e^(2*d*x + 2*c) - 1))/d
```

3.140.9 Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = x\,(a+b)-\frac{2\,a}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}$$

input
```
int(coth(c + d*x)^2*(a + b*tanh(c + d*x)^2),x)
```

output
```
x*(a + b) - (2*a)/(d*(exp(2*c + 2*d*x) - 1))
```

3.141 $\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

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3.141.1 Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{a\coth^2(c+dx)}{2d}+\frac{(a+b)\log(\sinh(c+dx))}{d}$$

output
```
-1/2*a*coth(d*x+c)^2/d+(a+b)*ln(sinh(d*x+c))/d
```

3.141.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{-a\coth^2(c+dx)+2(a+b)(\log(\cosh(c+dx))+\log(\tanh(c+dx)))}{2d}$$

input
```
Integrate[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-(a*Coth[c + d*x]^2) + 2*(a + b)*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]]
))/(2*d)
```

3.141.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 26, 4112, 26, 27, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i(a-b\tan(ic+idx)^2)}{\tan(ic+idx)^3}dx$$

$$\downarrow 26$$

$$-i\int \frac{a-b\tan(ic+idx)^2}{\tan(ic+idx)^3}dx$$

$$\downarrow 4112$$

$$-i\left(\int i(a+b)\coth(c+dx)dx-\frac{ia\coth^2(c+dx)}{2d}\right)$$

$$\downarrow 26$$

$$-i\left(i\int (a+b)\coth(c+dx)dx-\frac{ia\coth^2(c+dx)}{2d}\right)$$

$$\downarrow 27$$

$$-i\left(i(a+b)\int \coth(c+dx)dx-\frac{ia\coth^2(c+dx)}{2d}\right)$$

$$\downarrow 3042$$

$$-i\left(i(a+b)\int -i\tan\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{ia\coth^2(c+dx)}{2d}\right)$$

$$\downarrow 26$$

$$-i\left((a+b)\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx-\frac{ia\coth^2(c+dx)}{2d}\right)$$

$$\downarrow 3956$$

$$-i\left(\frac{i(a+b)\log(-i\sinh(c+dx))}{d}-\frac{ia\coth^2(c+dx)}{2d}\right)$$

input
```
Int[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-I)*(((-1/2*I)*a*Coth[c + d*x]^2)/d + (I*(a + b)*Log[(-I)*Sinh[c + d*x]])
/d)
```

3.141.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4112
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[
e + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[
a^2 + b^2, 0]
```

3.141.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

method	result	size
derivativedivides	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}\right)+b\ln(\sinh(dx+c))}{d}$	35
default	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}\right)+b\ln(\sinh(dx+c))}{d}$	35
parallelrisch	$\frac{(-2a-2b)\ln(1-\tanh(dx+c))+(2a+2b)\ln(\tanh(dx+c))-\coth(dx+c)^2a-2(a+b)xd}{2d}$	59
risch	$-ax-bx-\frac{2ac}{d}-\frac{2bc}{d}-\frac{2a\,\mathrm{e}^{2dx+2c}}{d\left(\mathrm{e}^{2dx+2c}-1\right)^2}+\frac{a\ln\left(\mathrm{e}^{2dx+2c}-1\right)}{d}+\frac{\ln\left(\mathrm{e}^{2dx+2c}-1\right)b}{d}$	86

input
```
int(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2)+b*ln(sinh(d*x+c)))
```

3.141.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(29) = 58.

Time = 0.27 (sec) , antiderivative size = 407, normalized size of antiderivative = 13.13

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx =$$

$$-\frac{(a+b)dx\cosh(dx+c)^4+4(a+b)dx\cosh(dx+c)\sinh(dx+c)^3+(a+b)dx\sinh(dx+c)^4+(a+b)$$

input
```
integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-((a + b)*d*x*cosh(d*x + c)^4 + 4*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^
3 + (a + b)*d*x*sinh(d*x + c)^4 + (a + b)*d*x - 2*((a + b)*d*x - a)*cosh(d
*x + c)^2 + 2*(3*(a + b)*d*x*cosh(d*x + c)^2 - (a + b)*d*x + a)*sinh(d*x +
 c)^2 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3
 + (a + b)*sinh(d*x + c)^4 - 2*(a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh
(d*x + c)^2 - a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (a + b
)*cosh(d*x + c))*sinh(d*x + c) + a + b)*log(2*sinh(d*x + c)/(cosh(d*x + c)
 - sinh(d*x + c))) + 4*((a + b)*d*x*cosh(d*x + c)^3 - ((a + b)*d*x - a)*co
sh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*
x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^
2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x
+ c) + d)
```

3.141.6 Sympy [F]

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)\coth^3\left(c+dx\right)dx$$

input
```
integrate(coth(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*coth(c + d*x)**3, x)
```

3.141.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(29) = 58$.

Time $= 0.20$ (sec) , antiderivative size $= 106$, normalized size of antiderivative $= 3.42$

$$\begin{aligned}&\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx \\ &= a\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right) \\ &\quad+\frac{b\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}\end{aligned}$$

input
```
integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
a*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d
*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b*log(e^(d*x
+ c) - e^(-d*x - c))/d
```

3.141.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{(dx+c)(a+b)-(a+b)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{2\,ae^{(2\,dx+2\,c)}}{\left(e^{(2\,dx+2\,c)}-1\right)^2}}{d}$$

input
```
integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-((d*x + c)*(a + b) - (a + b)*log(abs(e^(2*d*x + 2*c) - 1)) + 2*a*e^(2*d*x
 + 2*c)/(e^(2*d*x + 2*c) - 1)^2)/d
```

3.141.9 Mupad [B] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.45

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{\ln\left(\mathrm{e}^{2c}\,\mathrm{e}^{2dx}-1\right)\,(a+b)}{d}-\frac{2\,a}{d\,\left(\mathrm{e}^{2c+2dx}-1\right)}-\frac{2\,a}{d\,\left(\mathrm{e}^{4c+4dx}-2\,\mathrm{e}^{2c+2dx}+1\right)}-x\,(a+b)$$

input
```
int(coth(c + d*x)^3*(a + b*tanh(c + d*x)^2),x)
```

output
```
(log(exp(2*c)*exp(2*d*x) - 1)*(a + b))/d - (2*a)/(d*(exp(2*c + 2*d*x) - 1)
) - (2*a)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - x*(a + b)
```

3.142 $\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

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3.142.8 Giac [B] (verification not implemented) . . . 1076
3.142.9 Mupad [B] (verification not implemented) . . . 1077

3.142.1 Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = (a+b)x - \frac{(a+b)\coth(c+dx)}{d} - \frac{a\coth^3(c+dx)}{3d}$$

output
```
(a+b)*x-(a+b)*coth(d*x+c)/d-1/3*a*coth(d*x+c)^3/d
```

3.142.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.69

$$\begin{aligned}&\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx \\ &= -\frac{a\coth^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\tanh^2(c+dx)\right)}{3d} \\ &\quad -\frac{b\coth(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(c+dx)\right)}{d}\end{aligned}$$

input
```
Integrate[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-1/3*(a*Coth[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[c + d*x]^2])
/d - (b*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2])/d
```

3.142.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4112, 27, 3042, 25, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{a-b\tan(ic+idx)^2}{\tan(ic+idx)^4}dx$$

$$\downarrow 4112$$

$$\int (a+b)\coth^2(c+dx)dx - \frac{a\coth^3(c+dx)}{3d}$$

$$\downarrow 27$$

$$(a+b)\int \coth^2(c+dx)dx - \frac{a\coth^3(c+dx)}{3d}$$

$$\downarrow 3042$$

$$-\frac{a\coth^3(c+dx)}{3d} + (a+b)\int -\tan\left(ic+idx+\frac{\pi}{2}\right)^2 dx$$

$$\downarrow 25$$

$$-\frac{a\coth^3(c+dx)}{3d} - (a+b)\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2 dx$$

$$\downarrow 3954$$

$$-(a+b)\left(\frac{\coth(c+dx)}{d} - \int 1dx\right) - \frac{a\coth^3(c+dx)}{3d}$$

$$\downarrow 24$$

$$-(a+b)\left(\frac{\coth(c+dx)}{d} - x\right) - \frac{a\coth^3(c+dx)}{3d}$$

input
```
Int[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-1/3*(a*Coth[c + d*x]^3)/d - (a + b)*(-x + Coth[c + d*x]/d)
```

3.142.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4112
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[
e + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[
a^2 + b^2, 0]
```

3.142.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

method	result	size
parallelrisch	$\frac{-\coth(dx+c)^3a+(-3a-3b)\coth(dx+c)+3(a+b)xd}{3d}$	39
derivativedivides	$\frac{a\left(dx+c-\coth(dx+c)-\frac{\coth(dx+c)^3}{3}\right)+b(dx+c-\coth(dx+c))}{d}$	46
default	$\frac{a\left(dx+c-\coth(dx+c)-\frac{\coth(dx+c)^3}{3}\right)+b(dx+c-\coth(dx+c))}{d}$	46
risch	$ax+bx-\frac{2(6a\,e^{4dx+4c}+3b\,e^{4dx+4c}-6\,e^{2dx+2c}a-6b\,e^{2dx+2c}+4a+3b)}{3d(e^{2dx+2c}-1)^3}$	81

input
```
int(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-coth(d*x+c)^3*a+(-3*a-3*b)*coth(d*x+c)+3*(a+b)*x*d)/d
```

3.142.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(34) = 68$.

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 4.33

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx =$$

$$-\frac{(4\,a+3\,b)\cosh(dx+c)^3+3\,(4\,a+3\,b)\cosh(dx+c)\sinh(dx+c)^2-(3\,(a+b)dx+4\,a+3\,b)\sinh(dx$$
$$3\left(d\sinh(dx+c)^3+3\left(d\,c\right.\right.$$

input
```
integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/3*((4*a + 3*b)*cosh(d*x + c)^3 + 3*(4*a + 3*b)*cosh(d*x + c)*sinh(d*x +
 c)^2 - (3*(a + b)*d*x + 4*a + 3*b)*sinh(d*x + c)^3 - 3*b*cosh(d*x + c) +
3*(3*(a + b)*d*x - (3*(a + b)*d*x + 4*a + 3*b)*cosh(d*x + c)^2 + 4*a + 3*b
)*sinh(d*x + c))/(d*sinh(d*x + c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x +
 c))
```

3.142.6 Sympy [F]

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2(c+dx)\right)\coth^4(c+dx)\,dx$$

input
```
integrate(coth(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*coth(c + d*x)**4, x)
```

3.142.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(34) = 68$.

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.92

$$\begin{aligned}&\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx\\ &= \frac{1}{3}a\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right)\\ &\quad + b\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2dx-2c)}-1)}\right)\end{aligned}$$

input
```
integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/3*a*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(3
*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + b*(x +
c/d + 2/(d*(e^(-2*d*x - 2*c) - 1)))
```

3.142.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(34) = 68$.

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.39

$$\begin{aligned}&\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx\\ &= \frac{3\,(dx+c)(a+b)-\frac{2\left(6ae^{(4dx+4c)}+3be^{(4dx+4c)}-6ae^{(2dx+2c)}-6be^{(2dx+2c)}+4a+3b\right)}{\left(e^{(2dx+2c)}-1\right)^3}}{3d}\end{aligned}$$

input
```
integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*(a + b) - 2*(6*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) -
6*a*e^(2*d*x + 2*c) - 6*b*e^(2*d*x + 2*c) + 4*a + 3*b)/(e^(2*d*x + 2*c) -
1)^3)/d
```

3.142.9 Mupad [B] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 162, normalized size of antiderivative = 4.50

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{\frac{2b}{3d}-\frac{2\,\mathrm{e}^{2c+2dx}(2a+b)}{3d}}{\mathrm{e}^{4c+4dx}-2\,\mathrm{e}^{2c+2dx}+1} - \frac{\frac{2(2a+b)}{3d}-\frac{4b\,\mathrm{e}^{2c+2dx}}{3d}+\frac{2\,\mathrm{e}^{4c+4dx}(2a+b)}{3d}}{3\,\mathrm{e}^{2c+2dx}-3\,\mathrm{e}^{4c+4dx}+\mathrm{e}^{6c+6dx}-1} + x\,(a+b) - \frac{2\,(2\,a+b)}{3\,d\,(\mathrm{e}^{2c+2dx}-1)}$$

input
```
int(coth(c + d*x)^4*(a + b*tanh(c + d*x)^2),x)
```

output
```
((2*b)/(3*d) - (2*exp(2*c + 2*d*x)*(2*a + b))/(3*d))/(exp(4*c + 4*d*x) - 2
*exp(2*c + 2*d*x) + 1) - ((2*(2*a + b))/(3*d) - (4*b*exp(2*c + 2*d*x))/(3*
d) + (2*exp(4*c + 4*d*x)*(2*a + b))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c
 + 4*d*x) + exp(6*c + 6*d*x) - 1) + x*(a + b) - (2*(2*a + b))/(3*d*(exp(2*
c + 2*d*x) - 1))
```

3.143 $\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

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3.143.2 Mathematica [A] (verified) 1078
3.143.3 Rubi [C] (verified) 1079
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3.143.8 Giac [B] (verification not implemented) 1083
3.143.9 Mupad [B] (verification not implemented) 1084

3.143.1 Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{(a+b)\coth^2(c+dx)}{2d} - \frac{a\coth^4(c+dx)}{4d} + \frac{(a+b)\log(\sinh(c+dx))}{d}$$

output
```
-1/2*(a+b)*coth(d*x+c)^2/d-1/4*a*coth(d*x+c)^4/d+(a+b)*ln(sinh(d*x+c))/d
```

3.143.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{2(a+b)\coth^2(c+dx)+a\coth^4(c+dx)-4(a+b)(\log(\cosh(c+dx))+\log(\tanh(c+dx)))}{4d}$$

input
```
Integrate[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-1/4*(2*(a + b)*Coth[c + d*x]^2 + a*Coth[c + d*x]^4 - 4*(a + b)*(Log[Cosh[
c + d*x]] + Log[Tanh[c + d*x]]))/d
```

3.143.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 26, 4112, 26, 27, 3042, 26, 3954, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\downarrow 3042$$

$$\int \frac{i\left(a-b\tan(ic+idx)^2\right)}{\tan(ic+idx)^5}dx$$

$$\downarrow 26$$

$$i\int \frac{a-b\tan(ic+idx)^2}{\tan(ic+idx)^5}dx$$

$$\downarrow 4112$$

$$i\left(\int -i(a+b)\coth^3(c+dx)dx+\frac{ia\coth^4(c+dx)}{4d}\right)$$

$$\downarrow 26$$

$$i\left(\frac{ia\coth^4(c+dx)}{4d}-i\int (a+b)\coth^3(c+dx)dx\right)$$

$$\downarrow 27$$

$$i\left(\frac{ia\coth^4(c+dx)}{4d}-i(a+b)\int \coth^3(c+dx)dx\right)$$

$$\downarrow 3042$$

$$i\left(\frac{ia\coth^4(c+dx)}{4d}-i(a+b)\int i\tan\left(ic+idx+\frac{\pi}{2}\right)^3 dx\right)$$

$$\downarrow 26$$

$$i\left((a+b)\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^3 dx+\frac{ia\coth^4(c+dx)}{4d}\right)$$

$$\downarrow 3954$$

$$i\left((a+b)\left(\frac{i\coth^2(c+dx)}{2d}-\int i\coth(c+dx)dx\right)+\frac{ia\coth^4(c+dx)}{4d}\right)$$

$$\downarrow 26$$

$$i\left((a+b)\left(\frac{i\coth^2(c+dx)}{2d}-i\int \coth(c+dx)dx\right)+\frac{ia\coth^4(c+dx)}{4d}\right)$$

$$\downarrow 3042$$

$$i\left((a+b)\left(\frac{i\coth^2(c+dx)}{2d}-i\int -i\tan\left(ic+idx+\frac{\pi}{2}\right)dx\right)+\frac{ia\coth^4(c+dx)}{4d}\right)$$

$$\downarrow 26$$

$$i\left((a+b)\left(\frac{i\coth^2(c+dx)}{2d}-\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx\right)+\frac{ia\coth^4(c+dx)}{4d}\right)$$

$$\downarrow 3956$$

$$i\left((a+b)\left(\frac{i\coth^2(c+dx)}{2d}-\frac{i\log(-i\sinh(c+dx))}{d}\right)+\frac{ia\coth^4(c+dx)}{4d}\right)$$

input
```
Int[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2),x]
```

output
```
I*(((I/4)*a*Coth[c + d*x]^4)/d + (a + b)*(((I/2)*Coth[c + d*x]^2)/d - (I*L
og[(-I)*Sinh[c + d*x]])/d))
```

3.143.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4112
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[
e + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[
a^2 + b^2, 0]
```

3.143.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}-\frac{\coth(dx+c)^4}{4}\right)+b\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}\right)}{d}$
default	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}-\frac{\coth(dx+c)^4}{4}\right)+b\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}\right)}{d}$
parallelrisch	$\frac{(-4a-4b)\ln(1-\tanh(dx+c))+(4a+4b)\ln(\tanh(dx+c))-\coth(dx+c)^4a+(-2a-2b)\coth(dx+c)^2-4(a+b)xd}{4d}$
risch	$-ax-bx-\frac{2ac}{d}-\frac{2bc}{d}-\frac{2\,\mathrm{e}^{2dx+2c}\left(2a\,\mathrm{e}^{4dx+4c}+b\,\mathrm{e}^{4dx+4c}-2\,\mathrm{e}^{2dx+2c}a-2b\,\mathrm{e}^{2dx+2c}+2a+b\right)}{d\left(\mathrm{e}^{2dx+2c}-1\right)^4}+\frac{a\ln\left(\mathrm{e}^{2dx+2c}-1\right.}{d}$

input
```
int(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2-1/4*coth(d*x+c)^4)+b*(ln(sinh(d*
x+c))-1/2*coth(d*x+c)^2))
```

3.143.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1216 vs. 2(45) = 90.

Time = 0.27 (sec) , antiderivative size = 1216, normalized size of antiderivative = 24.82

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-((a + b)*d*x*cosh(d*x + c)^8 + 8*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^
7 + (a + b)*d*x*sinh(d*x + c)^8 - 2*(2*(a + b)*d*x - 2*a - b)*cosh(d*x + c
)^6 + 2*(14*(a + b)*d*x*cosh(d*x + c)^2 - 2*(a + b)*d*x + 2*a + b)*sinh(d*
x + c)^6 + 4*(14*(a + b)*d*x*cosh(d*x + c)^3 - 3*(2*(a + b)*d*x - 2*a - b)
*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*(a + b)*d*x - 2*a - 2*b)*cosh(d*x +
 c)^4 + 2*(35*(a + b)*d*x*cosh(d*x + c)^4 + 3*(a + b)*d*x - 15*(2*(a + b)*
d*x - 2*a - b)*cosh(d*x + c)^2 - 2*a - 2*b)*sinh(d*x + c)^4 + 8*(7*(a + b)
*d*x*cosh(d*x + c)^5 - 5*(2*(a + b)*d*x - 2*a - b)*cosh(d*x + c)^3 + (3*(a
 + b)*d*x - 2*a - 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 + (a + b)*d*x - 2*(2
*(a + b)*d*x - 2*a - b)*cosh(d*x + c)^2 + 2*(14*(a + b)*d*x*cosh(d*x + c)^
6 - 15*(2*(a + b)*d*x - 2*a - b)*cosh(d*x + c)^4 - 2*(a + b)*d*x + 6*(3*(a
 + b)*d*x - 2*a - 2*b)*cosh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)^2 - ((a +
b)*cosh(d*x + c)^8 + 8*(a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a + b)*sin
h(d*x + c)^8 - 4*(a + b)*cosh(d*x + c)^6 + 4*(7*(a + b)*cosh(d*x + c)^2 -
a - b)*sinh(d*x + c)^6 + 8*(7*(a + b)*cosh(d*x + c)^3 - 3*(a + b)*cosh(d*x
 + c))*sinh(d*x + c)^5 + 6*(a + b)*cosh(d*x + c)^4 + 2*(35*(a + b)*cosh(d*
x + c)^4 - 30*(a + b)*cosh(d*x + c)^2 + 3*a + 3*b)*sinh(d*x + c)^4 + 8*(7*
(a + b)*cosh(d*x + c)^5 - 10*(a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x
+ c))*sinh(d*x + c)^3 - 4*(a + b)*cosh(d*x + c)^2 + 4*(7*(a + b)*cosh(d*x
+ c)^6 - 15*(a + b)*cosh(d*x + c)^4 + 9*(a + b)*cosh(d*x + c)^2 - a - b...
```

3.143.6 Sympy [F]

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)\coth^5\left(c+dx\right)dx$$

input
```
integrate(coth(d*x+c)**5*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*coth(c + d*x)**5, x)
```

3.143.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(45) = 90$.

Time = 0.19 (sec) , antiderivative size = 206, normalized size of antiderivative = 4.20

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= a\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8\,d}}\right.$$
$$\left.+\,b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)\right.$$

input
```
integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
a*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2*
d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) -
 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + b*(x
+ c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x -
2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

3.143.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(45) = 90$.

Time = 0.34 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.90

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx =$$
$$-\frac{(dx+c)(a+b)-(a+b)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{2\left((2\,a+b)e^{(6\,dx+6\,c)}-2\,(a+b)e^{(4\,dx+4\,c)}+(2\,a+b)e^{(2\,dx+2\,c)}\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^4}}{d}$$

input
```
integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-((d*x + c)*(a + b) - (a + b)*log(abs(e^(2*d*x + 2*c) - 1)) + 2*((2*a + b)
*e^(6*d*x + 6*c) - 2*(a + b)*e^(4*d*x + 4*c) + (2*a + b)*e^(2*d*x + 2*c))/
(e^(2*d*x + 2*c) - 1)^4)/d
```

3.143.9 Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.61

$$\begin{aligned}
&\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx \\
&= \frac{\ln\left(\mathrm{e}^{2c}\,\mathrm{e}^{2dx}-1\right)\,(a+b)}{d} - \frac{2\,(2a+b)}{d\,\left(\mathrm{e}^{2c+2dx}-1\right)} - x\,(a+b) \\
&\quad - \frac{2\,(4a+b)}{d\,\left(\mathrm{e}^{4c+4dx}-2\,\mathrm{e}^{2c+2dx}+1\right)} - \frac{8a}{d\,\left(3\,\mathrm{e}^{2c+2dx}-3\,\mathrm{e}^{4c+4dx}+\mathrm{e}^{6c+6dx}-1\right)} \\
&\quad - \frac{4a}{d\,\left(6\,\mathrm{e}^{4c+4dx}-4\,\mathrm{e}^{2c+2dx}-4\,\mathrm{e}^{6c+6dx}+\mathrm{e}^{8c+8dx}+1\right)}
\end{aligned}$$

input
```
int(coth(c + d*x)^5*(a + b*tanh(c + d*x)^2),x)
```

output
```
(log(exp(2*c)*exp(2*d*x) - 1)*(a + b))/d - (2*(2*a + b))/(d*(exp(2*c + 2*d
*x) - 1)) - x*(a + b) - (2*(4*a + b))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2
*d*x) + 1)) - (8*a)/(d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c
+ 6*d*x) - 1)) - (4*a)/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp
(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1))
```

3.144 $\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

3.144.1 Optimal result 1085
3.144.2 Mathematica [B] (verified) 1086
3.144.3 Rubi [A] (verified) 1086
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3.144.5 Fricas [B] (verification not implemented) 1088
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3.144.7 Maxima [B] (verification not implemented) 1090
3.144.8 Giac [B] (verification not implemented) 1091
3.144.9 Mupad [B] (verification not implemented) 1091

3.144.1 Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = (a+b)^2x - \frac{(a+b)^2\tanh(c+dx)}{d} - \frac{(a+b)^2\tanh^3(c+dx)}{3d} - \frac{b(2a+b)\tanh^5(c+dx)}{5d} - \frac{b^2\tanh^7(c+dx)}{7d}$$

output

```
(a+b)^2*x-(a+b)^2*tanh(d*x+c)/d-1/3*(a+b)^2*tanh(d*x+c)^3/d-1/5*b*(2*a+b)*
tanh(d*x+c)^5/d-1/7*b^2*tanh(d*x+c)^7/d
```

3.144.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 190 vs. $2(83) = 166$.

Time $= 0.10$ (sec) , antiderivative size $= 190$, normalized size of antiderivative $= 2.29$

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{a^2\text{arctanh}(\tanh(c+dx))}{d} + \frac{2ab\text{arctanh}(\tanh(c+dx))}{d} + \frac{b^2\text{arctanh}(\tanh(c+dx))}{d} - \frac{a^2\tanh(c+dx)}{d} - \frac{2ab\tanh(c+dx)}{d} - \frac{b^2\tanh(c+dx)}{d} - \frac{a^2\tanh^3(c+dx)}{3d} - \frac{2ab\tanh^3(c+dx)}{3d} - \frac{b^2\tanh^3(c+dx)}{3d} - \frac{2ab\tanh^5(c+dx)}{5d} - \frac{b^2\tanh^5(c+dx)}{5d} - \frac{b^2\tanh^7(c+dx)}{7d}$$

input
```
Integrate[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(a^2*ArcTanh[Tanh[c + d*x]])/d + (2*a*b*ArcTanh[Tanh[c + d*x]])/d + (b^2*A
rcTanh[Tanh[c + d*x]])/d - (a^2*Tanh[c + d*x])/d - (2*a*b*Tanh[c + d*x])/d
 - (b^2*Tanh[c + d*x])/d - (a^2*Tanh[c + d*x]^3)/(3*d) - (2*a*b*Tanh[c + d
*x]^3)/(3*d) - (b^2*Tanh[c + d*x]^3)/(3*d) - (2*a*b*Tanh[c + d*x]^5)/(5*d)
 - (b^2*Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^7)/(7*d)
```

3.144.3 Rubi [A] (verified)

Time $= 0.30$ (sec) , antiderivative size $= 81$, normalized size of antiderivative $= 0.98$, number of steps used $= 5$, number of rules used $= 4$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used $= \{3042, 4153, 364, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$\downarrow \text{3042}$$

$$\int \tan(ic+idx)^4 \left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\downarrow 4153$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)} d\tanh(c+dx)}{d}$$

$$\downarrow 364$$

$$\frac{\int \left(-b^2\tanh^6(c+dx)-b(2a+b)\tanh^4(c+dx)-(a+b)^2\tanh^2(c+dx)-(a+b)^2+\frac{a^2+2ba+b^2}{1-\tanh^2(c+dx)}\right) d\tanh(c+$$

$$\downarrow 2009$$

$$\frac{(a+b)^2\text{arctanh}(\tanh(c+dx))-\frac{1}{5}b(2a+b)\tanh^5(c+dx)-\frac{1}{3}(a+b)^2\tanh^3(c+dx)-(a+b)^2\tanh(c+dx)-\frac{1}{7}$$

input
```
Int[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((a + b)^2*ArcTanh[Tanh[c + d*x]] - (a + b)^2*Tanh[c + d*x] - ((a + b)^2*T
anh[c + d*x]^3)/3 - (b*(2*a + b)*Tanh[c + d*x]^5)/5 - (b^2*Tanh[c + d*x]^7
)/7)/d
```

3.144.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.144.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.63

method	result
parallelrisch	$-\frac{15\tanh(dx+c)^7b^2+42\tanh(dx+c)^5ab+21\tanh(dx+c)^5b^2+35\tanh(dx+c)^3a^2+70\tanh(dx+c)^3ab+35b^2\tanh(dx+c)}{105d}$
derivativedivides	$\frac{-2ab\tanh(dx+c)-\frac{2\tanh(dx+c)^5ab}{5}-\frac{2\tanh(dx+c)^3ab}{3}-\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)-1)}{2}-\frac{\tanh(dx+c)^5b^2}{5}-\frac{\tanh(dx+c)^3a^2}{3}-}{d}$
default	$\frac{-2ab\tanh(dx+c)-\frac{2\tanh(dx+c)^5ab}{5}-\frac{2\tanh(dx+c)^3ab}{3}-\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)-1)}{2}-\frac{\tanh(dx+c)^5b^2}{5}-\frac{\tanh(dx+c)^3a^2}{3}-}{d}$
parts	$\frac{b^2\left(-\frac{\tanh(dx+c)^7}{7}-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{a^2\left(-\frac{\tanh(dx+c)}{3}\right.}{}$
risch	$a^2x+2abx+b^2x+\frac{4a^2\mathrm{e}^{12dx+12c}+12ab\,\mathrm{e}^{12dx+12c}+8b^2\mathrm{e}^{12dx+12c}+20a^2\mathrm{e}^{10dx+10c}+48ab\,\mathrm{e}^{10dx+10c}+24b^2\mathrm{e}^{10dx}}{}$

input

```
int(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output

```
-1/105*(15*tanh(d*x+c)^7*b^2+42*tanh(d*x+c)^5*a*b+21*tanh(d*x+c)^5*b^2+35*
tanh(d*x+c)^3*a^2+70*tanh(d*x+c)^3*a*b+35*b^2*tanh(d*x+c)^3-105*a^2*d*x-21
0*a*b*d*x-105*b^2*d*x+105*a^2*tanh(d*x+c)+210*a*b*tanh(d*x+c)+105*b^2*tanh
(d*x+c))/d
```

3.144.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 796 vs. 2(77) = 154.

Time = 0.26 (sec) , antiderivative size = 796, normalized size of antiderivative = 9.59

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$=\frac{\left(105\left(a^2+2ab+b^2\right)dx+140a^2+322ab+176b^2\right)\cosh(dx+c)^7+7\left(105\left(a^2+2ab+b^2\right)dx+140a^2+\right.}{}$$

input
```
integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/105*((105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*
x + c)^7 + 7*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*c
osh(d*x + c)*sinh(d*x + c)^6 - 2*(70*a^2 + 161*a*b + 88*b^2)*sinh(d*x + c)
^7 + 7*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*
x + c)^5 - 14*(3*(70*a^2 + 161*a*b + 88*b^2)*cosh(d*x + c)^2 + 40*a^2 + 71
*a*b + 28*b^2)*sinh(d*x + c)^5 + 35*((105*(a^2 + 2*a*b + b^2)*d*x + 140*a^
2 + 322*a*b + 176*b^2)*cosh(d*x + c)^3 + (105*(a^2 + 2*a*b + b^2)*d*x + 14
0*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 21*(105*(a^2 +
 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c)^3 - 14*(5*(
70*a^2 + 161*a*b + 88*b^2)*cosh(d*x + c)^4 + 10*(40*a^2 + 71*a*b + 28*b^2)
*cosh(d*x + c)^2 + 60*a^2 + 123*a*b + 84*b^2)*sinh(d*x + c)^3 + 7*(3*(105*
(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c)^5 + 1
0*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c
)^3 + 9*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d
*x + c))*sinh(d*x + c)^2 + 35*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322
*a*b + 176*b^2)*cosh(d*x + c) - 14*((70*a^2 + 161*a*b + 88*b^2)*cosh(d*x +
 c)^6 + 5*(40*a^2 + 71*a*b + 28*b^2)*cosh(d*x + c)^4 + 9*(20*a^2 + 41*a*b
+ 28*b^2)*cosh(d*x + c)^2 + 30*a^2 + 75*a*b)*sinh(d*x + c))/(d*cosh(d*x +
c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + 7*d*cosh(d*x + c)^5 + 35*(d*cos
h(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^4 + 21*d*cosh(d*x + c)^3 ...
```

3.144.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(70) = 140$.

Time $= 0.21$ (sec) , antiderivative size $= 165$, normalized size of antiderivative $= 1.99$

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$= \begin{cases} a^2x - \frac{a^2\tanh^3(c+dx)}{3d} - \frac{a^2\tanh(c+dx)}{d} + 2abx - \frac{2ab\tanh^5(c+dx)}{5d} - \frac{2ab\tanh^3(c+dx)}{3d} - \frac{2ab\tanh(c+dx)}{d} + b^2x - \frac{b^2\tanh^7}{7} \\ x\left(a+b\tanh^2(c)\right)^2\tanh^4(c) \end{cases}$$

input
```
integrate(tanh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Piecewise((a**2*x - a**2*tanh(c + d*x)**3/(3*d) - a**2*tanh(c + d*x)/d + 2
*a*b*x - 2*a*b*tanh(c + d*x)**5/(5*d) - 2*a*b*tanh(c + d*x)**3/(3*d) - 2*a
*b*tanh(c + d*x)/d + b**2*x - b**2*tanh(c + d*x)**7/(7*d) - b**2*tanh(c +
d*x)**5/(5*d) - b**2*tanh(c + d*x)**3/(3*d) - b**2*tanh(c + d*x)/d, Ne(d,
0)), (x*(a + b*tanh(c)**2)**2*tanh(c)**4, True))
```

3.144.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(77) = 154$.

Time $= 0.21$ (sec) , antiderivative size $= 369$, normalized size of antiderivative $= 4.45$

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$= \frac{1}{105}b^2\left(105\,x+\frac{105\,c}{d}-\frac{8\left(203\,e^{(-2\,dx-2\,c)}+609\,e^{(-4\,dx-4\,c)}+770\,e^{(-6\,dx-6\,c)}+770\,e^{(-8\,dx-8\,c)}+315\,e^{(-10}\right.\right.$$

$$\left.\left.+\frac{2}{15}ab\left(15\,x+\frac{15\,c}{d}-\frac{2\left(70\,e^{(-2\,dx-2\,c)}+140\,e^{(-4\,dx-4\,c)}+90\,e^{(-6\,dx-6\,c)}+45\,e^{(-8\,dx-8\,c)}+23\right)}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}\right)\right.$$

$$\left.+\frac{1}{3}a^2\left(3\,x+\frac{3\,c}{d}-\frac{4\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+2\right)}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)\right.$$

input
```
integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/105*b^2*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d*x - 4*c
) + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x - 10*c) +
 105*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
 + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e
^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 2/15*a*b*(15*x + 15*c/d -
2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c) + 45*e
^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^
(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 1/3*a^2*
(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2
*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

3.144.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(77) = 154$.

Time $= 0.38$ (sec) , antiderivative size $= 300$, normalized size of antiderivative $= 3.61$

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{105\,(a^2+2\,ab+b^2)(dx+c)+\frac{4\,(105\,a^2e^{(12\,dx+12\,c)}+315\,abe^{(12\,dx+12\,c)}+210\,b^2e^{(12\,dx+12\,c)}+525\,a^2e^{(10\,dx+10\,c)}+1260\,abe^{(10\,dx+10}}{}$$

input
```
integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/105*(105*(a^2 + 2*a*b + b^2)*(d*x + c) + 4*(105*a^2*e^(12*d*x + 12*c) +
315*a*b*e^(12*d*x + 12*c) + 210*b^2*e^(12*d*x + 12*c) + 525*a^2*e^(10*d*x
+ 10*c) + 1260*a*b*e^(10*d*x + 10*c) + 630*b^2*e^(10*d*x + 10*c) + 1120*a^
2*e^(8*d*x + 8*c) + 2555*a*b*e^(8*d*x + 8*c) + 1540*b^2*e^(8*d*x + 8*c) +
1330*a^2*e^(6*d*x + 6*c) + 3080*a*b*e^(6*d*x + 6*c) + 1540*b^2*e^(6*d*x +
6*c) + 945*a^2*e^(4*d*x + 4*c) + 2121*a*b*e^(4*d*x + 4*c) + 1218*b^2*e^(4*
d*x + 4*c) + 385*a^2*e^(2*d*x + 2*c) + 812*a*b*e^(2*d*x + 2*c) + 406*b^2*e
^(2*d*x + 2*c) + 70*a^2 + 161*a*b + 88*b^2)/(e^(2*d*x + 2*c) + 1)^7)/d
```

3.144.9 Mupad [B] (verification not implemented)

Time $= 0.17$ (sec) , antiderivative size $= 91$, normalized size of antiderivative $= 1.10$

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = x\left(a^2+2\,a\,b+b^2\right) - \frac{\tanh(c+d\,x)\,(a+b)^2}{d}$$
$$- \frac{\tanh(c+d\,x)^5\,(b^2+2\,a\,b)}{5\,d}$$
$$- \frac{b^2\tanh(c+d\,x)^7}{7\,d}$$
$$- \frac{\tanh(c+d\,x)^3\,(a^2+2\,a\,b+b^2)}{3\,d}$$

input
```
int(tanh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
x*(2*a*b + a^2 + b^2) - (tanh(c + d*x)*(a + b)^2)/d - (tanh(c + d*x)^5*(2*
a*b + b^2))/(5*d) - (b^2*tanh(c + d*x)^7)/(7*d) - (tanh(c + d*x)^3*(2*a*b
+ a^2 + b^2))/(3*d)
```

3.145 $\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

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3.145.1 Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{(a+b)^2\log(\cosh(c+dx))}{d} - \frac{(a+b)^2\tanh^2(c+dx)}{2d} - \frac{b(2a+b)\tanh^4(c+dx)}{4d} - \frac{b^2\tanh^6(c+dx)}{6d}$$

output
```
(a+b)^2*ln(cosh(d*x+c))/d-1/2*(a+b)^2*tanh(d*x+c)^2/d-1/4*b*(2*a+b)*tanh(d
*x+c)^4/d-1/6*b^2*tanh(d*x+c)^6/d
```

3.145.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{-12(a+b)^2\log(\cosh(c+dx)) + 6(a+b)^2\tanh^2(c+dx) + 3b(2a+b)\tanh^4(c+dx) + 2b^2\tanh^6(c+d}{12d}$$

input
```
Integrate[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-1/12*(-12*(a + b)^2*Log[Cosh[c + d*x]] + 6*(a + b)^2*Tanh[c + d*x]^2 + 3*
b*(2*a + b)*Tanh[c + d*x]^4 + 2*b^2*Tanh[c + d*x]^6)/d
```

3.145.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int i\tan(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\downarrow 26$$

$$i\int \tan(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\downarrow 4153$$

$$\frac{i\int -\frac{i\tanh^3(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\tanh^3(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 354$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{2d}$$

$$\downarrow 86$$

$$\frac{\int \left(-b^2\tanh^4(c+dx)-b(2a+b)\tanh^2(c+dx)-(a+b)^2-\frac{(a+b)^2}{\tanh^2(c+dx)-1}\right)d\tanh^2(c+dx)}{2d}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{2}b(2a+b)\tanh^4(c+dx)-(a+b)^2\tanh^2(c+dx)-(a+b)^2\log\left(1-\tanh^2(c+dx)\right)-\frac{1}{3}b^2\tanh^6(c+dx)}{2d}$$

input
```
Int[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-((a + b)^2*Log[1 - Tanh[c + d*x]^2]) - (a + b)^2*Tanh[c + d*x]^2 - (b*(2
*a + b)*Tanh[c + d*x]^4)/2 - (b^2*Tanh[c + d*x]^6)/3)/(2*d)
```

3.145.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.145.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.71

method	result
derivativedivides	$\frac{-\frac{\tanh(dx+c)^4 ab}{2}-\tanh(dx+c)^2 ab-\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)-1)}{2}-\frac{\tanh(dx+c)^4 b^2}{4}-\frac{a^2\tanh(dx+c)^2}{2}-\frac{b^2\tanh(dx+c)^2}{2}-\tan}{d}$
default	$\frac{-\frac{\tanh(dx+c)^4 ab}{2}-\tanh(dx+c)^2 ab-\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)-1)}{2}-\frac{\tanh(dx+c)^4 b^2}{4}-\frac{a^2\tanh(dx+c)^2}{2}-\frac{b^2\tanh(dx+c)^2}{2}-\tan}{d}$
parallelrisch	$-\frac{2\tanh(dx+c)^6 b^2+6\tanh(dx+c)^4 ab+3\tanh(dx+c)^4 b^2+12a^2 dx+24abdx+12b^2 dx+6a^2\tanh(dx+c)^2+12\tanh(dx+c)}{12d}$
parts	$\frac{a^2\left(-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b^2\left(-\frac{\tanh(dx+c)^6}{6}-\frac{\tanh(dx+c)^4}{4}-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(d}{d}$
risch	$-a^2x-2abx-b^2x-\frac{2a^2c}{d}-\frac{4abc}{d}-\frac{2cb^2}{d}+\frac{2\mathrm{e}^{2dx+2c}\left(3a^2\mathrm{e}^{8dx+8c}+12ab\,\mathrm{e}^{8dx+8c}+9b^2\mathrm{e}^{8dx+8c}+12a^2\mathrm{e}^{6dx}\right.}{}$

input
```
int(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*tanh(d*x+c)^4*a*b-tanh(d*x+c)^2*a*b-1/2*(a^2+2*a*b+b^2)*ln(tanh(
d*x+c)-1)-1/4*tanh(d*x+c)^4*b^2-1/2*a^2*tanh(d*x+c)^2-1/2*b^2*tanh(d*x+c)^
2-1/6*tanh(d*x+c)^6*b^2+1/2*(-a^2-2*a*b-b^2)*ln(tanh(d*x+c)+1))
```

3.145.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3441 vs. 2(70) = 140.

Time = 0.30 (sec) , antiderivative size = 3441, normalized size of antiderivative = 45.28

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^12 + 36*(a^2 + 2*a*b + b^2)*
d*x*cosh(d*x + c)*sinh(d*x + c)^11 + 3*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x +
c)^12 + 6*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x + c)^
10 + 6*(33*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + 3*(a^2 + 2*a*b + b^2)
*d*x - a^2 - 4*a*b - 3*b^2)*sinh(d*x + c)^10 + 60*(11*(a^2 + 2*a*b + b^2)*
d*x*cosh(d*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*co
sh(d*x + c))*sinh(d*x + c)^9 + 3*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*
a*b - 12*b^2)*cosh(d*x + c)^8 + 3*(495*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x +
c)^4 + 15*(a^2 + 2*a*b + b^2)*d*x + 90*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 -
4*a*b - 3*b^2)*cosh(d*x + c)^2 - 8*a^2 - 24*a*b - 12*b^2)*sinh(d*x + c)^8
+ 24*(99*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^5 + 30*(3*(a^2 + 2*a*b + b^
2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x + c)^3 + (15*(a^2 + 2*a*b + b^2)*d*
x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(15*(a^2 +
 2*a*b + b^2)*d*x - 9*a^2 - 24*a*b - 17*b^2)*cosh(d*x + c)^6 + 4*(693*(a^2
 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 315*(3*(a^2 + 2*a*b + b^2)*d*x - a^2
 - 4*a*b - 3*b^2)*cosh(d*x + c)^4 + 15*(a^2 + 2*a*b + b^2)*d*x + 21*(15*(a
^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c)^2 - 9*a^2 -
 24*a*b - 17*b^2)*sinh(d*x + c)^6 + 24*(99*(a^2 + 2*a*b + b^2)*d*x*cosh(d*
x + c)^7 + 63*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x +
 c)^5 + 7*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d...
```

3.145.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(65) = 130$.

Time $= 0.19$ (sec) , antiderivative size $= 170$, normalized size of antiderivative $= 2.24$

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \begin{cases} a^2x - \frac{a^2\log(\tanh(c+dx)+1)}{d} - \frac{a^2\tanh^2(c+dx)}{2d} + 2abx - \frac{2ab\log(\tanh(c+dx)+1)}{d} - \frac{ab\tanh^4(c+dx)}{2d} - \frac{ab\tanh^2(c+dx)}{d} + b^2 \\ x\left(a+b\tanh^2(c)\right)^2\tanh^3(c) \end{cases}$$

input
```
integrate(tanh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Piecewise((a**2*x - a**2*log(tanh(c + d*x) + 1)/d - a**2*tanh(c + d*x)**2/
(2*d) + 2*a*b*x - 2*a*b*log(tanh(c + d*x) + 1)/d - a*b*tanh(c + d*x)**4/(2
*d) - a*b*tanh(c + d*x)**2/d + b**2*x - b**2*log(tanh(c + d*x) + 1)/d - b*
*2*tanh(c + d*x)**6/(6*d) - b**2*tanh(c + d*x)**4/(4*d) - b**2*tanh(c + d*
x)**2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)**2*tanh(c)**3, True))
```

3.145.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(70) = 140$.

Time = 0.28 (sec) , antiderivative size = 333, normalized size of antiderivative = 4.38

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$= \frac{1}{3}b^2\left(3x + \frac{3c}{d} + \frac{3\log\left(e^{(-2dx-2c)}+1\right)}{d} + \frac{2\left(9e^{(-2dx-2c)} + 18e^{(-4dx-4c)} + 34e^{(-6dx-6c)} + 18e^{(}\right.}{d(6e^{(-2dx-2c)} + 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} + 15e^{(-8dx-8c)} -}$$

$$+2ab\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2dx-2c)}+1\right)}{d} + \frac{4\left(e^{(-2dx-2c)} + e^{(-4dx-4c)} + e^{(-6dx-6c)}\right)}{d(4e^{(-2dx-2c)} + 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)} + e^{(-8dx-8c)} + 1)}\right)$$

$$+a^2\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2dx-2c)}+1\right)}{d} + \frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)}\right)$$

input
```
integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/3*b^2*(3*x + 3*c/d + 3*log(e^(-2*d*x - 2*c) + 1)/d + 2*(9*e^(-2*d*x - 2*
c) + 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) + 18*e^(-8*d*x - 8*c) + 9*e
^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6
*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12
*c) + 1))) + 2*a*b*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x -
 2*c) + e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^
(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + a^2*(x + c
/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*
c) + e^(-4*d*x - 4*c) + 1)))
```

3.145.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(70) = 140$.

Time = 0.38 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.51

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx =$$

$$-\frac{3\left(a^2+2ab+b^2\right)(dx+c) - 3\left(a^2+2ab+b^2\right)\log\left(e^{(2dx+2c)}+1\right) - \frac{2\left(3\left(a^2+4ab+3b^2\right)e^{(10dx+10c)}+6\left(2a^2+6ab+\right.\right.}{3d}$$

input
```
integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/3*(3*(a^2 + 2*a*b + b^2)*(d*x + c) - 3*(a^2 + 2*a*b + b^2)*log(e^(2*d*x
 + 2*c) + 1) - 2*(3*(a^2 + 4*a*b + 3*b^2)*e^(10*d*x + 10*c) + 6*(2*a^2 + 6
*a*b + 3*b^2)*e^(8*d*x + 8*c) + 2*(9*a^2 + 24*a*b + 17*b^2)*e^(6*d*x + 6*c
) + 6*(2*a^2 + 6*a*b + 3*b^2)*e^(4*d*x + 4*c) + 3*(a^2 + 4*a*b + 3*b^2)*e^
(2*d*x + 2*c))/(e^(2*d*x + 2*c) + 1)^6)/d
```

3.145.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.32

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = x\left(a^2+2ab+b^2\right) - \frac{\tanh(c+dx)^4\left(b^2+2ab\right)}{4d} - \frac{\ln\left(\tanh(c+dx)+1\right)\left(a^2+2ab+b^2\right)}{d} - \frac{b^2\tanh(c+dx)^6}{6d} - \frac{\tanh(c+dx)^2\left(a^2+2ab+b^2\right)}{2d}$$

input
```
int(tanh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
x*(2*a*b + a^2 + b^2) - (tanh(c + d*x)^4*(2*a*b + b^2))/(4*d) - (log(tanh(
c + d*x) + 1)*(2*a*b + a^2 + b^2))/d - (b^2*tanh(c + d*x)^6)/(6*d) - (tanh
(c + d*x)^2*(2*a*b + a^2 + b^2))/(2*d)
```

3.146 $\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

3.146.1 Optimal result 1099
3.146.2 Mathematica [B] (verified) 1099
3.146.3 Rubi [A] (verified) 1100
3.146.4 Maple [A] (verified) 1102
3.146.5 Fricas [B] (verification not implemented) 1102
3.146.6 Sympy [B] (verification not implemented) 1103
3.146.7 Maxima [B] (verification not implemented) 1104
3.146.8 Giac [B] (verification not implemented) 1104
3.146.9 Mupad [B] (verification not implemented) 1105

3.146.1 Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = (a+b)^2x - \frac{(a+b)^2\tanh(c+dx)}{d} - \frac{b(2a+b)\tanh^3(c+dx)}{3d} - \frac{b^2\tanh^5(c+dx)}{5d}$$

output
```
(a+b)^2*x-(a+b)^2*tanh(d*x+c)/d-1/3*b*(2*a+b)*tanh(d*x+c)^3/d-1/5*b^2*tanh
(d*x+c)^5/d
```

3.146.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 137 vs. 2(63) = 126.

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.17

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{a^2\text{arctanh}(\tanh(c+dx))}{d} + \frac{2ab\text{arctanh}(\tanh(c+dx))}{d} + \frac{b^2\text{arctanh}(\tanh(c+dx))}{d} - \frac{a^2\tanh(c+dx)}{d} - \frac{2ab\tanh(c+dx)}{d} - \frac{b^2\tanh(c+dx)}{d} - \frac{2ab\tanh^3(c+dx)}{3d} - \frac{b^2\tanh^3(c+dx)}{3d} - \frac{b^2\tanh^5(c+dx)}{5d}$$

input
```
Integrate[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(a^2*ArcTanh[Tanh[c + d*x]])/d + (2*a*b*ArcTanh[Tanh[c + d*x]])/d + (b^2*A
rcTanh[Tanh[c + d*x]])/d - (a^2*Tanh[c + d*x])/d - (2*a*b*Tanh[c + d*x])/d
 - (b^2*Tanh[c + d*x])/d - (2*a*b*Tanh[c + d*x]^3)/(3*d) - (b^2*Tanh[c + d
*x]^3)/(3*d) - (b^2*Tanh[c + d*x]^5)/(5*d)
```

3.146.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 25, 4153, 25, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int -\tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\downarrow 25$$

$$-\int \tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\downarrow 4153$$

$$-\frac{\int -\frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)} d\tanh(c+dx)}{d}$$

$$\downarrow 25$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)} d\tanh(c+dx)}{d}$$

$$\downarrow 364$$

$$\frac{\int \left(-b^2\tanh^4(c+dx)-b(2a+b)\tanh^2(c+dx)-(a+b)^2+\frac{a^2+2ba+b^2}{1-\tanh^2(c+dx)}\right) d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$-\frac{-(a+b)^2\operatorname{arctanh}(\tanh(c+dx))+\frac{1}{3}b(2a+b)\tanh^3(c+dx)+(a+b)^2\tanh(c+dx)+\frac{1}{5}b^2\tanh^5(c+dx)}{d}$$

input
```
Int[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-((-((a + b)^2*ArcTanh[Tanh[c + d*x]]) + (a + b)^2*Tanh[c + d*x] + (b*(2*a
 + b)*Tanh[c + d*x]^3)/3 + (b^2*Tanh[c + d*x]^5)/5)/d)
```

3.146.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.146.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.54

method	result
parallelrisch	$-\frac{3\tanh(dx+c)^5b^2+10\tanh(dx+c)^3ab+5b^2\tanh(dx+c)^3-15a^2dx-30abdx-15b^2dx+15a^2\tanh(dx+c)+30ab\tanh(dx}{15d}$
derivativedivides	$\frac{-2ab\tanh(dx+c)-\frac{2\tanh(dx+c)^3ab}{3}-\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)-1)}{2}-\frac{b^2\tanh(dx+c)^3}{3}-b^2\tanh(dx+c)-a^2\tanh(dx+c)-\frac{}{}}{d}$
default	$\frac{-2ab\tanh(dx+c)-\frac{2\tanh(dx+c)^3ab}{3}-\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)-1)}{2}-\frac{b^2\tanh(dx+c)^3}{3}-b^2\tanh(dx+c)-a^2\tanh(dx+c)-\frac{}{}}{d}$
parts	$\frac{a^2\left(-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b^2\left(-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx}{d}$
risch	$a^2x+2abx+b^2x+\frac{2a^2e^{8dx+8c}+8ab\,e^{8dx+8c}+6b^2e^{8dx+8c}+8a^2e^{6dx+6c}+24ab\,e^{6dx+6c}+12b^2e^{6dx+6c}+12a^2e^{4dx}}{d(e^{2dx}$

input
```
int(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(3*tanh(d*x+c)^5*b^2+10*tanh(d*x+c)^3*a*b+5*b^2*tanh(d*x+c)^3-15*a^2
*d*x-30*a*b*d*x-15*b^2*d*x+15*a^2*tanh(d*x+c)+30*a*b*tanh(d*x+c)+15*b^2*ta
nh(d*x+c))/d
```

3.146.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(59) = 118$.

Time = 0.26 (sec) , antiderivative size = 483, normalized size of antiderivative = 7.67

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{\left(15\left(a^2+2\,ab+b^2\right)dx+15\,a^2+40\,ab+23\,b^2\right)\cosh(dx+c)^5+5\left(15\left(a^2+2\,ab+b^2\right)dx+15\,a^2+40\,ab+\right.}{}$$

input
```
integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/15*((15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c
)^5 + 5*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d*x +
 c)*sinh(d*x + c)^4 - (15*a^2 + 40*a*b + 23*b^2)*sinh(d*x + c)^5 + 5*(15*(
a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c)^3 - 5*(2*
(15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c)^2 + 9*a^2 + 16*a*b + 5*b^2)*sinh(
d*x + c)^3 + 5*(2*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*
cosh(d*x + c)^3 + 3*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2
)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2
 + 40*a*b + 23*b^2)*cosh(d*x + c) - 5*((15*a^2 + 40*a*b + 23*b^2)*cosh(d*x
 + c)^4 + 3*(9*a^2 + 16*a*b + 5*b^2)*cosh(d*x + c)^2 + 6*a^2 + 8*a*b + 10*
b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4
 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(
d*x + c)^2 + 10*d*cosh(d*x + c))
```

3.146.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(53) = 106$.

Time $= 0.15$ (sec) , antiderivative size $= 117$, normalized size of antiderivative $= 1.86$

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$= \begin{cases} a^2x - \frac{a^2\tanh(c+dx)}{d} + 2abx - \frac{2ab\tanh^3(c+dx)}{3d} - \frac{2ab\tanh(c+dx)}{d} + b^2x - \frac{b^2\tanh^5(c+dx)}{5d} - \frac{b^2\tanh^3(c+dx)}{3d} - \frac{b^2\tanh(c}{d} \\ x\left(a+b\tanh^2(c)\right)^2\tanh^2(c) \end{cases}$$

input
```
integrate(tanh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Piecewise((a**2*x - a**2*tanh(c + d*x)/d + 2*a*b*x - 2*a*b*tanh(c + d*x)**
3/(3*d) - 2*a*b*tanh(c + d*x)/d + b**2*x - b**2*tanh(c + d*x)**5/(5*d) - b
**2*tanh(c + d*x)**3/(3*d) - b**2*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*ta
nh(c)**2)**2*tanh(c)**2, True))
```

3.146.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(59) = 118$.

Time = 0.20 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.67

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{1}{15}b^2\left(15x+\frac{15c}{d}-\frac{2\left(70e^{(-2dx-2c)}+140e^{(-4dx-4c)}+90e^{(-6dx-6c)}+45e^{(-8dx-8c)}+23\right)}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}\right)$$
$$+\frac{2}{3}ab\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)$$
$$+a^2\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2dx-2c)}+1)}\right)$$

input
```
integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/15*b^2*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) +
90*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
 - 10*c) + 1))) + 2/3*a*b*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d
*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6
*c) + 1))) + a^2*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1)))
```

3.146.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(59) = 118$.

Time = 0.35 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.46

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{15\left(a^2+2ab+b^2\right)(dx+c)+\frac{2\left(15a^2e^{(8dx+8c)}+60abe^{(8dx+8c)}+45b^2e^{(8dx+8c)}+60a^2e^{(6dx+6c)}+180abe^{(6dx+6c)}+90b^2e^{(6dx+6c}\right.}{}}{15d}$$

input
```
integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/15*(15*(a^2 + 2*a*b + b^2)*(d*x + c) + 2*(15*a^2*e^(8*d*x + 8*c) + 60*a*
b*e^(8*d*x + 8*c) + 45*b^2*e^(8*d*x + 8*c) + 60*a^2*e^(6*d*x + 6*c) + 180*
a*b*e^(6*d*x + 6*c) + 90*b^2*e^(6*d*x + 6*c) + 90*a^2*e^(4*d*x + 4*c) + 22
0*a*b*e^(4*d*x + 4*c) + 140*b^2*e^(4*d*x + 4*c) + 60*a^2*e^(2*d*x + 2*c) +
 140*a*b*e^(2*d*x + 2*c) + 70*b^2*e^(2*d*x + 2*c) + 15*a^2 + 40*a*b + 23*b
^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.146.9 Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = x\left(a^2+2ab+b^2\right) - \frac{\tanh(c+dx)\,(a+b)^2}{d} - \frac{\tanh(c+dx)^3\left(b^2+2ab\right)}{3d} - \frac{b^2\tanh(c+dx)^5}{5d}$$

input
```
int(tanh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
x*(2*a*b + a^2 + b^2) - (tanh(c + d*x)*(a + b)^2)/d - (tanh(c + d*x)^3*(2*
a*b + b^2))/(3*d) - (b^2*tanh(c + d*x)^5)/(5*d)
```

3.147 $\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

3.147.1 Optimal result 1106
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3.147.1 Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{(a+b)^2\log(\cosh(c+dx))}{d} - \frac{b(a+b)\tanh^2(c+dx)}{2d} - \frac{\left(a+b\tanh^2(c+dx)\right)^2}{4d}$$

output
```
(a+b)^2*ln(cosh(d*x+c))/d-1/2*b*(a+b)*tanh(d*x+c)^2/d-1/4*(a+b*tanh(d*x+c)
^2)^2/d
```

3.147.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{-4(a+b)^2\log(\cosh(c+dx)) + 2b(2a+b)\tanh^2(c+dx) + b^2\tanh^4(c+dx)}{4d}$$

input
```
Integrate[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-1/4*(-4*(a + b)^2*Log[Cosh[c + d*x]] + 2*b*(2*a + b)*Tanh[c + d*x]^2 + b^
2*Tanh[c + d*x]^4)/d
```

3.147.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 4153, 26, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int -i\tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\downarrow 26$$

$$-i\int \tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\downarrow 4153$$

$$-\frac{i\int \frac{i\tanh(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)} d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\tanh(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)} d\tanh(c+dx)}{d}$$

$$\downarrow 353$$

$$\frac{\int \frac{\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)} d\tanh^2(c+dx)}{2d}$$

$$\downarrow 49$$

$$\frac{\int \left(\frac{(a+b)^2}{1-\tanh^2(c+dx)} - b(a+b) - b\left(b\tanh^2(c+dx)+a\right)\right) d\tanh^2(c+dx)}{2d}$$

$$\downarrow 2009$$

$$\frac{-b(a+b)\tanh^2(c+dx) - \frac{1}{2}\left(a+b\tanh^2(c+dx)\right)^2 + (a+b)^2\left(-\log\left(1-\tanh^2(c+dx)\right)\right)}{2d}$$

input
```
Int[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-((a + b)^2*Log[1 - Tanh[c + d*x]^2]) - b*(a + b)*Tanh[c + d*x]^2 - (a +
b*Tanh[c + d*x]^2)^2/2)/(2*d)
```

3.147.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.147.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{-\frac{\tanh(dx+c)^4 b^2}{4}-\tanh(dx+c)^2 ab-\frac{b^2\tanh(dx+c)^2}{2}-\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)-1)}{2}+\frac{\left(-a^2-2ab-b^2\right)\ln(\tanh(dx+c)+1)}{2}}{d}$
default	$\frac{-\frac{\tanh(dx+c)^4 b^2}{4}-\tanh(dx+c)^2 ab-\frac{b^2\tanh(dx+c)^2}{2}-\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)-1)}{2}+\frac{\left(-a^2-2ab-b^2\right)\ln(\tanh(dx+c)+1)}{2}}{d}$
parts	$\frac{a^2\ln(\cosh(dx+c))}{d}+\frac{b^2\left(-\frac{\tanh(dx+c)^4}{4}-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{2ab\left(-\frac{\tanh(dx+c)^2}{2}-$
parallelrisch	$-\frac{\tanh(dx+c)^4 b^2+4a^2dx+8abdx+4b^2dx+4\tanh(dx+c)^2 ab+2b^2\tanh(dx+c)^2+4\ln(1-\tanh(dx+c))a^2+8\ln(1-\tanh(d}{4d}$
risch	$-a^2x-2abx-b^2x-\frac{2a^2c}{d}-\frac{4abc}{d}-\frac{2cb^2}{d}+\frac{4b\,\mathrm{e}^{2dx+2c}\left(a\,\mathrm{e}^{4dx+4c}+b\,\mathrm{e}^{4dx+4c}+2\,\mathrm{e}^{2dx+2c}a+b\,\mathrm{e}^{2dx+2c}+a+\right.}{d\left(\mathrm{e}^{2dx+2c}+1\right)^4}$

input
```
int(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4*tanh(d*x+c)^4*b^2-tanh(d*x+c)^2*a*b-1/2*b^2*tanh(d*x+c)^2-1/2*(a
^2+2*a*b+b^2)*ln(tanh(d*x+c)-1)+1/2*(-a^2-2*a*b-b^2)*ln(tanh(d*x+c)+1))
```

3.147.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1638 vs. 2(53) = 106.

Time = 0.28 (sec) , antiderivative size = 1638, normalized size of antiderivative = 28.74

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*d*x*cosh
(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8 + 4*((
a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b +
b^2)*d*x*cosh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*sinh(d*x +
 c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*((a^2 + 2*a*b + b
^2)*d*x - a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*(a^2 + 2*a*b +
b^2)*d*x - 4*a*b - 2*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*d*x*
cosh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*x + 30*((a^2 + 2*a*b + b^2)*d*x
- a*b - b^2)*cosh(d*x + c)^2 - 4*a*b - 2*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2
+ 2*a*b + b^2)*d*x*cosh(d*x + c)^5 + 10*((a^2 + 2*a*b + b^2)*d*x - a*b - b
^2)*cosh(d*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x - 4*a*b - 2*b^2)*cosh(d*x
 + c))*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*x + 4*((a^2 + 2*a*b + b^2)*
d*x - a*b - b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x +
 c)^6 + 15*((a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d*x + c)^4 + (a^2 +
2*a*b + b^2)*d*x + 3*(3*(a^2 + 2*a*b + b^2)*d*x - 4*a*b - 2*b^2)*cosh(d*x
+ c)^2 - a*b - b^2)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8
 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^
2)*sinh(d*x + c)^8 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + 8*(7*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)...
```

3.147.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(48) = 96$.

Time $= 0.13$ (sec) , antiderivative size $= 122$, normalized size of antiderivative $= 2.14$

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \begin{cases} a^2x - \frac{a^2\log(\tanh(c+dx)+1)}{d} + 2abx - \frac{2ab\log(\tanh(c+dx)+1)}{d} - \frac{ab\tanh^2(c+dx)}{d} + b^2x - \frac{b^2\log(\tanh(c+dx)+1)}{d} - \frac{b^2\tanh^4}{4} \\ x\left(a+b\tanh^2(c)\right)^2\tanh(c) \end{cases}$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Piecewise((a**2*x - a**2*log(tanh(c + d*x) + 1)/d + 2*a*b*x - 2*a*b*log(ta
nh(c + d*x) + 1)/d - a*b*tanh(c + d*x)**2/d + b**2*x - b**2*log(tanh(c + d
*x) + 1)/d - b**2*tanh(c + d*x)**4/(4*d) - b**2*tanh(c + d*x)**2/(2*d), Ne
(d, 0)), (x*(a + b*tanh(c)**2)**2*tanh(c), True))
```

3.147.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(53) = 106$.

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.26

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$
$$= b^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)$$
$$+2\,ab\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$
$$+\frac{a^2\log\left(\cosh\left(dx+c\right)\right)}{d}$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
b^2*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d
*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c)
+ 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + 2*a*b*(x + c/d + log(e^(-
2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*
x - 4*c) + 1))) + a^2*log(cosh(d*x + c))/d
```

3.147.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(53) = 106$.

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.04

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx =$$
$$-\frac{(a^2+2\,ab+b^2)(dx+c)-(a^2+2\,ab+b^2)\log\left(e^{(2\,dx+2\,c)}+1\right)-\frac{4\left((ab+b^2)e^{(6\,dx+6\,c)}+(2\,ab+b^2)e^{(4\,dx+4\,c)}+(ab+b^2\right.}{\left(e^{(2\,dx+2\,c)}+1\right)^4}}{d}$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-((a^2 + 2*a*b + b^2)*(d*x + c) - (a^2 + 2*a*b + b^2)*log(e^(2*d*x + 2*c)
+ 1) - 4*((a*b + b^2)*e^(6*d*x + 6*c) + (2*a*b + b^2)*e^(4*d*x + 4*c) + (a
*b + b^2)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c) + 1)^4)/d
```

3.147.9 Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = x\left(a^2+2ab+b^2\right) - \frac{\tanh(c+dx)^2\left(b^2+2ab\right)}{2d} - \frac{\ln\left(\tanh(c+dx)+1\right)\left(a^2+2ab+b^2\right)}{d} - \frac{b^2\tanh(c+dx)^4}{4d}$$

input
```
int(tanh(c + d*x)*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
x*(2*a*b + a^2 + b^2) - (tanh(c + d*x)^2*(2*a*b + b^2))/(2*d) - (log(tanh(
c + d*x) + 1)*(2*a*b + a^2 + b^2))/d - (b^2*tanh(c + d*x)^4)/(4*d)
```

3.148 $\int \left(a + b\tanh^2(c+dx)\right)^2\,dx$

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3.148.1 Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \left(a + b\tanh^2(c+dx)\right)^2\,dx = (a+b)^2x - \frac{b(2a+b)\tanh(c+dx)}{d} - \frac{b^2\tanh^3(c+dx)}{3d}$$

output
```
(a+b)^2*x-b*(2*a+b)*tanh(d*x+c)/d-1/3*b^2*tanh(d*x+c)^3/d
```

3.148.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.51

$$\int \left(a + b\tanh^2(c+dx)\right)^2\,dx = \frac{\tanh(c+dx)\left(\frac{3(a+b)^2\operatorname{arctanh}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}} - b\left(6a + b\left(3+\tanh^2(c+dx)\right)\right)\right)}{3d}$$

input
```
Integrate[(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(Tanh[c + d*x]*((3*(a + b)^2*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c +
 d*x]^2] - b*(6*a + b*(3 + Tanh[c + d*x]^2))))/(3*d)
```

3.148.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4144, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a + b\tanh^2(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \left(a - b\tan(ic+idx)^2\right)^2 dx$$

$$\downarrow 4144$$

$$\frac{\int \frac{\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)} d\tanh(c+dx)}{d}$$

$$\downarrow 300$$

$$\frac{\int \left(\frac{(a+b)^2}{1-\tanh^2(c+dx)} - b^2\tanh^2(c+dx) - b(2a+b)\right) d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{(a+b)^2\text{arctanh}(\tanh(c+dx)) - b(2a+b)\tanh(c+dx) - \frac{1}{3}b^2\tanh^3(c+dx)}{d}$$

input
```
Int[(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((a + b)^2*ArcTanh[Tanh[c + d*x]] - b*(2*a + b)*Tanh[c + d*x] - (b^2*Tanh[
c + d*x]^3)/3)/d
```

3.148.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)  Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.148.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

method	result
parallelrisch	$-\frac{b^2\tanh(dx+c)^3-3a^2dx-6abdx-3b^2dx+6ab\tanh(dx+c)+3b^2\tanh(dx+c)}{3d}$
derivativedivides	$\frac{-\frac{b^2\tanh(dx+c)^3}{3}-2ab\tanh(dx+c)-b^2\tanh(dx+c)-\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)-1)}{2}+\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)+1)}{2}}{d}$
default	$\frac{-\frac{b^2\tanh(dx+c)^3}{3}-2ab\tanh(dx+c)-b^2\tanh(dx+c)-\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)-1)}{2}+\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)+1)}{2}}{d}$
risch	$a^2x+2abx+b^2x+\frac{4b\left(3a\,\mathrm{e}^{4dx+4c}+3b\,\mathrm{e}^{4dx+4c}+6\,\mathrm{e}^{2dx+2c}a+3b\,\mathrm{e}^{2dx+2c}+3a+2b\right)}{3d\left(\mathrm{e}^{2dx+2c}+1\right)^3}$
parts	$a^2x+\frac{b^2\left(-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{2ab\left(-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)}{d}\right.$

input
```
int((a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b^2*tanh(d*x+c)^3-3*a^2*d*x-6*a*b*d*x-3*b^2*d*x+6*a*b*tanh(d*x+c)+3*
b^2*tanh(d*x+c))/d
```

3.148.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(41) = 82$.

Time = 0.27 (sec) , antiderivative size = 201, normalized size of antiderivative = 4.67

$$\int \left(a + b\tanh^2(c+dx)\right)^2 dx$$

$$= \frac{\left(3\left(a^2+2ab+b^2\right)dx+6ab+4b^2\right)\cosh\left(dx+c\right)^3+3\left(3\left(a^2+2ab+b^2\right)dx+6ab+4b^2\right)\cosh\left(dx+c\right)\sin}{3\left(d\cosh\left(dx+\right.\right.}$$

input
```
integrate((a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/3*((3*(a^2 + 2*a*b + b^2)*d*x + 6*a*b + 4*b^2)*cosh(d*x + c)^3 + 3*(3*(a
^2 + 2*a*b + b^2)*d*x + 6*a*b + 4*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 - 2*(
3*a*b + 2*b^2)*sinh(d*x + c)^3 + 3*(3*(a^2 + 2*a*b + b^2)*d*x + 6*a*b + 4*
b^2)*cosh(d*x + c) - 6*((3*a*b + 2*b^2)*cosh(d*x + c)^2 + a*b)*sinh(d*x +
c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x
+ c))
```

3.148.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \left(a + b\tanh^2(c+dx)\right)^2 dx$$

$$= \begin{cases} a^2x + 2abx - \frac{2ab\tanh(c+dx)}{d} + b^2x - \frac{b^2\tanh^3(c+dx)}{3d} - \frac{b^2\tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a + b\tanh^2(c)\right)^2 & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*x - 2*a*b*tanh(c + d*x)/d + b**2*x - b**2*tanh(c
 + d*x)**3/(3*d) - b**2*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)*
*2, True))
```

3.148.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(41) = 82$.

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.65

$$\begin{aligned}&\int \left(a + b\tanh^2(c+dx)\right)^2\,dx\\ &= \frac{1}{3}b^2\left(3x + \frac{3c}{d} - \frac{4\left(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + 2\right)}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)}\right)\\ &\quad + 2ab\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2dx-2c)} + 1)}\right) + a^2x\end{aligned}$$

input
```
integrate((a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/3*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*
(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 2*a*b
*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^2*x
```

3.148.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(41) = 82$.

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.40

$$\begin{aligned}&\int \left(a + b\tanh^2(c+dx)\right)^2\,dx\\ &= \frac{3\left(a^2 + 2ab + b^2\right)(dx+c) + \frac{4\left(3abe^{(4dx+4c)} + 3b^2e^{(4dx+4c)} + 6abe^{(2dx+2c)} + 3b^2e^{(2dx+2c)} + 3ab + 2b^2\right)}{\left(e^{(2dx+2c)}+1\right)^3}}{3d}\end{aligned}$$

input
```
integrate((a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/3*(3*(a^2 + 2*a*b + b^2)*(d*x + c) + 4*(3*a*b*e^(4*d*x + 4*c) + 3*b^2*e^
(4*d*x + 4*c) + 6*a*b*e^(2*d*x + 2*c) + 3*b^2*e^(2*d*x + 2*c) + 3*a*b + 2*
b^2)/(e^(2*d*x + 2*c) + 1)^3)/d
```

3.148.9 Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \left(a+b\tanh^2(c+dx)\right)^2\,dx = x\left(a^2+2\,a\,b+b^2\right)-\frac{b^2\tanh(c+d\,x)^3}{3\,d}-\frac{b\tanh(c+d\,x)\,\left(2\,a+b\right)}{d}$$

input
```
int((a + b*tanh(c + d*x)^2)^2,x)
```

output
```
x*(2*a*b + a^2 + b^2) - (b^2*tanh(c + d*x)^3)/(3*d) - (b*tanh(c + d*x)*(2*
a + b))/d
```

3.149 $\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

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3.149.7 Maxima [B] (verification not implemented) 1123
3.149.8 Giac [B] (verification not implemented) 1124
3.149.9 Mupad [B] (verification not implemented) 1124

3.149.1 Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{(a+b)^2\log(\cosh(c+dx))}{d} + \frac{a^2\log(\tanh(c+dx))}{d} - \frac{b^2\tanh^2(c+dx)}{2d}$$

output
```
(a+b)^2*ln(cosh(d*x+c))/d+a^2*ln(tanh(d*x+c))/d-1/2*b^2*tanh(d*x+c)^2/d
```

3.149.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{2((a+b)^2\log(\cosh(c+dx)) + a^2\log(\tanh(c+dx))) - b^2\tanh^2(c+dx)}{2d}$$

input
```
Integrate[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(2*((a + b)^2*Log[Cosh[c + d*x]] + a^2*Log[Tanh[c + d*x]]) - b^2*Tanh[c +
d*x]^2)/(2*d)
```

3.149.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 4153, 26, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{i\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)}dx$$

$$\downarrow 26$$

$$i\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)}dx$$

$$\downarrow 4153$$

$$\frac{i\int -\frac{i\coth(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\coth(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 354$$

$$\frac{\int \frac{\coth(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{2d}$$

$$\downarrow 93$$

$$\frac{\int \left(\coth(c+dx)a^2-b^2-\frac{(a+b)^2}{\tanh^2(c+dx)-1}\right)d\tanh^2(c+dx)}{2d}$$

$$\downarrow 2009$$

$$\frac{a^2\log\left(\tanh^2(c+dx)\right)-(a+b)^2\log\left(1-\tanh^2(c+dx)\right)-b^2\tanh^2(c+dx)}{2d}$$

input
```
Int[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(a^2*Log[Tanh[c + d*x]^2] - (a + b)^2*Log[1 - Tanh[c + d*x]^2] - b^2*Tanh[
c + d*x]^2)/(2*d)
```

3.149.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.149.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20

method	result
parallelrisch	$\frac{-2(a+b)^2\ln(1-\tanh(dx+c))+2a^2\ln(\tanh(dx+c))-b^2\tanh(dx+c)^2-2dx(a+b)^2}{2d}$
derivativedivides	$-\frac{\frac{b^2\tanh(dx+c)^2}{2}+(\frac{1}{2}a^2+ab+\frac{1}{2}b^2)\ln(\tanh(dx+c)+1)+(\frac{1}{2}a^2+ab+\frac{1}{2}b^2)\ln(\tanh(dx+c)-1)-a^2\ln(\tanh(dx+c))}{d}$
default	$-\frac{\frac{b^2\tanh(dx+c)^2}{2}+(\frac{1}{2}a^2+ab+\frac{1}{2}b^2)\ln(\tanh(dx+c)+1)+(\frac{1}{2}a^2+ab+\frac{1}{2}b^2)\ln(\tanh(dx+c)-1)-a^2\ln(\tanh(dx+c))}{d}$
risch	$-a^2x-2abx-b^2x-\frac{4abc}{d}-\frac{2cb^2}{d}-\frac{2a^2c}{d}+\frac{2b^2e^{2dx+2c}}{d(e^{2dx+2c}+1)^2}+\frac{2ab\ln(e^{2dx+2c}+1)}{d}+\frac{\ln(e^{2dx+2c}+1)b^2}{d}$

input
```
int(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-2*(a+b)^2*ln(1-tanh(d*x+c))+2*a^2*ln(tanh(d*x+c))-b^2*tanh(d*x+c)^2-
2*d*x*(a+b)^2)/d
```

3.149.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. 2(47) = 94.

Time = 0.27 (sec) , antiderivative size = 668, normalized size of antiderivative = 13.63

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$

$$-\frac{(a^2+2\,ab+b^2)dx\cosh(dx+c)^4+4\,(a^2+2\,ab+b^2)dx\cosh(dx+c)\sinh(dx+c)^3+(a^2+2\,ab+b^2)d$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*d*x*cosh
(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^4 + (a^2
 + 2*a*b + b^2)*d*x + 2*((a^2 + 2*a*b + b^2)*d*x - b^2)*cosh(d*x + c)^2 +
2*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d*x - b
^2)*sinh(d*x + c)^2 - ((2*a*b + b^2)*cosh(d*x + c)^4 + 4*(2*a*b + b^2)*cos
h(d*x + c)*sinh(d*x + c)^3 + (2*a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a*b + b^
2)*cosh(d*x + c)^2 + 2*(3*(2*a*b + b^2)*cosh(d*x + c)^2 + 2*a*b + b^2)*sin
h(d*x + c)^2 + 2*a*b + b^2 + 4*((2*a*b + b^2)*cosh(d*x + c)^3 + (2*a*b + b
^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sin
h(d*x + c))) - (a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3
+ a^2*sinh(d*x + c)^4 + 2*a^2*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 +
 a^2)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*
sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*((
a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + ((a^2 + 2*a*b + b^2)*d*x - b^2)*c
osh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d
*x + c)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)
^2 + d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x
 + c) + d)
```

3.149.6 Sympy [F]

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2(c+dx)\right)^2 \coth(c+dx)\,dx$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x), x)
```

3.149.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(47) = 94$.

Time $= 0.27$ (sec) , antiderivative size $= 104$, normalized size of antiderivative $= 2.12$

$$\begin{aligned}
&\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx \\
&= b^2\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d} + \frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1\right)}\right) \\
&\quad + \frac{2\,ab\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d} + \frac{a^2\log\left(\sinh\left(dx+c\right)\right)}{d}
\end{aligned}$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
b^2*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-
2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 2*a*b*log(e^(d*x + c) + e^(-d*x -
 c))/d + a^2*log(sinh(d*x + c))/d
```

3.149.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(47) = 94$.

Time = 0.34 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.88

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{a^2\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)+(2\,ab+b^2)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)-\frac{2\,ab\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+}{e^{(2\,dx+2\,c}}}{2\,d}$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*(a^2*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) + (2*a*b + b^2)*log(e
^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) - (2*a*b*(e^(2*d*x + 2*c) + e^(-2*d
*x - 2*c)) + b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 4*a*b - 2*b^2)/(e^
(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2))/d
```

3.149.9 Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 210, normalized size of antiderivative = 4.29

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{2\,b^2}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)} - x\,(a+b)^2 - \frac{2\,b^2}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$
$$+ \frac{\ln\left(\mathrm{e}^{4\,c+4\,d\,x}-1\right)\,\left(d\,(b^2+2\,a\,b)+a^2\,d\right)}{2\,d^2}$$
$$+ \frac{\operatorname{atan}\left(\frac{\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\left(b^2\sqrt{-d^2}-a^2\sqrt{-d^2}+2\,a\,b\sqrt{-d^2}\right)}{d\sqrt{a^4-4\,a^3\,b+2\,a^2\,b^2+4\,a\,b^3+b^4}}\right)\,\sqrt{a^4-4\,a^3\,b+2\,a^2\,b^2+4\,a\,b^3+b^4}}{\sqrt{-d^2}}$$

input
```
int(coth(c + d*x)*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
(2*b^2)/(d*(exp(2*c + 2*d*x) + 1)) - x*(a + b)^2 - (2*b^2)/(d*(2*exp(2*c +
 2*d*x) + exp(4*c + 4*d*x) + 1)) + (log(exp(4*c + 4*d*x) - 1)*(d*(2*a*b +
b^2) + a^2*d))/(2*d^2) + (atan((exp(2*c)*exp(2*d*x)*(b^2*(-d^2)^(1/2) - a^
2*(-d^2)^(1/2) + 2*a*b*(-d^2)^(1/2)))/(d*(4*a*b^3 - 4*a^3*b + a^4 + b^4 +
2*a^2*b^2)^(1/2)))*(4*a*b^3 - 4*a^3*b + a^4 + b^4 + 2*a^2*b^2)^(1/2))/(-d^
2)^(1/2)
```

3.150 $\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

3.150.1 Optimal result 1126
3.150.2 Mathematica [C] (verified) 1126
3.150.3 Rubi [A] (verified) 1127
3.150.4 Maple [A] (verified) 1128
3.150.5 Fricas [B] (verification not implemented) 1129
3.150.6 Sympy [F] 1129
3.150.7 Maxima [A] (verification not implemented) 1129
3.150.8 Giac [A] (verification not implemented) 1130
3.150.9 Mupad [B] (verification not implemented) 1130

3.150.1 Optimal result

Integrand size = 23, antiderivative size = 36

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = (a+b)^2x - \frac{a^2\coth(c+dx)}{d} - \frac{b^2\tanh(c+dx)}{d}$$

output
```
(a+b)^2*x-a^2*coth(d*x+c)/d-b^2*tanh(d*x+c)/d
```

3.150.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.78

$$\begin{aligned}&\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx \\ &= 2abx + \frac{b^2\operatorname{arctanh}(\tanh(c+dx))}{d} \\ &\quad - \frac{a^2\coth(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(c+dx)\right)}{d} - \frac{b^2\tanh(c+dx)}{d}\end{aligned}$$

input
```
Integrate[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
2*a*b*x + (b^2*ArcTanh[Tanh[c + d*x]])/d - (a^2*Coth[c + d*x]*Hypergeometr
ic2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2])/d - (b^2*Tanh[c + d*x])/d
```

3.150.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 25, 4153, 25, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)^2}dx$$

$$\downarrow 4153$$

$$-\frac{\int -\frac{\coth^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 25$$

$$\frac{\int \frac{\coth^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 364$$

$$\frac{\int \left(-b^2+a^2\coth^2(c+dx)-\frac{(a+b)^2}{\tanh^2(c+dx)-1}\right)d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$-\frac{a^2\coth(c+dx)-(a+b)^2\text{arctanh}(\tanh(c+dx))+b^2\tanh(c+dx)}{d}$$

input
```
Int[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-((-((a + b)^2*ArcTanh[Tanh[c + d*x]]) + a^2*Coth[c + d*x] + b^2*Tanh[c +
d*x])/d)
```

3.150.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.150.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{-\coth(dx+c)a^2-b^2\tanh(dx+c)+dx(a+b)^2}{d}$	36
derivativedivides	$\frac{a^2(dx+c-\coth(dx+c))+2ab(dx+c)+b^2(dx+c-\tanh(dx+c))}{d}$	49
default	$\frac{a^2(dx+c-\coth(dx+c))+2ab(dx+c)+b^2(dx+c-\tanh(dx+c))}{d}$	49
risch	$a^2x+2abx+b^2x-\frac{2(a^2e^{2dx+2c}-e^{2dx+2c}b^2+a^2+b^2)}{d(e^{2dx+2c}+1)(e^{2dx+2c}-1)}$	82

input
```
int(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-coth(d*x+c)*a^2-b^2*tanh(d*x+c)+d*x*(a+b)^2)/d
```

3.150.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(36) = 72$.

Time $= 0.27$ (sec) , antiderivative size $= 97$, normalized size of antiderivative $= 2.69$

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{(a^2+b^2)\cosh(dx+c)^2 - 2\left((a^2+2ab+b^2)dx + a^2 + b^2\right)\cosh(dx+c)\sinh(dx+c) + (a^2+b^2)\sinh(}{2d\cosh(dx+c)\sinh(dx+c)}$$

input
```
integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*((a^2 + b^2)*cosh(d*x + c)^2 - 2*((a^2 + 2*a*b + b^2)*d*x + a^2 + b^2
)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*sinh(d*x + c)^2 + a^2 - b^2)/(
d*cosh(d*x + c)*sinh(d*x + c))
```

3.150.6 Sympy [F]

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2(c+dx)\right)^2 \coth^2(c+dx)\,dx$$

input
```
integrate(coth(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**2, x)
```

3.150.7 Maxima [A] (verification not implemented)

Time $= 0.20$ (sec) , antiderivative size $= 64$, normalized size of antiderivative $= 1.78$

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = b^2\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2\,dx-2\,c)}+1)}\right) + a^2\left(x + \frac{c}{d} + \frac{2}{d(e^{(-2\,dx-2\,c)}-1)}\right) + 2\,abx$$

input
```
integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
b^2*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^2*(x + c/d + 2/(d*(e^(-2*
d*x - 2*c) - 1))) + 2*a*b*x
```

3.150.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.97

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{(a^2+2\,ab+b^2)(dx+c)-\frac{2\left(a^2e^{(2\,dx+2\,c)}-b^2e^{(2\,dx+2\,c)}+a^2+b^2\right)}{e^{(4\,dx+4\,c)}-1}}{d}$$

input
```
integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
((a^2 + 2*a*b + b^2)*(d*x + c) - 2*(a^2*e^(2*d*x + 2*c) - b^2*e^(2*d*x + 2
*c) + a^2 + b^2)/(e^(4*d*x + 4*c) - 1))/d
```

3.150.9 Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.64

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = x\,(a+b)^2-\frac{\frac{2\,(a^2+b^2)}{d}+\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,(a^2-b^2)}{d}}{\mathrm{e}^{4\,c+4\,d\,x}-1}$$

input
```
int(coth(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
x*(a + b)^2 - ((2*(a^2 + b^2))/d + (2*exp(2*c + 2*d*x)*(a^2 - b^2))/d)/(ex
p(4*c + 4*d*x) - 1)
```

3.151 $\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

3.151.1 Optimal result 1131
3.151.2 Mathematica [A] (verified) 1131
3.151.3 Rubi [A] (warning: unable to verify) 1132
3.151.4 Maple [A] (verified) 1134
3.151.5 Fricas [B] (verification not implemented) 1134
3.151.6 Sympy [F] 1135
3.151.7 Maxima [B] (verification not implemented) 1135
3.151.8 Giac [B] (verification not implemented) 1136
3.151.9 Mupad [B] (verification not implemented) 1136

3.151.1 Optimal result

Integrand size = 23, antiderivative size = 52

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{a^2\coth^2(c+dx)}{2d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}+\frac{a(a+2b)\log(\tanh(c+dx))}{d}$$

output
```
-1/2*a^2*coth(d*x+c)^2/d+(a+b)^2*ln(cosh(d*x+c))/d+a*(a+2*b)*ln(tanh(d*x+c
))/d
```

3.151.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{-a^2\coth^2(c+dx)+2(a+b)^2\log(\cosh(c+dx))+2a(a+2b)\log(\tanh(c+dx))}{2d}$$

input
```
Integrate[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-(a^2*Coth[c + d*x]^2) + 2*(a + b)^2*Log[Cosh[c + d*x]] + 2*a*(a + 2*b)*L
og[Tanh[c + d*x]])/(2*d)
```

3.151.3 Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int -\frac{i\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)^3}dx$$

$$\downarrow 26$$

$$-i\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)^3}dx$$

$$\downarrow 4153$$

$$-\frac{i\int \frac{i\coth^3(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\coth^3(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 354$$

$$\frac{\int \frac{\coth^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{2d}$$

$$\downarrow 99$$

$$\frac{\int \left(-\frac{(a+b)^2}{\tanh^2(c+dx)-1}+a^2\coth^2(c+dx)+a(a+2b)\coth(c+dx)\right)d\tanh^2(c+dx)}{2d}$$

$$\downarrow 2009$$

$$\frac{a^2(-\coth(c+dx))+a(a+2b)\log\left(\tanh^2(c+dx)\right)-(a+b)^2\log\left(1-\tanh^2(c+dx)\right)}{2d}$$

input
```
Int[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-(a^2*Coth[c + d*x]) + a*(a + 2*b)*Log[Tanh[c + d*x]^2] - (a + b)^2*Log[1
 - Tanh[c + d*x]^2])/(2*d)
```

3.151.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.151.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.19

method	result
parallelrisch	$\frac{-2(a+b)^2\ln(1-\tanh(dx+c))+2a(a+2b)\ln(\tanh(dx+c))-\coth(dx+c)^2a^2-2dx(a+b)^2}{2d}$
derivativedivides	$-\frac{(\frac{1}{2}a^2+ab+\frac{1}{2}b^2)\ln(\tanh(dx+c)-1)+(\frac{1}{2}a^2+ab+\frac{1}{2}b^2)\ln(\tanh(dx+c)+1)+\frac{a^2}{2\tanh(dx+c)^2}-a(a+2b)\ln(\tanh(dx+c))}{d}$
default	$-\frac{(\frac{1}{2}a^2+ab+\frac{1}{2}b^2)\ln(\tanh(dx+c)-1)+(\frac{1}{2}a^2+ab+\frac{1}{2}b^2)\ln(\tanh(dx+c)+1)+\frac{a^2}{2\tanh(dx+c)^2}-a(a+2b)\ln(\tanh(dx+c))}{d}$
risch	$-a^2x-2abx-b^2x-\frac{2a^2c}{d}-\frac{4abc}{d}-\frac{2cb^2}{d}-\frac{2a^2e^{2dx+2c}}{d(e^{2dx+2c}-1)^2}+\frac{a^2\ln(e^{2dx+2c}-1)}{d}+\frac{2a\ln(e^{2dx+2c}-1)}{d}$

input
```
int(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-2*(a+b)^2*ln(1-tanh(d*x+c))+2*a*(a+2*b)*ln(tanh(d*x+c))-coth(d*x+c)^
2*a^2-2*d*x*(a+b)^2)/d
```

3.151.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(50) = 100$.

Time = 0.26 (sec) , antiderivative size = 677, normalized size of antiderivative = 13.02

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$

$$-\frac{(a^2+2\,ab+b^2)dx\cosh(dx+c)^4+4\,(a^2+2\,ab+b^2)dx\cosh(dx+c)\sinh(dx+c)^3+(a^2+2\,ab+b^2)d$$

input
```
integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*d*x*cosh
(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^4 + (a^2
 + 2*a*b + b^2)*d*x - 2*((a^2 + 2*a*b + b^2)*d*x - a^2)*cosh(d*x + c)^2 +
2*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d*x + a
^2)*sinh(d*x + c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x
+ c)^3 + b^2*sinh(d*x + c)^4 - 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x +
 c)^2 - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 - b^2*cosh(d*x
 + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c)))
 - ((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)*sinh(d*x
 + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 - 2*(a^2 + 2*a*b)*cosh(d*x + c)^2
+ 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 - a^2 - 2*a*b)*sinh(d*x + c)^2 + a^2
+ 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 - (a^2 + 2*a*b)*cosh(d*x + c))*
sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*((
a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 - ((a^2 + 2*a*b + b^2)*d*x - a^2)*c
osh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d
*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)
^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x
 + c) + d)
```

3.151.6 Sympy [F]

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2(c+dx)\right)^2 \coth^3(c+dx)\,dx$$

input
```
integrate(coth(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**3, x)
```

3.151.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(50) = 100$.

Time $= 0.20$ (sec) , antiderivative size $= 134$, normalized size of antiderivative $= 2.58$

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= a^2\left(x + \frac{c}{d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} - 1)}\right)$$
$$+ \frac{b^2\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d} + \frac{2\,ab\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}$$

input
```
integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
a^2*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2
*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b^2*log(e^(
d*x + c) + e^(-d*x - c))/d + 2*a*b*log(e^(d*x + c) - e^(-d*x - c))/d
```

3.151.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(50) = 100$.

Time $= 0.37$ (sec) , antiderivative size $= 141$, normalized size of antiderivative $= 2.71$

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{b^2\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)+\left(a^2+2\,ab\right)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)-\frac{a^2\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2}{e^{(2\,dx+2}}}{2\,d}$$

input
```
integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*(b^2*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) + (a^2 + 2*a*b)*log(e
^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) - (a^2*(e^(2*d*x + 2*c) + e^(-2*d*x
 - 2*c)) + 2*a*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a^2 - 4*a*b)/(e^
(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2))/d
```

3.151.9 Mupad [B] (verification not implemented)

Time $= 0.24$ (sec) , antiderivative size $= 211$, normalized size of antiderivative $= 4.06$

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{\ln\left(\mathrm{e}^{4\,c+4\,d\,x}-1\right)\,\left(d\left(a^2+2\,b\,a\right)+b^2\,d\right)}{2\,d^2}-\frac{2\,a^2}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}$$
$$-\frac{2\,a^2}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-x\,(a+b)^2$$
$$-\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\left(a^2\,\sqrt{-d^2}-b^2\,\sqrt{-d^2}+2\,a\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^4+4\,a^3\,b+2\,a^2\,b^2-4\,a\,b^3+b^4}}\right)\,\sqrt{a^4+4\,a^3\,b+2\,a^2\,b^2-4\,a\,b^3+b^4}}{\sqrt{-d^2}}$$

input
```
int(coth(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
(log(exp(4*c + 4*d*x) - 1)*(d*(2*a*b + a^2) + b^2*d))/(2*d^2) - (2*a^2)/(d
*(exp(2*c + 2*d*x) - 1)) - (2*a^2)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*
x) + 1)) - x*(a + b)^2 - (atan((exp(2*c)*exp(2*d*x)*(a^2*(-d^2)^(1/2) - b^
2*(-d^2)^(1/2) + 2*a*b*(-d^2)^(1/2)))/(d*(4*a^3*b - 4*a*b^3 + a^4 + b^4 +
2*a^2*b^2)^(1/2)))*(4*a^3*b - 4*a*b^3 + a^4 + b^4 + 2*a^2*b^2)^(1/2))/(-d^
2)^(1/2)
```

3.152 $\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

3.152.1 Optimal result 1138
3.152.2 Mathematica [A] (verified) 1138
3.152.3 Rubi [A] (verified) 1139
3.152.4 Maple [A] (verified) 1140
3.152.5 Fricas [B] (verification not implemented) 1141
3.152.6 Sympy [F] 1141
3.152.7 Maxima [B] (verification not implemented) 1141
3.152.8 Giac [B] (verification not implemented) 1142
3.152.9 Mupad [B] (verification not implemented) 1142

3.152.1 Optimal result

Integrand size = 23, antiderivative size = 43

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = (a+b)^2x - \frac{a(a+2b)\coth(c+dx)}{d} - \frac{a^2\coth^3(c+dx)}{3d}$$

output
```
(a+b)^2*x-a*(a+2*b)*coth(d*x+c)/d-1/3*a^2*coth(d*x+c)^3/d
```

3.152.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.51

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{\coth(c+dx)\left(a\left(3a+6b+a\coth^2(c+dx)\right)-3(a+b)^2\operatorname{arctanh}\left(\sqrt{\tanh^2(c+dx)}\right)\sqrt{\tanh^2(c+dx)}\right)}{3d}$$

input
```
Integrate[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-1/3*(Coth[c + d*x]*(a*(3*a + 6*b + a*Coth[c + d*x]^2) - 3*(a + b)^2*ArcTa
nh[Sqrt[Tanh[c + d*x]^2]]*Sqrt[Tanh[c + d*x]^2]))/d
```

3.152.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4153, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)^4}dx$$

$$\downarrow 4153$$

$$\frac{\int \frac{\coth^4(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 364$$

$$\frac{\int \left(a^2\coth^4(c+dx)+a(a+2b)\coth^2(c+dx)-\frac{(a+b)^2}{\tanh^2(c+dx)-1}\right)d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{3}a^2\coth^3(c+dx)+(a+b)^2\text{arctanh}(\tanh(c+dx))-a(a+2b)\coth(c+dx)}{d}$$

input
```
Int[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((a + b)^2*ArcTanh[Tanh[c + d*x]] - a*(a + 2*b)*Coth[c + d*x] - (a^2*Coth[
c + d*x]^3)/3)/d
```

3.152.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.152.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{-\coth(dx+c)^3a^2-3a(a+2b)\coth(dx+c)+3dx(a+b)^2}{3d}$	43
derivativedivides	$-\frac{\frac{a^2}{3\tanh(dx+c)^3}+\frac{a(a+2b)}{\tanh(dx+c)}+(\frac{1}{2}a^2+ab+\frac{1}{2}b^2)\ln(\tanh(dx+c)-1)+(-\frac{1}{2}a^2-ab-\frac{1}{2}b^2)\ln(\tanh(dx+c)+1)}{d}$	84
default	$-\frac{\frac{a^2}{3\tanh(dx+c)^3}+\frac{a(a+2b)}{\tanh(dx+c)}+(\frac{1}{2}a^2+ab+\frac{1}{2}b^2)\ln(\tanh(dx+c)-1)+(-\frac{1}{2}a^2-ab-\frac{1}{2}b^2)\ln(\tanh(dx+c)+1)}{d}$	84
risch	$a^2x+2abx+b^2x-\frac{4a(3a\,\mathrm{e}^{4dx+4c}+3b\,\mathrm{e}^{4dx+4c}-3\,\mathrm{e}^{2dx+2c}a-6b\,\mathrm{e}^{2dx+2c}+2a+3b)}{3d(\mathrm{e}^{2dx+2c}-1)^3}$	91

input
```
int(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-coth(d*x+c)^3*a^2-3*a*(a+2*b)*coth(d*x+c)+3*d*x*(a+b)^2)/d
```

3.152.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(41) = 82$.

Time $= 0.24$ (sec) , antiderivative size $= 197$, normalized size of antiderivative $= 4.58$

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx =$$

$$-\frac{2\left(2a^2+3ab\right)\cosh(dx+c)^3+6\left(2a^2+3ab\right)\cosh(dx+c)\sinh(dx+c)^2-\left(3\left(a^2+2ab+b^2\right)dx+4a\right.}{3\left(d\sin\right.}$$

input
```
integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(2*(2*a^2 + 3*a*b)*cosh(d*x + c)^3 + 6*(2*a^2 + 3*a*b)*cosh(d*x + c)*
sinh(d*x + c)^2 - (3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*sinh(d*x + c
)^3 - 6*a*b*cosh(d*x + c) + 3*(3*(a^2 + 2*a*b + b^2)*d*x - (3*(a^2 + 2*a*b
 + b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c)^2 + 4*a^2 + 6*a*b)*sinh(d*x + c
))/(d*sinh(d*x + c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

3.152.6 Sympy [F]

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx = \int \left(a+b\tanh^2(c+dx)\right)^2 \coth^4(c+dx)\, dx$$

input
```
integrate(coth(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**4, x)
```

3.152.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(41) = 82$.

Time $= 0.21$ (sec) , antiderivative size $= 114$, normalized size of antiderivative $= 2.65$

$$\begin{aligned}
&\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx \\
&= \frac{1}{3}a^2\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right) \\
&\quad + 2ab\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2dx-2c)}-1)}\right)+b^2x
\end{aligned}$$

input
```
integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/3*a^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*
(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 2*a*b
*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + b^2*x
```

3.152.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(41) = 82$.

Time $= 0.37$ (sec) , antiderivative size $= 103$, normalized size of antiderivative $= 2.40$

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{3\left(a^2+2ab+b^2\right)(dx+c) - \frac{4\left(3a^2e^{(4dx+4c)}+3abe^{(4dx+4c)}-3a^2e^{(2dx+2c)}-6abe^{(2dx+2c)}+2a^2+3ab\right)}{\left(e^{(2dx+2c)}-1\right)^3}}{3d}$$

input
```
integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/3*(3*(a^2 + 2*a*b + b^2)*(d*x + c) - 4*(3*a^2*e^(4*d*x + 4*c) + 3*a*b*e^
(4*d*x + 4*c) - 3*a^2*e^(2*d*x + 2*c) - 6*a*b*e^(2*d*x + 2*c) + 2*a^2 + 3*
a*b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

3.152.9 Mupad [B] (verification not implemented)

Time $= 0.15$ (sec) , antiderivative size $= 175$, normalized size of antiderivative $= 4.07$

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = x\left(a+b\right)^2 - \frac{\frac{4e^{2c+2dx}\left(a^2+ba\right)}{3d} - \frac{4ab}{3d}}{e^{4c+4dx}-2e^{2c+2dx}+1} - \frac{\frac{4\left(a^2+ba\right)}{3d} + \frac{4e^{4c+4dx}\left(a^2+ba\right)}{3d} - \frac{8abe^{2c+2dx}}{3d}}{3e^{2c+2dx}-3e^{4c+4dx}+e^{6c+6dx}-1} - \frac{4\left(a^2+ba\right)}{3d\left(e^{2c+2dx}-1\right)}$$

input
```
int(coth(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2,x)
```

output

```
x*(a + b)^2 - ((4*exp(2*c + 2*d*x)*(a*b + a^2))/(3*d) - (4*a*b)/(3*d))/(ex
p(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - ((4*(a*b + a^2))/(3*d) + (4*exp
(4*c + 4*d*x)*(a*b + a^2))/(3*d) - (8*a*b*exp(2*c + 2*d*x))/(3*d))/(3*exp(
2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - (4*(a*b + a^2)
)/(3*d*(exp(2*c + 2*d*x) - 1))
```

3.153 $\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

3.153.1 Optimal result 1144
3.153.2 Mathematica [A] (verified) 1144
3.153.3 Rubi [A] (warning: unable to verify) 1145
3.153.4 Maple [A] (verified) 1147
3.153.5 Fricas [B] (verification not implemented) 1147
3.153.6 Sympy [F] 1148
3.153.7 Maxima [B] (verification not implemented) 1149
3.153.8 Giac [A] (verification not implemented) 1149
3.153.9 Mupad [B] (verification not implemented) 1150

3.153.1 Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{a(a+2b)\coth^2(c+dx)}{2d} - \frac{a^2\coth^4(c+dx)}{4d} + \frac{(a+b)^2\log(\cosh(c+dx))}{d} + \frac{(a+b)^2\log(\tanh(c+dx))}{d}$$

output
```
-1/2*a*(a+2*b)*coth(d*x+c)^2/d-1/4*a^2*coth(d*x+c)^4/d+(a+b)^2*ln(cosh(d*x
+c))/d+(a+b)^2*ln(tanh(d*x+c))/d
```

3.153.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{2a(a+2b)\coth^2(c+dx)+a^2\coth^4(c+dx)-4(a+b)^2(\log(\cosh(c+dx))+\log(\tanh(c+dx)))}{4d}$$

input
```
Integrate[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-1/4*(2*a*(a + 2*b)*Coth[c + d*x]^2 + a^2*Coth[c + d*x]^4 - 4*(a + b)^2*(L
og[Cosh[c + d*x]] + Log[Tanh[c + d*x]]))/d
```

3.153.3 Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \frac{i\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)^5}dx$$

$$\downarrow 26$$

$$i\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)^5}dx$$

$$\downarrow 4153$$

$$\frac{i\int -\frac{i\coth^5(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\coth^5(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 354$$

$$\frac{\int \frac{\coth^3(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{2d}$$

$$\downarrow 99$$

$$\frac{\int \left(a^2\coth^3(c+dx)+a(a+2b)\coth^2(c+dx)+(a+b)^2\coth(c+dx)-\frac{(a+b)^2}{\tanh^2(c+dx)-1}\right)d\tanh^2(c+dx)}{2d}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{2}a^2\coth^2(c+dx)-a(a+2b)\coth(c+dx)+(a+b)^2\log\left(\tanh^2(c+dx)\right)-(a+b)^2\log\left(1-\tanh^2(c+dx)\right)}{2d}$$

input
```
Int[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-(a*(a + 2*b)*Coth[c + d*x]) - (a^2*Coth[c + d*x]^2)/2 + (a + b)^2*Log[Ta
nh[c + d*x]^2] - (a + b)^2*Log[1 - Tanh[c + d*x]^2])/(2*d)
```

3.153.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.153.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

method	result
parallelrisch	$\frac{-4(a+b)^2\ln(1-\tanh(dx+c))+4(a+b)^2\ln(\tanh(dx+c))-\coth(dx+c)^4a^2-2a\coth(dx+c)^2(a+2b)-4dx(a+b)^2}{4d}$
derivativedivides	$-\frac{(\frac{1}{2}a^2+ab+\frac{1}{2}b^2)\ln(\tanh(dx+c)+1)+(-a^2-2ab-b^2)\ln(\tanh(dx+c))+\frac{a^2}{4\tanh(dx+c)^4}+\frac{a(a+2b)}{2\tanh(dx+c)^2}+(\frac{1}{2}a^2+ab+\frac{1}{2}b^2$
default	$-\frac{(\frac{1}{2}a^2+ab+\frac{1}{2}b^2)\ln(\tanh(dx+c)+1)+(-a^2-2ab-b^2)\ln(\tanh(dx+c))+\frac{a^2}{4\tanh(dx+c)^4}+\frac{a(a+2b)}{2\tanh(dx+c)^2}+(\frac{1}{2}a^2+ab+\frac{1}{2}b^2$
risch	$-a^2x-2abx-b^2x-\frac{2a^2c}{d}-\frac{4abc}{d}-\frac{2cb^2}{d}-\frac{4a\,e^{2dx+2c}(a\,e^{4dx+4c}+b\,e^{4dx+4c}-e^{2dx+2c}a-2b\,e^{2dx+2c}+a+$

input
```
int(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-4*(a+b)^2*ln(1-tanh(d*x+c))+4*(a+b)^2*ln(tanh(d*x+c))-coth(d*x+c)^4*
a^2-2*a*coth(d*x+c)^2*(a+2*b)-4*d*x*(a+b)^2)/d
```

3.153.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1649 vs. 2(68) = 136.

Time = 0.27 (sec) , antiderivative size = 1649, normalized size of antiderivative = 22.90

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*d*x*cosh
(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8 - 4*((
a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b +
b^2)*d*x*cosh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d*x + a^2 + a*b)*sinh(d*x +
 c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 - 3*((a^2 + 2*a*b + b
^2)*d*x - a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*(a^2 + 2*a*b +
b^2)*d*x - 2*a^2 - 4*a*b)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*d*x*
cosh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*x - 30*((a^2 + 2*a*b + b^2)*d*x
- a^2 - a*b)*cosh(d*x + c)^2 - 2*a^2 - 4*a*b)*sinh(d*x + c)^4 + 8*(7*(a^2
+ 2*a*b + b^2)*d*x*cosh(d*x + c)^5 - 10*((a^2 + 2*a*b + b^2)*d*x - a^2 - a
*b)*cosh(d*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x - 2*a^2 - 4*a*b)*cosh(d*x
 + c))*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*x - 4*((a^2 + 2*a*b + b^2)*
d*x - a^2 - a*b)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x +
 c)^6 - 15*((a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^4 - (a^2 +
2*a*b + b^2)*d*x + 3*(3*(a^2 + 2*a*b + b^2)*d*x - 2*a^2 - 4*a*b)*cosh(d*x
+ c)^2 + a^2 + a*b)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8
 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^
2)*sinh(d*x + c)^8 - 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)^2 - a^2 - 2*a*b - b^2)*sinh(d*x + c)^6 + 8*(7*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)...
```

3.153.6 Sympy [F]

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^2\coth^5\left(c+dx\right)dx$$

input
```
integrate(coth(d*x+c)**5*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**5, x)
```

3.153.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(68) = 136$.

Time $= 0.19$ (sec) , antiderivative size $= 236$, normalized size of antiderivative $= 3.28$

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= a^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8}}\right.$$
$$\left.+2\,ab\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)\right.$$
$$\left.+\frac{b^2\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}\right)$$

input
```
integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
a^2*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-
2*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c)
 - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 2*a
*b*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*
d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b^2*log(e^(d
*x + c) - e^(-d*x - c))/d
```

3.153.8 Giac [A] (verification not implemented)

Time $= 0.42$ (sec) , antiderivative size $= 118$, normalized size of antiderivative $= 1.64$

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$
$$-\frac{(a^2+2\,ab+b^2)(dx+c)-(a^2+2\,ab+b^2)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{4\left((a^2+ab)e^{(6\,dx+6\,c)}-(a^2+2\,ab)e^{(4\,dx+4\,c)}+(a^2+\right.}{\left(e^{(2\,dx+2\,c)}-1\right)^4}}{d}$$

input
```
integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-((a^2 + 2*a*b + b^2)*(d*x + c) - (a^2 + 2*a*b + b^2)*log(abs(e^(2*d*x + 2
*c) - 1)) + 4*((a^2 + a*b)*e^(6*d*x + 6*c) - (a^2 + 2*a*b)*e^(4*d*x + 4*c)
 + (a^2 + a*b)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c) - 1)^4)/d
```

3.153.9 Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.74

$$\begin{aligned}
&\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx \\
&= \frac{\ln\left(\mathrm{e}^{2c}\,\mathrm{e}^{2dx}-1\right)\left(a^2+2ab+b^2\right)}{d} - x\,(a+b)^2 \\
&\quad - \frac{4\left(2a^2+ba\right)}{d\left(\mathrm{e}^{4c+4dx}-2\,\mathrm{e}^{2c+2dx}+1\right)} - \frac{8a^2}{d\left(3\,\mathrm{e}^{2c+2dx}-3\,\mathrm{e}^{4c+4dx}+\mathrm{e}^{6c+6dx}-1\right)} \\
&\quad - \frac{4a^2}{d\left(6\,\mathrm{e}^{4c+4dx}-4\,\mathrm{e}^{2c+2dx}-4\,\mathrm{e}^{6c+6dx}+\mathrm{e}^{8c+8dx}+1\right)} - \frac{4\left(a^2+ba\right)}{d\left(\mathrm{e}^{2c+2dx}-1\right)}
\end{aligned}$$

input
```
int(coth(c + d*x)^5*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
(log(exp(2*c)*exp(2*d*x) - 1)*(2*a*b + a^2 + b^2))/d - x*(a + b)^2 - (4*(a
*b + 2*a^2))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*a^2)/(d*
(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (4*a^2
)/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8
*c + 8*d*x) + 1)) - (4*(a*b + a^2))/(d*(exp(2*c + 2*d*x) - 1))
```

3.154 $\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

3.154.1 Optimal result 1151
3.154.2 Mathematica [C] (verified) 1151
3.154.3 Rubi [A] (verified) 1152
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3.154.5 Fricas [B] (verification not implemented) 1154
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3.154.8 Giac [B] (verification not implemented) 1156
3.154.9 Mupad [B] (verification not implemented) 1157

3.154.1 Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = (a+b)^2x - \frac{(a+b)^2\coth(c+dx)}{d} - \frac{a(a+2b)\coth^3(c+dx)}{3d} - \frac{a^2\coth^5(c+dx)}{5d}$$

output
```
(a+b)^2*x-(a+b)^2*coth(d*x+c)/d-1/3*a*(a+2*b)*coth(d*x+c)^3/d-1/5*a^2*coth
(d*x+c)^5/d
```

3.154.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.56

$$\begin{aligned}&\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx \\ &= -\frac{a^2\coth^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\tanh^2(c+dx)\right)}{5d} \\ &\quad - \frac{2ab\coth^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\tanh^2(c+dx)\right)}{3d} \\ &\quad - \frac{b^2\coth(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(c+dx)\right)}{d}\end{aligned}$$

input
```
Integrate[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-1/5*(a^2*Coth[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, Tanh[c + d*x]^2
])/d - (2*a*b*Coth[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[c + d*
x]^2])/(3*d) - (b^2*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c +
 d*x]^2])/d
```

3.154.3 Rubi [A] (verified)

Time $= 0.29$ (sec) , antiderivative size $= 65$, normalized size of antiderivative $= 1.03$, number of steps used $= 7$, number of rules used $= 6$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used $= \{3042, 25, 4153, 25, 364, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)^6}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)^6}dx$$

$$\downarrow 4153$$

$$-\frac{\int -\frac{\coth^6(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 25$$

$$\frac{\int \frac{\coth^6(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 364$$

$$\frac{\int \left(a^2\coth^6(c+dx)+a(a+2b)\coth^4(c+dx)+(a+b)^2\coth^2(c+dx)-\frac{(a+b)^2}{\tanh^2(c+dx)-1}\right)d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{5}a^2\coth^5(c+dx)-(a+b)^2\text{arctanh}(\tanh(c+dx))+\frac{1}{3}a(a+2b)\coth^3(c+dx)+(a+b)^2\coth(c+dx)}{d}$$

input
```
Int[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-((-((a + b)^2*ArcTanh[Tanh[c + d*x]]) + (a + b)^2*Coth[c + d*x] + (a*(a +
 2*b)*Coth[c + d*x]^3)/3 + (a^2*Coth[c + d*x]^5)/5)/d)
```

3.154.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.154.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

method	result
parallelrisch	$\frac{-3\coth(dx+c)^5a^2-5a\coth(dx+c)^3(a+2b)-15(a+b)^2\coth(dx+c)+15dx(a+b)^2}{15d}$
derivativedivides	$-\frac{(\frac{1}{2}a^2+ab+\frac{1}{2}b^2)\ln(\tanh(dx+c)-1)+(-\frac{1}{2}a^2-ab-\frac{1}{2}b^2)\ln(\tanh(dx+c)+1)-\frac{-a^2-2ab-b^2}{\tanh(dx+c)}+\frac{a^2}{5\tanh(dx+c)^5}+\frac{a(a+2b)}{3\tanh(dx+}$
default	$-\frac{(\frac{1}{2}a^2+ab+\frac{1}{2}b^2)\ln(\tanh(dx+c)-1)+(-\frac{1}{2}a^2-ab-\frac{1}{2}b^2)\ln(\tanh(dx+c)+1)-\frac{-a^2-2ab-b^2}{\tanh(dx+c)}+\frac{a^2}{5\tanh(dx+c)^5}+\frac{a(a+2b)}{3\tanh(dx+}$
risch	$a^2x+2abx+b^2x-\frac{2(45a^2e^{8dx+8c}+60abe^{8dx+8c}+15b^2e^{8dx+8c}-90a^2e^{6dx+6c}-180abe^{6dx+6c}-60b^2e^{6dx+6c}+$

input
```
int(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/15*(-3*coth(d*x+c)^5*a^2-5*a*coth(d*x+c)^3*(a+2*b)-15*(a+b)^2*coth(d*x+c
)+15*d*x*(a+b)^2)/d
```

3.154.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(59) = 118$.

Time = 0.26 (sec) , antiderivative size = 473, normalized size of antiderivative = 7.51

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$

$$-\frac{(23\,a^2+40\,ab+15\,b^2)\cosh(dx+c)^5+5\,(23\,a^2+40\,ab+15\,b^2)\cosh(dx+c)\sinh(dx+c)^4-(15\,(a^2+$$

input
```
integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/15*((23*a^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^5 + 5*(23*a^2 + 40*a*b + 1
5*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*(a^2 + 2*a*b + b^2)*d*x + 23*a^
2 + 40*a*b + 15*b^2)*sinh(d*x + c)^5 - 5*(5*a^2 + 16*a*b + 9*b^2)*cosh(d*x
 + c)^3 + 5*(15*(a^2 + 2*a*b + b^2)*d*x - 2*(15*(a^2 + 2*a*b + b^2)*d*x +
23*a^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^2 + 23*a^2 + 40*a*b + 15*b^2)*sinh
(d*x + c)^3 + 5*(2*(23*a^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^3 - 3*(5*a^2 +
 16*a*b + 9*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^2 + 4*a*b + 3*b^
2)*cosh(d*x + c) - 5*((15*(a^2 + 2*a*b + b^2)*d*x + 23*a^2 + 40*a*b + 15*b
^2)*cosh(d*x + c)^4 + 30*(a^2 + 2*a*b + b^2)*d*x - 3*(15*(a^2 + 2*a*b + b^
2)*d*x + 23*a^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^2 + 46*a^2 + 80*a*b + 30*
b^2)*sinh(d*x + c))/(d*sinh(d*x + c)^5 + 5*(2*d*cosh(d*x + c)^2 - d)*sinh(
d*x + c)^3 + 5*(d*cosh(d*x + c)^4 - 3*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x +
c))
```

3.154.6 Sympy [F]

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2(c+dx)\right)^2 \coth^6(c+dx)\,dx$$

input
```
integrate(coth(d*x+c)**6*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**6, x)
```

3.154.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(59) = 118$.

Time $= 0.19$ (sec) , antiderivative size $= 231$, normalized size of antiderivative $= 3.67$

$$\begin{aligned}
&\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx \\
&= \frac{1}{15}a^2\left(15x + \frac{15c}{d} - \frac{2\left(70e^{(-2dx-2c)} - 140e^{(-4dx-4c)} + 90e^{(-6dx-6c)} - 45e^{(-8dx-8c)} - 23\right)}{d(5e^{(-2dx-2c)} - 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} - 5e^{(-8dx-8c)} + e^{(-10dx-10c)} - 1)}\right) \\
&\quad + \frac{2}{3}ab\left(3x + \frac{3c}{d} - \frac{4\left(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} - 2\right)}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)}\right) \\
&\quad + b^2\left(x + \frac{c}{d} + \frac{2}{d(e^{(-2dx-2c)} - 1)}\right)
\end{aligned}$$

input
```
integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/15*a^2*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) - 140*e^(-4*d*x - 4*c) +
90*e^(-6*d*x - 6*c) - 45*e^(-8*d*x - 8*c) - 23)/(d*(5*e^(-2*d*x - 2*c) - 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x
 - 10*c) - 1))) + 2/3*a*b*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d
*x - 4*c) - 2)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6
*c) - 1))) + b^2*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1)))
```

3.154.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(59) = 118$.

Time $= 0.42$ (sec) , antiderivative size $= 218$, normalized size of antiderivative $= 3.46$

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$= \frac{15\left(a^2+2ab+b^2\right)(dx+c) - \frac{2\left(45a^2e^{(8dx+8c)}+60abe^{(8dx+8c)}+15b^2e^{(8dx+8c)}-90a^2e^{(6dx+6c)}-180abe^{(6dx+6c)}-60b^2e^{(6dx+6c}}{}}{15d}$$

input
```
integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/15*(15*(a^2 + 2*a*b + b^2)*(d*x + c) - 2*(45*a^2*e^(8*d*x + 8*c) + 60*a*
b*e^(8*d*x + 8*c) + 15*b^2*e^(8*d*x + 8*c) - 90*a^2*e^(6*d*x + 6*c) - 180*
a*b*e^(6*d*x + 6*c) - 60*b^2*e^(6*d*x + 6*c) + 140*a^2*e^(4*d*x + 4*c) + 2
20*a*b*e^(4*d*x + 4*c) + 90*b^2*e^(4*d*x + 4*c) - 70*a^2*e^(2*d*x + 2*c) -
 140*a*b*e^(2*d*x + 2*c) - 60*b^2*e^(2*d*x + 2*c) + 23*a^2 + 40*a*b + 15*b
^2)/(e^(2*d*x + 2*c) - 1)^5)/d
```

3.154.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 529, normalized size of antiderivative = 8.40

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\frac{2\left(b^2+2ab\right)}{5d}-\frac{2e^{2c+2dx}\left(3a^2+4ab+b^2\right)}{5d}}{e^{4c+4dx}-2e^{2c+2dx}+1}$$
$$+\frac{\frac{2\left(b^2+2ab\right)}{5d}+\frac{6e^{4c+4dx}\left(b^2+2ab\right)}{5d}-\frac{2e^{6c+6dx}\left(3a^2+4ab+b^2\right)}{5d}-\frac{2e^{2c+2dx}\left(5a^2+4ab+3b^2\right)}{5d}}{6e^{4c+4dx}-4e^{2c+2dx}-4e^{6c+6dx}+e^{8c+8dx}+1}+x\left(a+b\right)^2$$
$$-\frac{\frac{2\left(3a^2+4ab+b^2\right)}{5d}-\frac{8e^{2c+2dx}\left(b^2+2ab\right)}{5d}-\frac{8e^{6c+6dx}\left(b^2+2ab\right)}{5d}+\frac{2e^{8c+8dx}\left(3a^2+4ab+b^2\right)}{5d}+\frac{4e^{4c+4dx}\left(5a^2+4ab+3b^2\right)}{5d}}{5e^{2c+2dx}-10e^{4c+4dx}+10e^{6c+6dx}-5e^{8c+8dx}+e^{10c+10dx}-1}$$
$$-\frac{\frac{2\left(5a^2+4ab+3b^2\right)}{15d}-\frac{4e^{2c+2dx}\left(b^2+2ab\right)}{5d}+\frac{2e^{4c+4dx}\left(3a^2+4ab+b^2\right)}{5d}}{3e^{2c+2dx}-3e^{4c+4dx}+e^{6c+6dx}-1}-\frac{2\left(3a^2+4ab+b^2\right)}{5d\left(e^{2c+2dx}-1\right)}$$

input
```
int(coth(c + d*x)^6*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
((2*(2*a*b + b^2))/(5*d) - (2*exp(2*c + 2*d*x)*(4*a*b + 3*a^2 + b^2))/(5*d
))/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) + ((2*(2*a*b + b^2))/(5*d)
+ (6*exp(4*c + 4*d*x)*(2*a*b + b^2))/(5*d) - (2*exp(6*c + 6*d*x)*(4*a*b +
3*a^2 + b^2))/(5*d) - (2*exp(2*c + 2*d*x)*(4*a*b + 5*a^2 + 3*b^2))/(5*d))/
(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c +
8*d*x) + 1) + x*(a + b)^2 - ((2*(4*a*b + 3*a^2 + b^2))/(5*d) - (8*exp(2*c
+ 2*d*x)*(2*a*b + b^2))/(5*d) - (8*exp(6*c + 6*d*x)*(2*a*b + b^2))/(5*d) +
 (2*exp(8*c + 8*d*x)*(4*a*b + 3*a^2 + b^2))/(5*d) + (4*exp(4*c + 4*d*x)*(4
*a*b + 5*a^2 + 3*b^2))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) +
10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) - ((2*(
4*a*b + 5*a^2 + 3*b^2))/(15*d) - (4*exp(2*c + 2*d*x)*(2*a*b + b^2))/(5*d)
+ (2*exp(4*c + 4*d*x)*(4*a*b + 3*a^2 + b^2))/(5*d))/(3*exp(2*c + 2*d*x) -
3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - (2*(4*a*b + 3*a^2 + b^2))/(5*
d*(exp(2*c + 2*d*x) - 1))
```

3.155 $\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

3.155.1 Optimal result 1158
3.155.2 Mathematica [A] (verified) 1158
3.155.3 Rubi [A] (warning: unable to verify) 1159
3.155.4 Maple [A] (verified) 1161
3.155.5 Fricas [B] (verification not implemented) 1161
3.155.6 Sympy [F(-1)] 1162
3.155.7 Maxima [B] (verification not implemented) 1163
3.155.8 Giac [B] (verification not implemented) 1163
3.155.9 Mupad [B] (verification not implemented) 1164

3.155.1 Optimal result

Integrand size = 23, antiderivative size = 92

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{(a+b)^2\coth^2(c+dx)}{2d} - \frac{a(a+2b)\coth^4(c+dx)}{4d} - \frac{a^2\coth^6(c+dx)}{6d} + \frac{(a+b)^2\log(\cosh(c+dx))}{d} + \frac{(a+b)^2\log(\tanh(c+dx))}{d}$$

output
```
-1/2*(a+b)^2*coth(d*x+c)^2/d-1/4*a*(a+2*b)*coth(d*x+c)^4/d-1/6*a^2*coth(d*
x+c)^6/d+(a+b)^2*ln(cosh(d*x+c))/d+(a+b)^2*ln(tanh(d*x+c))/d
```

3.155.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.80

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{6(a+b)^2\coth^2(c+dx)+3a(a+2b)\coth^4(c+dx)+2a^2\coth^6(c+dx)-12(a+b)^2(\log(\cosh(c+dx)$$

input
```
Integrate[Coth[c + d*x]^7*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-1/12*(6*(a + b)^2*Coth[c + d*x]^2 + 3*a*(a + 2*b)*Coth[c + d*x]^4 + 2*a^2
*Coth[c + d*x]^6 - 12*(a + b)^2*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]]))
/d
```

3.155.3 Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int -\frac{i\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)^7}dx$$

$$\downarrow 26$$

$$-i\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)^7}dx$$

$$\downarrow 4153$$

$$-\frac{i\int \frac{i\coth^7(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\coth^7(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 354$$

$$\frac{\int \frac{\coth^4(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{2d}$$

$$\downarrow 99$$

$$\frac{\int \left(a^2\coth^4(c+dx)+a(a+2b)\coth^3(c+dx)+(a+b)^2\coth^2(c+dx)+(a+b)^2\coth(c+dx)-\frac{(a+b)^2}{\tanh^2(c+dx)-1}\right)c}{2d}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{3}a^2\coth^3(c+dx)-\frac{1}{2}a(a+2b)\coth^2(c+dx)-(a+b)^2\coth(c+dx)+(a+b)^2\log\left(\tanh^2(c+dx)\right)-(a+b)^2}{2d}$$

input
```
Int[Coth[c + d*x]^7*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-((a + b)^2*Coth[c + d*x]) - (a*(a + 2*b)*Coth[c + d*x]^2)/2 - (a^2*Coth[
c + d*x]^3)/3 + (a + b)^2*Log[Tanh[c + d*x]^2] - (a + b)^2*Log[1 - Tanh[c
+ d*x]^2])/(2*d)
```

3.155.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.155.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\frac{-12(a+b)^2\ln(1-\tanh(dx+c))+12(a+b)^2\ln(\tanh(dx+c))-2\coth(dx+c)^6a^2-3a\coth(dx+c)^4(a+2b)-6\coth(dx+c)^2(a}{12d}$
derivativedivides	$-\frac{(\frac{1}{2}a^2+ab+\frac{1}{2}b^2)\ln(\tanh(dx+c)-1)+(\frac{1}{2}a^2+ab+\frac{1}{2}b^2)\ln(\tanh(dx+c)+1)-\frac{-a^2-2ab-b^2}{2\tanh(dx+c)^2}+(-a^2-2ab-b^2)\ln(\tanh(dx}{d}$
default	$-\frac{(\frac{1}{2}a^2+ab+\frac{1}{2}b^2)\ln(\tanh(dx+c)-1)+(\frac{1}{2}a^2+ab+\frac{1}{2}b^2)\ln(\tanh(dx+c)+1)-\frac{-a^2-2ab-b^2}{2\tanh(dx+c)^2}+(-a^2-2ab-b^2)\ln(\tanh(dx}{d}$
risch	$-a^2x-2abx-b^2x-\frac{2a^2c}{d}-\frac{4abc}{d}-\frac{2cb^2}{d}-\frac{2e^{2dx+2c}(9a^2e^{8dx+8c}+12abe^{8dx+8c}+3b^2e^{8dx+8c}-18a^2e^{6d}}{}$

input

```
int(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output

```
1/12*(-12*(a+b)^2*ln(1-tanh(d*x+c))+12*(a+b)^2*ln(tanh(d*x+c))-2*coth(d*x+
c)^6*a^2-3*a*coth(d*x+c)^4*(a+2*b)-6*coth(d*x+c)^2*(a+b)^2-12*d*x*(a+b)^2)
/d
```

3.155.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3454 vs. $2(86) = 172$.

Time = 0.31 (sec) , antiderivative size = 3454, normalized size of antiderivative = 37.54

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input

```
integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output

```
-1/3*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^12 + 36*(a^2 + 2*a*b + b^2)*
d*x*cosh(d*x + c)*sinh(d*x + c)^11 + 3*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x +
c)^12 - 6*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^
10 + 6*(33*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 - 3*(a^2 + 2*a*b + b^2)
*d*x + 3*a^2 + 4*a*b + b^2)*sinh(d*x + c)^10 + 60*(11*(a^2 + 2*a*b + b^2)*
d*x*cosh(d*x + c)^3 - (3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*co
sh(d*x + c))*sinh(d*x + c)^9 + 3*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24
*a*b - 8*b^2)*cosh(d*x + c)^8 + 3*(495*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x +
c)^4 + 15*(a^2 + 2*a*b + b^2)*d*x - 90*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2
- 4*a*b - b^2)*cosh(d*x + c)^2 - 12*a^2 - 24*a*b - 8*b^2)*sinh(d*x + c)^8
+ 24*(99*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^5 - 30*(3*(a^2 + 2*a*b + b^
2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^3 + (15*(a^2 + 2*a*b + b^2)*d*
x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(15*(a^2 +
 2*a*b + b^2)*d*x - 17*a^2 - 24*a*b - 9*b^2)*cosh(d*x + c)^6 + 4*(693*(a^2
 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 - 315*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a
^2 - 4*a*b - b^2)*cosh(d*x + c)^4 - 15*(a^2 + 2*a*b + b^2)*d*x + 21*(15*(a
^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^2 + 17*a^2
+ 24*a*b + 9*b^2)*sinh(d*x + c)^6 + 24*(99*(a^2 + 2*a*b + b^2)*d*x*cosh(d*
x + c)^7 - 63*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x +
 c)^5 + 7*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d...
```

3.155.6 Sympy [F(-1)]

Timed out.

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Timed out}$$

input

```
integrate(coth(d*x+c)**7*(a+b*tanh(d*x+c)**2)**2,x)
```

output

```
Timed out
```

3.155.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. 2(86) = 172.

Time = 0.20 (sec) , antiderivative size = 390, normalized size of antiderivative = 4.24

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$= \frac{1}{3}a^2\left(3x+\frac{3c}{d}+\frac{3\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{3\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(9e^{(-2dx-2c)}-18e^{(-4dx-4c)}+\right.}{d(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6dx}}\right.$$

$$+2ab\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2dx-2c)}-e^{(-4dx-4c)}+e^{(-6dx-}\right.}{d(4e^{(-2dx-2c)}-6e^{(-4dx-4c)}+4e^{(-6dx-6c)}-e}\right.$$

$$+b^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)}-e^{(-4dx-4c)}-1)}\right)$$

input
```
integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/3*a^2*(3*x + 3*c/d + 3*log(e^(-d*x - c) + 1)/d + 3*log(e^(-d*x - c) - 1)
/d + 2*(9*e^(-2*d*x - 2*c) - 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) - 1
8*e^(-8*d*x - 8*c) + 9*e^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(
-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x -
 10*c) - e^(-12*d*x - 12*c) - 1))) + 2*a*b*(x + c/d + log(e^(-d*x - c) + 1
)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x
 - 6*c) - e^(-8*d*x - 8*c) - 1))) + b^2*(x + c/d + log(e^(-d*x - c) + 1)/d
 + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e
^(-4*d*x - 4*c) - 1)))
```

3.155.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(86) = 172.

Time = 0.46 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.09

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$

$$-\frac{3\left(a^2+2ab+b^2\right)(dx+c)-3\left(a^2+2ab+b^2\right)\log\left(\left|e^{(2dx+2c)}-1\right|\right)+\frac{2\left(3\left(3a^2+4ab+b^2\right)e^{(10dx+10c)}-6\left(3a^2+6a\right.\right.}{3d}$$

input
```
integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/3*(3*(a^2 + 2*a*b + b^2)*(d*x + c) - 3*(a^2 + 2*a*b + b^2)*log(abs(e^(2
*d*x + 2*c) - 1)) + 2*(3*(3*a^2 + 4*a*b + b^2)*e^(10*d*x + 10*c) - 6*(3*a^
2 + 6*a*b + 2*b^2)*e^(8*d*x + 8*c) + 2*(17*a^2 + 24*a*b + 9*b^2)*e^(6*d*x
+ 6*c) - 6*(3*a^2 + 6*a*b + 2*b^2)*e^(4*d*x + 4*c) + 3*(3*a^2 + 4*a*b + b^
2)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c) - 1)^6)/d
```

3.155.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 362, normalized size of antiderivative = 3.93

$$
\begin{aligned}
&\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx \\
&= \frac{\ln\left(\mathrm{e}^{2c}\,\mathrm{e}^{2dx}-1\right)\left(a^2+2ab+b^2\right)}{d} - \frac{2\left(3a^2+4ab+b^2\right)}{d\left(\mathrm{e}^{2c+2dx}-1\right)} \\
&\quad - \frac{32a^2}{3d\left(15\,\mathrm{e}^{4c+4dx}-6\,\mathrm{e}^{2c+2dx}-20\,\mathrm{e}^{6c+6dx}+15\,\mathrm{e}^{8c+8dx}-6\,\mathrm{e}^{10c+10dx}+\mathrm{e}^{12c+12dx}+1\right)} \\
&\quad - x(a+b)^2 - \frac{2\left(9a^2+8ab+b^2\right)}{d\left(\mathrm{e}^{4c+4dx}-2\,\mathrm{e}^{2c+2dx}+1\right)} - \frac{8\left(13a^2+6ba\right)}{3d\left(3\,\mathrm{e}^{2c+2dx}-3\,\mathrm{e}^{4c+4dx}+\mathrm{e}^{6c+6dx}-1\right)} \\
&\quad - \frac{4\left(11a^2+2ba\right)}{d\left(6\,\mathrm{e}^{4c+4dx}-4\,\mathrm{e}^{2c+2dx}-4\,\mathrm{e}^{6c+6dx}+\mathrm{e}^{8c+8dx}+1\right)} \\
&\quad - \frac{32a^2}{d\left(5\,\mathrm{e}^{2c+2dx}-10\,\mathrm{e}^{4c+4dx}+10\,\mathrm{e}^{6c+6dx}-5\,\mathrm{e}^{8c+8dx}+\mathrm{e}^{10c+10dx}-1\right)}
\end{aligned}
$$

input
```
int(coth(c + d*x)^7*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
(log(exp(2*c)*exp(2*d*x) - 1)*(2*a*b + a^2 + b^2))/d - (2*(4*a*b + 3*a^2 +
 b^2))/(d*(exp(2*c + 2*d*x) - 1)) - (32*a^2)/(3*d*(15*exp(4*c + 4*d*x) - 6
*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c
 + 10*d*x) + exp(12*c + 12*d*x) + 1)) - x*(a + b)^2 - (2*(8*a*b + 9*a^2 +
b^2))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*(6*a*b + 13*a^2
))/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1))
- (4*(2*a*b + 11*a^2))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp
(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (32*a^2)/(d*(5*exp(2*c + 2*d*x) -
 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c
 + 10*d*x) - 1))
```

3.156 $\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

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3.156.1 Optimal result

Integrand size = 23, antiderivative size = 114

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = (a+b)^3x - \frac{(a+b)^3\tanh(c+dx)}{d} - \frac{(a+b)^3\tanh^3(c+dx)}{3d} - \frac{b(3a^2+3ab+b^2)\tanh^5(c+dx)}{5d} - \frac{b^2(3a+b)\tanh^7(c+dx)}{7d} - \frac{b^3\tanh^9(c+dx)}{9d}$$

output
```
(a+b)^3*x-(a+b)^3*tanh(d*x+c)/d-1/3*(a+b)^3*tanh(d*x+c)^3/d-1/5*b*(3*a^2+3
*a*b+b^2)*tanh(d*x+c)^5/d-1/7*b^2*(3*a+b)*tanh(d*x+c)^7/d-1/9*b^3*tanh(d*x
+c)^9/d
```

3.156.2 Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\tanh(c+dx)\left(-315(a+b)^3 - 105(a+b)^3\tanh^2(c+dx) - 63b(3a^2+3ab+b^2)\tanh^4(c+dx) - 45b^2(3a\right.}{315d}$$

input
```
Integrate[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(Tanh[c + d*x]*(-315*(a + b)^3 - 105*(a + b)^3*Tanh[c + d*x]^2 - 63*b*(3*a
^2 + 3*a*b + b^2)*Tanh[c + d*x]^4 - 45*b^2*(3*a + b)*Tanh[c + d*x]^6 - 35*
b^3*Tanh[c + d*x]^8 + (315*(a + b)^3*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[
Tanh[c + d*x]^2]))/(315*d)
```

3.156.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4153, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\downarrow 3042$$

$$\int \tan(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)^3\,dx$$

$$\downarrow 4153$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}\,d\tanh(c+dx)}{d}$$

$$\downarrow 364$$

$$\frac{\int \left(-b^3\tanh^8(c+dx)-b^2(3a+b)\tanh^6(c+dx)-b\left(3a^2+3ba+b^2\right)\tanh^4(c+dx)-(a+b)^3\tanh^2(c+dx)-(\right.}{d}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{5}b\left(3a^2+3ab+b^2\right)\tanh^5(c+dx)+(a+b)^3\text{arctanh}(\tanh(c+dx))-\frac{1}{7}b^2(3a+b)\tanh^7(c+dx)-\frac{1}{3}(a+b)^3\tan}{d}$$

input
```
Int[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((a + b)^3*ArcTanh[Tanh[c + d*x]] - (a + b)^3*Tanh[c + d*x] - ((a + b)^3*T
anh[c + d*x]^3)/3 - (b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^5)/5 - (b^2*(3*
a + b)*Tanh[c + d*x]^7)/7 - (b^3*Tanh[c + d*x]^9)/9)/d
```

3.156.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.156.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(106) = 212$.

Time $= 0.17$ (sec) , antiderivative size $= 218$, normalized size of antiderivative $= 1.91$

method	result
parallelrisch	$-\frac{35\tanh(dx+c)^9b^3+135ab^2\tanh(dx+c)^7+45\tanh(dx+c)^7b^3+189a^2b\tanh(dx+c)^5+189\tanh(dx+c)^5ab^2+63b^3\tanh$
derivativedivides	$\frac{-3a^2b\tanh(dx+c)-3ab^2\tanh(dx+c)-\frac{3ab^2\tanh(dx+c)^7}{7}-\frac{3a^2b\tanh(dx+c)^5}{5}-\frac{3\tanh(dx+c)^5ab^2}{5}-\tanh(dx+c)^3a^2b-ab^2$
default	$\frac{-3a^2b\tanh(dx+c)-3ab^2\tanh(dx+c)-\frac{3ab^2\tanh(dx+c)^7}{7}-\frac{3a^2b\tanh(dx+c)^5}{5}-\frac{3\tanh(dx+c)^5ab^2}{5}-\tanh(dx+c)^3a^2b-ab^2$
parts	$\frac{b^3\left(-\frac{\tanh(dx+c)^9}{9}-\frac{\tanh(dx+c)^7}{7}-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{a}{-}$
risch	$a^3x+3ba^2x+3ab^2x+b^3x+\frac{\frac{352ab^2}{35}+10b^3e^{16dx+16c}+\frac{8a^3}{3}+108a^2be^{14dx+14c}+24ab^2e^{16dx+16c}+18a^2be^{1}}{}$

input
```
int(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/315*(35*tanh(d*x+c)^9*b^3+135*a*b^2*tanh(d*x+c)^7+45*tanh(d*x+c)^7*b^3+
189*a^2*b*tanh(d*x+c)^5+189*tanh(d*x+c)^5*a*b^2+63*b^3*tanh(d*x+c)^5+105*a
^3*tanh(d*x+c)^3+315*tanh(d*x+c)^3*a^2*b+315*a*b^2*tanh(d*x+c)^3+105*b^3*t
anh(d*x+c)^3-315*a^3*d*x-945*a^2*b*d*x-945*a*b^2*d*x-315*b^3*d*x+315*a^3*t
anh(d*x+c)+945*a^2*b*tanh(d*x+c)+945*a*b^2*tanh(d*x+c)+315*b^3*tanh(d*x+c)
)/d
```

3.156.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1563 vs. $2(106) = 212$.

Time $= 0.27$ (sec) , antiderivative size $= 1563$, normalized size of antiderivative $= 13.71$

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/315*((420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^9 + 9*(420*a^3 + 1449*a^2*b + 1584*a*b^
2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(
d*x + c)^8 - (420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3)*sinh(d*x + c)^9
 + 9*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 - 9*(280*a^3 + 819*a^2*b + 744*a*b^2 +
213*b^3 + 4*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^7 + 21*(4*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 31
5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + 3*(420*a^3 + 1449
*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*c
osh(d*x + c))*sinh(d*x + c)^6 + 36*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 56
3*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 9*(14*(
420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3)*cosh(d*x + c)^4 + 700*a^3 + 2
016*a^2*b + 2136*a*b^2 + 852*b^3 + 21*(280*a^3 + 819*a^2*b + 744*a*b^2 + 2
13*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 9*(14*(420*a^3 + 1449*a^2*b + 1
584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x +
c)^5 + 35*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + 20*(420*a^3 + 1449*a^2*b + 1584*
a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*
sinh(d*x + c)^4 + 84*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315...
```

3.156.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(99) = 198$.

Time $= 0.30$ (sec) , antiderivative size $= 260$, normalized size of antiderivative $= 2.28$

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \begin{cases} a^3x - \frac{a^3\tanh^3(c+dx)}{3d} - \frac{a^3\tanh(c+dx)}{d} + 3a^2bx - \frac{3a^2b\tanh^5(c+dx)}{5d} - \frac{a^2b\tanh^3(c+dx)}{d} - \frac{3a^2b\tanh(c+dx)}{d} + 3ab^2x - \frac{3}{} \\ x\left(a+b\tanh^2(c)\right)^3\tanh^4(c) \end{cases}$$

input
```
integrate(tanh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Piecewise((a**3*x - a**3*tanh(c + d*x)**3/(3*d) - a**3*tanh(c + d*x)/d + 3
*a**2*b*x - 3*a**2*b*tanh(c + d*x)**5/(5*d) - a**2*b*tanh(c + d*x)**3/d -
3*a**2*b*tanh(c + d*x)/d + 3*a*b**2*x - 3*a*b**2*tanh(c + d*x)**7/(7*d) -
3*a*b**2*tanh(c + d*x)**5/(5*d) - a*b**2*tanh(c + d*x)**3/d - 3*a*b**2*tan
h(c + d*x)/d + b**3*x - b**3*tanh(c + d*x)**9/(9*d) - b**3*tanh(c + d*x)**
7/(7*d) - b**3*tanh(c + d*x)**5/(5*d) - b**3*tanh(c + d*x)**3/(3*d) - b**3
*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**3*tanh(c)**4, True))
```

3.156.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(106) = 212$.

Time $= 0.21$ (sec) , antiderivative size $= 583$, normalized size of antiderivative $= 5.11$

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{1}{315}b^3\left(315x+\frac{315c}{d}-\frac{2\left(3492e^{(-2dx-2c)}+13968e^{(-4dx-4c)}+26292e^{(-6dx-6c)}+39438e^{(-8dx-8c)}+31\right.}{d(9e^{(-2dx-2c)}+36e^{(-4dx-4c)}+84e^{(-6dx-6c)}+126e^{(-8dx-8c)}+126e^{(-10}\right.$$

$$+\frac{1}{35}ab^2\left(105x+\frac{105c}{d}-\frac{8\left(203e^{(-2dx-2c)}+609e^{(-4dx-4c)}+770e^{(-6dx-6c)}+770e^{(-8dx-8c)}+315e^{(}\right.}{d(7e^{(-2dx-2c)}+21e^{(-4dx-4c)}+35e^{(-6dx-6c)}+35e^{(-8dx-8c)}+21e^{(-10dx-10c}\right.$$

$$+\frac{1}{5}a^2b\left(15x+\frac{15c}{d}-\frac{2\left(70e^{(-2dx-2c)}+140e^{(-4dx-4c)}+90e^{(-6dx-6c)}+45e^{(-8dx-8c)}+23\right)}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}\right)$$

$$+\frac{1}{3}a^3\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)$$

input
```
integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/315*b^3*(315*x + 315*c/d - 2*(3492*e^(-2*d*x - 2*c) + 13968*e^(-4*d*x -
4*c) + 26292*e^(-6*d*x - 6*c) + 39438*e^(-8*d*x - 8*c) + 31500*e^(-10*d*x
- 10*c) + 21000*e^(-12*d*x - 12*c) + 6300*e^(-14*d*x - 14*c) + 1575*e^(-16
*d*x - 16*c) + 563)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-
6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d
*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 1
8*c) + 1))) + 1/35*a*b^2*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*
e^(-4*d*x - 4*c) + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-1
0*d*x - 10*c) + 105*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e
^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*
x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 1/5*a^2*b*(
15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d
*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*
x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) +
 1))) + 1/3*a^3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c)
+ 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

3.156.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(106) = 212$.

Time $= 0.51$ (sec) , antiderivative size $= 534$, normalized size of antiderivative $= 4.68$

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{315\left(a^3+3\,a^2b+3\,ab^2+b^3\right)(dx+c)+\frac{2\left(630\,a^3e^{(16\,dx+16\,c)}+2835\,a^2be^{(16\,dx+16\,c)}+3780\,ab^2e^{(16\,dx+16\,c)}+1575\,b^3e^{(16\,dx+16\,c)}+\right.}{}$$

input
```
integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/315*(315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) + 2*(630*a^3*e^(16*d*
x + 16*c) + 2835*a^2*b*e^(16*d*x + 16*c) + 3780*a*b^2*e^(16*d*x + 16*c) +
1575*b^3*e^(16*d*x + 16*c) + 4410*a^3*e^(14*d*x + 14*c) + 17010*a^2*b*e^(1
4*d*x + 14*c) + 18900*a*b^2*e^(14*d*x + 14*c) + 6300*b^3*e^(14*d*x + 14*c)
 + 13650*a^3*e^(12*d*x + 12*c) + 48510*a^2*b*e^(12*d*x + 12*c) + 54180*a*b
^2*e^(12*d*x + 12*c) + 21000*b^3*e^(12*d*x + 12*c) + 24570*a^3*e^(10*d*x +
 10*c) + 85050*a^2*b*e^(10*d*x + 10*c) + 94500*a*b^2*e^(10*d*x + 10*c) + 3
1500*b^3*e^(10*d*x + 10*c) + 28350*a^3*e^(8*d*x + 8*c) + 97524*a^2*b*e^(8*
d*x + 8*c) + 105084*a*b^2*e^(8*d*x + 8*c) + 39438*b^3*e^(8*d*x + 8*c) + 21
630*a^3*e^(6*d*x + 6*c) + 73206*a^2*b*e^(6*d*x + 6*c) + 78876*a*b^2*e^(6*d
*x + 6*c) + 26292*b^3*e^(6*d*x + 6*c) + 10710*a^3*e^(4*d*x + 4*c) + 35154*
a^2*b*e^(4*d*x + 4*c) + 38124*a*b^2*e^(4*d*x + 4*c) + 13968*b^3*e^(4*d*x +
 4*c) + 3150*a^3*e^(2*d*x + 2*c) + 10206*a^2*b*e^(2*d*x + 2*c) + 10476*a*b
^2*e^(2*d*x + 2*c) + 3492*b^3*e^(2*d*x + 2*c) + 420*a^3 + 1449*a^2*b + 158
4*a*b^2 + 563*b^3)/(e^(2*d*x + 2*c) + 1)^9)/d
```

3.156.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.21

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = x\left(a^3+3a^2b+3ab^2+b^3\right) - \frac{\tanh(c+dx)\,(a+b)^3}{d} - \frac{\tanh(c+dx)^5\left(3a^2b+3ab^2+b^3\right)}{5d} - \frac{\tanh(c+dx)^7\left(b^3+3ab^2\right)}{7d} - \frac{b^3\tanh(c+dx)^9}{9d} - \frac{\tanh(c+dx)^3\left(a^3+3a^2b+3ab^2+b^3\right)}{3d}$$

input
```
int(tanh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c + d*x)*(a + b)^3)/d - (tanh(c
+ d*x)^5*(3*a*b^2 + 3*a^2*b + b^3))/(5*d) - (tanh(c + d*x)^7*(3*a*b^2 + b^
3))/(7*d) - (b^3*tanh(c + d*x)^9)/(9*d) - (tanh(c + d*x)^3*(3*a*b^2 + 3*a^
2*b + a^3 + b^3))/(3*d)
```

3.157 $\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

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3.157.1 Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{(a+b)^3\log(\cosh(c+dx))}{d} - \frac{(a+b)^3\tanh^2(c+dx)}{2d} - \frac{b(3a^2+3ab+b^2)\tanh^4(c+dx)}{4d} - \frac{b^2(3a+b)\tanh^6(c+dx)}{6d} - \frac{b^3\tanh^8(c+dx)}{8d}$$

output
```
(a+b)^3*ln(cosh(d*x+c))/d-1/2*(a+b)^3*tanh(d*x+c)^2/d-1/4*b*(3*a^2+3*a*b+b
^2)*tanh(d*x+c)^4/d-1/6*b^2*(3*a+b)*tanh(d*x+c)^6/d-1/8*b^3*tanh(d*x+c)^8/
d
```

3.157.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{2(a+b)^3\log(\cosh(c+dx)) - (a+b)^3\tanh^2(c+dx) - \frac{1}{2}b(3a^2+3ab+b^2)\tanh^4(c+dx) - \frac{1}{3}b^2(3a+b)\tanh^6(c+dx) - \frac{1}{4}b^3\tanh^8(c+dx)}{2d}$$

input
```
Integrate[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(2*(a + b)^3*Log[Cosh[c + d*x]] - (a + b)^3*Tanh[c + d*x]^2 - (b*(3*a^2 +
3*a*b + b^2)*Tanh[c + d*x]^4)/2 - (b^2*(3*a + b)*Tanh[c + d*x]^6)/3 - (b^3
*Tanh[c + d*x]^8)/4)/(2*d)
```

3.157.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\downarrow 3042$$

$$\int i\tan(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^3\,dx$$

$$\downarrow 26$$

$$i\int \tan(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^3\,dx$$

$$\downarrow 4153$$

$$\frac{i\int -\frac{i\tanh^3(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}\,d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\tanh^3(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}\,d\tanh(c+dx)}{d}$$

$$\downarrow 354$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}\,d\tanh^2(c+dx)}{2d}$$

$$\downarrow 86$$

$$\frac{\int \left(-b^3\tanh^6(c+dx) - b^2(3a+b)\tanh^4(c+dx) - b\left(3a^2+3ba+b^2\right)\tanh^2(c+dx) - (a+b)^3 - \frac{(a+b)^3}{\tanh^2(c+dx)-1}\right) d}{2d}$$

$$\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}b\left(3a^2+3ab+b^2\right)\tanh^4(c+dx) - \frac{1}{3}b^2(3a+b)\tanh^6(c+dx) - (a+b)^3\tanh^2(c+dx) - (a+b)^3\log\left(1-\tanh\right.}{2d}$$

input
```
Int[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-((a + b)^3*Log[1 - Tanh[c + d*x]^2]) - (a + b)^3*Tanh[c + d*x]^2 - (b*(3
*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^4)/2 - (b^2*(3*a + b)*Tanh[c + d*x]^6)/3
 - (b^3*Tanh[c + d*x]^8)/4)/(2*d)
```

3.157.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.157.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(99) = 198$.

Time = 0.14 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.92

method	result
derivativedivides	$\frac{-\frac{\tanh(dx+c)^6 a b^2}{2}-\frac{3\tanh(dx+c)^4 a^2 b}{4}-\frac{3\tanh(dx+c)^4 a b^2}{4}-\frac{3\tanh(dx+c)^2 a^2 b}{2}-\frac{3a b^2\tanh(dx+c)^2}{2}-\frac{\tanh(dx+c)^6 b^3}{6}-\frac{b^3\tanh(d}{4}$
default	$\frac{-\frac{\tanh(dx+c)^6 a b^2}{2}-\frac{3\tanh(dx+c)^4 a^2 b}{4}-\frac{3\tanh(dx+c)^4 a b^2}{4}-\frac{3\tanh(dx+c)^2 a^2 b}{2}-\frac{3a b^2\tanh(dx+c)^2}{2}-\frac{\tanh(dx+c)^6 b^3}{6}-\frac{b^3\tanh(d}{4}$
parts	$\frac{a^3\left(-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b^3\left(-\frac{\tanh(dx+c)^8}{8}-\frac{\tanh(dx+c)^6}{6}-\frac{\tanh(dx+c)^4}{4}-\frac{\tanh(dx+}{d}$
parallelrisch	$-\frac{3\tanh(dx+c)^8 b^3+12\tanh(dx+c)^6 a b^2+4\tanh(dx+c)^6 b^3+18\tanh(dx+c)^4 a^2 b+18\tanh(dx+c)^4 a b^2+6b^3\tanh(dx+c}{}$
risch	$-a^3x-3b a^2x-3a b^2x-b^3x-\frac{2a^3c}{d}-\frac{6bc a^2}{d}-\frac{6a b^2c}{d}-\frac{2b^3c}{d}+\frac{2\mathrm{e}^{2dx+2c}(27a b^2+3a^3+18a^2b\mathrm{e}^{12a}}{}$

input
```
int(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*tanh(d*x+c)^6*a*b^2-3/4*tanh(d*x+c)^4*a^2*b-3/4*tanh(d*x+c)^4*a*
b^2-3/2*tanh(d*x+c)^2*a^2*b-3/2*a*b^2*tanh(d*x+c)^2-1/6*tanh(d*x+c)^6*b^3-
1/4*b^3*tanh(d*x+c)^4-1/2*tanh(d*x+c)^2*a^3-1/2*b^3*tanh(d*x+c)^2-1/8*tanh
(d*x+c)^8*b^3+1/2*(-a^3-3*a^2*b-3*a*b^2-b^3)*ln(tanh(d*x+c)+1)-1/2*(a^3+3*
a^2*b+3*a*b^2+b^3)*ln(tanh(d*x+c)-1))
```

3.157.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7502 vs. 2(99) = 198.

Time = 0.34 (sec) , antiderivative size = 7502, normalized size of antiderivative = 70.11

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.157.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(94) = 188.

Time = 0.27 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.61

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \begin{cases} a^3x - \frac{a^3\log(\tanh(c+dx)+1)}{d} - \frac{a^3\tanh^2(c+dx)}{2d} + 3a^2bx - \frac{3a^2b\log(\tanh(c+dx)+1)}{d} - \frac{3a^2b\tanh^4(c+dx)}{4d} - \frac{3a^2b\tanh^2(c+dx)}{2d} \\ x\left(a+b\tanh^2(c)\right)^3\tanh^3(c) \end{cases}$$

input
```
integrate(tanh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Piecewise((a**3*x - a**3*log(tanh(c + d*x) + 1)/d - a**3*tanh(c + d*x)**2/
(2*d) + 3*a**2*b*x - 3*a**2*b*log(tanh(c + d*x) + 1)/d - 3*a**2*b*tanh(c +
 d*x)**4/(4*d) - 3*a**2*b*tanh(c + d*x)**2/(2*d) + 3*a*b**2*x - 3*a*b**2*l
og(tanh(c + d*x) + 1)/d - a*b**2*tanh(c + d*x)**6/(2*d) - 3*a*b**2*tanh(c
+ d*x)**4/(4*d) - 3*a*b**2*tanh(c + d*x)**2/(2*d) + b**3*x - b**3*log(tanh
(c + d*x) + 1)/d - b**3*tanh(c + d*x)**8/(8*d) - b**3*tanh(c + d*x)**6/(6*
d) - b**3*tanh(c + d*x)**4/(4*d) - b**3*tanh(c + d*x)**2/(2*d), Ne(d, 0)),
 (x*(a + b*tanh(c)**2)**3*tanh(c)**3, True))
```

3.157.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(99) = 198$.

Time = 0.29 (sec) , antiderivative size = 540, normalized size of antiderivative = 5.05

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= ab^2\left(3x+\frac{3c}{d}+\frac{3\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2\left(9e^{(-2dx-2c)}+18e^{(-4dx-4c)}+34e^{(-6dx-6c)}+18e^{(-}\right.\right.$$

$$+\frac{1}{3}b^3\left(3x+\frac{3c}{d}+\frac{3\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{8\left(3e^{(-2dx-2c)}+9e^{(-4dx-4c)}+25e^{(-6dx-6c)}+26\right.\right.$$

$$+3a^2b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{4\left(e^{(-2dx-2c)}+e^{(-4dx-4c)}+e^{(-6dx-6c)}\right)}{d(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1)}\right)$$

$$+a^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1)}\right)$$

input
```
integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
a*b^2*(3*x + 3*c/d + 3*log(e^(-2*d*x - 2*c) + 1)/d + 2*(9*e^(-2*d*x - 2*c)
 + 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) + 18*e^(-8*d*x - 8*c) + 9*e^(
-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d
*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c
) + 1))) + 1/3*b^3*(3*x + 3*c/d + 3*log(e^(-2*d*x - 2*c) + 1)/d + 8*(3*e^(
-2*d*x - 2*c) + 9*e^(-4*d*x - 4*c) + 25*e^(-6*d*x - 6*c) + 26*e^(-8*d*x -
8*c) + 25*e^(-10*d*x - 10*c) + 9*e^(-12*d*x - 12*c) + 3*e^(-14*d*x - 14*c)
)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*
e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-1
4*d*x - 14*c) + e^(-16*d*x - 16*c) + 1))) + 3*a^2*b*(x + c/d + log(e^(-2*d
*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + e^(-6*d*x - 6*
c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(
-8*d*x - 8*c) + 1))) + a^3*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-
2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))
```

3.157.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(99) = 198$.

Time = 0.46 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.89

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$

$$-\frac{3\left(a^3+3a^2b+3ab^2+b^3\right)(dx+c)-3\left(a^3+3a^2b+3ab^2+b^3\right)\log\left(e^{(2dx+2c)}+1\right)-\frac{2\left(3\left(a^3+6a^2b+9ab^2+4\right.\right.}{}$$

input
```
integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) - 3*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*log(e^(2*d*x + 2*c) + 1) - 2*(3*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b
^3)*e^(14*d*x + 14*c) + 18*(a^3 + 5*a^2*b + 6*a*b^2 + 2*b^3)*e^(12*d*x + 1
2*c) + (45*a^3 + 198*a^2*b + 237*a*b^2 + 100*b^3)*e^(10*d*x + 10*c) + 4*(1
5*a^3 + 63*a^2*b + 78*a*b^2 + 26*b^3)*e^(8*d*x + 8*c) + (45*a^3 + 198*a^2*
b + 237*a*b^2 + 100*b^3)*e^(6*d*x + 6*c) + 18*(a^3 + 5*a^2*b + 6*a*b^2 + 2
*b^3)*e^(4*d*x + 4*c) + 3*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*e^(2*d*x + 2*c
))/(e^(2*d*x + 2*c) + 1)^8)/d
```

3.157.9 Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.45

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= x\left(a^3+3a^2b+3ab^2+b^3\right)-\frac{\tanh(c+dx)^4\left(3a^2b+3ab^2+b^3\right)}{4d}$$

$$-\frac{\ln\left(\tanh(c+dx)+1\right)\left(a^3+3a^2b+3ab^2+b^3\right)}{d}-\frac{\tanh(c+dx)^6\left(b^3+3ab^2\right)}{6d}$$

$$-\frac{b^3\tanh(c+dx)^8}{8d}-\frac{\tanh(c+dx)^2\left(a^3+3a^2b+3ab^2+b^3\right)}{2d}$$

input
```
int(tanh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c + d*x)^4*(3*a*b^2 + 3*a^2*b +
b^3))/(4*d) - (log(tanh(c + d*x) + 1)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/d -
 (tanh(c + d*x)^6*(3*a*b^2 + b^3))/(6*d) - (b^3*tanh(c + d*x)^8)/(8*d) - (
tanh(c + d*x)^2*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(2*d)
```

3.158 $\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

3.158.1 Optimal result 1180
3.158.2 Mathematica [A] (verified) 1180
3.158.3 Rubi [A] (verified) 1181
3.158.4 Maple [A] (verified) 1183
3.158.5 Fricas [B] (verification not implemented) 1183
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3.158.9 Mupad [B] (verification not implemented) 1186

3.158.1 Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = (a+b)^3x - \frac{(a+b)^3\tanh(c+dx)}{d} - \frac{b(3a^2+3ab+b^2)\tanh^3(c+dx)}{3d} - \frac{b^2(3a+b)\tanh^5(c+dx)}{5d} - \frac{b^3\tanh^7(c+dx)}{7d}$$

output
```
(a+b)^3*x-(a+b)^3*tanh(d*x+c)/d-1/3*b*(3*a^2+3*a*b+b^2)*tanh(d*x+c)^3/d-1/
5*b^2*(3*a+b)*tanh(d*x+c)^5/d-1/7*b^3*tanh(d*x+c)^7/d
```

3.158.2 Mathematica [A] (verified)

Time = 2.00 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.15

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\tanh(c+dx)\left(-105(a+b)^3 - 35b(3a^2+3ab+b^2)\tanh^2(c+dx) - 21b^2(3a+b)\tanh^4(c+dx) - 15b^3\,\text{ta}\right.}{105d}$$

input
```
Integrate[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(Tanh[c + d*x]*(-105*(a + b)^3 - 35*b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^
2 - 21*b^2*(3*a + b)*Tanh[c + d*x]^4 - 15*b^3*Tanh[c + d*x]^6 + (105*(a +
b)^3*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c + d*x]^2]))/(105*d)
```

3.158.3 Rubi [A] (verified)

Time $= 0.30$ (sec) , antiderivative size $= 93$, normalized size of antiderivative $= 0.99$, number of steps used $= 7$, number of rules used $= 6$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used $= \{3042, 25, 4153, 25, 364, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$$

$$\downarrow 3042$$

$$\int -\tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^3 dx$$

$$\downarrow 25$$

$$-\int \tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^3 dx$$

$$\downarrow 4153$$

$$-\frac{\int -\frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)} d\tanh(c+dx)}{d}$$

$$\downarrow 25$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)} d\tanh(c+dx)}{d}$$

$$\downarrow 364$$

$$\frac{\int \left(-b^3\tanh^6(c+dx)-b^2(3a+b)\tanh^4(c+dx)-b\left(3a^2+3ba+b^2\right)\tanh^2(c+dx)-(a+b)^3+\frac{a^3+3ba^2+3b^2a+b^3}{1-\tanh^2(c+dx)}\right.}{d}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{3}b\left(3a^2+3ab+b^2\right)\tanh^3(c+dx)-(a+b)^3\operatorname{arctanh}(\tanh(c+dx))+\frac{1}{5}b^2(3a+b)\tanh^5(c+dx)+(a+b)^3\tanh}{d}$$

input
```
Int[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-((-((a + b)^3*ArcTanh[Tanh[c + d*x]]) + (a + b)^3*Tanh[c + d*x] + (b*(3*a
^2 + 3*a*b + b^2)*Tanh[c + d*x]^3)/3 + (b^2*(3*a + b)*Tanh[c + d*x]^5)/5 +
 (b^3*Tanh[c + d*x]^7)/7)/d)
```

3.158.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.158.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.74

method	result
parallelrisch	$-\frac{15\tanh(dx+c)^7b^3+63\tanh(dx+c)^5ab^2+21b^3\tanh(dx+c)^5+105\tanh(dx+c)^3a^2b+105ab^2\tanh(dx+c)^3+35b^3\tanh($
derivativedivides	$\frac{-3a^2b\tanh(dx+c)-3ab^2\tanh(dx+c)-\frac{3\tanh(dx+c)^5ab^2}{5}-\tanh(dx+c)^3a^2b-ab^2\tanh(dx+c)^3-\frac{b^3\tanh(dx+c)^5}{5}-\frac{b^3\tanh}{}$
default	$\frac{-3a^2b\tanh(dx+c)-3ab^2\tanh(dx+c)-\frac{3\tanh(dx+c)^5ab^2}{5}-\tanh(dx+c)^3a^2b-ab^2\tanh(dx+c)^3-\frac{b^3\tanh(dx+c)^5}{5}-\frac{b^3\tanh}{}$
parts	$\frac{a^3\left(-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b^3\left(-\frac{\tanh(dx+c)^7}{7}-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+\right.}{d}$
risch	$a^3x+3ba^2x+3ab^2x+b^3x+\frac{\frac{46ab^2}{5}+2a^3+12a^2be^{12dx+12c}+72ab^2e^{10dx+10c}+60a^2be^{10dx+10c}+18ab^2e^{12}}{}$

input
```
int(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/105*(15*tanh(d*x+c)^7*b^3+63*tanh(d*x+c)^5*a*b^2+21*b^3*tanh(d*x+c)^5+1
05*tanh(d*x+c)^3*a^2*b+105*a*b^2*tanh(d*x+c)^3+35*b^3*tanh(d*x+c)^3-105*a^
3*d*x-315*a^2*b*d*x-315*a*b^2*d*x-105*b^3*d*x+105*a^3*tanh(d*x+c)+315*a^2*
b*tanh(d*x+c)+315*a*b^2*tanh(d*x+c)+105*b^3*tanh(d*x+c))/d
```

3.158.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1036 vs. $2(88) = 176$.

Time = 0.27 (sec) , antiderivative size = 1036, normalized size of antiderivative = 11.02

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/105*((105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 + 7*(105*a^3 + 420*a^2*b + 483*a*b^2 +
176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x
+ c)^6 - (105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3)*sinh(d*x + c)^7 + 7*(
105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*d*x)*cosh(d*x + c)^5 - 7*(75*a^3 + 240*a^2*b + 213*a*b^2 + 56*b^3 +
3*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3)*cosh(d*x + c)^2)*sinh(d*x +
c)^5 + 35*((105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + (105*a^3 + 420*a^2*b + 483*a*b^2
+ 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d
*x + c)^4 + 21*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - 7*(5*(105*a^3 + 420*a^2*b + 4
83*a*b^2 + 176*b^3)*cosh(d*x + c)^4 + 135*a^3 + 360*a^2*b + 369*a*b^2 + 16
8*b^3 + 10*(75*a^3 + 240*a^2*b + 213*a*b^2 + 56*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^3 + 7*(3*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 + 10*(105*a^3 + 420*a^2*b +
 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x +
 c)^3 + 9*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(105*a^3 + 420*a
^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*c...
```

3.158.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(82) = 164$.

Time $= 0.22$ (sec) , antiderivative size $= 192$, normalized size of antiderivative $= 2.04$

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \begin{cases} a^3x - \frac{a^3\tanh(c+dx)}{d} + 3a^2bx - \frac{a^2b\tanh^3(c+dx)}{d} - \frac{3a^2b\tanh(c+dx)}{d} + 3ab^2x - \frac{3ab^2\tanh^5(c+dx)}{5d} - \frac{ab^2\tanh^3(c+dx)}{d} - \\ x\left(a+b\tanh^2(c)\right)^3\tanh^2(c) \end{cases}$$

input
```
integrate(tanh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Piecewise((a**3*x - a**3*tanh(c + d*x)/d + 3*a**2*b*x - a**2*b*tanh(c + d*
x)**3/d - 3*a**2*b*tanh(c + d*x)/d + 3*a*b**2*x - 3*a*b**2*tanh(c + d*x)**
5/(5*d) - a*b**2*tanh(c + d*x)**3/d - 3*a*b**2*tanh(c + d*x)/d + b**3*x -
b**3*tanh(c + d*x)**7/(7*d) - b**3*tanh(c + d*x)**5/(5*d) - b**3*tanh(c +
d*x)**3/(3*d) - b**3*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**3*
tanh(c)**2, True))
```

3.158.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(88) = 176$.

Time = 0.21 (sec) , antiderivative size = 400, normalized size of antiderivative = 4.26

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{1}{105}b^3\left(105x + \frac{105c}{d} - \frac{8\left(203e^{(-2dx-2c)} + 609e^{(-4dx-4c)} + 770e^{(-6dx-6c)} + 770e^{(-8dx-8c)} + 315e^{(-10}\right.}{d(7e^{(-2dx-2c)} + 21e^{(-4dx-4c)} + 35e^{(-6dx-6c)} + 35e^{(-8dx-8c)} + 21e^{(-10dx-10c)} +}\right)$$

$$+\frac{1}{5}ab^2\left(15x + \frac{15c}{d} - \frac{2\left(70e^{(-2dx-2c)} + 140e^{(-4dx-4c)} + 90e^{(-6dx-6c)} + 45e^{(-8dx-8c)} + 23\right)}{d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)}\right)$$

$$+a^2b\left(3x + \frac{3c}{d} - \frac{4\left(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + 2\right)}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)}\right)$$

$$+a^3\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2dx-2c)} + 1)}\right)$$

input
```
integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/105*b^3*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d*x - 4*c
) + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x - 10*c) +
 105*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
 + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e
^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 1/5*a*b^2*(15*x + 15*c/d -
 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c) + 45*
e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e
^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + a^2*b*(
3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*
d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + a^3*(x + c/d -
 2/(d*(e^(-2*d*x - 2*c) + 1)))
```

3.158.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(88) = 176$.

Time = 0.43 (sec) , antiderivative size = 418, normalized size of antiderivative = 4.45

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{105\left(a^3+3a^2b+3ab^2+b^3\right)(dx+c) + \frac{2\left(105a^3e^{(12dx+12c)}+630a^2be^{(12dx+12c)}+945ab^2e^{(12dx+12c)}+420b^3e^{(12dx+12c)}+630\right.}{}}{}$$

input
```
integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/105*(105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) + 2*(105*a^3*e^(12*d*
x + 12*c) + 630*a^2*b*e^(12*d*x + 12*c) + 945*a*b^2*e^(12*d*x + 12*c) + 42
0*b^3*e^(12*d*x + 12*c) + 630*a^3*e^(10*d*x + 10*c) + 3150*a^2*b*e^(10*d*x
 + 10*c) + 3780*a*b^2*e^(10*d*x + 10*c) + 1260*b^3*e^(10*d*x + 10*c) + 157
5*a^3*e^(8*d*x + 8*c) + 6720*a^2*b*e^(8*d*x + 8*c) + 7665*a*b^2*e^(8*d*x +
 8*c) + 3080*b^3*e^(8*d*x + 8*c) + 2100*a^3*e^(6*d*x + 6*c) + 7980*a^2*b*e
^(6*d*x + 6*c) + 9240*a*b^2*e^(6*d*x + 6*c) + 3080*b^3*e^(6*d*x + 6*c) + 1
575*a^3*e^(4*d*x + 4*c) + 5670*a^2*b*e^(4*d*x + 4*c) + 6363*a*b^2*e^(4*d*x
 + 4*c) + 2436*b^3*e^(4*d*x + 4*c) + 630*a^3*e^(2*d*x + 2*c) + 2310*a^2*b*
e^(2*d*x + 2*c) + 2436*a*b^2*e^(2*d*x + 2*c) + 812*b^3*e^(2*d*x + 2*c) + 1
05*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3)/(e^(2*d*x + 2*c) + 1)^7)/d
```

3.158.9 Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = x\left(a^3+3a^2b+3ab^2+b^3\right) - \frac{\tanh(c+dx)\,(a+b)^3}{d} - \frac{\tanh(c+dx)^3\left(3a^2b+3ab^2+b^3\right)}{3d} - \frac{\tanh(c+dx)^5\left(b^3+3ab^2\right)}{5d} - \frac{b^3\tanh(c+dx)^7}{7d}$$

input
```
int(tanh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c + d*x)*(a + b)^3)/d - (tanh(c
+ d*x)^3*(3*a*b^2 + 3*a^2*b + b^3))/(3*d) - (tanh(c + d*x)^5*(3*a*b^2 + b^
3))/(5*d) - (b^3*tanh(c + d*x)^7)/(7*d)
```

3.159 $\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

3.159.1 Optimal result 1187
3.159.2 Mathematica [A] (verified) 1187
3.159.3 Rubi [A] (verified) 1188
3.159.4 Maple [A] (verified) 1190
3.159.5 Fricas [B] (verification not implemented) 1190
3.159.6 Sympy [B] (verification not implemented) 1191
3.159.7 Maxima [B] (verification not implemented) 1192
3.159.8 Giac [B] (verification not implemented) 1192
3.159.9 Mupad [B] (verification not implemented) 1193

3.159.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{(a+b)^3\log(\cosh(c+dx))}{d} - \frac{b(a+b)^2\tanh^2(c+dx)}{2d} - \frac{(a+b)\left(a+b\tanh^2(c+dx)\right)^2}{4d} - \frac{\left(a+b\tanh^2(c+dx)\right)^3}{6d}$$

output
```
(a+b)^3*ln(cosh(d*x+c))/d-1/2*b*(a+b)^2*tanh(d*x+c)^2/d-1/4*(a+b)*(a+b*tan
h(d*x+c)^2)^2/d-1/6*(a+b*tanh(d*x+c)^2)^3/d
```

3.159.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{-2(a+b)^3\log(\cosh(c+dx)) + b(a+b)^2\tanh^2(c+dx) + \frac{1}{2}(a+b)\left(a+b\tanh^2(c+dx)\right)^2 + \frac{1}{3}\left(a+b\ta\right.}{2d}$$

input
```
Integrate[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-1/2*(-2*(a + b)^3*Log[Cosh[c + d*x]] + b*(a + b)^2*Tanh[c + d*x]^2 + ((a
+ b)*(a + b*Tanh[c + d*x]^2)^2)/2 + (a + b*Tanh[c + d*x]^2)^3/3)/d
```

3.159.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 4153, 26, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\downarrow 3042$$

$$\int -i\tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)^3\,dx$$

$$\downarrow 26$$

$$-i\int \tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)^3\,dx$$

$$\downarrow 4153$$

$$-\frac{i\int \frac{i\tanh(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\tanh(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 353$$

$$\frac{\int \frac{\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{2d}$$

$$\downarrow 49$$

$$\frac{\int \left(\frac{(a+b)^3}{1-\tanh^2(c+dx)}-b(a+b)^2-b\left(b\tanh^2(c+dx)+a\right)(a+b)-b\left(b\tanh^2(c+dx)+a\right)^2\right)d\tanh^2(c+dx)}{2d}$$

$$\downarrow 2009$$

$$\frac{-b(a+b)^2\tanh^2(c+dx)-\frac{1}{2}(a+b)\left(a+b\tanh^2(c+dx)\right)^2-\frac{1}{3}\left(a+b\tanh^2(c+dx)\right)^3+(a+b)^3\left(-\log\left(1-\tanh\right.\right.}{2d}$$

input
```
Int[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-((a + b)^3*Log[1 - Tanh[c + d*x]^2]) - b*(a + b)^2*Tanh[c + d*x]^2 - ((a
 + b)*(a + b*Tanh[c + d*x]^2)^2)/2 - (a + b*Tanh[c + d*x]^2)^3/3)/(2*d)
```

3.159.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.159.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.82

method	result
derivativedivides	$\frac{-\frac{3\tanh(dx+c)^4ab^2}{4}-\frac{3\tanh(dx+c)^2a^2b}{2}-\frac{3ab^2\tanh(dx+c)^2}{2}-\frac{b^3\tanh(dx+c)^4}{4}-\frac{b^3\tanh(dx+c)^2}{2}-\frac{\tanh(dx+c)^6b^3}{6}+\frac{(-a^3-3a^2b-$
default	$\frac{-\frac{3\tanh(dx+c)^4ab^2}{4}-\frac{3\tanh(dx+c)^2a^2b}{2}-\frac{3ab^2\tanh(dx+c)^2}{2}-\frac{b^3\tanh(dx+c)^4}{4}-\frac{b^3\tanh(dx+c)^2}{2}-\frac{\tanh(dx+c)^6b^3}{6}+\frac{(-a^3-3a^2b-$
parts	$\frac{a^3\ln(\cosh(dx+c))}{d}+\frac{b^3\left(-\frac{\tanh(dx+c)^6}{6}-\frac{\tanh(dx+c)^4}{4}-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{3a^2b($
parallelrisch	$-\frac{2\tanh(dx+c)^6b^3+9\tanh(dx+c)^4ab^2+3b^3\tanh(dx+c)^4+12a^3dx+36a^2bdx+36ab^2dx+12b^3dx+18\tanh(dx+c)^2a^2b-$
risch	$-a^3x-3ba^2x-3ab^2x-b^3x-\frac{2a^3c}{d}-\frac{6bca^2}{d}-\frac{6ab^2c}{d}-\frac{2b^3c}{d}+\frac{2b\,e^{2dx+2c}(9a^2e^{8dx+8c}+18abe^{8dx}$

input
```
int(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-3/4*tanh(d*x+c)^4*a*b^2-3/2*tanh(d*x+c)^2*a^2*b-3/2*a*b^2*tanh(d*x+c
)^2-1/4*b^3*tanh(d*x+c)^4-1/2*b^3*tanh(d*x+c)^2-1/6*tanh(d*x+c)^6*b^3+1/2*
(-a^3-3*a^2*b-3*a*b^2-b^3)*ln(tanh(d*x+c)+1)-1/2*(a^3+3*a^2*b+3*a*b^2+b^3)
*ln(tanh(d*x+c)-1))
```

3.159.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4298 vs. 2(77) = 154.

Time = 0.30 (sec) , antiderivative size = 4298, normalized size of antiderivative = 51.78

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^12 + 36*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^11 + 3*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*x*sinh(d*x + c)^12 - 18*(a^2*b + 2*a*b^2 + b^3 - (a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^10 + 18*(11*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 - a^2*b - 2*a*b^2 - b^3 + (a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x)*sinh(d*x + c)^10 + 60*(11*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*x*cosh(d*x + c)^3 - 3*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 - 9*(8*a^2*b + 12*a
*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^8 + 9*
(165*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^4 - 8*a^2*b - 12*a*
b^2 - 4*b^3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 90*(a^2*b + 2*a*b^2
+ b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c
)^8 + 72*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^5 - 30*(a^2
*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3
- (8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cos
h(d*x + c))*sinh(d*x + c)^7 - 4*(27*a^2*b + 36*a*b^2 + 17*b^3 - 15*(a^3 +
3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 + 4*(693*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*d*x*cosh(d*x + c)^6 - 945*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 27*a^2*b - 36*a*b^2 - 17*b^3
+ 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 63*(8*a^2*b + 12*a*b^2 + 4*b...
```

3.159.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(71) = 142$.

Time $= 0.19$ (sec) , antiderivative size $= 211$, normalized size of antiderivative $= 2.54$

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \begin{cases} a^3x - \frac{a^3\log(\tanh(c+dx)+1)}{d} + 3a^2bx - \frac{3a^2b\log(\tanh(c+dx)+1)}{d} - \frac{3a^2b\tanh^2(c+dx)}{2d} + 3ab^2x - \frac{3ab^2\log(\tanh(c+dx)+1)}{d} - \\ x\left(a+b\tanh^2(c)\right)^3\tanh(c) \end{cases}$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Piecewise((a**3*x - a**3*log(tanh(c + d*x) + 1)/d + 3*a**2*b*x - 3*a**2*b*
log(tanh(c + d*x) + 1)/d - 3*a**2*b*tanh(c + d*x)**2/(2*d) + 3*a*b**2*x -
3*a*b**2*log(tanh(c + d*x) + 1)/d - 3*a*b**2*tanh(c + d*x)**4/(4*d) - 3*a*
b**2*tanh(c + d*x)**2/(2*d) + b**3*x - b**3*log(tanh(c + d*x) + 1)/d - b**
3*tanh(c + d*x)**6/(6*d) - b**3*tanh(c + d*x)**4/(4*d) - b**3*tanh(c + d*x
)**2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)**3*tanh(c), True))
```

3.159.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(77) = 154$.

Time = 0.28 (sec) , antiderivative size = 351, normalized size of antiderivative = 4.23

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{1}{3}b^3\left(3x+\frac{3c}{d}+\frac{3\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2\left(9e^{(-2dx-2c)}+18e^{(-4dx-4c)}+34e^{(-6dx-6c)}+18e^{(}\right.\right.$$

$$\left.\left.+3ab^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{4\left(e^{(-2dx-2c)}+e^{(-4dx-4c)}+e^{(-6dx-6c)}\right)}{d(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1)}\right)\right.\right.$$

$$\left.\left.+3a^2b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1)}\right)\right.\right.$$

$$\left.\left.+\frac{a^3\log\left(\cosh\left(dx+c\right)\right)}{d}\right.\right.$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/3*b^3*(3*x + 3*c/d + 3*log(e^(-2*d*x - 2*c) + 1)/d + 2*(9*e^(-2*d*x - 2*
c) + 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) + 18*e^(-8*d*x - 8*c) + 9*e
^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6
*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12
*c) + 1))) + 3*a*b^2*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x
 - 2*c) + e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*
e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + 3*a^2*b*
(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*
x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a^3*log(cosh(d*x + c))/d
```

3.159.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(77) = 154$.

Time = 0.40 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.60

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$

$$-\frac{3\left(a^3+3a^2b+3ab^2+b^3\right)(dx+c)-3\left(a^3+3a^2b+3ab^2+b^3\right)\log\left(e^{(2dx+2c)}+1\right)-\frac{2\left(9\left(a^2b+2ab^2+b^3\right)e^{(1}}{}}{3}$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) - 3*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*log(e^(2*d*x + 2*c) + 1) - 2*(9*(a^2*b + 2*a*b^2 + b^3)*e^(10*
d*x + 10*c) + 18*(2*a^2*b + 3*a*b^2 + b^3)*e^(8*d*x + 8*c) + 2*(27*a^2*b +
 36*a*b^2 + 17*b^3)*e^(6*d*x + 6*c) + 18*(2*a^2*b + 3*a*b^2 + b^3)*e^(4*d*
x + 4*c) + 9*(a^2*b + 2*a*b^2 + b^3)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c) + 1
)^6)/d
```

3.159.9 Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.48

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= x\left(a^3+3a^2b+3ab^2+b^3\right) - \frac{\tanh(c+dx)^2\left(3a^2b+3ab^2+b^3\right)}{2d}$$
$$- \frac{\ln\left(\tanh(c+dx)+1\right)\left(a^3+3a^2b+3ab^2+b^3\right)}{d}$$
$$- \frac{\tanh(c+dx)^4\left(b^3+3ab^2\right)}{4d} - \frac{b^3\tanh(c+dx)^6}{6d}$$

input
```
int(tanh(c + d*x)*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c + d*x)^2*(3*a*b^2 + 3*a^2*b +
b^3))/(2*d) - (log(tanh(c + d*x) + 1)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/d -
 (tanh(c + d*x)^4*(3*a*b^2 + b^3))/(4*d) - (b^3*tanh(c + d*x)^6)/(6*d)
```

3.160 $\int \left(a + b\tanh^2(c+dx)\right)^3\,dx$

3.160.1 Optimal result 1194
3.160.2 Mathematica [A] (verified) 1194
3.160.3 Rubi [A] (verified) 1195
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3.160.1 Optimal result

Integrand size = 14, antiderivative size = 74

$$\int \left(a + b\tanh^2(c+dx)\right)^3\,dx = (a+b)^3x - \frac{b(3a^2+3ab+b^2)\tanh(c+dx)}{d} - \frac{b^2(3a+b)\tanh^3(c+dx)}{3d} - \frac{b^3\tanh^5(c+dx)}{5d}$$

output
```
(a+b)^3*x-b*(3*a^2+3*a*b+b^2)*tanh(d*x+c)/d-1/3*b^2*(3*a+b)*tanh(d*x+c)^3/
d-1/5*b^3*tanh(d*x+c)^5/d
```

3.160.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$\int \left(a + b\tanh^2(c+dx)\right)^3\,dx = \frac{\tanh(c+dx)\left(\frac{15(a+b)^3\operatorname{arctanh}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}} - b\left(45a^2+15ab\left(3+\tanh^2(c+dx)\right)+b^2\left(15+5\tanh^2(c+\right.\right.\right.}{15d}$$

input
```
Integrate[(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(Tanh[c + d*x]*((15*(a + b)^3*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c
+ d*x]^2] - b*(45*a^2 + 15*a*b*(3 + Tanh[c + d*x]^2) + b^2*(15 + 5*Tanh[c
+ d*x]^2 + 3*Tanh[c + d*x]^4))))/(15*d)
```

3.160.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4144, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a + b\tanh^2(c+dx)\right)^3 dx$$

$$\downarrow 3042$$

$$\int \left(a - b\tan(ic+idx)^2\right)^3 dx$$

$$\downarrow 4144$$

$$\frac{\int \frac{\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)} d\tanh(c+dx)}{d}$$

$$\downarrow 300$$

$$\frac{\int \left(-b^3\tanh^4(c+dx) - b^2(3a+b)\tanh^2(c+dx) - b\left(3a^2+3ba+b^2\right) + \frac{(a+b)^3}{1-\tanh^2(c+dx)}\right) d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{-b\left(3a^2+3ab+b^2\right)\tanh(c+dx) + (a+b)^3\text{arctanh}(\tanh(c+dx)) - \frac{1}{3}b^2(3a+b)\tanh^3(c+dx) - \frac{1}{5}b^3\tanh^5(c+d}{d}$$

input
```
Int[(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((a + b)^3*ArcTanh[Tanh[c + d*x]] - b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]
- (b^2*(3*a + b)*Tanh[c + d*x]^3)/3 - (b^3*Tanh[c + d*x]^5)/5)/d
```

3.160.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.160.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.51

method	result
parallelrisch	$-\frac{3b^3\tanh(dx+c)^5+15a\,b^2\tanh(dx+c)^3+5b^3\tanh(dx+c)^3-15a^3dx-45a^2bdx-45a\,b^2dx-15b^3dx+45a^2b\tanh(dx+c)-}{15d}$
derivativedivides	$\frac{-3a^2b\tanh(dx+c)-3a\,b^2\tanh(dx+c)^3-a\,b^2\tanh(dx+c)^3-\frac{\left(a^3+3a^2b+3a\,b^2+b^3\right)\ln(\tanh(dx+c)-1)}{2}-\frac{b^3\tanh(dx+c)^3}{3}-b^3\tan}{d}$
default	$\frac{-3a^2b\tanh(dx+c)-3a\,b^2\tanh(dx+c)^3-a\,b^2\tanh(dx+c)^3-\frac{\left(a^3+3a^2b+3a\,b^2+b^3\right)\ln(\tanh(dx+c)-1)}{2}-\frac{b^3\tanh(dx+c)^3}{3}-b^3\tan}{d}$
parts	$a^3x+\frac{b^3\left(-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{3a^2b\left(-\tanh(dx+c)\right.}{}$
risch	$a^3x+3b\,a^2x+3a\,b^2x+b^3x+\frac{2b(45a^2\mathrm{e}^{8dx+8c}+90ab\,\mathrm{e}^{8dx+8c}+45b^2\mathrm{e}^{8dx+8c}+180a^2\mathrm{e}^{6dx+6c}+270ab\,\mathrm{e}^{6dx+6c}}{}$

input
```
int((a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(3*b^3*tanh(d*x+c)^5+15*a*b^2*tanh(d*x+c)^3+5*b^3*tanh(d*x+c)^3-15*a
^3*d*x-45*a^2*b*d*x-45*a*b^2*d*x-15*b^3*d*x+45*a^2*b*tanh(d*x+c)+45*a*b^2*
tanh(d*x+c)+15*b^3*tanh(d*x+c))/d
```

3.160.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(70) = 140$.

Time $= 0.27$ (sec) , antiderivative size $= 567$, normalized size of antiderivative $= 7.66$

$$\int \left(a + b\tanh^2(c+dx)\right)^3 \, dx$$
$$= \frac{(45\,a^2b + 60\,ab^2 + 23\,b^3 + 15\,(a^3 + 3\,a^2b + 3\,ab^2 + b^3)dx)\cosh(dx+c)^5 + 5\,(45\,a^2b + 60\,ab^2 + 23\,b^3 + 15$$

input
```
integrate((a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/15*((45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d
*x)*cosh(d*x + c)^5 + 5*(45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^4 - (45*a^2*b + 60*a*b^2
 + 23*b^3)*sinh(d*x + c)^5 + 5*(45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - 5*(27*a^2*b + 24*a*b^2 + 5*
b^3 + 2*(45*a^2*b + 60*a*b^2 + 23*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 +
5*(2*(45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*
x)*cosh(d*x + c)^3 + 3*(45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(45*a^2*b + 60*a*
b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c) - 5*(
(45*a^2*b + 60*a*b^2 + 23*b^3)*cosh(d*x + c)^4 + 18*a^2*b + 12*a*b^2 + 10*
b^3 + 3*(27*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*c
osh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 +
 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d
*x + c))
```

3.160.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.70

$$\int \left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \begin{cases} a^3x+3a^2bx-\frac{3a^2b\tanh(c+dx)}{d}+3ab^2x-\frac{ab^2\tanh^3(c+dx)}{d}-\frac{3ab^2\tanh(c+dx)}{d}+b^3x-\frac{b^3\tanh^5(c+dx)}{5d}-\frac{b^3\tanh^3(c+d \\ x\left(a+b\tanh^2(c)\right)^3 \end{cases}$$

input
```
integrate((a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Piecewise((a**3*x + 3*a**2*b*x - 3*a**2*b*tanh(c + d*x)/d + 3*a*b**2*x - a
*b**2*tanh(c + d*x)**3/d - 3*a*b**2*tanh(c + d*x)/d + b**3*x - b**3*tanh(c
 + d*x)**5/(5*d) - b**3*tanh(c + d*x)**3/(3*d) - b**3*tanh(c + d*x)/d, Ne(
d, 0)), (x*(a + b*tanh(c)**2)**3, True))
```

3.160.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(70) = 140.

Time = 0.20 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.23

$$\int \left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{1}{15}b^3\left(15\,x+\frac{15\,c}{d}-\frac{2\left(70\,e^{(-2\,dx-2\,c)}+140\,e^{(-4\,dx-4\,c)}+90\,e^{(-6\,dx-6\,c)}+45\,e^{(-8\,dx-8\,c)}+23\right)}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}\right)$$
$$+ ab^2\left(3\,x+\frac{3\,c}{d}-\frac{4\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+2\right)}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)$$
$$+ 3\,a^2b\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2\,dx-2\,c)}+1)}\right)+a^3x$$

input
```
integrate((a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/15*b^3*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) +
90*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
 - 10*c) + 1))) + a*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x
 - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c
) + 1))) + 3*a^2*b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^3*x
```

3.160.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(70) = 140$.

Time = 0.31 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.26

$$\int \left(a + b\tanh^2(c+dx)\right)^3 dx$$

$$= \frac{15\left(a^3 + 3a^2b + 3ab^2 + b^3\right)(dx+c) + \frac{2\left(45a^2be^{(8dx+8c)} + 90ab^2e^{(8dx+8c)} + 45b^3e^{(8dx+8c)} + 180a^2be^{(6dx+6c)} + 270ab^2e^{(6dx+6}\right.}{}$$

input
```
integrate((a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/15*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) + 2*(45*a^2*b*e^(8*d*x
+ 8*c) + 90*a*b^2*e^(8*d*x + 8*c) + 45*b^3*e^(8*d*x + 8*c) + 180*a^2*b*e^(
6*d*x + 6*c) + 270*a*b^2*e^(6*d*x + 6*c) + 90*b^3*e^(6*d*x + 6*c) + 270*a^
2*b*e^(4*d*x + 4*c) + 330*a*b^2*e^(4*d*x + 4*c) + 140*b^3*e^(4*d*x + 4*c)
+ 180*a^2*b*e^(2*d*x + 2*c) + 210*a*b^2*e^(2*d*x + 2*c) + 70*b^3*e^(2*d*x
+ 2*c) + 45*a^2*b + 60*a*b^2 + 23*b^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.160.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

$$\int \left(a + b\tanh^2(c+dx)\right)^3 dx = x\left(a^3 + 3a^2b + 3ab^2 + b^3\right) - \frac{\tanh(c+dx)^3\left(b^3 + 3ab^2\right)}{3d}$$
$$- \frac{b^3\tanh(c+dx)^5}{5d} - \frac{b\tanh(c+dx)\left(3a^2 + 3ab + b^2\right)}{d}$$

input
```
int((a + b*tanh(c + d*x)^2)^3,x)
```

output
```
x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c + d*x)^3*(3*a*b^2 + b^3))/(3*d
) - (b^3*tanh(c + d*x)^5)/(5*d) - (b*tanh(c + d*x)*(3*a*b + 3*a^2 + b^2))/
d
```

3.161 $\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

3.161.1 Optimal result 1200
3.161.2 Mathematica [A] (verified) 1200
3.161.3 Rubi [A] (verified) 1201
3.161.4 Maple [A] (verified) 1203
3.161.5 Fricas [B] (verification not implemented) 1203
3.161.6 Sympy [F] 1204
3.161.7 Maxima [B] (verification not implemented) 1205
3.161.8 Giac [B] (verification not implemented) 1205
3.161.9 Mupad [B] (verification not implemented) 1206

3.161.1 Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{(a+b)^3\log(\cosh(c+dx))}{d}+\frac{a^3\log(\tanh(c+dx))}{d} - \frac{b^2(3a+b)\tanh^2(c+dx)}{2d}-\frac{b^3\tanh^4(c+dx)}{4d}$$

output
```
(a+b)^3*ln(cosh(d*x+c))/d+a^3*ln(tanh(d*x+c))/d-1/2*b^2*(3*a+b)*tanh(d*x+c
)^2/d-1/4*b^3*tanh(d*x+c)^4/d
```

3.161.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{2(a+b)^3\log(\cosh(c+dx))+2a^3\log(\tanh(c+dx))-b^2(3a+b)\tanh^2(c+dx)-\frac{1}{2}b^3\tanh^4(c+dx)}{2d}$$

input
```
Integrate[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(2*(a + b)^3*Log[Cosh[c + d*x]] + 2*a^3*Log[Tanh[c + d*x]] - b^2*(3*a + b)
*Tanh[c + d*x]^2 - (b^3*Tanh[c + d*x]^4)/2)/(2*d)
```

3.161.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 4153, 26, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{i\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)}dx$$

$$\downarrow 26$$

$$i\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)}dx$$

$$\downarrow 4153$$

$$\frac{i\int -\frac{i\coth(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\coth(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 354$$

$$\frac{\int \frac{\coth(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{2d}$$

$$\downarrow 93$$

$$\frac{\int \left(\coth(c+dx)a^3-b^3\tanh^2(c+dx)-b^2(3a+b)-\frac{(a+b)^3}{\tanh^2(c+dx)-1}\right)d\tanh^2(c+dx)}{2d}$$

$$\downarrow 2009$$

$$\frac{a^3\log\left(\tanh^2(c+dx)\right)-b^2(3a+b)\tanh^2(c+dx)-(a+b)^3\log\left(1-\tanh^2(c+dx)\right)-\frac{1}{2}b^3\tanh^4(c+dx)}{2d}$$

input
```
Int[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(a^3*Log[Tanh[c + d*x]^2] - (a + b)^3*Log[1 - Tanh[c + d*x]^2] - b^2*(3*a
+ b)*Tanh[c + d*x]^2 - (b^3*Tanh[c + d*x]^4)/2)/(2*d)
```

3.161.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.161.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11

method	result
parallelrisch	$\frac{-4(a+b)^3\ln(1-\tanh(dx+c))+4a^3\ln(\tanh(dx+c))-b^3\tanh(dx+c)^4+(-6ab^2-2b^3)\tanh(dx+c)^2-4dx(a+b)^3}{4d}$
derivativedivides	$-\frac{\frac{b^3\tanh(dx+c)^4}{4}+\frac{3ab^2\tanh(dx+c)^2}{2}+\frac{b^3\tanh(dx+c)^2}{2}+(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}ab^2+\frac{1}{2}b^3)\ln(\tanh(dx+c)+1)-a^3\ln(\tanh(dx+c)$
default	$-\frac{\frac{b^3\tanh(dx+c)^4}{4}+\frac{3ab^2\tanh(dx+c)^2}{2}+\frac{b^3\tanh(dx+c)^2}{2}+(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}ab^2+\frac{1}{2}b^3)\ln(\tanh(dx+c)+1)-a^3\ln(\tanh(dx+c)$
risch	$-a^3x-3ba^2x-3ab^2x-b^3x-\frac{6bca^2}{d}-\frac{6ab^2c}{d}-\frac{2b^3c}{d}-\frac{2a^3c}{d}+\frac{2b^2e^{2dx+2c}(3ae^{4dx+4c}+2be^{4dx+4}}{d(e^{2dx}$

input
```
int(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-4*(a+b)^3*ln(1-tanh(d*x+c))+4*a^3*ln(tanh(d*x+c))-b^3*tanh(d*x+c)^4+
(-6*a*b^2-2*b^3)*tanh(d*x+c)^2-4*d*x*(a+b)^3)/d
```

3.161.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2381 vs. $2(68) = 136$.

Time = 0.29 (sec) , antiderivative size = 2381, normalized size of antiderivative = 33.07

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^8 + 8*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x*sinh(d*x + c)^8 - 2*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 + 2*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
d*x*cosh(d*x + c)^2 - 3*a*b^2 - 2*b^3 + 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
d*x)*sinh(d*x + c)^6 + 4*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x
 + c)^3 - 3*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(
d*x + c))*sinh(d*x + c)^5 - 2*(6*a*b^2 + 2*b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^4 + 2*(35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*
cosh(d*x + c)^4 - 6*a*b^2 - 2*b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x
 - 15*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x +
 c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x
 + c)^5 - 5*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(
d*x + c)^3 - (6*a*b^2 + 2*b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cos
h(d*x + c))*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 2*(3*a
*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2 + 2*
(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^6 - 15*(3*a*b^2 + 2*
b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 3*a*b^2 - 2
*b^3 + 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 6*(6*a*b^2 + 2*b^3 - 3*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3...
```

3.161.6 Sympy [F]

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2(c+dx)\right)^3 \coth(c+dx)\,dx$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*coth(c + d*x), x)
```

3.161.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(68) = 136$.

Time $= 0.28$ (sec) , antiderivative size $= 214$, normalized size of antiderivative $= 2.97$

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= b^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)$$
$$+3\,ab^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$
$$+\frac{3\,a^2b\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d}+\frac{a^3\log\left(\sinh\left(dx+c\right)\right)}{d}$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
b^3*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d
*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c)
+ 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + 3*a*b^2*(x + c/d + log(e^
(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*
d*x - 4*c) + 1))) + 3*a^2*b*log(e^(d*x + c) + e^(-d*x - c))/d + a^3*log(si
nh(d*x + c))/d
```

3.161.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(68) = 136$.

Time $= 0.39$ (sec) , antiderivative size $= 267$, normalized size of antiderivative $= 3.71$

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{2\,a^3\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)+2\left(3\,a^2b+3\,ab^2+b^3\right)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)-\frac{9\,a^2b(e^{(2\,dx+2\,c}}{}$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/4*(2*a^3*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) + 2*(3*a^2*b + 3*a*
b^2 + b^3)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) - (9*a^2*b*(e^(2*d*
x + 2*c) + e^(-2*d*x - 2*c))^2 + 9*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*
c))^2 + 3*b^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 36*a^2*b*(e^(2*d*x
 + 2*c) + e^(-2*d*x - 2*c)) + 12*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))
  - 4*b^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 36*a^2*b - 12*a*b^2 - 4*b^
3)/(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2)^2)/d
```

3.161.9 Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 380, normalized size of antiderivative = 5.28

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\ln\left(\mathrm{e}^{4c+4dx}-1\right)\left(a^3d+d\left(3a^2b+3ab^2+b^3\right)\right)}{2d^2}$$
$$-x(a+b)^3+\frac{2\left(2b^3+3ab^2\right)}{d\left(\mathrm{e}^{2c+2dx}+1\right)}+\frac{8b^3}{d\left(3\,\mathrm{e}^{2c+2dx}+3\,\mathrm{e}^{4c+4dx}+\mathrm{e}^{6c+6dx}+1\right)}$$
$$+\frac{\operatorname{atan}\left(\frac{\mathrm{e}^{2c}\,\mathrm{e}^{2dx}\left(b^3\sqrt{-d^2}-a^3\sqrt{-d^2}+3ab^2\sqrt{-d^2}+3a^2b\sqrt{-d^2}\right)}{d\sqrt{a^6-6a^5b+3a^4b^2+16a^3b^3+15a^2b^4+6ab^5+b^6}}\right)\sqrt{a^6-6a^5b+3a^4b^2+16a^3b^3+15a^2b^4+6ab^5}}{\sqrt{-d^2}}$$
$$-\frac{2\left(4b^3+3ab^2\right)}{d\left(2\,\mathrm{e}^{2c+2dx}+\mathrm{e}^{4c+4dx}+1\right)}-\frac{4b^3}{d\left(4\,\mathrm{e}^{2c+2dx}+6\,\mathrm{e}^{4c+4dx}+4\,\mathrm{e}^{6c+6dx}+\mathrm{e}^{8c+8dx}+1\right)}$$

input
```
int(coth(c + d*x)*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
(log(exp(4*c + 4*d*x) - 1)*(a^3*d + d*(3*a*b^2 + 3*a^2*b + b^3)))/(2*d^2)
- x*(a + b)^3 + (2*(3*a*b^2 + 2*b^3))/(d*(exp(2*c + 2*d*x) + 1)) + (8*b^3)
/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (a
tan((exp(2*c)*exp(2*d*x)*(b^3*(-d^2)^(1/2) - a^3*(-d^2)^(1/2) + 3*a*b^2*(-
d^2)^(1/2) + 3*a^2*b*(-d^2)^(1/2)))/(d*(6*a*b^5 - 6*a^5*b + a^6 + b^6 + 15
*a^2*b^4 + 16*a^3*b^3 + 3*a^4*b^2)^(1/2)))*(6*a*b^5 - 6*a^5*b + a^6 + b^6
+ 15*a^2*b^4 + 16*a^3*b^3 + 3*a^4*b^2)^(1/2))/(-d^2)^(1/2) - (2*(3*a*b^2 +
 4*b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (4*b^3)/(d*(4*e
xp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*
x) + 1))
```

3.162 $\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

3.162.1 Optimal result 1207
3.162.2 Mathematica [A] (verified) 1207
3.162.3 Rubi [A] (verified) 1208
3.162.4 Maple [A] (verified) 1209
3.162.5 Fricas [B] (verification not implemented) 1210
3.162.6 Sympy [F] 1210
3.162.7 Maxima [B] (verification not implemented) 1211
3.162.8 Giac [B] (verification not implemented) 1211
3.162.9 Mupad [B] (verification not implemented) 1212

3.162.1 Optimal result

Integrand size = 23, antiderivative size = 59

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = (a+b)^3x - \frac{a^3\coth(c+dx)}{d} - \frac{b^2(3a+b)\tanh(c+dx)}{d} - \frac{b^3\tanh^3(c+dx)}{3d}$$

output
```
(a+b)^3*x-a^3*coth(d*x+c)/d-b^2*(3*a+b)*tanh(d*x+c)/d-1/3*b^3*tanh(d*x+c)^
3/d
```

3.162.2 Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.37

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\tanh(c+dx)\left(-3a^3\coth^2(c+dx)+3(a+b)^3\operatorname{arctanh}\left(\sqrt{\coth^2(c+dx)}\right)\sqrt{\coth^2(c+dx)}-b^2(9a+3b$$

input
```
Integrate[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(Tanh[c + d*x]*(-3*a^3*Coth[c + d*x]^2 + 3*(a + b)^3*ArcTanh[Sqrt[Coth[c +
 d*x]^2]]*Sqrt[Coth[c + d*x]^2] - b^2*(9*a + 3*b + b*Tanh[c + d*x]^2)))/(3
*d)
```

3.162.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 25, 4153, 25, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)^2}dx$$

$$\downarrow 4153$$

$$-\frac{\int -\frac{\coth^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 25$$

$$\frac{\int \frac{\coth^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 364$$

$$\frac{\int \left(\coth^2(c+dx)a^3-b^3\tanh^2(c+dx)-b^2(3a+b)-\frac{(a+b)^3}{\tanh^2(c+dx)-1}\right)d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$-\frac{a^3\coth(c+dx)-(a+b)^3\text{arctanh}(\tanh(c+dx))+b^2(3a+b)\tanh(c+dx)+\frac{1}{3}b^3\tanh^3(c+dx)}{d}$$

input
```
Int[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-((-((a + b)^3*ArcTanh[Tanh[c + d*x]]) + a^3*Coth[c + d*x] + b^2*(3*a + b)
*Tanh[c + d*x] + (b^3*Tanh[c + d*x]^3)/3)/d)
```

3.162.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.162.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\frac{-b^3\tanh(dx+c)^3+(-9ab^2-3b^3)\tanh(dx+c)-3\coth(dx+c)a^3+3dx(a+b)^3}{3d}$
derivativedivides	$-\frac{\frac{b^3\tanh(dx+c)^3}{3}+3ab^2\tanh(dx+c)+b^3\tanh(dx+c)+(-\frac{1}{2}a^3-\frac{3}{2}a^2b-\frac{3}{2}ab^2-\frac{1}{2}b^3)\ln(\tanh(dx+c)+1)+(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}$
default	$-\frac{\frac{b^3\tanh(dx+c)^3}{3}+3ab^2\tanh(dx+c)+b^3\tanh(dx+c)+(-\frac{1}{2}a^3-\frac{3}{2}a^2b-\frac{3}{2}ab^2-\frac{1}{2}b^3)\ln(\tanh(dx+c)+1)+(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}$
risch	$a^3x+3ba^2x+3ab^2x+b^3x-\frac{2(3a^3e^{6dx+6c}-9ab^2e^{6dx+6c}-6e^{6dx+6c}b^3+9a^3e^{4dx+4c}-9ab^2e^{4dx+4c}+9a^3e$
	$3d(e^{2dx+2c}+1)^3(e^{2dx+2c}$

input
```
int(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-b^3*tanh(d*x+c)^3+(-9*a*b^2-3*b^3)*tanh(d*x+c)-3*coth(d*x+c)*a^3+3*d
*x*(a+b)^3)/d
```

3.162.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(57) = 114$.

Time $= 0.26$ (sec) , antiderivative size $= 341$, normalized size of antiderivative $= 5.78$

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$
$$-\frac{(3\,a^3+9\,ab^2+4\,b^3)\cosh(dx+c)^4-4\,(3\,a^3+9\,ab^2+4\,b^3+3\,(a^3+3\,a^2b+3\,ab^2+b^3)dx)\cosh(dx+c$$

input
```
integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/12*((3*a^3 + 9*a*b^2 + 4*b^3)*cosh(d*x + c)^4 - 4*(3*a^3 + 9*a*b^2 + 4*
b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^3
 + (3*a^3 + 9*a*b^2 + 4*b^3)*sinh(d*x + c)^4 + 9*a^3 - 9*a*b^2 + 4*(3*a^3
 - b^3)*cosh(d*x + c)^2 + 2*(6*a^3 - 2*b^3 + 3*(3*a^3 + 9*a*b^2 + 4*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c)^2 - 4*((3*a^3 + 9*a*b^2 + 4*b^3 + 3*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + (3*a^3 + 9*a*b^2 + 4*b^3 + 3
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cos
h(d*x + c)*sinh(d*x + c)^3 + (d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*
x + c))
```

3.162.6 Sympy [F]

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2(c+dx)\right)^3\coth^2(c+dx)\,dx$$

input
```
integrate(coth(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*coth(c + d*x)**2, x)
```

3.162.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(57) = 114$.

Time = 0.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.49

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{1}{3}b^3\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)$$
$$+3ab^2\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2dx-2c)}+1)}\right)+a^3\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2dx-2c)}-1)}\right)+3a^2bx$$

input
```
integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/3*b^3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*
(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 3*a*b
^2*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^3*(x + c/d + 2/(d*(e^(-2*d
*x - 2*c) - 1))) + 3*a^2*b*x
```

3.162.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(57) = 114$.

Time = 0.43 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.29

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{3\left(a^3+3a^2b+3ab^2+b^3\right)(dx+c)-\frac{6a^3}{e^{(2dx+2c)}-1}+\frac{2\left(9ab^2e^{(4dx+4c)}+6b^3e^{(4dx+4c)}+18ab^2e^{(2dx+2c)}+6b^3e^{(2dx+2c)}+9ab^2-\right.}{\left(e^{(2dx+2c)}+1\right)^3}}{3d}$$

input
```
integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) - 6*a^3/(e^(2*d*x + 2*c)
- 1) + 2*(9*a*b^2*e^(4*d*x + 4*c) + 6*b^3*e^(4*d*x + 4*c) + 18*a*b^2*e^(2*
d*x + 2*c) + 6*b^3*e^(2*d*x + 2*c) + 9*a*b^2 + 4*b^3)/(e^(2*d*x + 2*c) + 1
)^3)/d
```

3.162.9 Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.69

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= x\,(a+b)^3 + \frac{\frac{2ab^2}{d} + \frac{2e^{2c+2dx}\left(2b^3+3ab^2\right)}{3d}}{2e^{2c+2dx}+e^{4c+4dx}+1} + \frac{\frac{2\left(2b^3+3ab^2\right)}{3d} + \frac{2e^{4c+4dx}\left(2b^3+3ab^2\right)}{3d} + \frac{4ab^2e^{2c+2dx}}{d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}$$
$$- \frac{2a^3}{d\left(e^{2c+2dx}-1\right)} + \frac{2\left(2b^3+3ab^2\right)}{3d\left(e^{2c+2dx}+1\right)}$$

input
```
int(coth(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
x*(a + b)^3 + ((2*a*b^2)/d + (2*exp(2*c + 2*d*x)*(3*a*b^2 + 2*b^3))/(3*d))
/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + ((2*(3*a*b^2 + 2*b^3))/(3*d
) + (2*exp(4*c + 4*d*x)*(3*a*b^2 + 2*b^3))/(3*d) + (4*a*b^2*exp(2*c + 2*d*
x))/d)/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) -
(2*a^3)/(d*(exp(2*c + 2*d*x) - 1)) + (2*(3*a*b^2 + 2*b^3))/(3*d*(exp(2*c +
 2*d*x) + 1))
```

3.163 $\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

3.163.1 Optimal result 1213
3.163.2 Mathematica [A] (verified) 1213
3.163.3 Rubi [A] (warning: unable to verify) 1214
3.163.4 Maple [A] (verified) 1216
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3.163.8 Giac [B] (verification not implemented) 1218
3.163.9 Mupad [B] (verification not implemented) 1219

3.163.1 Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{a^3\coth^2(c+dx)}{2d} + \frac{(a+b)^3\log(\cosh(c+dx))}{d} + \frac{a^2(a+3b)\log(\tanh(c+dx))}{d} - \frac{b^3\tanh^2(c+dx)}{2d}$$

output
```
-1/2*a^3*coth(d*x+c)^2/d+(a+b)^3*ln(cosh(d*x+c))/d+a^2*(a+3*b)*ln(tanh(d*x
+c))/d-1/2*b^3*tanh(d*x+c)^2/d
```

3.163.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{a^3\coth^2(c+dx) - 2(a+b)^3\log(\cosh(c+dx)) - 2a^2(a+3b)\log(\tanh(c+dx)) + b^3\tanh^2(c+dx)}{2d}$$

input
```
Integrate[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-1/2*(a^3*Coth[c + d*x]^2 - 2*(a + b)^3*Log[Cosh[c + d*x]] - 2*a^2*(a + 3*
b)*Log[Tanh[c + d*x]] + b^3*Tanh[c + d*x]^2)/d
```

3.163.3 Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)^3}dx$$

$$\downarrow 26$$

$$-i\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)^3}dx$$

$$\downarrow 4153$$

$$-\frac{i\int \frac{i\coth^3(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\coth^3(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 354$$

$$\frac{\int \frac{\coth^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{2d}$$

$$\downarrow 99$$

$$\frac{\int \left(\coth^2(c+dx)a^3+(a+3b)\coth(c+dx)a^2-b^3-\frac{(a+b)^3}{\tanh^2(c+dx)-1}\right)d\tanh^2(c+dx)}{2d}$$

$$\downarrow 2009$$

$$\frac{a^3(-\coth(c+dx))+a^2(a+3b)\log\left(\tanh^2(c+dx)\right)-(a+b)^3\log\left(1-\tanh^2(c+dx)\right)-b^3\tanh^2(c+dx)}{2d}$$

input
```
Int[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-(a^3*Coth[c + d*x]) + a^2*(a + 3*b)*Log[Tanh[c + d*x]^2] - (a + b)^3*Log
[1 - Tanh[c + d*x]^2] - b^3*Tanh[c + d*x]^2)/(2*d)
```

3.163.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.163.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

method	result
parallelrisch	$\frac{-2(a+b)^3\ln(1-\tanh(dx+c))+2a^2(a+3b)\ln(\tanh(dx+c))-\coth(dx+c)^2a^3-b^3\tanh(dx+c)^2-2dx(a+b)^3}{2d}$
derivativedivides	$-\frac{\frac{b^3\tanh(dx+c)^2}{2}+(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}ab^2+\frac{1}{2}b^3)\ln(\tanh(dx+c)+1)+\frac{a^3}{2\tanh(dx+c)^2}-a^2(a+3b)\ln(\tanh(dx+c))+(\frac{1}{2}a^3+\frac{3}{2}a$
default	$-\frac{\frac{b^3\tanh(dx+c)^2}{2}+(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}ab^2+\frac{1}{2}b^3)\ln(\tanh(dx+c)+1)+\frac{a^3}{2\tanh(dx+c)^2}-a^2(a+3b)\ln(\tanh(dx+c))+(\frac{1}{2}a^3+\frac{3}{2}a$
risch	$-a^3x-3ba^2x-3ab^2x-b^3x-\frac{6ab^2c}{d}-\frac{2b^3c}{d}-\frac{2a^3c}{d}-\frac{6bca^2}{d}-\frac{2e^{2dx+2c}(a^3e^{4dx+4c}-e^{4dx+4c}b^3+$

input
```
int(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-2*(a+b)^3*ln(1-tanh(d*x+c))+2*a^2*(a+3*b)*ln(tanh(d*x+c))-coth(d*x+c
)^2*a^3-b^3*tanh(d*x+c)^2-2*d*x*(a+b)^3)/d
```

3.163.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1686 vs. 2(68) = 136.

Time = 0.27 (sec) , antiderivative size = 1686, normalized size of antiderivative = 23.42

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^8 + 8*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x*sinh(d*x + c)^8 + 2*(a^3 - b^3)*cosh(d*x + c)^6 + 2*(14*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 + a^3 - b^3)*sinh(d*x + c)
^6 + 4*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^3 + 3*(a^3 -
b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(2*a^3 + 2*b^3 - (a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 2*(35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3
)*d*x*cosh(d*x + c)^4 + 2*a^3 + 2*b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*
x + 15*(a^3 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^5 + 5*(a^3 - b^3)*cosh(d*x + c)^3 + (2*
a^3 + 2*b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x
 + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 2*(a^3 - b^3)*cosh(d*x + c
)^2 + 2*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^6 + 15*(a^3
- b^3)*cosh(d*x + c)^4 + a^3 - b^3 + 6*(2*a^3 + 2*b^3 - (a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a*b^2 + b^3)*cos
h(d*x + c)^8 + 56*(3*a*b^2 + b^3)*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*(3*
a*b^2 + b^3)*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*(3*a*b^2 + b^3)*cosh(d*x
+ c)*sinh(d*x + c)^7 + (3*a*b^2 + b^3)*sinh(d*x + c)^8 - 2*(3*a*b^2 + b^3)
*cosh(d*x + c)^4 + 2*(35*(3*a*b^2 + b^3)*cosh(d*x + c)^4 - 3*a*b^2 - b^3)*
sinh(d*x + c)^4 + 8*(7*(3*a*b^2 + b^3)*cosh(d*x + c)^5 - (3*a*b^2 + b^3...
```

3.163.6 Sympy [F]

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2(c+dx)\right)^3 \coth^3(c+dx)\,dx$$

input
```
integrate(coth(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*coth(c + d*x)**3, x)
```

3.163.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(68) = 136$.

Time $= 0.29$ (sec) , antiderivative size $= 203$, normalized size of antiderivative $= 2.82$

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= a^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$
$$+ b^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$
$$+\frac{3\,ab^2\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d}+\frac{3\,a^2b\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}$$

input
```
integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
a^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2
*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b^3*(x + c/
d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c
) + e^(-4*d*x - 4*c) + 1))) + 3*a*b^2*log(e^(d*x + c) + e^(-d*x - c))/d +
3*a^2*b*log(e^(d*x + c) - e^(-d*x - c))/d
```

3.163.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(68) = 136$.

Time $= 0.47$ (sec) , antiderivative size $= 274$, normalized size of antiderivative $= 3.81$

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{2\left(3\,ab^2+b^3\right)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)+2\left(a^3+3\,a^2b\right)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)-\frac{a^3\left(e^{(2\,dx+2\,c}\right.}{}}{}$$

input
```
integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/4*(2*(3*a*b^2 + b^3)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) + 2*(a^
3 + 3*a^2*b)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) - (a^3*(e^(2*d*x
+ 2*c) + e^(-2*d*x - 2*c))^2 + 3*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)
)^2 + 3*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + b^3*(e^(2*d*x + 2*c
) + e^(-2*d*x - 2*c))^2 + 8*a^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 8*b
^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 12*a^3 - 12*a^2*b - 12*a*b^2 + 1
2*b^3)/((e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 - 4))/d
```

3.163.9 Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 327, normalized size of antiderivative = 4.54

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\ln\left(\mathrm{e}^{4c+4dx}-1\right)\left(d\,a^3+3\,d\,a^2\,b+3\,d\,a\,b^2+d\,b^3\right)}{2\,d^2}$$
$$-\frac{\frac{4\left(a^3+b^3\right)}{d}+\frac{2\,\mathrm{e}^{2c+2dx}\left(a^3-b^3\right)}{d}}{\mathrm{e}^{4c+4dx}-1}-\frac{\frac{4\left(a^3+b^3\right)}{d}+\frac{4\,\mathrm{e}^{2c+2dx}\left(a^3-b^3\right)}{d}}{\mathrm{e}^{8c+8dx}-2\,\mathrm{e}^{4c+4dx}+1}$$
$$-\frac{\operatorname{atan}\left(\frac{\mathrm{e}^{2c}\,\mathrm{e}^{2dx}\left(a^3\sqrt{-d^2}-b^3\sqrt{-d^2}-3\,a\,b^2\sqrt{-d^2}+3\,a^2\,b\sqrt{-d^2}\right)}{d\sqrt{a^6+6\,a^5\,b+3\,a^4\,b^2-20\,a^3\,b^3+3\,a^2\,b^4+6\,a\,b^5+b^6}}\right)\sqrt{a^6+6\,a^5\,b+3\,a^4\,b^2-20\,a^3\,b^3+3\,a^2\,b^4+6\,a\,b^5+}}{\sqrt{-d^2}}$$
$$-x\,(a+b)^3$$

input
```
int(coth(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
(log(exp(4*c + 4*d*x) - 1)*(a^3*d + b^3*d + 3*a*b^2*d + 3*a^2*b*d))/(2*d^2
) - ((4*(a^3 + b^3))/d + (2*exp(2*c + 2*d*x)*(a^3 - b^3))/d)/(exp(4*c + 4*
d*x) - 1) - ((4*(a^3 + b^3))/d + (4*exp(2*c + 2*d*x)*(a^3 - b^3))/d)/(exp(
8*c + 8*d*x) - 2*exp(4*c + 4*d*x) + 1) - (atan((exp(2*c)*exp(2*d*x)*(a^3*(
-d^2)^(1/2) - b^3*(-d^2)^(1/2) - 3*a*b^2*(-d^2)^(1/2) + 3*a^2*b*(-d^2)^(1/
2)))/(d*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 3*a^2*b^4 - 20*a^3*b^3 + 3*a^4*b^
2)^(1/2)))*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 3*a^2*b^4 - 20*a^3*b^3 + 3*a^4
*b^2)^(1/2))/(-d^2)^(1/2) - x*(a + b)^3
```

3.164 $\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

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3.164.1 Optimal result

Integrand size = 23, antiderivative size = 59

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = (a+b)^3x - \frac{a^2(a+3b)\coth(c+dx)}{d} - \frac{a^3\coth^3(c+dx)}{3d} - \frac{b^3\tanh(c+dx)}{d}$$

output
```
(a+b)^3*x-a^2*(a+3*b)*coth(d*x+c)/d-1/3*a^3*coth(d*x+c)^3/d-b^3*tanh(d*x+c
)/d
```

3.164.2 Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\left(-3b^3-3a^2(a+3b)\coth^2(c+dx)-a^3\coth^4(c+dx)+3(a+b)^3\text{arctanh}\left(\sqrt{\coth^2(c+dx)}\right)\sqrt{\coth^2(c}\right.}{3d}$$

input
```
Integrate[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((-3*b^3 - 3*a^2*(a + 3*b)*Coth[c + d*x]^2 - a^3*Coth[c + d*x]^4 + 3*(a +
b)^3*ArcTanh[Sqrt[Coth[c + d*x]^2]]*Sqrt[Coth[c + d*x]^2])*Tanh[c + d*x])/
(3*d)
```

3.164.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4153, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)^4}dx$$

$$\downarrow 4153$$

$$\frac{\int \frac{\coth^4(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 364$$

$$\frac{\int \left(a^3\coth^4(c+dx)+a^2(a+3b)\coth^2(c+dx)-b^3-\frac{(a+b)^3}{\tanh^2(c+dx)-1}\right)d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{3}a^3\coth^3(c+dx)-a^2(a+3b)\coth(c+dx)+(a+b)^3\text{arctanh}(\tanh(c+dx))+b^3(-\tanh(c+dx))}{d}$$

input
```
Int[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((a + b)^3*ArcTanh[Tanh[c + d*x]] - a^2*(a + 3*b)*Coth[c + d*x] - (a^3*Cot
h[c + d*x]^3)/3 - b^3*Tanh[c + d*x])/d
```

3.164.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.164.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

method	result
parallelrisch	$\frac{-\coth(dx+c)^3a^3-3a^2(a+3b)\coth(dx+c)-3b^3\tanh(dx+c)+3dx(a+b)^3}{3d}$
derivativedivides	$-\frac{\frac{\coth(dx+c)^3a^3}{3}+\coth(dx+c)a^3+3a^2b\coth(dx+c)+\left(-\frac{1}{2}a^3-\frac{3}{2}a^2b-\frac{3}{2}ab^2-\frac{1}{2}b^3\right)\ln(\coth(dx+c)+1)+\frac{b^3}{\coth(dx+c)}+\left(\frac{1}{2}a^3\right.}{d}$
default	$-\frac{\frac{\coth(dx+c)^3a^3}{3}+\coth(dx+c)a^3+3a^2b\coth(dx+c)+\left(-\frac{1}{2}a^3-\frac{3}{2}a^2b-\frac{3}{2}ab^2-\frac{1}{2}b^3\right)\ln(\coth(dx+c)+1)+\frac{b^3}{\coth(dx+c)}+\left(\frac{1}{2}a^3\right.}{d}$
risch	$a^3x+3ba^2x+3ab^2x+b^3x-\frac{2(6a^3e^{6dx+6c}+9a^2be^{6dx+6c}-3e^{6dx+6c}b^3-9a^2be^{4dx+4c}+9e^{4dx+4c}b^3-2a^3e}{3d\left(e^{2dx+2c}-1\right)^3\left(e^{2dx+2c}\right.}$

input
```
int(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-coth(d*x+c)^3*a^3-3*a^2*(a+3*b)*coth(d*x+c)-3*b^3*tanh(d*x+c)+3*d*x*
(a+b)^3)/d
```

3.164.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(57) = 114$.

Time $= 0.27$ (sec) , antiderivative size $= 341$, normalized size of antiderivative $= 5.78$

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$

$$-\frac{(4\,a^3+9\,a^2b+3\,b^3)\cosh(dx+c)^4-4\,(4\,a^3+9\,a^2b+3\,b^3+3\,(a^3+3\,a^2b+3\,ab^2+b^3)dx)\cosh(dx+c$$

input
```
integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/12*((4*a^3 + 9*a^2*b + 3*b^3)*cosh(d*x + c)^4 - 4*(4*a^3 + 9*a^2*b + 3*
b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^3
 + (4*a^3 + 9*a^2*b + 3*b^3)*sinh(d*x + c)^4 - 9*a^2*b + 9*b^3 + 4*(a^3 -
3*b^3)*cosh(d*x + c)^2 + 2*(2*a^3 - 6*b^3 + 3*(4*a^3 + 9*a^2*b + 3*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c)^2 - 4*((4*a^3 + 9*a^2*b + 3*b^3 + 3*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - (4*a^3 + 9*a^2*b + 3*b^3 + 3
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cos
h(d*x + c)*sinh(d*x + c)^3 + (d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*
x + c))
```

3.164.6 Sympy [F]

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2(c+dx)\right)^3\coth^4(c+dx)\,dx$$

input
```
integrate(coth(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*coth(c + d*x)**4, x)
```

3.164.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(57) = 114$.

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.49

$$\begin{aligned}&\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx\\&=\frac{1}{3}a^3\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right)\\&\quad+b^3\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2dx-2c)}+1)}\right)+3a^2b\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2dx-2c)}-1)}\right)+3ab^2x\end{aligned}$$

input
```
integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/3*a^3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*
(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + b^3*(
x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + 3*a^2*b*(x + c/d + 2/(d*(e^(-2*d
*x - 2*c) - 1))) + 3*a*b^2*x
```

3.164.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(57) = 114$.

Time = 0.49 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.29

$$\begin{aligned}&\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx\\&=\frac{3\left(a^3+3a^2b+3ab^2+b^3\right)(dx+c)+\frac{6b^3}{e^{(2dx+2c)}+1}-\frac{2\left(6a^3e^{(4dx+4c)}+9a^2be^{(4dx+4c)}-6a^3e^{(2dx+2c)}-18a^2be^{(2dx+2c)}+4a^3+\right.}{\left(e^{(2dx+2c)}-1\right)^3}}{3d}\end{aligned}$$

input
```
integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) + 6*b^3/(e^(2*d*x + 2*c)
+ 1) - 2*(6*a^3*e^(4*d*x + 4*c) + 9*a^2*b*e^(4*d*x + 4*c) - 6*a^3*e^(2*d*x
 + 2*c) - 18*a^2*b*e^(2*d*x + 2*c) + 4*a^3 + 9*a^2*b)/(e^(2*d*x + 2*c) - 1
)^3)/d
```

3.164.9 Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.71

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= x\,(a+b)^3 + \frac{\frac{2a^2b}{d} - \frac{2\,\mathrm{e}^{2c+2dx}\,(2a^3+3ba^2)}{3d}}{\mathrm{e}^{4c+4dx} - 2\,\mathrm{e}^{2c+2dx} + 1} - \frac{\frac{2\,(2a^3+3ba^2)}{3d} + \frac{2\,\mathrm{e}^{4c+4dx}\,(2a^3+3ba^2)}{3d} - \frac{4a^2b\,\mathrm{e}^{2c+2dx}}{d}}{3\,\mathrm{e}^{2c+2dx} - 3\,\mathrm{e}^{4c+4dx} + \mathrm{e}^{6c+6dx} - 1}$$
$$+ \frac{2b^3}{d\,(\mathrm{e}^{2c+2dx}+1)} - \frac{2\,(2a^3+3ba^2)}{3d\,(\mathrm{e}^{2c+2dx}-1)}$$

input
```
int(coth(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
x*(a + b)^3 + ((2*a^2*b)/d - (2*exp(2*c + 2*d*x)*(3*a^2*b + 2*a^3))/(3*d))
/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - ((2*(3*a^2*b + 2*a^3))/(3*d
) + (2*exp(4*c + 4*d*x)*(3*a^2*b + 2*a^3))/(3*d) - (4*a^2*b*exp(2*c + 2*d*
x))/d)/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) +
(2*b^3)/(d*(exp(2*c + 2*d*x) + 1)) - (2*(3*a^2*b + 2*a^3))/(3*d*(exp(2*c +
 2*d*x) - 1))
```

3.165 $\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

3.165.1 Optimal result 1226
3.165.2 Mathematica [A] (verified) 1226
3.165.3 Rubi [A] (warning: unable to verify) 1227
3.165.4 Maple [A] (verified) 1229
3.165.5 Fricas [B] (verification not implemented) 1229
3.165.6 Sympy [F(-1)] 1230
3.165.7 Maxima [B] (verification not implemented) 1231
3.165.8 Giac [B] (verification not implemented) 1231
3.165.9 Mupad [B] (verification not implemented) 1232

3.165.1 Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{a^2(a+3b)\coth^2(c+dx)}{2d} - \frac{a^3\coth^4(c+dx)}{4d} + \frac{(a+b)^3\log(\cosh(c+dx))}{d} + \frac{a(a^2+3ab+3b^2)\log(\tanh(c+dx))}{d}$$

output
```
-1/2*a^2*(a+3*b)*coth(d*x+c)^2/d-1/4*a^3*coth(d*x+c)^4/d+(a+b)^3*ln(cosh(d
*x+c))/d+a*(a^2+3*a*b+3*b^2)*ln(tanh(d*x+c))/d
```

3.165.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{-a^2(a+3b)\coth^2(c+dx) - \frac{1}{2}a^3\coth^4(c+dx) + 2(a+b)^3\log(\sinh(c+dx)) - 2b^3\log(\tanh(c+dx))}{2d}$$

input
```
Integrate[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-(a^2*(a + 3*b)*Coth[c + d*x]^2) - (a^3*Coth[c + d*x]^4)/2 + 2*(a + b)^3*
Log[Sinh[c + d*x]] - 2*b^3*Log[Tanh[c + d*x]])/(2*d)
```

3.165.3 Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\downarrow 3042$$

$$\int \frac{i\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)^5}dx$$

$$\downarrow 26$$

$$i\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)^5}dx$$

$$\downarrow 4153$$

$$\frac{i\int -\frac{i\coth^5(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\coth^5(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 354$$

$$\frac{\int \frac{\coth^3(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{2d}$$

$$\downarrow 99$$

$$\frac{\int \left(-\frac{(a+b)^3}{\tanh^2(c+dx)-1}+a^3\coth^3(c+dx)+a^2(a+3b)\coth^2(c+dx)+a\left(a^2+3ba+3b^2\right)\coth(c+dx)\right)d\tanh^2(c+d}{2d}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{2}a^3\coth^2(c+dx)+a\left(a^2+3ab+3b^2\right)\log\left(\tanh^2(c+dx)\right)-a^2(a+3b)\coth(c+dx)-(a+b)^3\log\left(1-\tanh^2(\right.}{2d}$$

input
```
Int[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-(a^2*(a + 3*b)*Coth[c + d*x]) - (a^3*Coth[c + d*x]^2)/2 + a*(a^2 + 3*a*b
 + 3*b^2)*Log[Tanh[c + d*x]^2] - (a + b)^3*Log[1 - Tanh[c + d*x]^2])/(2*d)
```

3.165.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.165.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.08

method	result
parallelrisch	$\frac{-4(a+b)^3\ln(1-\tanh(dx+c))+4(a^3+3a^2b+3ab^2)\ln(\tanh(dx+c))-\coth(dx+c)^4a^3-2\coth(dx+c)^2a^2(a+3b)-4dx(a+b$
derivativedivides	$-\frac{\frac{a^3}{4\tanh(dx+c)^4}-a(a^2+3ab+3b^2)\ln(\tanh(dx+c))+\frac{a^2(a+3b)}{2\tanh(dx+c)^2}+(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}ab^2+\frac{1}{2}b^3)\ln(\tanh(dx+c)+1)+(\frac{1}{2}a$
default	$-\frac{\frac{a^3}{4\tanh(dx+c)^4}-a(a^2+3ab+3b^2)\ln(\tanh(dx+c))+\frac{a^2(a+3b)}{2\tanh(dx+c)^2}+(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}ab^2+\frac{1}{2}b^3)\ln(\tanh(dx+c)+1)+(\frac{1}{2}a$
risch	$-a^3x-3ba^2x-3ab^2x-b^3x-\frac{2a^3c}{d}-\frac{6bca^2}{d}-\frac{6ab^2c}{d}-\frac{2b^3c}{d}-\frac{2a^2e^{2dx+2c}(2ae^{4dx+4c}+3be^{4dx+$

input
```
int(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-4*(a+b)^3*ln(1-tanh(d*x+c))+4*(a^3+3*a^2*b+3*a*b^2)*ln(tanh(d*x+c))-
coth(d*x+c)^4*a^3-2*coth(d*x+c)^2*a^2*(a+3*b)-4*d*x*(a+b)^3)/d
```

3.165.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2393 vs. 2(79) = 158.

Time = 0.29 (sec) , antiderivative size = 2393, normalized size of antiderivative = 28.83

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^8 + 8*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x*sinh(d*x + c)^8 + 2*(2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 + 2*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
d*x*cosh(d*x + c)^2 + 2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
d*x)*sinh(d*x + c)^6 + 4*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x
 + c)^3 + 3*(2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(
d*x + c))*sinh(d*x + c)^5 - 2*(2*a^3 + 6*a^2*b - 3*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^4 + 2*(35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*
cosh(d*x + c)^4 - 2*a^3 - 6*a^2*b + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x
 + 15*(2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x +
 c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x
 + c)^5 + 5*(2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(
d*x + c)^3 - (2*a^3 + 6*a^2*b - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cos
h(d*x + c))*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 2*(2*a
^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2 + 2*
(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^6 + 15*(2*a^3 + 3*a^
2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 2*a^3 + 3*a
^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 6*(2*a^3 + 6*a^2*b - 3*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - (b^...
```

3.165.6 Sympy [F(-1)]

Timed out.

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Timed out}$$

input
```
integrate(coth(d*x+c)**5*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.165.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(79) = 158$.

Time = 0.21 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.18

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= a^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8}}\right.$$

$$+3\,a^2b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$

$$\left.+\frac{b^3\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d}+\frac{3\,ab^2\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}\right.$$

input
```
integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
a^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-
2*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c)
 - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 3*a
^2*b*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-
2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b^3*log(e^
(d*x + c) + e^(-d*x - c))/d + 3*a*b^2*log(e^(d*x + c) - e^(-d*x - c))/d
```

3.165.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(79) = 158$.

Time = 0.52 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.22

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{2\,b^3\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)+2\left(a^3+3\,a^2b+3\,ab^2\right)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)-\frac{3\,a^3\left(e^{(2\,dx+2\,c)}\right.}{}}{}$$

input
```
integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/4*(2*b^3*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) + 2*(a^3 + 3*a^2*b
+ 3*a*b^2)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) - (3*a^3*(e^(2*d*x
+ 2*c) + e^(-2*d*x - 2*c))^2 + 9*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)
)^2 + 9*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 4*a^3*(e^(2*d*x + 2
*c) + e^(-2*d*x - 2*c)) - 12*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) -
36*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 4*a^3 - 12*a^2*b + 36*a*b^
2)/(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2)^2)/d
```

3.165.9 Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.59

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\ln\left(\mathrm{e}^{4c+4dx}-1\right)\left(b^3d+d\left(a^3+3a^2b+3ab^2\right)\right)}{2d^2}$$
$$-x(a+b)^3-\frac{2\left(2a^3+3ba^2\right)}{d\left(\mathrm{e}^{2c+2dx}-1\right)}-\frac{8a^3}{d\left(3\mathrm{e}^{2c+2dx}-3\mathrm{e}^{4c+4dx}+\mathrm{e}^{6c+6dx}-1\right)}$$
$$-\frac{\operatorname{atan}\left(\frac{\mathrm{e}^{2c}\mathrm{e}^{2dx}\left(a^3\sqrt{-d^2}-b^3\sqrt{-d^2}+3ab^2\sqrt{-d^2}+3a^2b\sqrt{-d^2}\right)}{d\sqrt{a^6+6a^5b+15a^4b^2+16a^3b^3+3a^2b^4-6ab^5+b^6}}\right)\sqrt{a^6+6a^5b+15a^4b^2+16a^3b^3+3a^2b^4-6ab^5}}{\sqrt{-d^2}}$$
$$-\frac{2\left(4a^3+3ba^2\right)}{d\left(\mathrm{e}^{4c+4dx}-2\mathrm{e}^{2c+2dx}+1\right)}-\frac{4a^3}{d\left(6\mathrm{e}^{4c+4dx}-4\mathrm{e}^{2c+2dx}-4\mathrm{e}^{6c+6dx}+\mathrm{e}^{8c+8dx}+1\right)}$$

input
```
int(coth(c + d*x)^5*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
(log(exp(4*c + 4*d*x) - 1)*(b^3*d + d*(3*a*b^2 + 3*a^2*b + a^3)))/(2*d^2)
- x*(a + b)^3 - (2*(3*a^2*b + 2*a^3))/(d*(exp(2*c + 2*d*x) - 1)) - (8*a^3)
/(d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (a
tan((exp(2*c)*exp(2*d*x)*(a^3*(-d^2)^(1/2) - b^3*(-d^2)^(1/2) + 3*a*b^2*(-
d^2)^(1/2) + 3*a^2*b*(-d^2)^(1/2)))/(d*(6*a^5*b - 6*a*b^5 + a^6 + b^6 + 3*
a^2*b^4 + 16*a^3*b^3 + 15*a^4*b^2)^(1/2)))*(6*a^5*b - 6*a*b^5 + a^6 + b^6
+ 3*a^2*b^4 + 16*a^3*b^3 + 15*a^4*b^2)^(1/2))/(-d^2)^(1/2) - (2*(3*a^2*b +
 4*a^3))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (4*a^3)/(d*(6*e
xp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*
x) + 1))
```

3.166 $\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

3.166.1 Optimal result 1233
3.166.2 Mathematica [A] (verified) 1233
3.166.3 Rubi [A] (verified) 1234
3.166.4 Maple [A] (verified) 1236
3.166.5 Fricas [B] (verification not implemented) 1236
3.166.6 Sympy [F(-1)] 1237
3.166.7 Maxima [B] (verification not implemented) 1237
3.166.8 Giac [B] (verification not implemented) 1238
3.166.9 Mupad [B] (verification not implemented) 1239

3.166.1 Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = (a+b)^3x - \frac{a(a^2+3ab+3b^2)\coth(c+dx)}{d} - \frac{a^2(a+3b)\coth^3(c+dx)}{3d} - \frac{a^3\coth^5(c+dx)}{5d}$$

output
```
(a+b)^3*x-a*(a^2+3*a*b+3*b^2)*coth(d*x+c)/d-1/3*a^2*(a+3*b)*coth(d*x+c)^3/
d-1/5*a^3*coth(d*x+c)^5/d
```

3.166.2 Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{a\coth(c+dx)\left(15(a^2+3ab+3b^2)+5a(a+3b)\coth^2(c+dx)+3a^2\coth^4(c+dx)\right)}{15d} + \frac{(a+b)^3\operatorname{arctanh}\left(\sqrt{\tanh^2(c+dx)}\right)\tanh(c+dx)}{d\sqrt{\tanh^2(c+dx)}}$$

input
```
Integrate[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-1/15*(a*Coth[c + d*x]*(15*(a^2 + 3*a*b + 3*b^2) + 5*a*(a + 3*b)*Coth[c +
d*x]^2 + 3*a^2*Coth[c + d*x]^4))/d + ((a + b)^3*ArcTanh[Sqrt[Tanh[c + d*x]
^2]]*Tanh[c + d*x])/(d*Sqrt[Tanh[c + d*x]^2])
```

3.166.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 25, 4153, 25, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)^6}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)^6}dx$$

$$\downarrow 4153$$

$$-\frac{\int -\frac{\coth^6(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 25$$

$$\frac{\int \frac{\coth^6(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 364$$

$$\frac{\int \left(a^3\coth^6(c+dx)+a^2(a+3b)\coth^4(c+dx)+a\left(a^2+3ba+3b^2\right)\coth^2(c+dx)-\frac{(a+b)^3}{\tanh^2(c+dx)-1}\right)d\tanh(c+dx}{d}$$

$$\downarrow 2009$$

$$-\frac{\frac{1}{5}a^3\coth^5(c+dx)+a\left(a^2+3ab+3b^2\right)\coth(c+dx)+\frac{1}{3}a^2(a+3b)\coth^3(c+dx)-(a+b)^3\text{arctanh}(\tanh(c+dx}{d}$$

input
```
Int[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-((-((a + b)^3*ArcTanh[Tanh[c + d*x]]) + a*(a^2 + 3*a*b + 3*b^2)*Coth[c +
d*x] + (a^2*(a + 3*b)*Coth[c + d*x]^3)/3 + (a^3*Coth[c + d*x]^5)/5)/d)
```

3.166.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.166.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

method	result
parallelrisch	$\frac{-3\coth(dx+c)^5a^3-5a^2\coth(dx+c)^3(a+3b)+\left(-15a^3-45a^2b-45ab^2\right)\coth(dx+c)+15dx(a+b)^3}{15d}$
derivativedivides	$-\frac{\left(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}ab^2+\frac{1}{2}b^3\right)\ln(\tanh(dx+c)-1)+\frac{a^3}{5\tanh(dx+c)^5}+\frac{a\left(a^2+3ab+3b^2\right)}{\tanh(dx+c)}+\frac{a^2(a+3b)}{3\tanh(dx+c)^3}+\left(-\frac{1}{2}a^3-\frac{3}{2}a^2b-\frac{3}{2}ab^2$
default	$-\frac{\left(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}ab^2+\frac{1}{2}b^3\right)\ln(\tanh(dx+c)-1)+\frac{a^3}{5\tanh(dx+c)^5}+\frac{a\left(a^2+3ab+3b^2\right)}{\tanh(dx+c)}+\frac{a^2(a+3b)}{3\tanh(dx+c)^3}+\left(-\frac{1}{2}a^3-\frac{3}{2}a^2b-\frac{3}{2}ab^2$
risch	$a^3x+3ba^2x+3ab^2x+b^3x-\frac{2a(45a^2e^{8dx+8c}+90abe^{8dx+8c}+45b^2e^{8dx+8c}-90a^2e^{6dx+6c}-270abe^{6dx+6c}-$

input
```
int(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/15*(-3*coth(d*x+c)^5*a^3-5*a^2*coth(d*x+c)^3*(a+3*b)+(-15*a^3-45*a^2*b-4
5*a*b^2)*coth(d*x+c)+15*d*x*(a+b)^3)/d
```

3.166.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. $2(70) = 140$.

Time = 0.26 (sec) , antiderivative size = 557, normalized size of antiderivative = 7.53

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$

$$-\frac{(23\,a^3+60\,a^2b+45\,ab^2)\cosh(dx+c)^5+5\,(23\,a^3+60\,a^2b+45\,ab^2)\cosh(dx+c)\sinh(dx+c)^4-(23$$

input
```
integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/15*((23*a^3 + 60*a^2*b + 45*a*b^2)*cosh(d*x + c)^5 + 5*(23*a^3 + 60*a^2
*b + 45*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 - (23*a^3 + 60*a^2*b + 45*a*b
^2 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*sinh(d*x + c)^5 - 5*(5*a^3 +
24*a^2*b + 27*a*b^2)*cosh(d*x + c)^3 + 5*(23*a^3 + 60*a^2*b + 45*a*b^2 + 1
5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 2*(23*a^3 + 60*a^2*b + 45*a*b^2 +
15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^3 +
 5*(2*(23*a^3 + 60*a^2*b + 45*a*b^2)*cosh(d*x + c)^3 - 3*(5*a^3 + 24*a^2*b
 + 27*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^3 + 6*a^2*b + 9*a*b^
2)*cosh(d*x + c) - 5*((23*a^3 + 60*a^2*b + 45*a*b^2 + 15*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 46*a^3 + 120*a^2*b + 90*a*b^2 + 30*(
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 3*(23*a^3 + 60*a^2*b + 45*a*b^2 + 15*
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*si
nh(d*x + c)^5 + 5*(2*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*cosh(d*
x + c)^4 - 3*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))
```

3.166.6 Sympy [F(-1)]

Timed out.

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Timed out}$$

input
```
integrate(coth(d*x+c)**6*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.166.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(70) = 140$.

Time $= 0.21$ (sec) , antiderivative size $= 239$, normalized size of antiderivative $= 3.23$

$$\begin{aligned}
&\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx \\
&= \frac{1}{15}a^3\left(15x+\frac{15c}{d}-\frac{2\left(70e^{(-2dx-2c)}-140e^{(-4dx-4c)}+90e^{(-6dx-6c)}-45e^{(-8dx-8c)}-23\right)}{d(5e^{(-2dx-2c)}-10e^{(-4dx-4c)}+10e^{(-6dx-6c)}-5e^{(-8dx-8c)}+e^{(-10dx-10c)}-1)}\right) \\
&\quad + a^2b\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right) \\
&\quad + 3ab^2\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2dx-2c)}-1)}\right)+b^3x
\end{aligned}$$

input
```
integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/15*a^3*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) - 140*e^(-4*d*x - 4*c) +
90*e^(-6*d*x - 6*c) - 45*e^(-8*d*x - 8*c) - 23)/(d*(5*e^(-2*d*x - 2*c) - 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x
 - 10*c) - 1))) + a^2*b*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x
 - 4*c) - 2)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c
) - 1))) + 3*a*b^2*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + b^3*x
```

3.166.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(70) = 140$.

Time $= 0.57$ (sec) , antiderivative size $= 241$, normalized size of antiderivative $= 3.26$

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{15\left(a^3+3\,a^2b+3\,ab^2+b^3\right)(dx+c) - \frac{2\left(45\,a^3e^{(8\,dx+8\,c)}+90\,a^2be^{(8\,dx+8\,c)}+45\,ab^2e^{(8\,dx+8\,c)}-90\,a^3e^{(6\,dx+6\,c)}-270\,a^2be^{(6\,dx+6\,c)}\right.}{}}{}$$

input
```
integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/15*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) - 2*(45*a^3*e^(8*d*x +
8*c) + 90*a^2*b*e^(8*d*x + 8*c) + 45*a*b^2*e^(8*d*x + 8*c) - 90*a^3*e^(6*d
*x + 6*c) - 270*a^2*b*e^(6*d*x + 6*c) - 180*a*b^2*e^(6*d*x + 6*c) + 140*a^
3*e^(4*d*x + 4*c) + 330*a^2*b*e^(4*d*x + 4*c) + 270*a*b^2*e^(4*d*x + 4*c)
- 70*a^3*e^(2*d*x + 2*c) - 210*a^2*b*e^(2*d*x + 2*c) - 180*a*b^2*e^(2*d*x
+ 2*c) + 23*a^3 + 60*a^2*b + 45*a*b^2)/(e^(2*d*x + 2*c) - 1)^5)/d
```

3.166.9 Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 568, normalized size of antiderivative = 7.68

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = x\,(a+b)^3$$
$$-\frac{\frac{6\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d}+\frac{6\,e^{8\,c+8\,d\,x}\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d}-\frac{24\,e^{2\,c+2\,d\,x}\,(a^2\,b+a\,b^2)}{5\,d}-\frac{24\,e^{6\,c+6\,d\,x}\,(a^2\,b+a\,b^2)}{5\,d}+\frac{4\,e^{4\,c+4\,d\,x}\,(5\,a^3+6\,a^2\,b+9\,a$$
$$\frac{}{5\,e^{2\,c+2\,d\,x}-10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}-5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}-1}$$
$$-\frac{\frac{2\,(5\,a^3+6\,a^2\,b+9\,a\,b^2)}{15\,d}+\frac{6\,e^{4\,c+4\,d\,x}\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d}-\frac{12\,e^{2\,c+2\,d\,x}\,(a^2\,b+a\,b^2)}{5\,d}}{3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1}$$
$$+\frac{\frac{6\,(a^2\,b+a\,b^2)}{5\,d}-\frac{6\,e^{6\,c+6\,d\,x}\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d}+\frac{18\,e^{4\,c+4\,d\,x}\,(a^2\,b+a\,b^2)}{5\,d}-\frac{2\,e^{2\,c+2\,d\,x}\,(5\,a^3+6\,a^2\,b+9\,a\,b^2)}{5\,d}}{6\,e^{4\,c+4\,d\,x}-4\,e^{2\,c+2\,d\,x}-4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1}$$
$$+\frac{\frac{6\,(a^2\,b+a\,b^2)}{5\,d}-\frac{6\,e^{2\,c+2\,d\,x}\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d}}{e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1}-\frac{6\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d\,\left(e^{2\,c+2\,d\,x}-1\right)}$$

input
```
int(coth(c + d*x)^6*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
x*(a + b)^3 - ((6*(a*b^2 + 2*a^2*b + a^3))/(5*d) + (6*exp(8*c + 8*d*x)*(a*
b^2 + 2*a^2*b + a^3))/(5*d) - (24*exp(2*c + 2*d*x)*(a*b^2 + a^2*b))/(5*d)
- (24*exp(6*c + 6*d*x)*(a*b^2 + a^2*b))/(5*d) + (4*exp(4*c + 4*d*x)*(9*a*b
^2 + 6*a^2*b + 5*a^3))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) +
10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) - ((2*(
9*a*b^2 + 6*a^2*b + 5*a^3))/(15*d) + (6*exp(4*c + 4*d*x)*(a*b^2 + 2*a^2*b
+ a^3))/(5*d) - (12*exp(2*c + 2*d*x)*(a*b^2 + a^2*b))/(5*d))/(3*exp(2*c +
2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) + ((6*(a*b^2 + a^2*b))
/(5*d) - (6*exp(6*c + 6*d*x)*(a*b^2 + 2*a^2*b + a^3))/(5*d) + (18*exp(4*c
+ 4*d*x)*(a*b^2 + a^2*b))/(5*d) - (2*exp(2*c + 2*d*x)*(9*a*b^2 + 6*a^2*b +
 5*a^3))/(5*d))/(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d
*x) + exp(8*c + 8*d*x) + 1) + ((6*(a*b^2 + a^2*b))/(5*d) - (6*exp(2*c + 2*
d*x)*(a*b^2 + 2*a^2*b + a^3))/(5*d))/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x
) + 1) - (6*(a*b^2 + 2*a^2*b + a^3))/(5*d*(exp(2*c + 2*d*x) - 1))
```

3.167 $\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

3.167.1 Optimal result 1240
3.167.2 Mathematica [A] (verified) 1240
3.167.3 Rubi [A] (warning: unable to verify) 1241
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3.167.1 Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{a(a^2+3ab+3b^2)\coth^2(c+dx)}{2d} - \frac{a^2(a+3b)\coth^4(c+dx)}{4d} - \frac{a^3\coth^6(c+dx)}{6d} + \frac{(a+b)^3\log(\cosh(c+dx))}{d} + \frac{(a+b)^3\log(\tanh(c+dx))}{d}$$

output
```
-1/2*a*(a^2+3*a*b+3*b^2)*coth(d*x+c)^2/d-1/4*a^2*(a+3*b)*coth(d*x+c)^4/d-1
/6*a^3*coth(d*x+c)^6/d+(a+b)^3*ln(cosh(d*x+c))/d+(a+b)^3*ln(tanh(d*x+c))/d
```

3.167.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{a(a+b)^2\coth^2(c+dx) + \frac{1}{2}(a+b)\left(b+a\coth^2(c+dx)\right)^2 + \frac{1}{3}\left(b+a\coth^2(c+dx)\right)^3 - 2(a+b)^3\log(\sin$$

input
```
Integrate[Coth[c + d*x]^7*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-1/2*(a*(a + b)^2*Coth[c + d*x]^2 + ((a + b)*(b + a*Coth[c + d*x]^2)^2)/2
+ (b + a*Coth[c + d*x]^2)^3/3 - 2*(a + b)^3*Log[Sinh[c + d*x]])/d
```

3.167.3 Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$$

$$\downarrow 3042$$

$$\int -\frac{i\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)^7}dx$$

$$\downarrow 26$$

$$-i\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)^7}dx$$

$$\downarrow 4153$$

$$-\frac{i\int \frac{i\coth^7(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\coth^7(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\downarrow 354$$

$$\frac{\int \frac{\coth^4(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{2d}$$

$$\downarrow 99$$

$$\frac{\int \left(a^3\coth^4(c+dx)+a^2(a+3b)\coth^3(c+dx)+a\left(a^2+3ba+3b^2\right)\coth^2(c+dx)+(a+b)^3\coth(c+dx)-\frac{(}{\tanh}\right.}{2d}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{3}a^3\coth^3(c+dx)-a\left(a^2+3ab+3b^2\right)\coth(c+dx)-\frac{1}{2}a^2(a+3b)\coth^2(c+dx)+(a+b)^3\log\left(\tanh^2(c+dx)\right)}{2d}$$

input
```
Int[Coth[c + d*x]^7*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-(a*(a^2 + 3*a*b + 3*b^2)*Coth[c + d*x]) - (a^2*(a + 3*b)*Coth[c + d*x]^2
)/2 - (a^3*Coth[c + d*x]^3)/3 + (a + b)^3*Log[Tanh[c + d*x]^2] - (a + b)^3
*Log[1 - Tanh[c + d*x]^2])/(2*d)
```

3.167.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.167.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.03

method	result
parallelrisch	$\frac{-12(a+b)^3\ln(1-\tanh(dx+c))+12(a+b)^3\ln(\tanh(dx+c))-2\coth(dx+c)^6a^3-3a^2\coth(dx+c)^4(a+3b)+(-6a^3-18a^2b-$
derivativedivides	$-\frac{(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}ab^2+\frac{1}{2}b^3)\ln(\tanh(dx+c)+1)+(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}ab^2+\frac{1}{2}b^3)\ln(\tanh(dx+c)-1)+(-a^3-3a^2b-3ab^2-b^3$
default	$-\frac{(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}ab^2+\frac{1}{2}b^3)\ln(\tanh(dx+c)+1)+(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}ab^2+\frac{1}{2}b^3)\ln(\tanh(dx+c)-1)+(-a^3-3a^2b-3ab^2-b^3$
risch	$-a^3x-3ba^2x-3ab^2x-b^3x-\frac{2a^3c}{d}-\frac{6bca^2}{d}-\frac{6ab^2c}{d}-\frac{2b^3c}{d}-\frac{2a\,e^{2dx+2c}(9a^2e^{8dx+8c}+18ab\,e^{8dx}$

input
```
int(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*(-12*(a+b)^3*ln(1-tanh(d*x+c))+12*(a+b)^3*ln(tanh(d*x+c))-2*coth(d*x+
c)^6*a^3-3*a^2*coth(d*x+c)^4*(a+3*b)+(-6*a^3-18*a^2*b-18*a*b^2)*coth(d*x+c
)^2-12*d*x*(a+b)^3)/d
```

3.167.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4305 vs. $2(97) = 194$.

Time = 0.31 (sec) , antiderivative size = 4305, normalized size of antiderivative = 41.80

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output

```
-1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^12 + 36*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^11 + 3*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*x*sinh(d*x + c)^12 + 18*(a^3 + 2*a^2*b + a*b^2 - (a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^10 + 18*(11*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 + a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x)*sinh(d*x + c)^10 + 60*(11*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*x*cosh(d*x + c)^3 + 3*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 - 9*(4*a^3 + 12*a^2
*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^8 + 9*
(165*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^4 - 4*a^3 - 12*a^2*
b - 8*a*b^2 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 90*(a^3 + 2*a^2*b +
a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c
)^8 + 72*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^5 + 30*(a^3
 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3
- (4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cos
h(d*x + c))*sinh(d*x + c)^7 + 4*(17*a^3 + 36*a^2*b + 27*a*b^2 - 15*(a^3 +
3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 + 4*(693*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*d*x*cosh(d*x + c)^6 + 945*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 17*a^3 + 36*a^2*b + 27*a*b^2
- 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 63*(4*a^3 + 12*a^2*b + 8*a*b...
```

3.167.6 Sympy [F(-1)]

Timed out.

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Timed out}$$

input

```
integrate(coth(d*x+c)**7*(a+b*tanh(d*x+c)**2)**3,x)
```

output

```
Timed out
```

3.167.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(97) = 194$.

Time = 0.21 (sec) , antiderivative size = 420, normalized size of antiderivative = 4.08

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{1}{3}a^3\left(3x+\frac{3c}{d}+\frac{3\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{3\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(9e^{(-2dx-2c)}-18e^{(-4dx-4c)}+\right.}{d(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6dx}}\right.$$

$$+3a^2b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2dx-2c)}-e^{(-4dx-4c)}+e^{(-6dx}\right.}{d(4e^{(-2dx-2c)}-6e^{(-4dx-4c)}+4e^{(-6dx-6c)}-}\right.$$

$$+3ab^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)}-e^{(-4dx-4c)}-1)}\right)$$

$$+\frac{b^3\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}$$

input
```
integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/3*a^3*(3*x + 3*c/d + 3*log(e^(-d*x - c) + 1)/d + 3*log(e^(-d*x - c) - 1)
/d + 2*(9*e^(-2*d*x - 2*c) - 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) - 1
8*e^(-8*d*x - 8*c) + 9*e^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(
-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x -
 10*c) - e^(-12*d*x - 12*c) - 1))) + 3*a^2*b*(x + c/d + log(e^(-d*x - c) +
 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) +
 e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d
*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 3*a*b^2*(x + c/d + log(e^(-d*x - c)
+ 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*
c) - e^(-4*d*x - 4*c) - 1))) + b^3*log(e^(d*x + c) - e^(-d*x - c))/d
```

3.167.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(97) = 194$.

Time = 0.62 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.11

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$

$$-\frac{3\left(a^3+3a^2b+3ab^2+b^3\right)(dx+c)-3\left(a^3+3a^2b+3ab^2+b^3\right)\log\left(\left|e^{(2dx+2c)}-1\right|\right)+\frac{2\left(9\left(a^3+2a^2b+ab^2\right)e\right.}{}}{}$$

input
```
integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) - 3*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*log(abs(e^(2*d*x + 2*c) - 1)) + 2*(9*(a^3 + 2*a^2*b + a*b^2)*e
^(10*d*x + 10*c) - 18*(a^3 + 3*a^2*b + 2*a*b^2)*e^(8*d*x + 8*c) + 2*(17*a^
3 + 36*a^2*b + 27*a*b^2)*e^(6*d*x + 6*c) - 18*(a^3 + 3*a^2*b + 2*a*b^2)*e^
(4*d*x + 4*c) + 9*(a^3 + 2*a^2*b + a*b^2)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c
) - 1)^6)/d
```

3.167.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 380, normalized size of antiderivative = 3.69

$$
\begin{aligned}
&\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx \\
&= \frac{\ln\left(\mathrm{e}^{2c}\,\mathrm{e}^{2dx}-1\right)\,\left(a^3+3a^2b+3ab^2+b^3\right)}{d} \\
&\quad - \frac{4\left(11a^3+3ba^2\right)}{d\left(6\,\mathrm{e}^{4c+4dx}-4\,\mathrm{e}^{2c+2dx}-4\,\mathrm{e}^{6c+6dx}+\mathrm{e}^{8c+8dx}+1\right)} \\
&\quad - \frac{32a^3}{3d\left(15\,\mathrm{e}^{4c+4dx}-6\,\mathrm{e}^{2c+2dx}-20\,\mathrm{e}^{6c+6dx}+15\,\mathrm{e}^{8c+8dx}-6\,\mathrm{e}^{10c+10dx}+\mathrm{e}^{12c+12dx}+1\right)} \\
&\quad - \frac{6\left(3a^3+4a^2b+ab^2\right)}{d\left(\mathrm{e}^{4c+4dx}-2\,\mathrm{e}^{2c+2dx}+1\right)} - \frac{6\left(a^3+2a^2b+ab^2\right)}{d\left(\mathrm{e}^{2c+2dx}-1\right)} \\
&\quad - \frac{8\left(13a^3+9ba^2\right)}{3d\left(3\,\mathrm{e}^{2c+2dx}-3\,\mathrm{e}^{4c+4dx}+\mathrm{e}^{6c+6dx}-1\right)} \\
&\quad - \frac{32a^3}{d\left(5\,\mathrm{e}^{2c+2dx}-10\,\mathrm{e}^{4c+4dx}+10\,\mathrm{e}^{6c+6dx}-5\,\mathrm{e}^{8c+8dx}+\mathrm{e}^{10c+10dx}-1\right)} - x\left(a+b\right)^3
\end{aligned}
$$

input
```
int(coth(c + d*x)^7*(a + b*tanh(c + d*x)^2)^3,x)
```

output

```
(log(exp(2*c)*exp(2*d*x) - 1)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/d - (4*(3*a
^2*b + 11*a^3))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c +
6*d*x) + exp(8*c + 8*d*x) + 1)) - (32*a^3)/(3*d*(15*exp(4*c + 4*d*x) - 6*e
xp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c +
 10*d*x) + exp(12*c + 12*d*x) + 1)) - (6*(a*b^2 + 4*a^2*b + 3*a^3))/(d*(ex
p(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (6*(a*b^2 + 2*a^2*b + a^3))/(d
*(exp(2*c + 2*d*x) - 1)) - (8*(9*a^2*b + 13*a^3))/(3*d*(3*exp(2*c + 2*d*x)
 - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (32*a^3)/(d*(5*exp(2*c +
2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) +
exp(10*c + 10*d*x) - 1)) - x*(a + b)^3
```

3.168 $\int \left(a + b\tanh^2(c+dx)\right)^4 dx$

3.168.1 Optimal result 1248
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3.168.1 Optimal result

Integrand size = 14, antiderivative size = 110

$$\int \left(a + b\tanh^2(c+dx)\right)^4 dx = (a+b)^4 x - \frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d} - \frac{b^2(6a^2+4ab+b^2)\tanh^3(c+dx)}{3d} - \frac{b^3(4a+b)\tanh^5(c+dx)}{5d} - \frac{b^4\tanh^7(c+dx)}{7d}$$

output
```
(a+b)^4*x-b*(2*a+b)*(2*a^2+2*a*b+b^2)*tanh(d*x+c)/d-1/3*b^2*(6*a^2+4*a*b+b
^2)*tanh(d*x+c)^3/d-1/5*b^3*(4*a+b)*tanh(d*x+c)^5/d-1/7*b^4*tanh(d*x+c)^7/
d
```

3.168.2 Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.16

$$\int \left(a + b\tanh^2(c+dx)\right)^4 dx = \frac{\tanh(c+dx)\left(\frac{105(a+b)^4\operatorname{arctanh}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}} - b\left(105(4a^3+6a^2b+4ab^2+b^3)+35b(6a^2+4ab+b^2)\tan\right.\right.}{105d}$$

input
```
Integrate[(a + b*Tanh[c + d*x]^2)^4,x]
```

output
```
(Tanh[c + d*x]*((105*(a + b)^4*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c
 + d*x]^2] - b*(105*(4*a^3 + 6*a^2*b + 4*a*b^2 + b^3) + 35*b*(6*a^2 + 4*a*
b + b^2)*Tanh[c + d*x]^2 + 21*b^2*(4*a + b)*Tanh[c + d*x]^4 + 15*b^3*Tanh[
c + d*x]^6)))/(105*d)
```

3.168.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4144, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a + b\tanh^2(c+dx)\right)^4 dx$$

$$\downarrow 3042$$

$$\int \left(a - b\tan(ic+idx)^2\right)^4 dx$$

$$\downarrow 4144$$

$$\frac{\int \frac{\left(b\tanh^2(c+dx)+a\right)^4}{1-\tanh^2(c+dx)} d\tanh(c+dx)}{d}$$

$$\downarrow 300$$

$$\frac{\int \left(-b^4\tanh^6(c+dx) - b^3(4a+b)\tanh^4(c+dx) - b^2\left(6a^2+4ba+b^2\right)\tanh^2(c+dx) - b(2a+b)\left(2a^2+2ba+b^2\right.\right.}{d}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{3}b^2\left(6a^2+4ab+b^2\right)\tanh^3(c+dx) - b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx) + (a+b)^4\text{arctanh}(\tanh(c+dx))}{d}$$

input
```
Int[(a + b*Tanh[c + d*x]^2)^4,x]
```

output
```
((a + b)^4*ArcTanh[Tanh[c + d*x]] - b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tanh
[c + d*x] - (b^2*(6*a^2 + 4*a*b + b^2)*Tanh[c + d*x]^3)/3 - (b^3*(4*a + b)
*Tanh[c + d*x]^5)/5 - (b^4*Tanh[c + d*x]^7)/7)/d
```

3.168.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)  Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.168.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.63

method	result
parallelrisch	$-\frac{15b^4\tanh(dx+c)^7+84ab^3\tanh(dx+c)^5+21b^4\tanh(dx+c)^5+210a^2b^2\tanh(dx+c)^3+140ab^3\tanh(dx+c)^3+35b^4\tanh$
derivativedivides	$\frac{-4a^3b\tanh(dx+c)-6a^2b^2\tanh(dx+c)-4ab^3\tanh(dx+c)-\frac{4ab^3\tanh(dx+c)^5}{5}-2a^2b^2\tanh(dx+c)^3-\frac{4ab^3\tanh(dx+c)^3}{3}-$
default	$\frac{-4a^3b\tanh(dx+c)-6a^2b^2\tanh(dx+c)-4ab^3\tanh(dx+c)-\frac{4ab^3\tanh(dx+c)^5}{5}-2a^2b^2\tanh(dx+c)^3-\frac{4ab^3\tanh(dx+c)^3}{3}-$
parts	$xa^4+\frac{b^4\left(-\frac{\tanh(dx+c)^7}{7}-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{4ab^3($
risch	$xa^4+4ba^3x+6a^2b^2x+4ab^3x+b^4x+\frac{8b(161ab^2+105a^3+315a^2be^{12dx+12c}+1575a^2be^{10dx+10c}+315$

input
```
int((a+b*tanh(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/105*(15*b^4*tanh(d*x+c)^7+84*a*b^3*tanh(d*x+c)^5+21*b^4*tanh(d*x+c)^5+2
10*a^2*b^2*tanh(d*x+c)^3+140*a*b^3*tanh(d*x+c)^3+35*b^4*tanh(d*x+c)^3-105*
a^4*d*x-420*a^3*b*d*x-630*a^2*b^2*d*x-420*a*b^3*d*x-105*b^4*d*x+420*a^3*b*
tanh(d*x+c)+630*a^2*b^2*tanh(d*x+c)+420*a*b^3*tanh(d*x+c)+105*b^4*tanh(d*x
+c))/d
```

3.168.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1176 vs. $2(104) = 208$.

Time $= 0.28$ (sec) , antiderivative size $= 1176$, normalized size of antiderivative $= 10.69$

$$\int \left(a + b\tanh^2(c + dx)\right)^4 dx = \text{Too large to display}$$

input
```
integrate((a+b*tanh(d*x+c)^2)^4,x, algorithm="fricas")
```

output
```
1/105*((420*a^3*b + 840*a^2*b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b
 + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^7 + 7*(420*a^3*b + 840*a^
2*b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4)*d*x)*cosh(d*x + c)*sinh(d*x + c)^6 - 4*(105*a^3*b + 210*a^2*b^2 + 161*
a*b^3 + 44*b^4)*sinh(d*x + c)^7 + 7*(420*a^3*b + 840*a^2*b^2 + 644*a*b^3 +
 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x +
 c)^5 - 28*(75*a^3*b + 120*a^2*b^2 + 71*a*b^3 + 14*b^4 + 3*(105*a^3*b + 21
0*a^2*b^2 + 161*a*b^3 + 44*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 35*((42
0*a^3*b + 840*a^2*b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b
^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^3 + (420*a^3*b + 840*a^2*b^2 + 644*
a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cos
h(d*x + c))*sinh(d*x + c)^4 + 21*(420*a^3*b + 840*a^2*b^2 + 644*a*b^3 + 17
6*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)
^3 - 28*(5*(105*a^3*b + 210*a^2*b^2 + 161*a*b^3 + 44*b^4)*cosh(d*x + c)^4
+ 135*a^3*b + 180*a^2*b^2 + 123*a*b^3 + 42*b^4 + 10*(75*a^3*b + 120*a^2*b^
2 + 71*a*b^3 + 14*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 7*(3*(420*a^3*b
+ 840*a^2*b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a
*b^3 + b^4)*d*x)*cosh(d*x + c)^5 + 10*(420*a^3*b + 840*a^2*b^2 + 644*a*b^3
 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x
 + c)^3 + 9*(420*a^3*b + 840*a^2*b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 +...
```

3.168.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(99) = 198$.

Time = 0.23 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.90

$$\int \left(a + b\tanh^2(c+dx)\right)^4 dx$$

$$= \begin{cases} a^4x + 4a^3bx - \frac{4a^3b\tanh(c+dx)}{d} + 6a^2b^2x - \frac{2a^2b^2\tanh^3(c+dx)}{d} - \frac{6a^2b^2\tanh(c+dx)}{d} + 4ab^3x - \frac{4ab^3\tanh^5(c+dx)}{5d} - \frac{4ab}{} \\ x\left(a + b\tanh^2(c)\right)^4 \end{cases}$$

input
```
integrate((a+b*tanh(d*x+c)**2)**4,x)
```

output
```
Piecewise((a**4*x + 4*a**3*b*x - 4*a**3*b*tanh(c + d*x)/d + 6*a**2*b**2*x
- 2*a**2*b**2*tanh(c + d*x)**3/d - 6*a**2*b**2*tanh(c + d*x)/d + 4*a*b**3*
x - 4*a*b**3*tanh(c + d*x)**5/(5*d) - 4*a*b**3*tanh(c + d*x)**3/(3*d) - 4*
a*b**3*tanh(c + d*x)/d + b**4*x - b**4*tanh(c + d*x)**7/(7*d) - b**4*tanh(
c + d*x)**5/(5*d) - b**4*tanh(c + d*x)**3/(3*d) - b**4*tanh(c + d*x)/d, Ne
(d, 0)), (x*(a + b*tanh(c)**2)**4, True))
```

3.168.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(104) = 208$.

Time = 0.21 (sec) , antiderivative size = 410, normalized size of antiderivative = 3.73

$$\int \left(a + b\tanh^2(c+dx)\right)^4 dx$$

$$= \frac{1}{105}b^4\left(105x + \frac{105c}{d} - \frac{8\left(203e^{(-2dx-2c)} + 609e^{(-4dx-4c)} + 770e^{(-6dx-6c)} + 770e^{(-8dx-8c)} + 315e^{(-1}{d(7e^{(-2dx-2c)} + 21e^{(-4dx-4c)} + 35e^{(-6dx-6c)} + 35e^{(-8dx-8c)} + 21e^{(-10dx-10c)} +}\right.$$

$$+ \frac{4}{15}ab^3\left(15x + \frac{15c}{d} - \frac{2\left(70e^{(-2dx-2c)} + 140e^{(-4dx-4c)} + 90e^{(-6dx-6c)} + 45e^{(-8dx-8c)} + 23\right)}{d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)}\right.$$

$$+ 2a^2b^2\left(3x + \frac{3c}{d} - \frac{4\left(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + 2\right)}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)}\right)$$

$$+ 4a^3b\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2dx-2c)} + 1)}\right) + a^4x$$

input
```
integrate((a+b*tanh(d*x+c)^2)^4,x, algorithm="maxima")
```

output
```
1/105*b^4*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d*x - 4*c
) + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x - 10*c) +
 105*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
 + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e
^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 4/15*a*b^3*(15*x + 15*c/d
- 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c) + 45
*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*
e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 2*a^2*
b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e
^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 4*a^3*b*(
x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^4*x
```

3.168.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(104) = 208$.

Time $= 0.31$ (sec) , antiderivative size $= 447$, normalized size of antiderivative $= 4.06$

$$\int \left(a + b\tanh^2(c+dx)\right)^4 dx$$

$$= \frac{105\,(a^4 + 4\,a^3 b + 6\,a^2 b^2 + 4\,ab^3 + b^4)(dx + c) + \frac{8\,(105\,a^3 b e^{(12\,dx+12\,c)} + 315\,a^2 b^2 e^{(12\,dx+12\,c)} + 315\,ab^3 e^{(12\,dx+12\,c)} + 105\,b^4 e^{(12}}{}$$

input
```
integrate((a+b*tanh(d*x+c)^2)^4,x, algorithm="giac")
```

output
```
1/105*(105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(d*x + c) + 8*(105*
a^3*b*e^(12*d*x + 12*c) + 315*a^2*b^2*e^(12*d*x + 12*c) + 315*a*b^3*e^(12*
d*x + 12*c) + 105*b^4*e^(12*d*x + 12*c) + 630*a^3*b*e^(10*d*x + 10*c) + 15
75*a^2*b^2*e^(10*d*x + 10*c) + 1260*a*b^3*e^(10*d*x + 10*c) + 315*b^4*e^(1
0*d*x + 10*c) + 1575*a^3*b*e^(8*d*x + 8*c) + 3360*a^2*b^2*e^(8*d*x + 8*c)
+ 2555*a*b^3*e^(8*d*x + 8*c) + 770*b^4*e^(8*d*x + 8*c) + 2100*a^3*b*e^(6*d
*x + 6*c) + 3990*a^2*b^2*e^(6*d*x + 6*c) + 3080*a*b^3*e^(6*d*x + 6*c) + 77
0*b^4*e^(6*d*x + 6*c) + 1575*a^3*b*e^(4*d*x + 4*c) + 2835*a^2*b^2*e^(4*d*x
 + 4*c) + 2121*a*b^3*e^(4*d*x + 4*c) + 609*b^4*e^(4*d*x + 4*c) + 630*a^3*b
*e^(2*d*x + 2*c) + 1155*a^2*b^2*e^(2*d*x + 2*c) + 812*a*b^3*e^(2*d*x + 2*c
) + 203*b^4*e^(2*d*x + 2*c) + 105*a^3*b + 210*a^2*b^2 + 161*a*b^3 + 44*b^4
)/(e^(2*d*x + 2*c) + 1)^7)/d
```

3.168.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.21

$$\int \left(a+b\tanh^2(c+dx)\right)^4\,dx = x\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right) - \frac{\tanh(c+dx)^3\left(6a^2b^2+4ab^3+b^4\right)}{3d} - \frac{\tanh(c+dx)^5\left(b^4+4ab^3\right)}{5d} - \frac{b^4\tanh(c+dx)^7}{7d} - \frac{b\tanh(c+dx)\left(4a^3+6a^2b+4ab^2+b^3\right)}{d}$$

input
```
int((a + b*tanh(c + d*x)^2)^4,x)
```

output
```
x*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2) - (tanh(c + d*x)^3*(4*a*b^3
+ b^4 + 6*a^2*b^2))/(3*d) - (tanh(c + d*x)^5*(4*a*b^3 + b^4))/(5*d) - (b^4
*tanh(c + d*x)^7)/(7*d) - (b*tanh(c + d*x)*(4*a*b^2 + 6*a^2*b + 4*a^3 + b^
3))/d
```

3.169 $\int \left(a + b\tanh^2(c+dx)\right)^5 \, dx$

3.169.1 Optimal result 1255
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3.169.1 Optimal result

Integrand size = 14, antiderivative size = 160

$$\begin{aligned}\int \left(a+b\tanh^2(c+dx)\right)^5 \, dx = (a+b)^5 x &- \frac{b(5a^4+10a^3b+10a^2b^2+5ab^3+b^4)\tanh(c+dx)}{d} \\ &- \frac{b^2(10a^3+10a^2b+5ab^2+b^3)\tanh^3(c+dx)}{3d} \\ &- \frac{b^3(10a^2+5ab+b^2)\tanh^5(c+dx)}{5d} \\ &- \frac{b^4(5a+b)\tanh^7(c+dx)}{7d} - \frac{b^5\tanh^9(c+dx)}{9d}\end{aligned}$$

output
```
(a+b)^5*x-b*(5*a^4+10*a^3*b+10*a^2*b^2+5*a*b^3+b^4)*tanh(d*x+c)/d-1/3*b^2*
(10*a^3+10*a^2*b+5*a*b^2+b^3)*tanh(d*x+c)^3/d-1/5*b^3*(10*a^2+5*a*b+b^2)*t
anh(d*x+c)^5/d-1/7*b^4*(5*a+b)*tanh(d*x+c)^7/d-1/9*b^5*tanh(d*x+c)^9/d
```

3.169.2 Mathematica [A] (verified)

Time = 2.72 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int \left(a + b\tanh^2(c+dx)\right)^5 \, dx$$

$$= \frac{\tanh(c+dx)\left(\frac{315(a+b)^5\operatorname{arctanh}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}} - b\left(315(5a^4+10a^3b+10a^2b^2+5ab^3+b^4)+105b(10a^3+\right.}{}$$

input
```
Integrate[(a + b*Tanh[c + d*x]^2)^5,x]
```

output
```
(Tanh[c + d*x]*((315*(a + b)^5*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c
 + d*x]^2] - b*(315*(5*a^4 + 10*a^3*b + 10*a^2*b^2 + 5*a*b^3 + b^4) + 105*
b*(10*a^3 + 10*a^2*b + 5*a*b^2 + b^3)*Tanh[c + d*x]^2 + 63*b^2*(10*a^2 + 5
*a*b + b^2)*Tanh[c + d*x]^4 + 45*b^3*(5*a + b)*Tanh[c + d*x]^6 + 35*b^4*Ta
nh[c + d*x]^8)))/(315*d)
```

3.169.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4144, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a + b\tanh^2(c+dx)\right)^5 dx$$

$$\downarrow 3042$$

$$\int \left(a - b\tan(ic+idx)^2\right)^5 dx$$

$$\downarrow 4144$$

$$\frac{\int \frac{(b\tanh^2(c+dx)+a)^5}{1-\tanh^2(c+dx)} d\tanh(c+dx)}{d}$$

$$\downarrow 300$$

$$\frac{\int \left(-b^5\tanh^8(c+dx) - b^4(5a+b)\tanh^6(c+dx) - b^3\left(10a^2+5ba+b^2\right)\tanh^4(c+dx) - b^2\left(10a^3+10ba^2+5b^2a\right.\right.}{d}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{5}b^3\left(10a^2+5ab+b^2\right)\tanh^5(c+dx) - \frac{1}{3}b^2\left(10a^3+10a^2b+5ab^2+b^3\right)\tanh^3(c+dx) - b\left(5a^4+10a^3b+10a^2b^2\right.}{d}$$

input
```
Int[(a + b*Tanh[c + d*x]^2)^5,x]
```

output
```
((a + b)^5*ArcTanh[Tanh[c + d*x]] - b*(5*a^4 + 10*a^3*b + 10*a^2*b^2 + 5*a
*b^3 + b^4)*Tanh[c + d*x] - (b^2*(10*a^3 + 10*a^2*b + 5*a*b^2 + b^3)*Tanh[
c + d*x]^3)/3 - (b^3*(10*a^2 + 5*a*b + b^2)*Tanh[c + d*x]^5)/5 - (b^4*(5*a
 + b)*Tanh[c + d*x]^7)/7 - (b^5*Tanh[c + d*x]^9)/9)/d
```

3.169.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.169.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.64

method	result
parallelrisch	$-\frac{3150a^3b^2\tanh(dx+c)+1575a\,b^4\tanh(dx+c)+525a\,b^4\tanh(dx+c)^3+3150a^2b^3\tanh(dx+c)+225a\,b^4\tanh(dx+c)^7+63}{}$
derivativedivides	$\frac{-5a^4b\tanh(dx+c)-10a^3b^2\tanh(dx+c)-10a^2b^3\tanh(dx+c)-5a\,b^4\tanh(dx+c)-\frac{5a\,b^4\tanh(dx+c)^7}{7}-b^5\tanh(dx+c)-\frac{b^5}{}}{}$
default	$\frac{-5a^4b\tanh(dx+c)-10a^3b^2\tanh(dx+c)-10a^2b^3\tanh(dx+c)-5a\,b^4\tanh(dx+c)-\frac{5a\,b^4\tanh(dx+c)^7}{7}-b^5\tanh(dx+c)-\frac{b^5}{}}{}$
parts	$a^5x+\frac{b^5\left(-\frac{\tanh(dx+c)^9}{9}-\frac{\tanh(dx+c)^7}{7}-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1}{2}\right.}{d}$
risch	$a^5x+5b\,a^4x+10a^3b^2x+10b^3a^2x+5a\,b^4x+b^5x+\frac{2b(31500a^3b\,\mathrm{e}^{2dx+2c}+34020a^2b^2\mathrm{e}^{2dx+2c}+1746}{}$

input
```
int((a+b*tanh(d*x+c)^2)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/315*(3150*a^3*b^2*tanh(d*x+c)+1575*a*b^4*tanh(d*x+c)+525*a*b^4*tanh(d*x
+c)^3+3150*a^2*b^3*tanh(d*x+c)+225*a*b^4*tanh(d*x+c)^7+630*a^2*b^3*tanh(d*
x+c)^5+315*a*b^4*tanh(d*x+c)^5+1050*a^3*b^2*tanh(d*x+c)^3-3150*a^2*b^3*d*x
-315*a^5*d*x-315*b^5*d*x+63*b^5*tanh(d*x+c)^5+105*b^5*tanh(d*x+c)^3+45*b^5
*tanh(d*x+c)^7+315*b^5*tanh(d*x+c)+35*b^5*tanh(d*x+c)^9-1575*a^4*b*d*x-315
0*a^3*b^2*d*x-1575*a*b^4*d*x+1050*a^2*b^3*tanh(d*x+c)^3+1575*a^4*b*tanh(d*
x+c))/d
```

3.169.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2133 vs. $2(152) = 304$.

Time $= 0.30$ (sec) , antiderivative size $= 2133$, normalized size of antiderivative $= 13.33$

$$\int \left(a + b\tanh^2(c + dx)\right)^5 \, dx = \text{Too large to display}$$

input
```
integrate((a+b*tanh(d*x+c)^2)^5,x, algorithm="fricas")
```

output

```
1/315*((1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 +
315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*
x + c)^9 + 9*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*
b^5 + 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*c
osh(d*x + c)*sinh(d*x + c)^8 - (1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 +
 2640*a*b^4 + 563*b^5)*sinh(d*x + c)^9 + 9*(1575*a^4*b + 4200*a^3*b^2 + 48
30*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a
^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x + c)^7 - 9*(1225*a^4*b + 2800*a^3*b^
2 + 2730*a^2*b^3 + 1240*a*b^4 + 213*b^5 + 4*(1575*a^4*b + 4200*a^3*b^2 + 4
830*a^2*b^3 + 2640*a*b^4 + 563*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 21*
(4*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*
(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x +
c)^3 + 3*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5
+ 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(
d*x + c))*sinh(d*x + c)^6 + 36*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 +
 2640*a*b^4 + 563*b^5 + 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a
*b^4 + b^5)*d*x)*cosh(d*x + c)^5 - 9*(3500*a^4*b + 7000*a^3*b^2 + 6720*a^2
*b^3 + 3560*a*b^4 + 852*b^5 + 14*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3
 + 2640*a*b^4 + 563*b^5)*cosh(d*x + c)^4 + 21*(1225*a^4*b + 2800*a^3*b^2 +
 2730*a^2*b^3 + 1240*a*b^4 + 213*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^5 ...
```

3.169.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(148) = 296$.

Time $= 0.32$ (sec) , antiderivative size $= 308$, normalized size of antiderivative $= 1.92$

$$\int \left(a + b\tanh^2(c + dx)\right)^5 \, dx$$

$$= \begin{cases} a^5 x + 5a^4 bx - \frac{5a^4 b\tanh(c+dx)}{d} + 10a^3 b^2 x - \frac{10a^3 b^2 \tanh^3(c+dx)}{3d} - \frac{10a^3 b^2 \tanh(c+dx)}{d} + 10a^2 b^3 x - \frac{2a^2 b^3 \tanh^5(c+dx)}{d} \\ x\left(a + b\tanh^2(c)\right)^5 \end{cases}$$

input

```
integrate((a+b*tanh(d*x+c)**2)**5,x)
```

output

```
Piecewise((a**5*x + 5*a**4*b*x - 5*a**4*b*tanh(c + d*x)/d + 10*a**3*b**2*x
 - 10*a**3*b**2*tanh(c + d*x)**3/(3*d) - 10*a**3*b**2*tanh(c + d*x)/d + 10
*a**2*b**3*x - 2*a**2*b**3*tanh(c + d*x)**5/d - 10*a**2*b**3*tanh(c + d*x)
**3/(3*d) - 10*a**2*b**3*tanh(c + d*x)/d + 5*a*b**4*x - 5*a*b**4*tanh(c +
d*x)**7/(7*d) - a*b**4*tanh(c + d*x)**5/d - 5*a*b**4*tanh(c + d*x)**3/(3*d
) - 5*a*b**4*tanh(c + d*x)/d + b**5*x - b**5*tanh(c + d*x)**9/(9*d) - b**5
*tanh(c + d*x)**7/(7*d) - b**5*tanh(c + d*x)**5/(5*d) - b**5*tanh(c + d*x)
**3/(3*d) - b**5*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**5, Tru
e))
```

3.169.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(152) = 304$.

Time $= 0.21$ (sec) , antiderivative size $= 624$, normalized size of antiderivative $= 3.90$

$$\int \left(a + b\tanh^2(c+dx)\right)^5 dx$$

$$= \frac{1}{315} b^5 \left(315\,x + \frac{315\,c}{d} - \frac{2\left(3492\,e^{(-2\,dx-2\,c)} + 13968\,e^{(-4\,dx-4\,c)} + 26292\,e^{(-6\,dx-6\,c)} + 39438\,e^{(-8\,dx-8\,c)} + 31\right.}{d(9\,e^{(-2\,dx-2\,c)} + 36\,e^{(-4\,dx-4\,c)} + 84\,e^{(-6\,dx-6\,c)} + 126\,e^{(-8\,dx-8\,c)} + 126\,e^{(-10}\right.$$

$$+ \frac{1}{21} ab^4 \left(105\,x + \frac{105\,c}{d} - \frac{8\left(203\,e^{(-2\,dx-2\,c)} + 609\,e^{(-4\,dx-4\,c)} + 770\,e^{(-6\,dx-6\,c)} + 770\,e^{(-8\,dx-8\,c)} + 315\,e^{(}\right.}{d(7\,e^{(-2\,dx-2\,c)} + 21\,e^{(-4\,dx-4\,c)} + 35\,e^{(-6\,dx-6\,c)} + 35\,e^{(-8\,dx-8\,c)} + 21\,e^{(-10\,dx-10\,c)}\right.$$

$$+ \frac{2}{3} a^2b^3 \left(15\,x + \frac{15\,c}{d} - \frac{2\left(70\,e^{(-2\,dx-2\,c)} + 140\,e^{(-4\,dx-4\,c)} + 90\,e^{(-6\,dx-6\,c)} + 45\,e^{(-8\,dx-8\,c)} + 23\right)}{d(5\,e^{(-2\,dx-2\,c)} + 10\,e^{(-4\,dx-4\,c)} + 10\,e^{(-6\,dx-6\,c)} + 5\,e^{(-8\,dx-8\,c)} + e^{(-10\,dx-10\,c)} + 1)}\right.$$

$$+ \frac{10}{3} a^3b^2 \left(3\,x + \frac{3\,c}{d} - \frac{4\left(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + 2\right)}{d(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} + 1)}\right)$$

$$+ 5\,a^4b \left(x + \frac{c}{d} - \frac{2}{d(e^{(-2\,dx-2\,c)} + 1)}\right) + a^5x$$

input

```
integrate((a+b*tanh(d*x+c)^2)^5,x, algorithm="maxima")
```

output
```
1/315*b^5*(315*x + 315*c/d - 2*(3492*e^(-2*d*x - 2*c) + 13968*e^(-4*d*x -
4*c) + 26292*e^(-6*d*x - 6*c) + 39438*e^(-8*d*x - 8*c) + 31500*e^(-10*d*x
- 10*c) + 21000*e^(-12*d*x - 12*c) + 6300*e^(-14*d*x - 14*c) + 1575*e^(-16
*d*x - 16*c) + 563)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-
6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d
*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 1
8*c) + 1))) + 1/21*a*b^4*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*
e^(-4*d*x - 4*c) + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-1
0*d*x - 10*c) + 105*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e
^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*
x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 2/3*a^2*b^3
*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6
*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*
d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c)
 + 1))) + 10/3*a^3*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x
- 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c)
 + 1))) + 5*a^4*b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^5*x
```

3.169.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(152) = 304$.

Time $= 0.36$ (sec) , antiderivative size $= 721$, normalized size of antiderivative $= 4.51$

$$\int \left(a + b\tanh^2(c + dx)\right)^5 \, dx$$

$$= \frac{315\,(a^5 + 5\,a^4 b + 10\,a^3 b^2 + 10\,a^2 b^3 + 5\,a b^4 + b^5)(dx + c) + \frac{2\,(1575\,a^4 b e^{(16\,dx + 16\,c)} + 6300\,a^3 b^2 e^{(16\,dx + 16\,c)} + 9450\,a^2 b^3 e^{(16}}{}$$

input
```
integrate((a+b*tanh(d*x+c)^2)^5,x, algorithm="giac")
```

output
```
1/315*(315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*(d*x
+ c) + 2*(1575*a^4*b*e^(16*d*x + 16*c) + 6300*a^3*b^2*e^(16*d*x + 16*c) +
9450*a^2*b^3*e^(16*d*x + 16*c) + 6300*a*b^4*e^(16*d*x + 16*c) + 1575*b^5*e
^(16*d*x + 16*c) + 12600*a^4*b*e^(14*d*x + 14*c) + 44100*a^3*b^2*e^(14*d*x
 + 14*c) + 56700*a^2*b^3*e^(14*d*x + 14*c) + 31500*a*b^4*e^(14*d*x + 14*c)
 + 6300*b^5*e^(14*d*x + 14*c) + 44100*a^4*b*e^(12*d*x + 12*c) + 136500*a^3
*b^2*e^(12*d*x + 12*c) + 161700*a^2*b^3*e^(12*d*x + 12*c) + 90300*a*b^4*e^
(12*d*x + 12*c) + 21000*b^5*e^(12*d*x + 12*c) + 88200*a^4*b*e^(10*d*x + 10
*c) + 245700*a^3*b^2*e^(10*d*x + 10*c) + 283500*a^2*b^3*e^(10*d*x + 10*c)
+ 157500*a*b^4*e^(10*d*x + 10*c) + 31500*b^5*e^(10*d*x + 10*c) + 110250*a^
4*b*e^(8*d*x + 8*c) + 283500*a^3*b^2*e^(8*d*x + 8*c) + 325080*a^2*b^3*e^(8
*d*x + 8*c) + 175140*a*b^4*e^(8*d*x + 8*c) + 39438*b^5*e^(8*d*x + 8*c) + 8
8200*a^4*b*e^(6*d*x + 6*c) + 216300*a^3*b^2*e^(6*d*x + 6*c) + 244020*a^2*b
^3*e^(6*d*x + 6*c) + 131460*a*b^4*e^(6*d*x + 6*c) + 26292*b^5*e^(6*d*x + 6
*c) + 44100*a^4*b*e^(4*d*x + 4*c) + 107100*a^3*b^2*e^(4*d*x + 4*c) + 11718
0*a^2*b^3*e^(4*d*x + 4*c) + 63540*a*b^4*e^(4*d*x + 4*c) + 13968*b^5*e^(4*d
*x + 4*c) + 12600*a^4*b*e^(2*d*x + 2*c) + 31500*a^3*b^2*e^(2*d*x + 2*c) +
34020*a^2*b^3*e^(2*d*x + 2*c) + 17460*a*b^4*e^(2*d*x + 2*c) + 3492*b^5*e^(
2*d*x + 2*c) + 1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563
*b^5)/(e^(2*d*x + 2*c) + 1)^9)/d
```

3.169.9 Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.18

$$\int \left(a+b\tanh^2(c+dx)\right)^5\,dx = x\left(a^5+5\,a^4\,b+10\,a^3\,b^2+10\,a^2\,b^3+5\,a\,b^4+b^5\right) - \frac{\tanh(c+d\,x)^3\left(10\,a^3\,b^2+10\,a^2\,b^3+5\,a\,b^4+b^5\right)}{3\,d} - \frac{\tanh(c+d\,x)^5\left(10\,a^2\,b^3+5\,a\,b^4+b^5\right)}{5\,d} - \frac{\tanh(c+d\,x)^7\left(b^5+5\,a\,b^4\right)}{7\,d} - \frac{b^5\tanh(c+d\,x)^9}{9\,d} - \frac{b\tanh(c+d\,x)\left(5\,a^4+10\,a^3\,b+10\,a^2\,b^2+5\,a\,b^3+b^4\right)}{d}$$

input
```
int((a + b*tanh(c + d*x)^2)^5,x)
```

output

```
x*(5*a*b^4 + 5*a^4*b + a^5 + b^5 + 10*a^2*b^3 + 10*a^3*b^2) - (tanh(c + d*
x)^3*(5*a*b^4 + b^5 + 10*a^2*b^3 + 10*a^3*b^2))/(3*d) - (tanh(c + d*x)^5*(
5*a*b^4 + b^5 + 10*a^2*b^3))/(5*d) - (tanh(c + d*x)^7*(5*a*b^4 + b^5))/(7*
d) - (b^5*tanh(c + d*x)^9)/(9*d) - (b*tanh(c + d*x)*(5*a*b^3 + 10*a^3*b +
5*a^4 + b^4 + 10*a^2*b^2))/d
```

3.170 $\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

3.170.1 Optimal result 1264
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3.170.3 Rubi [A] (verified) 1265
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3.170.1 Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)d} + \frac{a^2\log\left(a+b\tanh^2(c+dx)\right)}{2b^2(a+b)d} - \frac{\tanh^2(c+dx)}{2bd}$$

output
```
ln(cosh(d*x+c))/(a+b)/d+1/2*a^2*ln(a+b*tanh(d*x+c)^2)/b^2/(a+b)/d-1/2*tanh
(d*x+c)^2/b/d
```

3.170.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{-\frac{2\log(\cosh(c+dx))}{a+b} - \frac{a^2\log\left(a+b\tanh^2(c+dx)\right)}{b^2(a+b)} + \frac{\tanh^2(c+dx)}{b}}{2d}$$

input
```
Integrate[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2),x]
```

output
```
-1/2*((-2*Log[Cosh[c + d*x]])/(a + b) - (a^2*Log[a + b*Tanh[c + d*x]^2])/(
b^2*(a + b)) + Tanh[c + d*x]^2/b)/d
```

3.170.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\tan(ic+idx)^5}{a-b\tan(ic+idx)^2}dx$$

$$\downarrow 26$$

$$-i\int \frac{\tan(ic+idx)^5}{a-b\tan(ic+idx)^2}dx$$

$$\downarrow 4153$$

$$-\frac{i\int \frac{i\tanh^5(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\tanh^5(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh(c+dx)}{d}$$

$$\downarrow 354$$

$$\frac{\int \frac{\tanh^4(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh^2(c+dx)}{2d}$$

$$\downarrow 93$$

$$\frac{\int \left(\frac{a^2}{b(a+b)(b\tanh^2(c+dx)+a)}-\frac{1}{b}-\frac{1}{(a+b)(\tanh^2(c+dx)-1)}\right)d\tanh^2(c+dx)}{2d}$$

$$\downarrow 2009$$

$$\frac{\frac{a^2\log(a+b\tanh^2(c+dx))}{b^2(a+b)}-\frac{\log(1-\tanh^2(c+dx))}{a+b}-\frac{\tanh^2(c+dx)}{b}}{2d}$$

input
```
Int[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-(Log[1 - Tanh[c + d*x]^2]/(a + b)) + (a^2*Log[a + b*Tanh[c + d*x]^2])/(b
^2*(a + b)) - Tanh[c + d*x]^2/b)/(2*d)
```

3.170.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.170.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.20

method	result
parallelrisch	$-\frac{2b^2dx+\tanh(dx+c)^2ab+b^2\tanh(dx+c)^2+2\ln(1-\tanh(dx+c))b^2-a^2\ln\left(a+b\tanh(dx+c)^2\right)}{2b^2d(a+b)}$
derivativedivides	$\frac{-\frac{\tanh(dx+c)^2}{2b}+\frac{a^2\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)b^2}-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}-\frac{\ln(\tanh(dx+c)+1)}{2a+2b}}{d}$
default	$\frac{-\frac{\tanh(dx+c)^2}{2b}+\frac{a^2\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)b^2}-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}-\frac{\ln(\tanh(dx+c)+1)}{2a+2b}}{d}$
risch	$\frac{x}{a+b}-\frac{2a^2x}{b^2(a+b)}-\frac{2a^2c}{b^2d(a+b)}+\frac{2ax}{b^2}+\frac{2ac}{b^2d}-\frac{2x}{b}-\frac{2c}{bd}+\frac{2\,\mathrm{e}^{2dx+2c}}{bd\left(\mathrm{e}^{2dx+2c}+1\right)^2}+\frac{a^2\ln\left(\mathrm{e}^{4dx+4c}+\frac{2(a-b)\mathrm{e}^{2dx+2c}}{a+b}$

input
```
int(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*b^2*d*x+tanh(d*x+c)^2*a*b+b^2*tanh(d*x+c)^2+2*ln(1-tanh(d*x+c))*b^
2-a^2*ln(a+b*tanh(d*x+c)^2))/b^2/d/(a+b)
```

3.170.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 742 vs. $2(62) = 124$.

Time = 0.30 (sec) , antiderivative size = 742, normalized size of antiderivative = 11.24

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx =$$

$$-\frac{2\,b^2dx\cosh(dx+c)^4+8\,b^2dx\cosh(dx+c)\sinh(dx+c)^3+2\,b^2dx\sinh(dx+c)^4+2\,b^2dx+4\,(b^2dx-}{}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output

```
-1/2*(2*b^2*d*x*cosh(d*x + c)^4 + 8*b^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3
+ 2*b^2*d*x*sinh(d*x + c)^4 + 2*b^2*d*x + 4*(b^2*d*x - a*b - b^2)*cosh(d*x
 + c)^2 + 4*(3*b^2*d*x*cosh(d*x + c)^2 + b^2*d*x - a*b - b^2)*sinh(d*x + c
)^2 - (a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sin
h(d*x + c)^4 + 2*a^2*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2)*sin
h(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x
+ c))*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(c
osh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 2*((a
^2 - b^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 +
(a^2 - b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 -
b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 -
b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*cos
h(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(b^2*d*x*cosh(d*x + c)^3 +
 (b^2*d*x - a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))/((a*b^2 + b^3)*d*cosh
(d*x + c)^4 + 4*(a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b^2 + b
^3)*d*sinh(d*x + c)^4 + 2*(a*b^2 + b^3)*d*cosh(d*x + c)^2 + 2*(3*(a*b^2 +
b^3)*d*cosh(d*x + c)^2 + (a*b^2 + b^3)*d)*sinh(d*x + c)^2 + (a*b^2 + b^3)*
d + 4*((a*b^2 + b^3)*d*cosh(d*x + c)^3 + (a*b^2 + b^3)*d*cosh(d*x + c))*si
nh(d*x + c))
```

3.170.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(53) = 106$.

Time = 10.07 (sec) , antiderivative size = 415, normalized size of antiderivative = 6.29

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$= \begin{cases} \tilde{\infty} x \tanh^3(c) \\ \frac{x - \frac{\log(\tanh(c+dx)+1)}{d} - \frac{\tanh^4(c+dx)}{4d} - \frac{\tanh^2(c+dx)}{2d}}{a} \\ \frac{4dx\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd} - \frac{4dx}{2bd\tanh^2(c+dx)-2bd} - \frac{4\log(\tanh(c+dx)+1)\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd} + \frac{4\log(\tanh(c+dx)+1)}{2bd\tanh^2(c+dx)-2bd} - \frac{\tanh^4(c+dx)}{2bd\tanh^2(c+dx)-2} \\ \frac{x\tanh^5(c)}{a+b\tanh^2(c)} \\ \frac{a^2\log\left(-\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ab^2d+2b^3d} + \frac{a^2\log\left(\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ab^2d+2b^3d} - \frac{ab\tanh^2(c+dx)}{2ab^2d+2b^3d} + \frac{2b^2dx}{2ab^2d+2b^3d} - \frac{2b^2\log(\tanh(c+dx)+1)}{2ab^2d+2b^3d} - \frac{b^2\tan}{2ab} \end{cases}$$

input

```
integrate(tanh(d*x+c)**5/(a+b*tanh(d*x+c)**2),x)
```

output
```
Piecewise((zoo*x*tanh(c)**3, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - log(ta
nh(c + d*x) + 1)/d - tanh(c + d*x)**4/(4*d) - tanh(c + d*x)**2/(2*d))/a, E
q(b, 0)), (4*d*x*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 4*d*x
/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 4*log(tanh(c + d*x) + 1)*tanh(c + d*x)
**2/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + 4*log(tanh(c + d*x) + 1)/(2*b*d*tan
h(c + d*x)**2 - 2*b*d) - tanh(c + d*x)**4/(2*b*d*tanh(c + d*x)**2 - 2*b*d)
 + 2/(2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x*tanh(c)**5/(a + b*ta
nh(c)**2), Eq(d, 0)), (a**2*log(-sqrt(-a/b) + tanh(c + d*x))/(2*a*b**2*d +
 2*b**3*d) + a**2*log(sqrt(-a/b) + tanh(c + d*x))/(2*a*b**2*d + 2*b**3*d)
- a*b*tanh(c + d*x)**2/(2*a*b**2*d + 2*b**3*d) + 2*b**2*d*x/(2*a*b**2*d +
2*b**3*d) - 2*b**2*log(tanh(c + d*x) + 1)/(2*a*b**2*d + 2*b**3*d) - b**2*t
anh(c + d*x)**2/(2*a*b**2*d + 2*b**3*d), True))
```

3.170.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(62) = 124$.

Time $= 0.28$ (sec) , antiderivative size $= 133$, normalized size of antiderivative $= 2.02$

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{a^2\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(ab^2+b^3)d} + \frac{dx+c}{(a+b)d} + \frac{2\,e^{(-2\,dx-2\,c)}}{(2\,be^{(-2\,dx-2\,c)}+be^{(-4\,dx-4\,c)}+b)d} - \frac{(a-b)\log\left(e^{(-2\,dx-2\,c)}+1\right)}{b^2d}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*a^2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)
/((a*b^2 + b^3)*d) + (d*x + c)/((a + b)*d) + 2*e^(-2*d*x - 2*c)/((2*b*e^(-
2*d*x - 2*c) + b*e^(-4*d*x - 4*c) + b)*d) - (a - b)*log(e^(-2*d*x - 2*c) +
 1)/(b^2*d)
```

3.170.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(62) = 124$.

Time = 0.35 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.00

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{\frac{a^2\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{ab^2+b^3} - \frac{2\,(dx+c)}{a+b} - \frac{2\,(a-b)\log\left(e^{(2\,dx+2\,c)}+1\right)}{b^2} + \frac{4\,e^{(2\,dx+2\,c)}}{b\left(e^{(2\,dx+2\,c)}+1\right)^2}}{2\,d}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2*(a^2*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) -
 2*b*e^(2*d*x + 2*c) + a + b)/(a*b^2 + b^3) - 2*(d*x + c)/(a + b) - 2*(a -
 b)*log(e^(2*d*x + 2*c) + 1)/b^2 + 4*e^(2*d*x + 2*c)/(b*(e^(2*d*x + 2*c) +
 1)^2))/d
```

3.170.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= -\frac{b^2\left(\ln\left(\tanh(c+d\,x)+1\right) - d\,x + \frac{\tanh(c+d\,x)^2}{2}\right) - \frac{a^2\,\ln\left(b\tanh(c+d\,x)^2+a\right)}{2} + \frac{a\,b\tanh(c+d\,x)^2}{2}}{b^2\,d\,(a+b)}$$

input
```
int(tanh(c + d*x)^5/(a + b*tanh(c + d*x)^2),x)
```

output
```
-(b^2*(log(tanh(c + d*x) + 1) - d*x + tanh(c + d*x)^2/2) - (a^2*log(a + b*
tanh(c + d*x)^2))/2 + (a*b*tanh(c + d*x)^2)/2)/(b^2*d*(a + b))
```

3.171 $\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.171.1 Optimal result

Integrand size = 23, antiderivative size = 59

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}(a+b)d} - \frac{\tanh(c+dx)}{bd}$$

output
```
x/(a+b)+a^(3/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/b^(3/2)/(a+b)/d-tanh(d
*x+c)/b/d
```

3.171.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{c+dx}{(a+b)d} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}(a+b)d} - \frac{\tanh(c+dx)}{bd}$$

input
```
Integrate[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(c + d*x)/((a + b)*d) + (a^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/
(b^(3/2)*(a + b)*d) - Tanh[c + d*x]/(b*d)
```

3.171.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4153, 381, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(ic+idx)^4}{a-b\tan(ic+idx)^2}\,dx$$

$$\downarrow 4153$$

$$\frac{\int \frac{\tanh^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}\,d\tanh(c+dx)}{d}$$

$$\downarrow 381$$

$$\frac{\frac{\int \frac{a-(a-b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}\,d\tanh(c+dx)}{b} - \frac{\tanh(c+dx)}{b}}{d}$$

$$\downarrow 397$$

$$\frac{\frac{a^2\int \frac{1}{b\tanh^2(c+dx)+a}\,d\tanh(c+dx)}{a+b} + \frac{b\int \frac{1}{1-\tanh^2(c+dx)}\,d\tanh(c+dx)}{a+b}}{b} - \frac{\tanh(c+dx)}{b}}{d}$$

$$\downarrow 218$$

$$\frac{\frac{b\int \frac{1}{1-\tanh^2(c+dx)}\,d\tanh(c+dx)}{a+b} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)}}{b} - \frac{\tanh(c+dx)}{b}}{d}$$

$$\downarrow 219$$

$$\frac{\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)} + \frac{b\operatorname{arctanh}(\tanh(c+dx))}{a+b}}{b} - \frac{\tanh(c+dx)}{b}}{d}$$

input
```
Int[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(((a^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[b]*(a + b)) + (b
*ArcTanh[Tanh[c + d*x]])/(a + b))/b - Tanh[c + d*x]/b)/d
```

3.171.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.171.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.47

method	result	size
derivativedivides	$\frac{-\frac{\tanh(dx+c)}{b}+\frac{a^2\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)b\sqrt{ab}}-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}+\frac{\ln(\tanh(dx+c)+1)}{2a+2b}}{d}$	87
default	$\frac{-\frac{\tanh(dx+c)}{b}+\frac{a^2\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)b\sqrt{ab}}-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}+\frac{\ln(\tanh(dx+c)+1)}{2a+2b}}{d}$	87
risch	$\frac{x}{a+b}+\frac{2}{bd\left(e^{2dx+2c}+1\right)}+\frac{\sqrt{-ab}\,a\ln\left(e^{2dx+2c}+\frac{2\sqrt{-ab}+a-b}{a+b}\right)}{2b^2(a+b)d}-\frac{\sqrt{-ab}\,a\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}-a+b}{a+b}\right)}{2b^2(a+b)d}$	131

input
```
int(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b*tanh(d*x+c)+1/(a+b)/b*a^2/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)
^(1/2))-1/(2*a+2*b)*ln(tanh(d*x+c)-1)+1/(2*a+2*b)*ln(tanh(d*x+c)+1))
```

3.171.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(51) = 102$.

Time = 0.29 (sec) , antiderivative size = 777, normalized size of antiderivative = 13.17

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$= \left[\frac{2\,bdx\cosh(dx+c)^2+4\,bdx\cosh(dx+c)\sinh(dx+c)+2\,bdx\sinh(dx+c)^2+2\,bdx+\left(a\cosh(dx+c\right.}{\right.}\right.$$

input
```
integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output

```
[1/2*(2*b*d*x*cosh(d*x + c)^2 + 4*b*d*x*cosh(d*x + c)*sinh(d*x + c) + 2*b*
d*x*sinh(d*x + c)^2 + 2*b*d*x + (a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sin
h(d*x + c) + a*sinh(d*x + c)^2 + a)*sqrt(-a/b)*log(((a^2 + 2*a*b + b^2)*co
sh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2
 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*
b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x +
c))*sinh(d*x + c) + 4*((a*b + b^2)*cosh(d*x + c)^2 + 2*(a*b + b^2)*cosh(d*
x + c)*sinh(d*x + c) + (a*b + b^2)*sinh(d*x + c)^2 + a*b - b^2)*sqrt(-a/b)
)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a
+ b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x +
 c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh
(d*x + c))*sinh(d*x + c) + a + b)) + 4*a + 4*b)/((a*b + b^2)*d*cosh(d*x +
c)^2 + 2*(a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a*b + b^2)*d*sinh(d*
x + c)^2 + (a*b + b^2)*d), (b*d*x*cosh(d*x + c)^2 + 2*b*d*x*cosh(d*x + c)*
sinh(d*x + c) + b*d*x*sinh(d*x + c)^2 + b*d*x + (a*cosh(d*x + c)^2 + 2*a*c
osh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a)*sqrt(a/b)*arctan(1/2*(
(a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*
sinh(d*x + c)^2 + a - b)*sqrt(a/b)/a) + 2*a + 2*b)/((a*b + b^2)*d*cosh(d*x
 + c)^2 + 2*(a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a*b + b^2)*d*s...
```

3.171.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(48) = 96$.

Time = 4.76 (sec) , antiderivative size = 428, normalized size of antiderivative = 7.25

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$= \begin{cases} \tilde{\infty} x \tanh^2(c) \\ \frac{x - \frac{\tanh^3(c+dx)}{3d} - \frac{\tanh(c+dx)}{d}}{a} \\ \frac{x - \frac{\tanh(c+dx)}{d}}{b} \\ \frac{3dx\tanh^2(c+dx)}{2bd\tanh^2(c+dx) - 2bd} - \frac{3dx}{2bd\tanh^2(c+dx) - 2bd} - \frac{2\tanh^3(c+dx)}{2bd\tanh^2(c+dx) - 2bd} + \frac{3\tanh(c+dx)}{2bd\tanh^2(c+dx) - 2bd} \\ \frac{x\tanh^4(c)}{a + b\tanh^2(c)} \\ \frac{a^2\log\left(-\sqrt{-\frac{a}{b}} + \tanh(c+dx)\right)}{2ab^2d\sqrt{-\frac{a}{b}} + 2b^3d\sqrt{-\frac{a}{b}}} - \frac{a^2\log\left(\sqrt{-\frac{a}{b}} + \tanh(c+dx)\right)}{2ab^2d\sqrt{-\frac{a}{b}} + 2b^3d\sqrt{-\frac{a}{b}}} - \frac{2ab\sqrt{-\frac{a}{b}}\tanh(c+dx)}{2ab^2d\sqrt{-\frac{a}{b}} + 2b^3d\sqrt{-\frac{a}{b}}} + \frac{2b^2dx\sqrt{-\frac{a}{b}}}{2ab^2d\sqrt{-\frac{a}{b}} + 2b^3d\sqrt{-\frac{a}{b}}} - \frac{2b^2\sqrt{-\frac{a}{b}}\tanh}{2ab^2d\sqrt{-\frac{a}{b}} + 2} \end{cases}$$

input

```
integrate(tanh(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

output
```
Piecewise((zoo*x*tanh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - tanh(c
 + d*x)**3/(3*d) - tanh(c + d*x)/d)/a, Eq(b, 0)), ((x - tanh(c + d*x)/d)/b
, Eq(a, 0)), (3*d*x*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 3*
d*x/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 2*tanh(c + d*x)**3/(2*b*d*tanh(c +
d*x)**2 - 2*b*d) + 3*tanh(c + d*x)/(2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a,
 -b)), (x*tanh(c)**4/(a + b*tanh(c)**2), Eq(d, 0)), (a**2*log(-sqrt(-a/b)
+ tanh(c + d*x))/(2*a*b**2*d*sqrt(-a/b) + 2*b**3*d*sqrt(-a/b)) - a**2*log(
sqrt(-a/b) + tanh(c + d*x))/(2*a*b**2*d*sqrt(-a/b) + 2*b**3*d*sqrt(-a/b))
- 2*a*b*sqrt(-a/b)*tanh(c + d*x)/(2*a*b**2*d*sqrt(-a/b) + 2*b**3*d*sqrt(-a
/b)) + 2*b**2*d*x*sqrt(-a/b)/(2*a*b**2*d*sqrt(-a/b) + 2*b**3*d*sqrt(-a/b))
 - 2*b**2*sqrt(-a/b)*tanh(c + d*x)/(2*a*b**2*d*sqrt(-a/b) + 2*b**3*d*sqrt(
-a/b)), True))
```

3.171.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(51) = 102$.

Time = 0.39 (sec) , antiderivative size = 509, normalized size of antiderivative = 8.63

$$\begin{aligned}\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = &-\frac{(a-b)\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{8\,(ab+b^2)d}\\ &+\frac{(a-b)\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{8\,(ab+b^2)d}\\ &+\frac{(a^2-6\,ab+b^2)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{16\,(ab+b^2)\sqrt{ab}d}\\ &+\frac{(a-b)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{4\sqrt{ab}bd}\\ &-\frac{(a^2-6\,ab+b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{16\,(ab+b^2)\sqrt{ab}d}\\ &-\frac{3\,(a+b)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{8\sqrt{ab}bd}\\ &-\frac{(a-b)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{4\sqrt{ab}bd}\\ &-\frac{\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{4\,bd}\\ &+\frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{4\,bd}\\ &+\frac{3\,\log\left(e^{(2\,dx+2\,c)}+1\right)}{4\,bd}-\frac{3\,\log\left(e^{(-2\,dx-2\,c)}+1\right)}{4\,bd}\\ &+\frac{5}{8\,(be^{(2\,dx+2\,c)}+b)d}-\frac{11}{8\,(be^{(-2\,dx-2\,c)}+b)d}\end{aligned}$$

input
```
integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/8*(a - b)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a +
 b)/((a*b + b^2)*d) + 1/8*(a - b)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)
*e^(-4*d*x - 4*c) + a + b)/((a*b + b^2)*d) + 1/16*(a^2 - 6*a*b + b^2)*arct
an(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a*b + b^2)*sqrt(a*b)
*d) + 1/4*(a - b)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/
(sqrt(a*b)*b*d) - 1/16*(a^2 - 6*a*b + b^2)*arctan(1/2*((a + b)*e^(-2*d*x -
 2*c) + a - b)/sqrt(a*b))/((a*b + b^2)*sqrt(a*b)*d) - 3/8*(a + b)*arctan(1
/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*b*d) - 1/4*(a
- b)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*b
*d) - 1/4*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)
/(b*d) + 1/4*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a
 + b)/(b*d) + 3/4*log(e^(2*d*x + 2*c) + 1)/(b*d) - 3/4*log(e^(-2*d*x - 2*c
) + 1)/(b*d) + 5/8/((b*e^(2*d*x + 2*c) + b)*d) - 11/8/((b*e^(-2*d*x - 2*c)
 + b)*d)
```

3.171.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.47

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{a^2\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{(ab+b^2)\sqrt{ab}} + \frac{dx+c}{a+b} + \frac{2}{b\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

input
```
integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
(a^2*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))
/((a*b + b^2)*sqrt(a*b)) + (d*x + c)/(a + b) + 2/(b*(e^(2*d*x + 2*c) + 1))
)/d
```

3.171.9 Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b} - \frac{\tanh(c+d\,x)}{b\,d} + \frac{a^2\,\mathrm{atan}\left(\frac{b\tanh(c+d\,x)}{\sqrt{a\,b}}\right)}{b\,d\,\sqrt{a\,b}\,(a+b)}$$

input
```
int(tanh(c + d*x)^4/(a + b*tanh(c + d*x)^2),x)
```

output
```
x/(a + b) - tanh(c + d*x)/(b*d) + (a^2*atan((b*tanh(c + d*x))/(a*b)^(1/2))
)/(b*d*(a*b)^(1/2)*(a + b))
```

3.172 $\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.172.1 Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)d} - \frac{a\log\left(a+b\tanh^2(c+dx)\right)}{2b(a+b)d}$$

output
```
ln(cosh(d*x+c))/(a+b)/d-1/2*a*ln(a+b*tanh(d*x+c)^2)/b/(a+b)/d
```

3.172.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{\log(\cosh(c+dx))}{a+b} - \frac{a\log\left(a+b\tanh^2(c+dx)\right)}{2b(a+b)}}{d}$$

input
```
Integrate[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(Log[Cosh[c + d*x]]/(a + b) - (a*Log[a + b*Tanh[c + d*x]^2])/(2*b*(a + b))
)/d
```

3.172.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{i\tan(ic+idx)^3}{a-b\tan(ic+idx)^2}dx$$

$$\downarrow 26$$

$$i\int \frac{\tan(ic+idx)^3}{a-b\tan(ic+idx)^2}dx$$

$$\downarrow 4153$$

$$\frac{i\int -\frac{i\tanh^3(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\tanh^3(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh(c+dx)}{d}$$

$$\downarrow 354$$

$$\frac{\int \frac{\tanh^2(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh^2(c+dx)}{2d}$$

$$\downarrow 86$$

$$\frac{\int \left(-\frac{a}{(a+b)(b\tanh^2(c+dx)+a)}-\frac{1}{(a+b)(\tanh^2(c+dx)-1)}\right)d\tanh^2(c+dx)}{2d}$$

$$\downarrow 2009$$

$$\frac{-\frac{\log(1-\tanh^2(c+dx))}{a+b}-\frac{a\log(a+b\tanh^2(c+dx))}{b(a+b)}}{2d}$$

input
```
Int[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-(Log[1 - Tanh[c + d*x]^2]/(a + b)) - (a*Log[a + b*Tanh[c + d*x]^2])/(b*(
a + b)))/(2*d)
```

3.172.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.172.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

method	result	size
parallelrisch	$-\frac{2dxb+2\ln(1-\tanh(dx+c))b+a\ln\left(a+b\tanh(dx+c)^2\right)}{2bd(a+b)}$	49
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}-\frac{\ln(\tanh(dx+c)+1)}{2a+2b}-\frac{a\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)b}}{d}$	70
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}-\frac{\ln(\tanh(dx+c)+1)}{2a+2b}-\frac{a\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)b}}{d}$	70
risch	$\frac{x}{a+b}-\frac{2x}{b}-\frac{2c}{bd}+\frac{2ax}{b(a+b)}+\frac{2ac}{bd(a+b)}+\frac{\ln(e^{2dx+2c}+1)}{bd}-\frac{a\ln\left(e^{4dx+4c}+\frac{2(a-b)e^{2dx+2c}}{a+b}+1\right)}{2bd(a+b)}$	117

input
```
int(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*d*x*b+2*ln(1-tanh(d*x+c))*b+a*ln(a+b*tanh(d*x+c)^2))/b/d/(a+b)
```

3.172.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(44) = 88$.

Time = 0.30 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.57

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{2\,bdx + a\log\left(\frac{2\left((a+b)\cosh(dx+c)^2+(a+b)\sinh(dx+c)^2+a-b\right)}{\cosh(dx+c)^2-2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right) - 2\,(a+b)\log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{2\,(ab+b^2)d}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(2*b*d*x + a*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2
 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2)) - 2*(a + b)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/((a
*b + b^2)*d)
```

3.172.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(36) = 72.

Time = 2.86 (sec) , antiderivative size = 306, normalized size of antiderivative = 6.65

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \begin{cases} \tilde{\infty} x \tanh(c) \\ \frac{x - \frac{\log(\tanh(c+dx)+1)}{d} - \frac{\tanh^2(c+dx)}{2d}}{a} \\ \frac{2dx\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd} - \frac{2dx}{2bd\tanh^2(c+dx)-2bd} - \frac{2\log(\tanh(c+dx)+1)\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd} + \frac{2\log(\tanh(c+dx)+1)}{2bd\tanh^2(c+dx)-2bd} + \frac{1}{2bd\tanh^2(c+dx)-2} \\ \frac{x\tanh^3(c)}{a+b\tanh^2(c)} \\ -\frac{a\log\left(-\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2abd+2b^2d} - \frac{a\log\left(\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2abd+2b^2d} + \frac{2bdx}{2abd+2b^2d} - \frac{2b\log(\tanh(c+dx)+1)}{2abd+2b^2d} \end{cases}$$

input
```
integrate(tanh(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

output
```
Piecewise((zoo*x*tanh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - log(tanh(
c + d*x) + 1)/d - tanh(c + d*x)**2/(2*d))/a, Eq(b, 0)), (2*d*x*tanh(c + d*
x)**2/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 2*d*x/(2*b*d*tanh(c + d*x)**2 - 2
*b*d) - 2*log(tanh(c + d*x) + 1)*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2
- 2*b*d) + 2*log(tanh(c + d*x) + 1)/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + 1/(
2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x*tanh(c)**3/(a + b*tanh(c)*
*2), Eq(d, 0)), (-a*log(-sqrt(-a/b) + tanh(c + d*x))/(2*a*b*d + 2*b**2*d)
- a*log(sqrt(-a/b) + tanh(c + d*x))/(2*a*b*d + 2*b**2*d) + 2*b*d*x/(2*a*b*
d + 2*b**2*d) - 2*b*log(tanh(c + d*x) + 1)/(2*a*b*d + 2*b**2*d), True))
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.78

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{a\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{2(ab+b^2)d} + \frac{dx+c}{(a+b)d} + \frac{\log\left(e^{(-2dx-2c)}+1\right)}{bd}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/2*a*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/
((a*b + b^2)*d) + (d*x + c)/((a + b)*d) + log(e^(-2*d*x - 2*c) + 1)/(b*d)
```

3.172.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(44) = 88$.

Time = 0.35 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.09

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{a\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{ab+b^2}+\frac{2\,(dx+c)}{a+b}-\frac{2\,\log\left(e^{(2\,dx+2\,c)}+1\right)}{b}}{2\,d}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/2*(a*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) -
2*b*e^(2*d*x + 2*c) + a + b)/(a*b + b^2) + 2*(d*x + c)/(a + b) - 2*log(e^(
2*d*x + 2*c) + 1)/b)/d
```

3.172.9 Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{a\,\ln\left(b\tanh(c+d\,x)^2+a\right)}{2}+b\left(\ln\left(\tanh(c+d\,x)+1\right)-d\,x\right)}{b\,d\,(a+b)}$$

input
```
int(tanh(c + d*x)^3/(a + b*tanh(c + d*x)^2),x)
```

output
```
-((a*log(a + b*tanh(c + d*x)^2))/2 + b*(log(tanh(c + d*x) + 1) - d*x))/(b*
d*(a + b))
```

3.173 $\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.173.2 Mathematica [A] (verified) 1286
3.173.3 Rubi [A] (verified) 1287
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3.173.1 Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)d}$$

output
```
x/(a+b)-arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*a^(1/2)/(a+b)/d/b^(1/2)
```

3.173.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}} + \operatorname{arctanh}(\tanh(c+dx))}{(a+b)d}$$

input
```
Integrate[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-((Sqrt[a]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[b]) + ArcTanh[Ta
nh[c + d*x]])/((a + b)*d)
```

3.173.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 25, 4153, 25, 383, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\tan(ic+idx)^2}{a-b\tan(ic+idx)^2}\,dx$$

$$\downarrow 25$$

$$-\int \frac{\tan(ic+idx)^2}{a-b\tan(ic+idx)^2}\,dx$$

$$\downarrow 4153$$

$$-\frac{\int -\frac{\tanh^2(c+dx)}{(1-\tanh^2(c+dx))\,(b\tanh^2(c+dx)+a)}\,d\tanh(c+dx)}{d}$$

$$\downarrow 25$$

$$\frac{\int \frac{\tanh^2(c+dx)}{(1-\tanh^2(c+dx))\,(b\tanh^2(c+dx)+a)}\,d\tanh(c+dx)}{d}$$

$$\downarrow 383$$

$$-\frac{\frac{a\int \frac{1}{b\tanh^2(c+dx)+a}\,d\tanh(c+dx)}{a+b} - \frac{\int \frac{1}{1-\tanh^2(c+dx)}\,d\tanh(c+dx)}{a+b}}{d}$$

$$\downarrow 218$$

$$-\frac{\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)} - \frac{\int \frac{1}{1-\tanh^2(c+dx)}\,d\tanh(c+dx)}{a+b}}{d}$$

$$\downarrow 219$$

$$-\frac{\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)} - \frac{\operatorname{arctanh}(\tanh(c+dx))}{a+b}}{d}$$

input
```
Int[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
-(((Sqrt[a]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[b]*(a + b)) - A
rcTanh[Tanh[c + d*x]]/(a + b))/d)
```

3.173.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.173.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.57

method	result	size
derivativedivides	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2a+2b}-\frac{a\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}}{d}$	72
default	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2a+2b}-\frac{a\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}}{d}$	72
risch	$\frac{x}{a+b}+\frac{\sqrt{-ab}\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}-a+b}{a+b}\right)}{2b(a+b)d}-\frac{\sqrt{-ab}\ln\left(e^{2dx+2c}+\frac{2\sqrt{-ab}+a-b}{a+b}\right)}{2b(a+b)d}$	108

input
```
int(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(2*a+2*b)*ln(tanh(d*x+c)+1)-a/(a+b)/(a*b)^(1/2)*arctan(b*tanh(d*x+c
)/(a*b)^(1/2))-1/(2*a+2*b)*ln(tanh(d*x+c)-1))
```

3.173.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(38) = 76.

Time = 0.29 (sec) , antiderivative size = 486, normalized size of antiderivative = 10.57

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \left[\frac{2\,dx+\sqrt{-\frac{a}{b}}\log\left(\frac{(a^2+2\,ab+b^2)\cosh(dx+c)^4+4\,(a^2+2\,ab+b^2)\cosh(dx+c)\sinh(dx+c)^3+(a^2+2\,ab+b^2)\sinh(dx+c)^4+2\,(a^2-b^2)\cosh(}{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx+}\right.\right.$$

input
```
integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(2*d*x + sqrt(-a/b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2
 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d
*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*
a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*
((a*b + b^2)*cosh(d*x + c)^2 + 2*(a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) +
 (a*b + b^2)*sinh(d*x + c)^2 + a*b - b^2)*sqrt(-a/b))/((a + b)*cosh(d*x +
c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 +
 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*
x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x +
c) + a + b)))/((a + b)*d), (d*x - sqrt(a/b)*arctan(1/2*((a + b)*cosh(d*x +
 c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 +
a - b)*sqrt(a/b)/a))/((a + b)*d)]
```

3.173.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(37) = 74$.

Time = 2.08 (sec) , antiderivative size = 253, normalized size of antiderivative = 5.50

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$= \begin{cases} \tilde{\infty} x & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{x-\frac{\tanh(c+dx)}{d}}{a} & \text{for } b=0 \\ \frac{x}{b} & \text{for } a=0 \\ \frac{dx\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd} - \frac{dx}{2bd\tanh^2(c+dx)-2bd} + \frac{\tanh(c+dx)}{2bd\tanh^2(c+dx)-2bd} & \text{for } a=-b \\ \frac{x\tanh^2(c)}{a+b\tanh^2(c)} & \text{for } d=0 \\ -\frac{a\log\left(-\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2abd\sqrt{-\frac{a}{b}}+2b^2d\sqrt{-\frac{a}{b}}} + \frac{a\log\left(\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2abd\sqrt{-\frac{a}{b}}+2b^2d\sqrt{-\frac{a}{b}}} + \frac{2bdx\sqrt{-\frac{a}{b}}}{2abd\sqrt{-\frac{a}{b}}+2b^2d\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - tanh(c + d*x)/d)/
a, Eq(b, 0)), (x/b, Eq(a, 0)), (d*x*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)*
*2 - 2*b*d) - d*x/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + tanh(c + d*x)/(2*b*d*
tanh(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x*tanh(c)**2/(a + b*tanh(c)**2), E
q(d, 0)), (-a*log(-sqrt(-a/b) + tanh(c + d*x))/(2*a*b*d*sqrt(-a/b) + 2*b**
2*d*sqrt(-a/b)) + a*log(sqrt(-a/b) + tanh(c + d*x))/(2*a*b*d*sqrt(-a/b) + 
2*b**2*d*sqrt(-a/b)) + 2*b*d*x*sqrt(-a/b)/(2*a*b*d*sqrt(-a/b) + 2*b**2*d*s
qrt(-a/b)), True))
```

3.173.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(38) = 76$.

Time $= 0.30$ (sec) , antiderivative size $= 215$, normalized size of antiderivative $= 4.67$

$$\begin{aligned}\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = &-\frac{(a-b)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{4\sqrt{ab}(a+b)d}\\ &+\frac{\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}d}\\ &+\frac{(a-b)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{4\sqrt{ab}(a+b)d}\\ &+\frac{\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{4\,(a+b)d}\\ &-\frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{4\,(a+b)d}\end{aligned}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/4*(a - b)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(sqrt
(a*b)*(a + b)*d) + 1/2*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(
a*b))/(sqrt(a*b)*d) + 1/4*(a - b)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a
 - b)/sqrt(a*b))/(sqrt(a*b)*(a + b)*d) + 1/4*log((a + b)*e^(4*d*x + 4*c) +
 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a + b)*d) - 1/4*log(2*(a - b)*e^(-2*
d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a + b)*d)
```

3.173.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.41

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{a\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}(a+b)} - \frac{dx+c}{a+b}}{d}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-(a*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/
(sqrt(a*b)*(a + b)) - (d*x + c)/(a + b))/d
```

3.173.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b} - \frac{a\,\mathrm{atan}\left(\frac{b\tanh(c+dx)}{\sqrt{a\,b}}\right)}{d\sqrt{a\,b}\,(a+b)}$$

input
```
int(tanh(c + d*x)^2/(a + b*tanh(c + d*x)^2),x)
```

output
```
x/(a + b) - (a*atan((b*tanh(c + d*x))/(a*b)^(1/2)))/(d*(a*b)^(1/2)*(a + b)
)
```

3.174 $\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

3.174.1 Optimal result 1293
3.174.2 Mathematica [A] (verified) 1293
3.174.3 Rubi [A] (verified) 1294
3.174.4 Maple [A] (verified) 1296
3.174.5 Fricas [B] (verification not implemented) 1296
3.174.6 Sympy [B] (verification not implemented) 1297
3.174.7 Maxima [A] (verification not implemented) 1297
3.174.8 Giac [A] (verification not implemented) 1298
3.174.9 Mupad [B] (verification not implemented) 1298

3.174.1 Optimal result

Integrand size = 21, antiderivative size = 42

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)d}$$

output
```
ln(cosh(d*x+c))/(a+b)/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)/d
```

3.174.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{2\log(\cosh(c+dx)) + \log\left(a+b\tanh^2(c+dx)\right)}{2ad+2bd}$$

input
```
Integrate[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(2*Log[Cosh[c + d*x]] + Log[a + b*Tanh[c + d*x]^2])/(2*a*d + 2*b*d)
```

3.174.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 4153, 26, 353, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\tan(ic+idx)}{a-b\tan(ic+idx)^2}dx$$

$$\downarrow 26$$

$$-i\int \frac{\tan(ic+idx)}{a-b\tan(ic+idx)^2}dx$$

$$\downarrow 4153$$

$$-\frac{i\int \frac{i\tanh(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\tanh(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh(c+dx)}{d}$$

$$\downarrow 353$$

$$\frac{\int \frac{1}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh^2(c+dx)}{2d}$$

$$\downarrow 47$$

$$\frac{\frac{\int \frac{1}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{a+b}+\frac{b\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh^2(c+dx)}{a+b}}{2d}$$

$$\downarrow 16$$

$$\frac{\frac{\log(a+b\tanh^2(c+dx))}{a+b}-\frac{\log(1-\tanh^2(c+dx))}{a+b}}{2d}$$

input
```
Int[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-(Log[1 - Tanh[c + d*x]^2]/(a + b)) + Log[a + b*Tanh[c + d*x]^2]/(a + b))
/(2*d)
```

3.174.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.174.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

method	result	size
parallelrisch	$-\frac{2dx+2\ln(1-\tanh(dx+c))-\ln\left(a+b\tanh(dx+c)^2\right)}{2d(a+b)}$	44
risch	$-\frac{x}{a+b}-\frac{2c}{d(a+b)}+\frac{\ln\left(e^{4dx+4c}+\frac{2(a-b)e^{2dx+2c}}{a+b}+1\right)}{2d(a+b)}$	64
derivativedivides	$\frac{\frac{\ln\left(a+b\tanh(dx+c)^2\right)}{2a+2b}-\frac{\ln(\tanh(dx+c)+1)}{2a+2b}-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}}{d}$	66
default	$\frac{\frac{\ln\left(a+b\tanh(dx+c)^2\right)}{2a+2b}-\frac{\ln(\tanh(dx+c)+1)}{2a+2b}-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}}{d}$	66

input
```
int(tanh(d*x+c)/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*d*x+2*ln(1-tanh(d*x+c))-ln(a+b*tanh(d*x+c)^2))/d/(a+b)
```

3.174.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(40) = 80$.

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.95

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{2\,dx-\log\left(\frac{2\left((a+b)\cosh(dx+c)^2+(a+b)\sinh(dx+c)^2+a-b\right)}{\cosh(dx+c)^2-2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right)}{2\,(a+b)d}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(2*d*x - log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a
 - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2))
)/((a + b)*d)
```

3.174.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(32) = 64$.

Time = 1.93 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.48

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$= \begin{cases} \frac{\tilde{\infty} x}{\tanh(c)} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{x - \frac{\log(\tanh(c+dx)+1)}{d}}{a} & \text{for } b=0 \\ \frac{1}{2bd\tanh^2(c+dx) - 2bd} & \text{for } a=-b \\ \frac{x\tanh(c)}{a+b\tanh^2(c)} & \text{for } d=0 \\ \frac{2dx}{2ad+2bd} + \frac{\log\left(-\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ad+2bd} + \frac{\log\left(\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ad+2bd} - \frac{2\log(\tanh(c+dx)+1)}{2ad+2bd} & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

output
```
Piecewise((zoo*x/tanh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - log(tanh(
c + d*x) + 1)/d)/a, Eq(b, 0)), (1/(2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a,
-b)), (x*tanh(c)/(a + b*tanh(c)**2), Eq(d, 0)), (2*d*x/(2*a*d + 2*b*d) + l
og(-sqrt(-a/b) + tanh(c + d*x))/(2*a*d + 2*b*d) + log(sqrt(-a/b) + tanh(c
+ d*x))/(2*a*d + 2*b*d) - 2*log(tanh(c + d*x) + 1)/(2*a*d + 2*b*d), True))
```

3.174.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{dx+c}{(a+b)d} + \frac{\log\left(2(a-b)e^{(-2dx-2c)} + (a+b)e^{(-4dx-4c)} + a + b\right)}{2(a+b)d}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
(d*x + c)/((a + b)*d) + 1/2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4
*d*x - 4*c) + a + b)/((a + b)*d)
```

3.174.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{2\,(a+b)d}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2*log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) +
e^(-2*d*x - 2*c)) + 2*a - 2*b))/((a + b)*d)
```

3.174.9 Mupad [B] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b} - \frac{\ln\left(\tanh(c+d\,x)+1\right) - \frac{\ln\left(b\tanh(c+d\,x)^2+a\right)}{2}}{d\,(a+b)}$$

input
```
int(tanh(c + d*x)/(a + b*tanh(c + d*x)^2),x)
```

output
```
x/(a + b) - (log(tanh(c + d*x) + 1) - log(a + b*tanh(c + d*x)^2)/2)/(d*(a
+ b))
```

3.175 $\int \frac{1}{a+b\tanh^2(c+dx)}\,dx$

3.175.1 Optimal result 1299
3.175.2 Mathematica [A] (verified) 1299
3.175.3 Rubi [A] (verified) 1300
3.175.4 Maple [A] (verified) 1301
3.175.5 Fricas [B] (verification not implemented) 1302
3.175.6 Sympy [B] (verification not implemented) 1303
3.175.7 Maxima [A] (verification not implemented) 1303
3.175.8 Giac [A] (verification not implemented) 1304
3.175.9 Mupad [B] (verification not implemented) 1304

3.175.1 Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{1}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b} + \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)d}$$

output
```
x/(a+b)+arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/(a+b)/d/a^(1/2)
```

3.175.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \frac{1}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}} - \log(1-\tanh(c+dx)) + \log(1+\tanh(c+dx))}{2ad+2bd}$$

input
```
Integrate[(a + b*Tanh[c + d*x]^2)^(-1),x]
```

output
```
((2*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a] - Log[1 - Tan
h[c + d*x]] + Log[1 + Tanh[c + d*x]])/(2*a*d + 2*b*d)
```

3.175.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4143, 3042, 4158, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a + b\tanh^2(c+dx)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{a - b\tan(ic+idx)^2}dx$$
$$\downarrow 4143$$
$$\frac{b\int \frac{\text{sech}^2(c+dx)}{b\tanh^2(c+dx)+a}dx}{a+b} + \frac{x}{a+b}$$
$$\downarrow 3042$$
$$\frac{x}{a+b} + \frac{b\int \frac{\sec(ic+idx)^2}{a-b\tan(ic+idx)^2}dx}{a+b}$$
$$\downarrow 4158$$
$$\frac{b\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{d(a+b)} + \frac{x}{a+b}$$
$$\downarrow 218$$
$$\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a+b)} + \frac{x}{a+b}$$

input
```
Int[(a + b*Tanh[c + d*x]^2)^(-1),x]
```

output
```
x/(a + b) + (Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a
+ b)*d)
```

3.175.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4143
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/(a -
 b), x] - Simp[b/(a - b)   Int[Sec[e + f*x]^2/(a + b*Tan[e + f*x]^2), x], x
] /; FreeQ[{a, b, e, f}, x] && NeQ[a, b]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.175.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.58

method	result	size
derivativedivides	$\frac{\frac{b\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}+\frac{\ln(\tanh(dx+c)+1)}{2a+2b}}{d}$	71
default	$\frac{\frac{b\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}+\frac{\ln(\tanh(dx+c)+1)}{2a+2b}}{d}$	71
risch	$\frac{x}{a+b}+\frac{\sqrt{-ab}\ln\left(e^{2dx+2c}+\frac{2\sqrt{-ab}+a-b}{a+b}\right)}{2a(a+b)d}-\frac{\sqrt{-ab}\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}-a+b}{a+b}\right)}{2a(a+b)d}$	108

input
```
int(1/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b/(a+b)/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2))-1/(2*a+2*b)*ln(
tanh(d*x+c)-1)+1/(2*a+2*b)*ln(tanh(d*x+c)+1))
```

3.175.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(37) = 74$.

Time = 0.28 (sec) , antiderivative size = 484, normalized size of antiderivative = 10.76

$$\int \frac{1}{a + b\tanh^2(c + dx)}\,dx$$

$$= \left[\frac{2\,dx + \sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+2\,ab+b^2)\cosh(dx+c)^4+4\,(a^2+2\,ab+b^2)\cosh(dx+c)\sinh(dx+c)^3+(a^2+2\,ab+b^2)\sinh(dx+c)^4+2\,(a^2-b^2)\cosh(}{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx+c)\sinh(dx+c)^3+(a+b)\sinh(dx+c)^4+2\,(a-b)\cosh(dx+c)^2+2\,(3\,(a+b)\cosh(dx+c)^2+a-b)\sinh(dx+c)^2+4\,((a+b)\cosh(dx+c)^3+(a-b)\cosh(dx+c))\sinh(dx+c)+a+b}\right)}{d}, \right.$$

input
```
integrate(1/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(2*d*x + sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2
 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d
*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*
a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*
((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) +
 (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x +
c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 +
 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*
x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x +
c) + a + b)))/((a + b)*d), (d*x + sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x +
 c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 +
a - b)*sqrt(b/a)/b))/((a + b)*d)]
```

3.175.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(37) = 74$.

Time = 2.00 (sec) , antiderivative size = 240, normalized size of antiderivative = 5.33

$$\int \frac{1}{a + b\tanh^2(c + dx)}\,dx$$

$$= \begin{cases} \frac{\tilde{\infty} x}{\tanh^2(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{x}{a} & \text{for } b = 0 \\ \frac{x - \frac{1}{d\tanh(c+dx)}}{b} & \text{for } a = 0 \\ -\frac{dx\tanh^2(c+dx)}{2bd\tanh^2(c+dx) - 2bd} + \frac{dx}{2bd\tanh^2(c+dx) - 2bd} + \frac{\tanh(c+dx)}{2bd\tanh^2(c+dx) - 2bd} & \text{for } a = -b \\ \frac{x}{a + b\tanh^2(c)} & \text{for } d = 0 \\ \frac{2dx\sqrt{-\frac{a}{b}}}{2ad\sqrt{-\frac{a}{b}} + 2bd\sqrt{-\frac{a}{b}}} + \frac{\log\left(-\sqrt{-\frac{a}{b}} + \tanh(c+dx)\right)}{2ad\sqrt{-\frac{a}{b}} + 2bd\sqrt{-\frac{a}{b}}} - \frac{\log\left(\sqrt{-\frac{a}{b}} + \tanh(c+dx)\right)}{2ad\sqrt{-\frac{a}{b}} + 2bd\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*tanh(d*x+c)**2),x)
```

output
```
Piecewise((zoo*x/tanh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a, Eq(b,
0)), ((x - 1/(d*tanh(c + d*x)))/b, Eq(a, 0)), (-d*x*tanh(c + d*x)**2/(2*b*
d*tanh(c + d*x)**2 - 2*b*d) + d*x/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + tanh(
c + d*x)/(2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x/(a + b*tanh(c)**
2), Eq(d, 0)), (2*d*x*sqrt(-a/b)/(2*a*d*sqrt(-a/b) + 2*b*d*sqrt(-a/b)) + l
og(-sqrt(-a/b) + tanh(c + d*x))/(2*a*d*sqrt(-a/b) + 2*b*d*sqrt(-a/b)) - lo
g(sqrt(-a/b) + tanh(c + d*x))/(2*a*d*sqrt(-a/b) + 2*b*d*sqrt(-a/b)), True)
)
```

3.175.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int \frac{1}{a + b\tanh^2(c + dx)}\,dx = -\frac{b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)} + a - b}{2\sqrt{ab}}\right)}{\sqrt{ab}(a+b)d} + \frac{dx + c}{(a+b)d}$$

input
```
integrate(1/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a
+ b)*d) + (d*x + c)/((a + b)*d)
```

3.175.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.40

$$\int \frac{1}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}(a+b)}+\frac{dx+c}{a+b}}{d}$$

input
```
integrate(1/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
(b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(
sqrt(a*b)*(a + b)) + (d*x + c)/(a + b))/d
```

3.175.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{1}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b}+\frac{b\,\mathrm{atan}\left(\frac{b\tanh(c+d\,x)}{\sqrt{a\,b}}\right)}{d\sqrt{a\,b}\,(a+b)}$$

input
```
int(1/(a + b*tanh(c + d*x)^2),x)
```

output
```
x/(a + b) + (b*atan((b*tanh(c + d*x))/(a*b)^(1/2)))/(d*(a*b)^(1/2)*(a + b)
)
```

3.176 $\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.176.1 Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)d} + \frac{\log(\tanh(c+dx))}{ad} - \frac{b\log\left(a+b\tanh^2(c+dx)\right)}{2a(a+b)d}$$

output
```
ln(cosh(d*x+c))/(a+b)/d+ln(tanh(d*x+c))/a/d-1/2*b*ln(a+b*tanh(d*x+c)^2)/a/
(a+b)/d
```

3.176.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{\log(\cosh(c+dx))}{a+b} + \frac{\log(\tanh(c+dx))}{a} - \frac{b\log\left(a+b\tanh^2(c+dx)\right)}{2a(a+b)}}{d}$$

input
```
Integrate[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(Log[Cosh[c + d*x]]/(a + b) + Log[Tanh[c + d*x]]/a - (b*Log[a + b*Tanh[c +
 d*x]^2])/(2*a*(a + b)))/d
```

3.176.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 4153, 26, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{i}{\tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\downarrow 26$$

$$i\int \frac{1}{\tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\downarrow 4153$$

$$\frac{i\int -\frac{i\coth(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\coth(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{d}$$

$$\downarrow 354$$

$$\frac{\int \frac{\coth(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)}d\tanh^2(c+dx)}{2d}$$

$$\downarrow 93$$

$$\frac{\int \left(-\frac{b^2}{a(a+b)\left(b\tanh^2(c+dx)+a\right)}+\frac{\coth(c+dx)}{a}-\frac{1}{(a+b)\left(\tanh^2(c+dx)-1\right)}\right)d\tanh^2(c+dx)}{2d}$$

$$\downarrow 2009$$

$$\frac{-\frac{\log\left(1-\tanh^2(c+dx)\right)}{a+b}-\frac{b\log\left(a+b\tanh^2(c+dx)\right)}{a(a+b)}+\frac{\log\left(\tanh^2(c+dx)\right)}{a}}{2d}$$

input
```
Int[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(Log[Tanh[c + d*x]^2]/a - Log[1 - Tanh[c + d*x]^2]/(a + b) - (b*Log[a + b*
Tanh[c + d*x]^2])/(a*(a + b)))/(2*d)
```

3.176.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.176.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

method	result	size
parallelrisch	$\frac{-b\ln\left(a+b\tanh(dx+c)^2\right)-2\ln(1-\tanh(dx+c))a+(2a+2b)\ln(\tanh(dx+c))-2adx}{2da(a+b)}$	65
derivativedivides	$-\frac{\frac{b\ln\left(a+b\tanh(dx+c)^2\right)}{2a(a+b)}+\frac{\ln(\tanh(dx+c)-1)}{2a+2b}-\frac{\ln(\tanh(dx+c))}{a}+\frac{\ln(\tanh(dx+c)+1)}{2a+2b}}{d}$	81
default	$-\frac{\frac{b\ln\left(a+b\tanh(dx+c)^2\right)}{2a(a+b)}+\frac{\ln(\tanh(dx+c)-1)}{2a+2b}-\frac{\ln(\tanh(dx+c))}{a}+\frac{\ln(\tanh(dx+c)+1)}{2a+2b}}{d}$	81
risch	$\frac{x}{a+b}-\frac{2x}{a}-\frac{2c}{da}+\frac{2bx}{a(a+b)}+\frac{2bc}{da(a+b)}+\frac{\ln\left(e^{2dx+2c}-1\right)}{da}-\frac{b\ln\left(e^{4dx+4c}+\frac{2(a-b)e^{2dx+2c}}{a+b}+1\right)}{2da(a+b)}$	117

input
```
int(coth(d*x+c)/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-b*ln(a+b*tanh(d*x+c)^2)-2*ln(1-tanh(d*x+c))*a+(2*a+2*b)*ln(tanh(d*x+
c))-2*a*d*x)/d/a/(a+b)
```

3.176.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(58) = 116.

Time = 0.31 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.97

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{2\,adx+b\log\left(\frac{2\left((a+b)\cosh(dx+c)^2+(a+b)\sinh(dx+c)^2+a-b\right)}{\cosh(dx+c)^2-2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right)-2\,(a+b)\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{2\,(a^2+ab)d}$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(2*a*d*x + b*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2
 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2)) - 2*(a + b)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/((a
^2 + a*b)*d)
```

3.176.6 Sympy [F]

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(coth(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

3.176.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.68

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{b\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^2+ab)d} + \frac{dx+c}{(a+b)d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{ad} + \frac{\log\left(e^{(-dx-c)}-1\right)}{ad}$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/2*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/
((a^2 + a*b)*d) + (d*x + c)/((a + b)*d) + log(e^(-d*x - c) + 1)/(a*d) + lo
g(e^(-d*x - c) - 1)/(a*d)
```

3.176.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{b\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{a^2+ab} + \frac{2\,(dx+c)}{a+b} - \frac{2\,\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{a}}{2\,d}$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/2*(b*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) -
2*b*e^(2*d*x + 2*c) + a + b)/(a^2 + a*b) + 2*(d*x + c)/(a + b) - 2*log(abs
(e^(2*d*x + 2*c) - 1))/a)/d
```

3.176.9 Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.23

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{\ln\left(12\,a\,b^2+4\,a^2\,b+9\,b^3-9\,b^3\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-12\,a\,b^2\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-4\,a^2\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\right)}{a\,d}$$
$$-\frac{b\,\ln\left(5\,a\,b+2\,a^2+3\,b^2+4\,a^2\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+2\,a^2\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}-6\,b^2\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+3\,b^2\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}+2\,a\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+5\,a\right.}{2\,d\,a^2+2\,b\,d\,a}$$
$$-\frac{x}{a+b}$$

input
```
int(coth(c + d*x)/(a + b*tanh(c + d*x)^2),x)
```

output
```
log(12*a*b^2 + 4*a^2*b + 9*b^3 - 9*b^3*exp(2*c)*exp(2*d*x) - 12*a*b^2*exp(
2*c)*exp(2*d*x) - 4*a^2*b*exp(2*c)*exp(2*d*x))/(a*d) - (b*log(5*a*b + 2*a^
2 + 3*b^2 + 4*a^2*exp(2*c)*exp(2*d*x) + 2*a^2*exp(4*c)*exp(4*d*x) - 6*b^2*
exp(2*c)*exp(2*d*x) + 3*b^2*exp(4*c)*exp(4*d*x) + 2*a*b*exp(2*c)*exp(2*d*x
) + 5*a*b*exp(4*c)*exp(4*d*x)))/(2*a^2*d + 2*a*b*d) - x/(a + b)
```

3.177 $\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.177.1 Optimal result

Integrand size = 23, antiderivative size = 60

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b} - \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a+b)d} - \frac{\coth(c+dx)}{ad}$$

output
```
x/(a+b)-b^(3/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/(a+b)/d-coth(d
*x+c)/a/d
```

3.177.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{c+dx}{(a+b)d} - \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a+b)d} - \frac{\coth(c+dx)}{ad}$$

input
```
Integrate[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(c + d*x)/((a + b)*d) - (b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/
(a^(3/2)*(a + b)*d) - Coth[c + d*x]/(a*d)
```

3.177.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 25, 4153, 25, 382, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\downarrow 4153$$

$$-\frac{\int -\frac{\coth^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{d}$$

$$\downarrow 25$$

$$\frac{\int \frac{\coth^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{d}$$

$$\downarrow 382$$

$$-\frac{\frac{\coth(c+dx)}{a}-\frac{\int \frac{b\tanh^2(c+dx)+a-b}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}}{d}$$

$$\downarrow 397$$

$$-\frac{\frac{\coth(c+dx)}{a}-\frac{\frac{a\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}-\frac{b^2\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{a}}{d}$$

$$\downarrow 218$$

$$-\frac{\frac{\coth(c+dx)}{a}-\frac{\frac{a\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}-\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a}}{d}$$

$$\downarrow 219$$

$$-\frac{-\frac{\coth(c+dx)}{a}-\frac{\frac{a\operatorname{arctanh}(\tanh(c+dx))}{a+b}-\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a}}{d}$$

input
```
Int[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
-((-((-((b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)
)) + (a*ArcTanh[Tanh[c + d*x]])/(a + b))/a) + Coth[c + d*x]/a)/d)
```

3.177.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.177.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.48

method	result	size
derivativedivides	$-\frac{\frac{1}{a\tanh(dx+c)}+\frac{b^2\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{a(a+b)\sqrt{ab}}-\frac{\ln(\tanh(dx+c)+1)}{2a+2b}+\frac{\ln(\tanh(dx+c)-1)}{2a+2b}}{d}$	89
default	$-\frac{\frac{1}{a\tanh(dx+c)}+\frac{b^2\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{a(a+b)\sqrt{ab}}-\frac{\ln(\tanh(dx+c)+1)}{2a+2b}+\frac{\ln(\tanh(dx+c)-1)}{2a+2b}}{d}$	89
risch	$\frac{x}{a+b}-\frac{2}{da\left(e^{2dx+2c}-1\right)}+\frac{\sqrt{-ab}\,b\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}-a+b}{a+b}\right)}{2a^2(a+b)d}-\frac{\sqrt{-ab}\,b\ln\left(e^{2dx+2c}+\frac{2\sqrt{-ab}+a-b}{a+b}\right)}{2a^2(a+b)d}$	131

input
```
int(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(1/a/tanh(d*x+c)+b^2/a/(a+b)/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(
1/2))-1/(2*a+2*b)*ln(tanh(d*x+c)+1)+1/(2*a+2*b)*ln(tanh(d*x+c)-1))
```

3.177.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(52) = 104.

Time = 0.31 (sec) , antiderivative size = 784, normalized size of antiderivative = 13.07

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$= \left[\frac{2\,adx\cosh(dx+c)^2 + 4\,adx\cosh(dx+c)\sinh(dx+c) + 2\,adx\sinh(dx+c)^2 - 2\,adx + \left(b\cosh(dx+c\right.}{} \right.$$

input
```
integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*d*x*cosh(d*x + c)^2 + 4*a*d*x*cosh(d*x + c)*sinh(d*x + c) + 2*a*
d*x*sinh(d*x + c)^2 - 2*a*d*x + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sin
h(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*co
sh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2
 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*
b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x +
c))*sinh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*
x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a)
)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a
+ b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x +
 c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh
(d*x + c))*sinh(d*x + c) + a + b)) - 4*a - 4*b)/((a^2 + a*b)*d*cosh(d*x +
c)^2 + 2*(a^2 + a*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*d*sinh(d*
x + c)^2 - (a^2 + a*b)*d), (a*d*x*cosh(d*x + c)^2 + 2*a*d*x*cosh(d*x + c)*
sinh(d*x + c) + a*d*x*sinh(d*x + c)^2 - a*d*x - (b*cosh(d*x + c)^2 + 2*b*c
osh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(b/a)*arctan(1/2*(
(a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*
sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) - 2*a - 2*b)/((a^2 + a*b)*d*cosh(d*x
 + c)^2 + 2*(a^2 + a*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*d*s...
```

3.177.6 Sympy [F]

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(coth(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(coth(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

3.177.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(52) = 104$.

Time = 0.35 (sec) , antiderivative size = 329, normalized size of antiderivative = 5.48

$$\begin{aligned}\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = &-\frac{b\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{4\,(a^2+ab)d}\\ &+\frac{b\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{4\,(a^2+ab)d}\\ &+\frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{4\,(a^2+ab)\sqrt{ab}d}\\ &-\frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{4\,(a^2+ab)\sqrt{ab}d}\\ &+\frac{b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}ad}\\ &+\frac{\log\left(e^{(2\,dx+2\,c)}-1\right)}{2\,ad}-\frac{\log\left(e^{(-2\,dx-2\,c)}-1\right)}{2\,ad}\\ &-\frac{1}{2\,(ae^{(2\,dx+2\,c)}-a)d}+\frac{3}{2\,(ae^{(-2\,dx-2\,c)}-a)d}\end{aligned}$$

input
```
integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((
a^2 + a*b)*d) + 1/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x -
 4*c) + a + b)/((a^2 + a*b)*d) + 1/4*(a*b - b^2)*arctan(1/2*((a + b)*e^(2*
d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + a*b)*sqrt(a*b)*d) - 1/4*(a*b - b^2)
*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^2 + a*b)*sqr
t(a*b)*d) + 1/2*b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))
/(sqrt(a*b)*a*d) + 1/2*log(e^(2*d*x + 2*c) - 1)/(a*d) - 1/2*log(e^(-2*d*x
- 2*c) - 1)/(a*d) - 1/2/((a*e^(2*d*x + 2*c) - a)*d) + 3/2/((a*e^(-2*d*x -
2*c) - a)*d)
```

3.177.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.48

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{b^2\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{(a^2+ab)\sqrt{ab}} - \frac{dx+c}{a+b} + \frac{2}{a\left(e^{(2\,dx+2\,c)}-1\right)}}{d}$$

input
```
integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-(b^2*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b)
)/((a^2 + a*b)*sqrt(a*b)) - (d*x + c)/(a + b) + 2/(a*(e^(2*d*x + 2*c) - 1)
))/d
```

3.177.9 Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 402, normalized size of antiderivative = 6.70

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b}$$
$$-\frac{\operatorname{atan}\left(\left(e^{2c}e^{2dx}\left(\frac{4b^2}{a\,d\,(a+b)^3\,(a^2+b\,a)\,\sqrt{b^3}}+\frac{\left(a^3\,d\,\sqrt{b^3}-a\,b^2\,d\,\sqrt{b^3}\right)(a-b)}{b^2\,(a+b)^2\,(a^2+b\,a)\,\sqrt{a^5\,d^2+2\,a^4\,b\,d^2+a^3\,b^2\,d^2}\,\sqrt{a^3\,d^2\,(a+b)^2}}\right)+\frac{(a-b)\left(a^3\right.}{b^2\,(a+b)^2\,(a^2+b\,a)}\right.}{\sqrt{a}}$$
$$-\frac{2}{a\,d\,\left(e^{2c+2dx}-1\right)}$$

input
```
int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2),x)
```

output
```
x/(a + b) - (atan((exp(2*c)*exp(2*d*x)*((4*b^2)/(a*d*(a + b)^3*(a*b + a^2)
*(b^3)^(1/2)) + ((a^3*d*(b^3)^(1/2) - a*b^2*d*(b^3)^(1/2))*(a - b))/(b^2*(
a + b)^2*(a*b + a^2)*(a^5*d^2 + 2*a^4*b*d^2 + a^3*b^2*d^2)^(1/2)*(a^3*d^2*
(a + b)^2)^(1/2))) + ((a - b)*(a^3*d*(b^3)^(1/2) + a*b^2*d*(b^3)^(1/2) + 2
*a^2*b*d*(b^3)^(1/2)))/(b^2*(a + b)^2*(a*b + a^2)*(a^5*d^2 + 2*a^4*b*d^2 +
 a^3*b^2*d^2)^(1/2)*(a^3*d^2*(a + b)^2)^(1/2)))*((a^3*(a^5*d^2 + 2*a^4*b*d
^2 + a^3*b^2*d^2)^(1/2))/2 + (a*b^2*(a^5*d^2 + 2*a^4*b*d^2 + a^3*b^2*d^2)^
(1/2))/2 + a^2*b*(a^5*d^2 + 2*a^4*b*d^2 + a^3*b^2*d^2)^(1/2)))*(b^3)^(1/2)
)/(a^5*d^2 + 2*a^4*b*d^2 + a^3*b^2*d^2)^(1/2) - 2/(a*d*(exp(2*c + 2*d*x) -
 1))
```

3.178 $\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.178.9 Mupad [B] (verification not implemented) 1323

3.178.1 Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\coth^2(c+dx)}{2ad} + \frac{\log(\cosh(c+dx))}{(a+b)d} + \frac{(a-b)\log(\tanh(c+dx))}{a^2d} + \frac{b^2\log\left(a+b\tanh^2(c+dx)\right)}{2a^2(a+b)d}$$

output
```
-1/2*coth(d*x+c)^2/a/d+ln(cosh(d*x+c))/(a+b)/d+(a-b)*ln(tanh(d*x+c))/a^2/d
+1/2*b^2*ln(a+b*tanh(d*x+c)^2)/a^2/(a+b)/d
```

3.178.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{\coth^2(c+dx)}{a} - \frac{b^2\log(b+a\coth^2(c+dx))}{a^2(a+b)} - \frac{2\log(\sinh(c+dx))}{a+b}}{2d}$$

input
```
Integrate[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
-1/2*(Coth[c + d*x]^2/a - (b^2*Log[b + a*Coth[c + d*x]^2])/(a^2*(a + b)) -
 (2*Log[Sinh[c + d*x]])/(a + b))/d
```

3.178.3 Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i}{\tan(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\downarrow 26$$

$$-i\int \frac{1}{\tan(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\downarrow 4153$$

$$-\frac{i\int \frac{i\coth^3(c+dx)}{(1-\tanh^2(c+dx))\,(b\tanh^2(c+dx)+a)}\,d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\coth^3(c+dx)}{(1-\tanh^2(c+dx))\,(b\tanh^2(c+dx)+a)}\,d\tanh(c+dx)}{d}$$

$$\downarrow 354$$

$$\frac{\int \frac{\coth^2(c+dx)}{(1-\tanh^2(c+dx))\,(b\tanh^2(c+dx)+a)}\,d\tanh^2(c+dx)}{2d}$$

$$\downarrow 93$$

$$\frac{\int \left(\frac{b^3}{a^2(a+b)\,(b\tanh^2(c+dx)+a)}+\frac{\coth^2(c+dx)}{a}+\frac{(a-b)\coth(c+dx)}{a^2}-\frac{1}{(a+b)\,(\tanh^2(c+dx)-1)}\right)d\tanh^2(c+dx)}{2d}$$

$$\downarrow 2009$$

$$\frac{\frac{b^2\log(a+b\tanh^2(c+dx))}{a^2(a+b)}+\frac{(a-b)\log(\tanh^2(c+dx))}{a^2}-\frac{\log(1-\tanh^2(c+dx))}{a+b}-\frac{\coth(c+dx)}{a}}{2d}$$

input
```
Int[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-(Coth[c + d*x]/a) + ((a - b)*Log[Tanh[c + d*x]^2])/a^2 - Log[1 - Tanh[c
+ d*x]^2]/(a + b) + (b^2*Log[a + b*Tanh[c + d*x]^2])/(a^2*(a + b)))/(2*d)
```

3.178.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.178.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{b^2\ln\left(a+b\tanh(dx+c)^2\right)-2\ln(1-\tanh(dx+c))a^2+\left(2a^2-2b^2\right)\ln(\tanh(dx+c))-\coth(dx+c)^2a(a+b)-2a^2dx}{2d\,a^2(a+b)}$
derivativedivides	$-\frac{-\frac{b^2\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)a^2}+\frac{(-a+b)\ln(\tanh(dx+c))}{a^2}+\frac{1}{2a\tanh(dx+c)^2}+\frac{\ln(\tanh(dx+c)+1)}{2a+2b}+\frac{\ln(\tanh(dx+c)-1)}{2a+2b}}{d}$
default	$-\frac{-\frac{b^2\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)a^2}+\frac{(-a+b)\ln(\tanh(dx+c))}{a^2}+\frac{1}{2a\tanh(dx+c)^2}+\frac{\ln(\tanh(dx+c)+1)}{2a+2b}+\frac{\ln(\tanh(dx+c)-1)}{2a+2b}}{d}$
risch	$\frac{x}{a+b}-\frac{2x}{a}-\frac{2c}{da}+\frac{2bx}{a^2}+\frac{2bc}{a^2d}-\frac{2b^2x}{a^2(a+b)}-\frac{2b^2c}{d\,a^2(a+b)}-\frac{2\,\mathrm{e}^{2dx+2c}}{da\left(\mathrm{e}^{2dx+2c}-1\right)^2}+\frac{\ln\left(\mathrm{e}^{2dx+2c}-1\right)}{da}-\frac{\ln\left(\mathrm{e}^{2dx+2c}\right.}{d\,a^2}$

input
```
int(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b^2*ln(a+b*tanh(d*x+c)^2)-2*ln(1-tanh(d*x+c))*a^2+(2*a^2-2*b^2)*ln(ta
nh(d*x+c))-coth(d*x+c)^2*a*(a+b)-2*a^2*d*x)/d/a^2/(a+b)
```

3.178.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 747 vs. $2(81) = 162$.

Time = 0.34 (sec) , antiderivative size = 747, normalized size of antiderivative = 8.79

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx =$$

$$-\frac{2\,a^2dx\cosh(dx+c)^4+8\,a^2dx\cosh(dx+c)\sinh(dx+c)^3+2\,a^2dx\sinh(dx+c)^4+2\,a^2dx-4\,(a^2dx-}{}$$

input
```
integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(2*a^2*d*x*cosh(d*x + c)^4 + 8*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3
+ 2*a^2*d*x*sinh(d*x + c)^4 + 2*a^2*d*x - 4*(a^2*d*x - a^2 - a*b)*cosh(d*x
 + c)^2 + 4*(3*a^2*d*x*cosh(d*x + c)^2 - a^2*d*x + a^2 + a*b)*sinh(d*x + c
)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sin
h(d*x + c)^4 - 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 - b^2)*sin
h(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x
+ c))*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(c
osh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*((a
^2 - b^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 +
(a^2 - b^2)*sinh(d*x + c)^4 - 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 -
b^2)*cosh(d*x + c)^2 - a^2 + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 -
b^2)*cosh(d*x + c)^3 - (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*sin
h(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(a^2*d*x*cosh(d*x + c)^3 -
 (a^2*d*x - a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + a^2*b)*d*cosh
(d*x + c)^4 + 4*(a^3 + a^2*b)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + a^2
*b)*d*sinh(d*x + c)^4 - 2*(a^3 + a^2*b)*d*cosh(d*x + c)^2 + 2*(3*(a^3 + a^
2*b)*d*cosh(d*x + c)^2 - (a^3 + a^2*b)*d)*sinh(d*x + c)^2 + (a^3 + a^2*b)*
d + 4*((a^3 + a^2*b)*d*cosh(d*x + c)^3 - (a^3 + a^2*b)*d*cosh(d*x + c))*si
nh(d*x + c))
```

3.178.6 Sympy [F]

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(coth(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(coth(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

3.178.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.87

$$\begin{aligned}\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = {} & \frac{b^2\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^3+a^2b)d} \\ & + \frac{dx+c}{(a+b)d} + \frac{2\,e^{(-2\,dx-2\,c)}}{(2\,ae^{(-2\,dx-2\,c)}-ae^{(-4\,dx-4\,c)}-a)d} \\ & + \frac{(a-b)\log\left(e^{(-dx-c)}+1\right)}{a^2d} + \frac{(a-b)\log\left(e^{(-dx-c)}-1\right)}{a^2d}\end{aligned}$$

input
```
integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*b^2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)
/((a^3 + a^2*b)*d) + (d*x + c)/((a + b)*d) + 2*e^(-2*d*x - 2*c)/((2*a*e^(-
2*d*x - 2*c) - a*e^(-4*d*x - 4*c) - a)*d) + (a - b)*log(e^(-d*x - c) + 1)/
(a^2*d) + (a - b)*log(e^(-d*x - c) - 1)/(a^2*d)
```

3.178.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.56

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{\frac{b^2\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{a^3+a^2b} - \frac{2\,(dx+c)}{a+b} + \frac{2\,(a-b)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{a^2} - \frac{4\,e^{(2\,dx+2\,c)}}{a\left(e^{(2\,dx+2\,c)}-1\right)^2}}{2\,d}$$

input
```
integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2*(b^2*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) -
 2*b*e^(2*d*x + 2*c) + a + b)/(a^3 + a^2*b) - 2*(d*x + c)/(a + b) + 2*(a -
 b)*log(abs(e^(2*d*x + 2*c) - 1))/a^2 - 4*e^(2*d*x + 2*c)/(a*(e^(2*d*x + 2
*c) - 1)^2))/d
```

3.178.9 Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.68

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{b^2\,\ln\left(3\,a\,b^2-2\,a^2\,b-2\,a^3+3\,b^3-4\,a^3\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-2\,a^3\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}-6\,b^3\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+3\,b^3\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}+6\,a\,b^2\,\mathrm{e}^{2\,c}\,\mathrm{e}\right.}{2\,d\,a^3+2\,b\,d\,a^2}$$
$$-\frac{x}{a+b}-\frac{2}{a\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$
$$+\frac{\ln\left(4\,a^4\,b+9\,b^5-12\,a^2\,b^3-9\,b^5\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-4\,a^4\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+12\,a^2\,b^3\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\right)\,(a-b)}{a^2\,d}$$
$$-\frac{2\,(a^2+b\,a)}{a^2\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)\,(a+b)}$$

input
```
int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2),x)
```

output
```
(b^2*log(3*a*b^2 - 2*a^2*b - 2*a^3 + 3*b^3 - 4*a^3*exp(2*c)*exp(2*d*x) - 2
*a^3*exp(4*c)*exp(4*d*x) - 6*b^3*exp(2*c)*exp(2*d*x) + 3*b^3*exp(4*c)*exp(
4*d*x) + 6*a*b^2*exp(2*c)*exp(2*d*x) + 4*a^2*b*exp(2*c)*exp(2*d*x) + 3*a*b
^2*exp(4*c)*exp(4*d*x) - 2*a^2*b*exp(4*c)*exp(4*d*x)))/(2*a^3*d + 2*a^2*b*
d) - x/(a + b) - 2/(a*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) + (lo
g(4*a^4*b + 9*b^5 - 12*a^2*b^3 - 9*b^5*exp(2*c)*exp(2*d*x) - 4*a^4*b*exp(2
*c)*exp(2*d*x) + 12*a^2*b^3*exp(2*c)*exp(2*d*x))*(a - b))/(a^2*d) - (2*(a*
b + a^2))/(a^2*d*(exp(2*c + 2*d*x) - 1)*(a + b))
```

3.179 $\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

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3.179.2 Mathematica [A] (verified) . . . 1325
3.179.3 Rubi [A] (verified) . . . 1326
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3.179.9 Mupad [B] (verification not implemented) . . . 1332

3.179.1 Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b} + \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a+b)d} - \frac{(a-b)\coth(c+dx)}{a^2 d} - \frac{\coth^3(c+dx)}{3ad}$$

output
```
x/(a+b)+b^(5/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/(a+b)/d-(a-b)*
coth(d*x+c)/a^2/d-1/3*coth(d*x+c)^3/a/d
```

3.179.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.11

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{6\left(c+dx+\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}}\right)}{a+b} - \frac{(-2a+3b+(4a-3b)\cosh(2(c+dx)))\coth(c+dx)\operatorname{csch}^2(c+dx)}{a^2}}{6d}$$

input
```
Integrate[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
((6*(c + d*x + (b^(5/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(5/2)))
/(a + b) - ((-2*a + 3*b + (4*a - 3*b)*Cosh[2*(c + d*x)])*Coth[c + d*x]*Csc
h[c + d*x]^2)/a^2)/(6*d)
```

3.179.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4153, 382, 27, 445, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\downarrow 4153$$

$$\frac{\int \frac{\coth^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{d}$$

$$\downarrow 382$$

$$\frac{\frac{\int \frac{3\coth^2(c+dx)\left(b\tanh^2(c+dx)+a-b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{3a} - \frac{\coth^3(c+dx)}{3a}}{d}$$

$$\downarrow 27$$

$$\frac{\frac{\int \frac{\coth^2(c+dx)\left(b\tanh^2(c+dx)+a-b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a} - \frac{\coth^3(c+dx)}{3a}}{d}$$

$$\downarrow 445$$

$$\frac{-\frac{\frac{\int -\frac{a^2-ba+b^2+(a-b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a} - \frac{(a-b)\coth(c+dx)}{a}}{a} - \frac{\coth^3(c+dx)}{3a}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{a^2-ba+b^2+(a-b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}-\frac{(a-b)\coth(c+dx)}{a}}{a}-\frac{\coth^3(c+dx)}{3a}}{d}$$

$$\downarrow 397$$

$$\frac{\frac{a^2\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{b^3\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{a}-\frac{(a-b)\coth(c+dx)}{a}}{a}-\frac{\coth^3(c+dx)}{3a}}{d}$$

$$\downarrow 218$$

$$\frac{\frac{\frac{a^2\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a}-\frac{(a-b)\coth(c+dx)}{a}}{a}-\frac{\coth^3(c+dx)}{3a}}{d}$$

$$\downarrow 219$$

$$\frac{\frac{\frac{a^2\text{arctanh}(\tanh(c+dx))}{a+b}+\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a}-\frac{(a-b)\coth(c+dx)}{a}}{a}-\frac{\coth^3(c+dx)}{3a}}{d}$$

input
```
Int[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-1/3*Coth[c + d*x]^3/a + (((b^(5/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a
]])/(Sqrt[a]*(a + b)) + (a^2*ArcTanh[Tanh[c + d*x]])/(a + b))/a - ((a - b)
*Coth[c + d*x])/a)/a)/d
```

3.179.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.179.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.33

method	result
derivativedivides	$-\frac{-\frac{b^3 \arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)a^2\sqrt{ab}}-\frac{-a+b}{a^2\tanh(dx+c)}+\frac{1}{3a\tanh(dx+c)^3}-\frac{\ln(\tanh(dx+c)+1)}{2a+2b}+\frac{\ln(\tanh(dx+c)-1)}{2a+2b}}{d}$
default	$-\frac{-\frac{b^3 \arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)a^2\sqrt{ab}}-\frac{-a+b}{a^2\tanh(dx+c)}+\frac{1}{3a\tanh(dx+c)^3}-\frac{\ln(\tanh(dx+c)+1)}{2a+2b}+\frac{\ln(\tanh(dx+c)-1)}{2a+2b}}{d}$
risch	$\frac{x}{a+b}-\frac{2\left(6a\,e^{4dx+4c}-3b\,e^{4dx+4c}-6\,e^{2dx+2c}a+6b\,e^{2dx+2c}+4a-3b\right)}{3d\,a^2\left(e^{2dx+2c}-1\right)^3}+\frac{\sqrt{-ab}\,b^2\ln\left(e^{2dx+2c}+\frac{2\sqrt{-ab}+a-b}{a+b}\right)}{2a^3(a+b)d}-\frac{\sqrt{-a}$

input
```
int(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-b^3/(a+b)/a^2/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2))-1/a^2*(
-a+b)/tanh(d*x+c)+1/3/a/tanh(d*x+c)^3-1/(2*a+2*b)*ln(tanh(d*x+c)+1)+1/(2*a
+2*b)*ln(tanh(d*x+c)-1))
```

3.179.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1023 vs. 2(72) = 144.

Time = 0.31 (sec) , antiderivative size = 2368, normalized size of antiderivative = 28.88

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/6*(6*a^2*d*x*cosh(d*x + c)^6 + 36*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^5
 + 6*a^2*d*x*sinh(d*x + c)^6 - 6*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(
d*x + c)^4 + 6*(15*a^2*d*x*cosh(d*x + c)^2 - 3*a^2*d*x - 4*a^2 - 2*a*b + 2
*b^2)*sinh(d*x + c)^4 - 6*a^2*d*x + 24*(5*a^2*d*x*cosh(d*x + c)^3 - (3*a^2
*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(3*a^2*d*
x + 4*a^2 - 4*b^2)*cosh(d*x + c)^2 + 6*(15*a^2*d*x*cosh(d*x + c)^4 + 3*a^2
*d*x - 6*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^2 + 4*a^2 - 4*b
^2)*sinh(d*x + c)^2 + 3*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*
x + c)^5 + b^2*sinh(d*x + c)^6 - 3*b^2*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x
 + c)^2 - b^2)*sinh(d*x + c)^4 + 3*b^2*cosh(d*x + c)^2 + 4*(5*b^2*cosh(d*x
 + c)^3 - 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^2*cosh(d*x + c)^4
- 6*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 - b^2 + 6*(b^2*cosh(d*x + c
)^5 - 2*b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)
*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2
)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*
sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)
^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x +
c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x +
c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh...
```

3.179.6 Sympy [F]

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(coth(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(coth(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)
```

3.179.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1038 vs. 2(72) = 144.

Time = 0.42 (sec) , antiderivative size = 1038, normalized size of antiderivative = 12.66

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/8*(a*b - b^2)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) +
 a + b)/((a^3 + a^2*b)*d) + 1/8*(a*b - b^2)*log(2*(a - b)*e^(-2*d*x - 2*c)
 + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^3 + a^2*b)*d) + 1/16*(a^2*b - 6*a
*b^2 + b^3)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^3
 + a^2*b)*sqrt(a*b)*d) - 1/16*(a^2*b - 6*a*b^2 + b^3)*arctan(1/2*((a + b)*e
^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^3 + a^2*b)*sqrt(a*b)*d) - 1/24*(3*
(12*a - b)*e^(4*d*x + 4*c) - 6*(9*a - b)*e^(2*d*x + 2*c) + 22*a - 3*b)/((a
^2*e^(6*d*x + 6*c) - 3*a^2*e^(4*d*x + 4*c) + 3*a^2*e^(2*d*x + 2*c) - a^2)*
d) - 1/6*(3*(4*a - b)*e^(4*d*x + 4*c) - 6*(2*a - b)*e^(2*d*x + 2*c) + 4*a
 - 3*b)/((a^2*e^(6*d*x + 6*c) - 3*a^2*e^(4*d*x + 4*c) + 3*a^2*e^(2*d*x + 2*
c) - a^2)*d) - 1/24*(6*(9*a - b)*e^(-2*d*x - 2*c) - 3*(12*a - b)*e^(-4*d*x
 - 4*c) - 22*a + 3*b)/((3*a^2*e^(-2*d*x - 2*c) - 3*a^2*e^(-4*d*x - 4*c) +
a^2*e^(-6*d*x - 6*c) - a^2)*d) - 1/6*(6*(2*a - b)*e^(-2*d*x - 2*c) - 3*(4*
a - b)*e^(-4*d*x - 4*c) - 4*a + 3*b)/((3*a^2*e^(-2*d*x - 2*c) - 3*a^2*e^(-
4*d*x - 4*c) + a^2*e^(-6*d*x - 6*c) - a^2)*d) + 1/4*(6*(a + b)*e^(-2*d*x -
 2*c) - 3*b*e^(-4*d*x - 4*c) - 2*a - 3*b)/((3*a^2*e^(-2*d*x - 2*c) - 3*a^2
*e^(-4*d*x - 4*c) + a^2*e^(-6*d*x - 6*c) - a^2)*d) + 1/4*b*log((a + b)*e^(
4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/(a^2*d) - 1/4*b*log(2*(a
 - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/(a^2*d) + 1/4*(
2*a - b)*log(e^(2*d*x + 2*c) - 1)/(a^2*d) - 1/2*b*log(e^(2*d*x + 2*c) -...
```

3.179.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(72) = 144.

Time = 0.35 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.79

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{\frac{3\,b^3\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{(a^3+a^2b)\sqrt{ab}} + \frac{3\,(dx+c)}{a+b} - \frac{2\left(6\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}+6\,be^{(2\,dx+2\,c)}+4\,a-3\,b\right)}{a^2\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

input
```
integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/3*(3*b^3*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt
(a*b))/((a^3 + a^2*b)*sqrt(a*b)) + 3*(d*x + c)/(a + b) - 2*(6*a*e^(4*d*x +
 4*c) - 3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) + 6*b*e^(2*d*x + 2*c) +
4*a - 3*b)/(a^2*(e^(2*d*x + 2*c) - 1)^3))/d
```

3.179.9 Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 519, normalized size of antiderivative = 6.33

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b}$$
$$+\frac{\operatorname{atan}\left(\left(e^{2c}e^{2dx}\left(\frac{4b^3}{a^2d(a+b)^3(a^3+ba^2)\sqrt{b^5}}+\frac{\left(a^4d\sqrt{b^5}-a^2b^2d\sqrt{b^5}\right)(a-b)}{b^3(a+b)^2(a^3+ba^2)\sqrt{a^7d^2+2a^6bd^2+a^5b^2d^2}\sqrt{a^5d^2(a+b)^2}}\right)+\frac{(a-b)}{b^3(a+b)^2(a^3+b}\right.\right.}{}$$
$$-\frac{8}{3ad\left(3e^{2c+2dx}-3e^{4c+4dx}+e^{6c+6dx}-1\right)}$$
$$-\frac{4(a^2+ba)}{a^2d(a+b)\left(e^{4c+4dx}-2e^{2c+2dx}+1\right)}-\frac{2(2a^2+ab-b^2)}{a^2d\left(e^{2c+2dx}-1\right)(a+b)}$$

input
```
int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2),x)
```

output
```
x/(a + b) + (atan((exp(2*c)*exp(2*d*x)*((4*b^3)/(a^2*d*(a + b)^3*(a^2*b +
a^3)*(b^5)^(1/2)) + ((a^4*d*(b^5)^(1/2) - a^2*b^2*d*(b^5)^(1/2))*(a - b))/
(b^3*(a + b)^2*(a^2*b + a^3)*(a^7*d^2 + 2*a^6*b*d^2 + a^5*b^2*d^2)^(1/2)*(
a^5*d^2*(a + b)^2)^(1/2))) + ((a - b)*(a^4*d*(b^5)^(1/2) + 2*a^3*b*d*(b^5)
^(1/2) + a^2*b^2*d*(b^5)^(1/2)))/(b^3*(a + b)^2*(a^2*b + a^3)*(a^7*d^2 + 2
*a^6*b*d^2 + a^5*b^2*d^2)^(1/2)*(a^5*d^2*(a + b)^2)^(1/2)))*((a^4*(a^7*d^2
 + 2*a^6*b*d^2 + a^5*b^2*d^2)^(1/2))/2 + a^3*b*(a^7*d^2 + 2*a^6*b*d^2 + a^
5*b^2*d^2)^(1/2) + (a^2*b^2*(a^7*d^2 + 2*a^6*b*d^2 + a^5*b^2*d^2)^(1/2))/2
))*(b^5)^(1/2))/(a^7*d^2 + 2*a^6*b*d^2 + a^5*b^2*d^2)^(1/2) - 8/(3*a*d*(3*
exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (4*(a*b +
 a^2))/(a^2*d*(a + b)*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (2*(a
*b + 2*a^2 - b^2))/(a^2*d*(exp(2*c + 2*d*x) - 1)*(a + b))
```

3.180 $\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

3.180.1 Optimal result 1333
3.180.2 Mathematica [A] (verified) 1333
3.180.3 Rubi [A] (verified) 1334
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3.180.8 Giac [B] (verification not implemented) 1338
3.180.9 Mupad [B] (verification not implemented) 1339

3.180.1 Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)^2 d} - \frac{a(a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{2b^2(a+b)^2 d} - \frac{a^2}{2b^2(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
ln(cosh(d*x+c))/(a+b)^2/d-1/2*a*(a+2*b)*ln(a+b*tanh(d*x+c)^2)/b^2/(a+b)^2/
d-1/2*a^2/b^2/(a+b)/d/(a+b*tanh(d*x+c)^2)
```

3.180.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{-2\log(\cosh(c+dx)) + \frac{a(a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{b^2} + \frac{a^2(a+b)}{b^2\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)^2 d}$$

input
```
Integrate[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-1/2*(-2*Log[Cosh[c + d*x]] + (a*(a + 2*b)*Log[a + b*Tanh[c + d*x]^2])/b^2
 + (a^2*(a + b))/(b^2*(a + b*Tanh[c + d*x]^2)))/((a + b)^2*d)
```

3.180.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\tan(ic+idx)^5}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\downarrow 26$$

$$-i\int \frac{\tan(ic+idx)^5}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\downarrow 4153$$

$$-\frac{i\int \frac{i\tanh^5(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\tanh^5(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 354$$

$$\frac{\int \frac{\tanh^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh^2(c+dx)}{2d}$$

$$\downarrow 99$$

$$\frac{\int \left(\frac{a^2}{b(a+b)\left(b\tanh^2(c+dx)+a\right)^2}-\frac{(a+2b)a}{b(a+b)^2\left(b\tanh^2(c+dx)+a\right)}-\frac{1}{(a+b)^2\left(\tanh^2(c+dx)-1\right)}\right)d\tanh^2(c+dx)}{2d}$$

$$\downarrow 2009$$

$$\frac{-\frac{a^2}{b^2(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{a(a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{b^2(a+b)^2}-\frac{\log\left(1-\tanh^2(c+dx)\right)}{(a+b)^2}}{2d}$$

input
```
Int[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-(Log[1 - Tanh[c + d*x]^2]/(a + b)^2) - (a*(a + 2*b)*Log[a + b*Tanh[c + d
*x]^2])/(b^2*(a + b)^2) - a^2/(b^2*(a + b)*(a + b*Tanh[c + d*x]^2)))/(2*d)
```

3.180.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.180.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{-\frac{a\left(\frac{(a+2b)\ln\left(a+b\tanh(dx+c)^2\right)}{b^2}+\frac{a(a+b)}{b^2\left(a+b\tanh(dx+c)^2\right)}\right)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}}{d}$
default	$\frac{-\frac{a\left(\frac{(a+2b)\ln\left(a+b\tanh(dx+c)^2\right)}{b^2}+\frac{a(a+b)}{b^2\left(a+b\tanh(dx+c)^2\right)}\right)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}}{d}$
parallelrisch	$-\frac{2x\tanh(dx+c)^2b^3d+2\ln\left(a+b\tanh(dx+c)^2\right)\tanh(dx+c)^2ab^2+\ln\left(a+b\tanh(dx+c)^2\right)a^3+a^3+a^2b+2ab^2dx+2\ln(1-}{2\left(a+b\tanh\right.}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{2x}{b^2}-\frac{2c}{b^2d}+\frac{2a^2x}{b^2(a^2+2ab+b^2)}+\frac{2a^2c}{b^2d(a^2+2ab+b^2)}+\frac{4ax}{b(a^2+2ab+b^2)}+\frac{4ac}{bd(a^2+2ab+b^2)}-\frac{}{bd(a+}$

input
```
int(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*a/(a+b)^2*((a+2*b)/b^2*ln(a+b*tanh(d*x+c)^2)+a*(a+b)/b^2/(a+b*ta
nh(d*x+c)^2))-1/2/(a+b)^2*ln(tanh(d*x+c)+1)-1/2/(a+b)^2*ln(tanh(d*x+c)-1))
```

3.180.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1141 vs. 2(79) = 158.

Time = 0.33 (sec) , antiderivative size = 1141, normalized size of antiderivative = 13.75

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(a*b^2 + b^3)*d*x*cosh(d*x + c)^4 + 8*(a*b^2 + b^3)*d*x*cosh(d*x +
 c)*sinh(d*x + c)^3 + 2*(a*b^2 + b^3)*d*x*sinh(d*x + c)^4 + 2*(a*b^2 + b^3
)*d*x + 4*(a^2*b + (a*b^2 - b^3)*d*x)*cosh(d*x + c)^2 + 4*(3*(a*b^2 + b^3)
*d*x*cosh(d*x + c)^2 + a^2*b + (a*b^2 - b^3)*d*x)*sinh(d*x + c)^2 + ((a^3
+ 3*a^2*b + 2*a*b^2)*cosh(d*x + c)^4 + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(d*
x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 2*a*b^2)*sinh(d*x + c)^4 + a^3 +
 3*a^2*b + 2*a*b^2 + 2*(a^3 + a^2*b - 2*a*b^2)*cosh(d*x + c)^2 + 2*(a^3 +
a^2*b - 2*a*b^2 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(d*x + c)^2)*sinh(d*x +
c)^2 + 4*((a^3 + 3*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + (a^3 + a^2*b - 2*a*b
^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)
*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c)
 + sinh(d*x + c)^2)) - 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4
+ 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3
+ 2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)^2 + 2*(a^3 + a^2*b - a*b^2 -
 b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + a^2*b - a*b^
2 - b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c)
- sinh(d*x + c))) + 8*((a*b^2 + b^3)*d*x*cosh(d*x + c)^3 + (a^2*b + (a*b^2
 - b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^3*b^2 + 3*a^2*b^3 + 3*a*...
```

3.180.6 Sympy [A] (verification not implemented)

Time = 74.91 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{a^2\left(\begin{cases}\frac{\tanh^2(c+dx)}{a^2} & \text{for } b=0\\ -\frac{1}{b(a+b\tanh^2(c+dx))} & \text{otherwise}\end{cases}\right)}{2bd\,(a+b)} - \frac{a(a+2b)\left(\begin{cases}\frac{\tanh^2(c+dx)}{a} & \text{for } b=0\\ \frac{\log\left(a+b\tanh^2(c+dx)\right)}{b} & \text{otherwise}\end{cases}\right)}{2bd\,(a+b)^2} - \frac{\log\left(\tanh^2(c+dx)-1\right)}{2d\,(a+b)^2}$$

input
```
integrate(tanh(d*x+c)**5/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
a**2*Piecewise((tanh(c + d*x)**2/a**2, Eq(b, 0)), (-1/(b*(a + b*tanh(c + d
*x)**2)), True))/(2*b*d*(a + b)) - a*(a + 2*b)*Piecewise((tanh(c + d*x)**2
/a, Eq(b, 0)), (log(a + b*tanh(c + d*x)**2)/b, True))/(2*b*d*(a + b)**2) -
 log(tanh(c + d*x)**2 - 1)/(2*d*(a + b)**2)
```

3.180.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(79) = 158$.

Time = 0.30 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.61

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx =$$
$$-\frac{2\,a^2 e^{(-2\,dx-2\,c)}}{(a^3b+3\,a^2b^2+3\,ab^3+b^4+2\,(a^3b+a^2b^2-ab^3-b^4)e^{(-2\,dx-2\,c)}+(a^3b+3\,a^2b^2+3\,ab^3+b^4)e^{(-4\,dx-4\,c)}}$$
$$-\frac{(a^2+2\,ab)\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^2b^2+2\,ab^3+b^4)d}$$
$$+\frac{dx+c}{(a^2+2\,ab+b^2)d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{b^2d}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-2*a^2*e^(-2*d*x - 2*c)/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4 + 2*(a^3*b + a
^2*b^2 - a*b^3 - b^4)*e^(-2*d*x - 2*c) + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^
4)*e^(-4*d*x - 4*c))*d) - 1/2*(a^2 + 2*a*b)*log(2*(a - b)*e^(-2*d*x - 2*c)
 + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^2*b^2 + 2*a*b^3 + b^4)*d) + (d*x
+ c)/((a^2 + 2*a*b + b^2)*d) + log(e^(-2*d*x - 2*c) + 1)/(b^2*d)
```

3.180.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(79) = 158$.

Time = 0.43 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.34

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx =$$
$$-\frac{\frac{(a^2+2\,ab)\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{a^2b^2+2\,ab^3+b^4}+\frac{2\,(dx+c)}{a^2+2\,ab+b^2}+\frac{4\,a^2e^{(2\,dx+2\,c)}}{(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2}}{2\,d}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((a^2 + 2*a*b)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d
*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^2*b^2 + 2*a*b^3 + b^4) + 2*(d*
x + c)/(a^2 + 2*a*b + b^2) + 4*a^2*e^(2*d*x + 2*c)/((a*e^(4*d*x + 4*c) + b
*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*(a +
 b)^2*b) - 2*log(e^(2*d*x + 2*c) + 1)/b^2)/d
```

3.180.9 Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.05

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= -\frac{a^2}{2\left(d\,a^2\,b^2+d\,a\,b^3\tanh\left(c+d\,x\right)^2+d\,a\,b^3+d\,b^4\tanh\left(c+d\,x\right)^2\right)}$$
$$-\frac{\ln\left(\tanh(c+d\,x)^2-1\right)}{2\left(d\,a^2+2\,d\,a\,b+d\,b^2\right)}-\frac{a^2\,\ln\left(b\tanh(c+d\,x)^2+a\right)}{2\left(d\,a^2\,b^2+2\,d\,a\,b^3+d\,b^4\right)}-\frac{a\,b\,\ln\left(b\tanh(c+d\,x)^2+a\right)}{d\,a^2\,b^2+2\,d\,a\,b^3+d\,b^4}$$

input
```
int(tanh(c + d*x)^5/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
- a^2/(2*(a^2*b^2*d + b^4*d*tanh(c + d*x)^2 + a*b^3*d + a*b^3*d*tanh(c + d
*x)^2)) - log(tanh(c + d*x)^2 - 1)/(2*(a^2*d + b^2*d + 2*a*b*d)) - (a^2*lo
g(a + b*tanh(c + d*x)^2))/(2*(b^4*d + a^2*b^2*d + 2*a*b^3*d)) - (a*b*log(a
 + b*tanh(c + d*x)^2))/(b^4*d + a^2*b^2*d + 2*a*b^3*d)
```

3.181 $\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

3.181.1 Optimal result 1340
3.181.2 Mathematica [A] (verified) 1340
3.181.3 Rubi [A] (verified) 1341
3.181.4 Maple [A] (verified) 1343
3.181.5 Fricas [B] (verification not implemented) 1343
3.181.6 Sympy [F(-1)] 1344
3.181.7 Maxima [B] (verification not implemented) 1345
3.181.8 Giac [B] (verification not implemented) 1345
3.181.9 Mupad [B] (verification not implemented) 1346

3.181.1 Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{x}{(a+b)^2} - \frac{\sqrt{a}(a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2b^{3/2}(a+b)^2 d} + \frac{a\tanh(c+dx)}{2b(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
x/(a+b)^2-1/2*(a+3*b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*a^(1/2)/b^(3/2)/
(a+b)^2/d+1/2*a*tanh(d*x+c)/b/(a+b)/d/(a+b*tanh(d*x+c)^2)
```

3.181.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{2(c+dx) - \frac{\sqrt{a}(a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}} + \frac{a(a+b)\sinh(2(c+dx))}{b(a-b+(a+b)\cosh(2(c+dx)))}}{2(a+b)^2 d}$$

input
```
Integrate[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(2*(c + d*x) - (Sqrt[a]*(a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])
/b^(3/2) + (a*(a + b)*Sinh[2*(c + d*x)])/(b*(a - b + (a + b)*Cosh[2*(c + d
*x)])))/(2*(a + b)^2*d)
```

3.181.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4153, 372, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(ic+idx)^4}{\left(a-b\tan(ic+idx)^2\right)^2}\,dx$$

$$\downarrow 4153$$

$$\frac{\int \frac{\tanh^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}\,d\tanh(c+dx)}{d}$$

$$\downarrow 372$$

$$\frac{\frac{a\tanh(c+dx)}{2b(a+b)\left(a+b\tanh^2(c+dx)\right)} - \frac{\int \frac{a-(a+2b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}\,d\tanh(c+dx)}{2b(a+b)}}{d}$$

$$\downarrow 397$$

$$\frac{\frac{a\tanh(c+dx)}{2b(a+b)\left(a+b\tanh^2(c+dx)\right)} - \frac{\frac{a(a+3b)\int \frac{1}{b\tanh^2(c+dx)+a}\,d\tanh(c+dx)}{a+b} - \frac{2b\int \frac{1}{1-\tanh^2(c+dx)}\,d\tanh(c+dx)}{a+b}}{2b(a+b)}}{d}$$

$$\downarrow 218$$

$$\frac{\frac{a\tanh(c+dx)}{2b(a+b)\left(a+b\tanh^2(c+dx)\right)} - \frac{\frac{\sqrt{a}(a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)} - \frac{2b\int \frac{1}{1-\tanh^2(c+dx)}\,d\tanh(c+dx)}{a+b}}{2b(a+b)}}{d}$$

$$\downarrow 219$$

$$\frac{\frac{a\tanh(c+dx)}{2b(a+b)(a+b\tanh^2(c+dx))}-\frac{\frac{\sqrt{a}(a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)}-\frac{2b\operatorname{arctanh}(\tanh(c+dx))}{a+b}}{2b(a+b)}}{d}$$

input
```
Int[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-1/2*((Sqrt[a]*(a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[b
]*(a + b)) - (2*b*ArcTanh[Tanh[c + d*x]])/(a + b))/(b*(a + b)) + (a*Tanh[c
 + d*x])/(2*b*(a + b)*(a + b*Tanh[c + d*x]^2)))/d
```

3.181.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.181.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{a\left(-\frac{(a+b)\tanh(dx+c)}{2b\left(a+b\tanh(dx+c)^2\right)}+\frac{(a+3b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2b\sqrt{ab}}\right)}{(a+b)^2}+\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}}{d}$
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{a\left(-\frac{(a+b)\tanh(dx+c)}{2b\left(a+b\tanh(dx+c)^2\right)}+\frac{(a+3b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2b\sqrt{ab}}\right)}{(a+b)^2}+\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}}{d}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{a\left(\mathrm{e}^{2dx+2c}a-b\,\mathrm{e}^{2dx+2c}+a+b\right)}{d(a+b)^2b\left(a\,\mathrm{e}^{4dx+4c}+b\,\mathrm{e}^{4dx+4c}+2\,\mathrm{e}^{2dx+2c}a-2b\,\mathrm{e}^{2dx+2c}+a+b\right)}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2dx+2c}-\frac{2\sqrt{-ab}-a+b}{a+b}\right)a}{4b^2(a+b)^2d}$

input
```
int(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/(a+b)^2*ln(tanh(d*x+c)-1)-a/(a+b)^2*(-1/2*(a+b)/b*tanh(d*x+c)/(a
+b*tanh(d*x+c)^2)+1/2*(a+3*b)/b/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/
2)))+1/2/(a+b)^2*ln(tanh(d*x+c)+1))
```

3.181.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(77) = 154$.

Time = 0.32 (sec) , antiderivative size = 1950, normalized size of antiderivative = 21.91

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*(a*b + b^2)*d*x*cosh(d*x + c)^4 + 16*(a*b + b^2)*d*x*cosh(d*x + c)
*sinh(d*x + c)^3 + 4*(a*b + b^2)*d*x*sinh(d*x + c)^4 + 4*(a*b + b^2)*d*x +
 4*(2*(a*b - b^2)*d*x - a^2 + a*b)*cosh(d*x + c)^2 + 4*(6*(a*b + b^2)*d*x*
cosh(d*x + c)^2 + 2*(a*b - b^2)*d*x - a^2 + a*b)*sinh(d*x + c)^2 + ((a^2 +
 4*a*b + 3*b^2)*cosh(d*x + c)^4 + 4*(a^2 + 4*a*b + 3*b^2)*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^2 + 4*a*b + 3*b^2)*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b - 3
*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 4*a*b + 3*b^2)*cosh(d*x + c)^2 + a^2 +
 2*a*b - 3*b^2)*sinh(d*x + c)^2 + a^2 + 4*a*b + 3*b^2 + 4*((a^2 + 4*a*b +
3*b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c
))*sqrt(-a/b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 +
 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2
+ a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*
cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a*b + b^2
)*cosh(d*x + c)^2 + 2*(a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b + b^2
)*sinh(d*x + c)^2 + a*b - b^2)*sqrt(-a/b))/((a + b)*cosh(d*x + c)^4 + 4*(a
 + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*
cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 +
4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)
) - 4*a^2 - 4*a*b + 8*(2*(a*b + b^2)*d*x*cosh(d*x + c)^3 + (2*(a*b - b^...
```

3.181.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

3.181.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1010 vs. 2(77) = 154.

Time = 0.52 (sec) , antiderivative size = 1010, normalized size of antiderivative = 11.35

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/32*(a^3 + 9*a^2*b - 9*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(2*d*x + 2*c)
+ a - b)/sqrt(a*b))/((a^3*b + 2*a^2*b^2 + a*b^3)*sqrt(a*b)*d) + 1/32*(a^3
+ 9*a^2*b - 9*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/s
qrt(a*b))/((a^3*b + 2*a^2*b^2 + a*b^3)*sqrt(a*b)*d) - 1/16*(a^3 - 5*a^2*b
- 5*a*b^2 + b^3 + (a^3 - 15*a^2*b + 15*a*b^2 - b^3)*e^(2*d*x + 2*c))/((a^4
*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + (a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^
4)*e^(4*d*x + 4*c) + 2*(a^4*b + a^3*b^2 - a^2*b^3 - a*b^4)*e^(2*d*x + 2*c)
)*d) + 1/16*(a^3 - 5*a^2*b - 5*a*b^2 + b^3 + (a^3 - 15*a^2*b + 15*a*b^2 -
b^3)*e^(-2*d*x - 2*c))/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + 2*(a^4*b
+ a^3*b^2 - a^2*b^3 - a*b^4)*e^(-2*d*x - 2*c) + (a^4*b + 3*a^3*b^2 + 3*a^2
*b^3 + a*b^4)*e^(-4*d*x - 4*c))*d) - 1/4*(a^2 - b^2 + (a^2 - 6*a*b + b^2)*
e^(2*d*x + 2*c))/((a^3*b + 2*a^2*b^2 + a*b^3 + (a^3*b + 2*a^2*b^2 + a*b^3)
*e^(4*d*x + 4*c) + 2*(a^3*b - a*b^3)*e^(2*d*x + 2*c))*d) + 1/4*(a^2 - b^2
+ (a^2 - 6*a*b + b^2)*e^(-2*d*x - 2*c))/((a^3*b + 2*a^2*b^2 + a*b^3 + 2*(a
^3*b - a*b^3)*e^(-2*d*x - 2*c) + (a^3*b + 2*a^2*b^2 + a*b^3)*e^(-4*d*x - 4
*c))*d) + 3/8*((a - b)*e^(-2*d*x - 2*c) + a + b)/((a^2*b + a*b^2 + 2*(a^2*
b - a*b^2)*e^(-2*d*x - 2*c) + (a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d) + 1/4*l
og((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^2 + 2*
a*b + b^2)*d) - 1/4*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4
*c) + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/8*(a + b)*arctan(1/2*((a + b)*...
```

3.181.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(77) = 154.

Time = 0.38 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.19

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx =$$

$$-\frac{\frac{(a^2+3\,ab)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{(a^2b+2\,ab^2+b^3)\sqrt{ab}} - \frac{2\,(dx+c)}{a^2+2\,ab+b^2} + \frac{2\left(a^2e^{(2\,dx+2\,c)}-abe^{(2\,dx+2\,c)}+a^2+ab\right)}{(a^2b+2\,ab^2+b^3)\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2}\right.}}{2\,d}$$

input
```
integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((a^2 + 3*a*b)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a
- b)/sqrt(a*b))/((a^2*b + 2*a*b^2 + b^3)*sqrt(a*b)) - 2*(d*x + c)/(a^2 + 2
*a*b + b^2) + 2*(a^2*e^(2*d*x + 2*c) - a*b*e^(2*d*x + 2*c) + a^2 + a*b)/((
a^2*b + 2*a*b^2 + b^3)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d
*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)))/d
```

3.181.9 Mupad [B] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 1655, normalized size of antiderivative = 18.60

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(tanh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
log(tanh(c + d*x) + 1)/(2*a^2*d + 2*b^2*d + 4*a*b*d) - log(tanh(c + d*x) -
 1)/(2*d*(a + b)^2) - (atan((((a + 3*b)*(-a*b^3)^(1/2)*((tanh(c + d*x)*(6*
a^3*b + a^4 + 4*b^4 + 9*a^2*b^2))/(2*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2))
+ ((a + 3*b)*(-a*b^3)^(1/2)*((2*a*b^6*d^2 + 8*a^2*b^5*d^2 + 12*a^3*b^4*d^2
 + 8*a^4*b^3*d^2 + 2*a^5*b^2*d^2)/(b^4*d^3 + 3*a*b^3*d^3 + a^3*b*d^3 + 3*a
^2*b^2*d^3) - (tanh(c + d*x)*(a + 3*b)*(-a*b^3)^(1/2)*(16*b^8*d^2 + 48*a*b
^7*d^2 + 32*a^2*b^6*d^2 - 32*a^3*b^5*d^2 - 48*a^4*b^4*d^2 - 16*a^5*b^3*d^2
))/(8*(b^5*d + a^2*b^3*d + 2*a*b^4*d)*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2))
))/(4*(b^5*d + a^2*b^3*d + 2*a*b^4*d)))*1i)/(4*(b^5*d + a^2*b^3*d + 2*a*b^
4*d)) + ((a + 3*b)*(-a*b^3)^(1/2)*((tanh(c + d*x)*(6*a^3*b + a^4 + 4*b^4 +
 9*a^2*b^2))/(2*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2)) - ((a + 3*b)*(-a*b^3)
^(1/2)*((2*a*b^6*d^2 + 8*a^2*b^5*d^2 + 12*a^3*b^4*d^2 + 8*a^4*b^3*d^2 + 2*
a^5*b^2*d^2)/(b^4*d^3 + 3*a*b^3*d^3 + a^3*b*d^3 + 3*a^2*b^2*d^3) + (tanh(c
 + d*x)*(a + 3*b)*(-a*b^3)^(1/2)*(16*b^8*d^2 + 48*a*b^7*d^2 + 32*a^2*b^6*d
^2 - 32*a^3*b^5*d^2 - 48*a^4*b^4*d^2 - 16*a^5*b^3*d^2))/(8*(b^5*d + a^2*b^
3*d + 2*a*b^4*d)*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2))))/(4*(b^5*d + a^2*b^
3*d + 2*a*b^4*d)))*1i)/(4*(b^5*d + a^2*b^3*d + 2*a*b^4*d)))/((3*a*b^2 + (5
*a^2*b)/2 + a^3/2)/(b^4*d^3 + 3*a*b^3*d^3 + a^3*b*d^3 + 3*a^2*b^2*d^3) - (
(a + 3*b)*(-a*b^3)^(1/2)*((tanh(c + d*x)*(6*a^3*b + a^4 + 4*b^4 + 9*a^2*b^
2))/(2*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2)) + ((a + 3*b)*(-a*b^3)^(1/2)...
```

3.182 $\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

3.182.1 Optimal result 1347
3.182.2 Mathematica [A] (verified) 1347
3.182.3 Rubi [A] (verified) 1348
3.182.4 Maple [A] (verified) 1350
3.182.5 Fricas [B] (verification not implemented) 1350
3.182.6 Sympy [A] (verification not implemented) 1351
3.182.7 Maxima [B] (verification not implemented) 1352
3.182.8 Giac [B] (verification not implemented) 1352
3.182.9 Mupad [B] (verification not implemented) 1353

3.182.1 Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)^2 d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^2 d} + \frac{a}{2b(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
ln(cosh(d*x+c))/(a+b)^2/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^2/d+1/2*a/b/(a+b
)/d/(a+b*tanh(d*x+c)^2)
```

3.182.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{2\log(\cosh(c+dx)) + \log\left(a+b\tanh^2(c+dx)\right) + \frac{a(a+b)}{b\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)^2 d}$$

input
```
Integrate[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(2*Log[Cosh[c + d*x]] + Log[a + b*Tanh[c + d*x]^2] + (a*(a + b))/(b*(a + b
*Tanh[c + d*x]^2)))/(2*(a + b)^2*d)
```

3.182.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{i\tan(ic+idx)^3}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\downarrow 26$$

$$i\int \frac{\tan(ic+idx)^3}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\downarrow 4153$$

$$\frac{i\int -\frac{i\tanh^3(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\tanh^3(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 354$$

$$\frac{\int \frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh^2(c+dx)}{2d}$$

$$\downarrow 86$$

$$\frac{\int \left(-\frac{a}{(a+b)\left(b\tanh^2(c+dx)+a\right)^2}-\frac{1}{(a+b)^2\left(\tanh^2(c+dx)-1\right)}+\frac{b}{(a+b)^2\left(b\tanh^2(c+dx)+a\right)}\right)d\tanh^2(c+dx)}{2d}$$

$$\downarrow 2009$$

$$\frac{\frac{a}{b(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\log\left(1-\tanh^2(c+dx)\right)}{(a+b)^2}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{(a+b)^2}}{2d}$$

input
```
Int[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-(Log[1 - Tanh[c + d*x]^2]/(a + b)^2) + Log[a + b*Tanh[c + d*x]^2]/(a + b
)^2 + a/(b*(a + b)*(a + b*Tanh[c + d*x]^2)))/(2*d)
```

3.182.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.182.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{-\frac{a(a+b)}{b\left(a+b\tanh(dx+c)^2\right)}-\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)^2}}{d}$
default	$\frac{-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{-\frac{a(a+b)}{b\left(a+b\tanh(dx+c)^2\right)}-\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)^2}}{d}$
parallelrisch	$-\frac{2abdx-ab+2\ln(1-\tanh(dx+c))ab-a^2-b^2\ln\left(a+b\tanh(dx+c)^2\right)\tanh(dx+c)^2-\ln\left(a+b\tanh(dx+c)^2\right)ab+2x\tanh(}{2\left(a+b\tanh(dx+c)^2\right)d(a+b)^2b}$
risch	$-\frac{x}{a^2+2ab+b^2}-\frac{2c}{d(a^2+2ab+b^2)}+\frac{2a\,e^{2dx+2c}}{d(a+b)^2\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)}+\frac{\ln\left(e^{4dx+4c}+}{2d(a^2}$

input
```
int(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/(a+b)^2*ln(tanh(d*x+c)+1)-1/2/(a+b)^2*ln(tanh(d*x+c)-1)-1/2/(a+b
)^2*(-a*(a+b)/b/(a+b*tanh(d*x+c)^2)-ln(a+b*tanh(d*x+c)^2)))
```

3.182.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. 2(68) = 136.

Time = 0.29 (sec) , antiderivative size = 629, normalized size of antiderivative = 8.74

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx =$$

$$-\frac{2\,(a+b)dx\cosh(dx+c)^4+8\,(a+b)dx\cosh(dx+c)\sinh(dx+c)^3+2\,(a+b)dx\sinh(dx+c)^4+2\,(a}{2\left((a^3+3\,a^2b+}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(a + b)*d*x*cosh(d*x + c)^4 + 8*(a + b)*d*x*cosh(d*x + c)*sinh(d*x
 + c)^3 + 2*(a + b)*d*x*sinh(d*x + c)^4 + 2*(a + b)*d*x + 4*((a - b)*d*x -
 a)*cosh(d*x + c)^2 + 4*(3*(a + b)*d*x*cosh(d*x + c)^2 + (a - b)*d*x - a)*
sinh(d*x + c)^2 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(
d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a
 + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^
3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*log(2*((a + b)*cosh(d*x
+ c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 8*((a + b)*d*x*cosh(d*x + c)^3 + ((
a - b)*d*x - a)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*d*cosh(d*x + c)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)
*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*sinh(d*x + c)^4 + 2*(
a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*cosh(d*x + c)^2 + (a^3 + a^2*b - a*b^2 - b^3)*d)*sinh(d*x + c)
^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3
)*d*cosh(d*x + c)^3 + (a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*
x + c))
```

3.182.6 Sympy [A] (verification not implemented)

Time = 42.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{a\left(\begin{cases}\frac{\tanh^2(c+dx)}{a^2} & \text{for } b=0\\ -\frac{1}{b(a+b\tanh^2(c+dx))} & \text{otherwise}\end{cases}\right)}{2d(a+b)} + \frac{b\left(\begin{cases}\frac{\tanh^2(c+dx)}{a} & \text{for } b=0\\ \frac{\log\left(a+b\tanh^2(c+dx)\right)}{b} & \text{otherwise}\end{cases}\right)}{2d(a+b)^2} - \frac{\log\left(\tanh^2(c+dx)-1\right)}{2d(a+b)^2}$$

input
```
integrate(tanh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
-a*Piecewise((tanh(c + d*x)**2/a**2, Eq(b, 0)), (-1/(b*(a + b*tanh(c + d*x
)**2)), True))/(2*d*(a + b)) + b*Piecewise((tanh(c + d*x)**2/a, Eq(b, 0)),
 (log(a + b*tanh(c + d*x)**2)/b, True))/(2*d*(a + b)**2) - log(tanh(c + d*
x)**2 - 1)/(2*d*(a + b)**2)
```

3.182.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(68) = 136$.

Time = 0.22 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.36

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{2\,a e^{(-2\,dx-2\,c)}}{(a^3+3\,a^2b+3\,ab^2+b^3+2\,(a^3+a^2b-ab^2-b^3)e^{(-2\,dx-2\,c)}+(a^3+3\,a^2b+3\,ab^2+b^3)e^{(-4\,dx-4\,c)})d} + \frac{dx+c}{(a^2+2\,ab+b^2)d} + \frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^2+2\,ab+b^2)d}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
2*a*e^(-2*d*x - 2*c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2*b - a*
b^2 - b^3)*e^(-2*d*x - 2*c) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-4*d*x -
4*c))*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d) + 1/2*log(2*(a - b)*e^(-2*d*x
 - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d)
```

3.182.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(68) = 136$.

Time = 0.36 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.07

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{\frac{\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{a^2+2\,ab+b^2} - \frac{e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2}{\left(a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right)}}{2\,d}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*(log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) +
 e^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^2 + 2*a*b + b^2) - (e^(2*d*x + 2*c) +
e^(-2*d*x - 2*c) - 2)/((a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d
*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2*b)*(a + b)))/d
```

3.182.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.92

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx =$$
$$-\frac{-a^2+ab\left(-1+\operatorname{atan}\left(\frac{a\tanh(c+dx)^2\,1\mathrm{i}+b\tanh(c+dx)^2\,1\mathrm{i}}{2a-a\tanh(c+dx)^2+b\tanh(c+dx)^2}\right)2\mathrm{i}\right)+b^2\tanh(c+dx)^2\operatorname{atan}\left(\frac{a\tanh(c+dx)^2\,1\mathrm{i}+b\tanh(c+dx)^2\,1\mathrm{i}}{2a-a\tanh(c+dx)^2+b\tanh(c+dx)^2}\right)2\mathrm{i}}{2\,d\,a^3\,b+2\,d\,a^2\,b^2\tanh\left(c+dx\right)^2+4\,d\,a^2\,b^2+4\,d\,a\,b^3\tanh\left(c+dx\right)^2+2\,d\,a\,b^3+2\,d\,b^4\tanh\left(c+dx\right)^2+2\,d\,b^4}$$

input
```
int(tanh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
-(a*b*(atan((a*tanh(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/(2*a - a*tanh(c
+ d*x)^2 + b*tanh(c + d*x)^2))*2i - 1) - a^2 + b^2*tanh(c + d*x)^2*atan((a
*tanh(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/(2*a - a*tanh(c + d*x)^2 + b*t
anh(c + d*x)^2))*2i)/(4*a^2*b^2*d + 2*b^4*d*tanh(c + d*x)^2 + 2*a*b^3*d +
2*a^3*b*d + 2*a^2*b^2*d*tanh(c + d*x)^2 + 4*a*b^3*d*tanh(c + d*x)^2)
```

3.183 $\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

3.183.1 Optimal result 1354
3.183.2 Mathematica [A] (verified) 1354
3.183.3 Rubi [A] (verified) 1355
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3.183.5 Fricas [B] (verification not implemented) 1358
3.183.6 Sympy [F(-1)] 1358
3.183.7 Maxima [B] (verification not implemented) 1359
3.183.8 Giac [B] (verification not implemented) 1360
3.183.9 Mupad [B] (verification not implemented) 1361

3.183.1 Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{x}{(a+b)^2} - \frac{(a-b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}(a+b)^2 d} - \frac{\tanh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
x/(a+b)^2-1/2*(a-b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/(a+b)^2/d/a^(1/2)/
b^(1/2)-1/2*tanh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c)^2)
```

3.183.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{2(c+dx) + \frac{(-a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} - \frac{(a+b)\sinh(2(c+dx))}{a-b+(a+b)\cosh(2(c+dx))}}{2(a+b)^2 d}$$

input
```
Integrate[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(2*(c + d*x) + ((-a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]
*Sqrt[b]) - ((a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]
))/(2*(a + b)^2*d)
```

3.183.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 25, 4153, 25, 373, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\tan(ic+idx)^2}{\left(a-b\tan(ic+idx)^2\right)^2}\,dx$$

$$\downarrow 25$$

$$-\int \frac{\tan(ic+idx)^2}{\left(a-b\tan(ic+idx)^2\right)^2}\,dx$$

$$\downarrow 4153$$

$$-\frac{\int -\frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}\,d\tanh(c+dx)}{d}$$

$$\downarrow 25$$

$$\frac{\int \frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}\,d\tanh(c+dx)}{d}$$

$$\downarrow 373$$

$$-\frac{\frac{\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)} - \frac{\int \frac{\tanh^2(c+dx)+1}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}\,d\tanh(c+dx)}{2(a+b)}}{d}$$

$$\downarrow 397$$

$$-\frac{\frac{\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)} - \frac{\frac{2\int \frac{1}{1-\tanh^2(c+dx)}\,d\tanh(c+dx)}{a+b} - \frac{(a-b)\int \frac{1}{b\tanh^2(c+dx)+a}\,d\tanh(c+dx)}{a+b}}{2(a+b)}}{d}$$

$$\downarrow 218$$

$$-\frac{\frac{\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)} - \frac{\frac{2\int \frac{1}{1-\tanh^2(c+dx)}\,d\tanh(c+dx)}{a+b} - \frac{(a-b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(a+b)}}{2(a+b)}}{d}$$

$$\downarrow 219$$

$$-\frac{\frac{\tanh(c+dx)}{2(a+b)(a+b\tanh^2(c+dx))}-\frac{\frac{2\operatorname{arctanh}(\tanh(c+dx))}{a+b}-\frac{(a-b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(a+b)}}{2(a+b)}}{d}$$

input
```
Int[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-((-1/2*(-(((a - b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*Sqrt
[b]*(a + b))) + (2*ArcTanh[Tanh[c + d*x]])/(a + b))/(a + b) + Tanh[c + d*x
]/(2*(a + b)*(a + b*Tanh[c + d*x]^2)))/d)
```

3.183.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.183.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}-\frac{\frac{\left(\frac{b}{2}+\frac{a}{2}\right)\tanh(dx+c)}{a+b\tanh(dx+c)^2}+\frac{(a-b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}}{d}$
default	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}-\frac{\frac{\left(\frac{b}{2}+\frac{a}{2}\right)\tanh(dx+c)}{a+b\tanh(dx+c)^2}+\frac{(a-b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}}{d}$
risch	$\frac{x}{a^2+2ab+b^2}+\frac{e^{2dx+2c}a-be^{2dx+2c}+a+b}{d(a+b)^2\left(ae^{4dx+4c}+be^{4dx+4c}+2e^{2dx+2c}a-2be^{2dx+2c}+a+b\right)}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}+2ab}{(a+b)\sqrt{-ab}}\right)a}{4\sqrt{-ab}\,(a+b)^2d}$

input
```
int(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/(a+b)^2*ln(tanh(d*x+c)+1)-1/(a+b)^2*((1/2*b+1/2*a)*tanh(d*x+c)/(a
+b*tanh(d*x+c)^2)+1/2*(a-b)/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2)))
-1/2/(a+b)^2*ln(tanh(d*x+c)-1))
```

3.183.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. 2(73) = 146.

Time = 0.31 (sec) , antiderivative size = 2025, normalized size of antiderivative = 23.82

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 + 16*(a^2*b + a*b^2)*d*x*cosh(
d*x + c)*sinh(d*x + c)^3 + 4*(a^2*b + a*b^2)*d*x*sinh(d*x + c)^4 + 4*a^2*b
 + 4*a*b^2 + 4*(a^2*b + a*b^2)*d*x + 4*(a^2*b - a*b^2 + 2*(a^2*b - a*b^2)*
d*x)*cosh(d*x + c)^2 + 4*(6*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^2 + a^2*b -
a*b^2 + 2*(a^2*b - a*b^2)*d*x)*sinh(d*x + c)^2 + ((a^2 - b^2)*cosh(d*x + c
)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - b^2)*sinh(d*x +
 c)^4 + 2*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - b^2)*cosh(d*x
+ c)^2 + a^2 - 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 - b^2)*c
osh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a
*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 -
b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^
2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x +
 c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c
)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a
- b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d
*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a
+ b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3
 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 8*(2*(a^2*b + a*b^2)*d
*x*cosh(d*x + c)^3 + (a^2*b - a*b^2 + 2*(a^2*b - a*b^2)*d*x)*cosh(d*x +...
```

3.183.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

3.183.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 614 vs. 2(73) = 146.

Time = 0.42 (sec) , antiderivative size = 614, normalized size of antiderivative = 7.22

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{(a^2-4ab-b^2)\arctan\left(\frac{(a+b)e^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{8(a^3+2a^2b+ab^2)\sqrt{ab}d}$$
$$+\frac{(a^2-4ab-b^2)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{8(a^3+2a^2b+ab^2)\sqrt{ab}d}$$
$$+\frac{a^2-b^2+(a^2-6ab+b^2)e^{(2dx+2c)}}{4(a^4+3a^3b+3a^2b^2+ab^3+(a^4+3a^3b+3a^2b^2+ab^3)e^{(4dx+4c)}+2(a^4+a^3b-a^2b^2-ab^3)e^{(2dx+2c)})}$$
$$-\frac{a^2-b^2+(a^2-6ab+b^2)e^{(-2dx-2c)}}{4(a^4+3a^3b+3a^2b^2+ab^3+2(a^4+a^3b-a^2b^2-ab^3)e^{(-2dx-2c)}+(a^4+3a^3b+3a^2b^2+ab^3)e^{(-4dx-4}}$$
$$-\frac{(a-b)e^{(-2dx-2c)}+a+b}{2(a^3+2a^2b+ab^2+2(a^3-ab^2)e^{(-2dx-2c)}+(a^3+2a^2b+ab^2)e^{(-4dx-4c)})d}$$
$$+\frac{\log\left((a+b)e^{(4dx+4c)}+2(a-b)e^{(2dx+2c)}+a+b\right)}{4(a^2+2ab+b^2)d}$$
$$-\frac{\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{4(a^2+2ab+b^2)d}+\frac{\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{4\sqrt{ab}ad}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/8*(a^2 - 4*a*b - b^2)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt
(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b)*d) + 1/8*(a^2 - 4*a*b - b^2)*arc
tan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^3 + 2*a^2*b + a*
b^2)*sqrt(a*b)*d) + 1/4*(a^2 - b^2 + (a^2 - 6*a*b + b^2)*e^(2*d*x + 2*c))/
((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*
e^(4*d*x + 4*c) + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*e^(2*d*x + 2*c))*d) -
1/4*(a^2 - b^2 + (a^2 - 6*a*b + b^2)*e^(-2*d*x - 2*c))/((a^4 + 3*a^3*b + 3
*a^2*b^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*e^(-2*d*x - 2*c) + (a
^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*e^(-4*d*x - 4*c))*d) - 1/2*((a - b)*e^(-
2*d*x - 2*c) + a + b)/((a^3 + 2*a^2*b + a*b^2 + 2*(a^3 - a*b^2)*e^(-2*d*x
- 2*c) + (a^3 + 2*a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d) + 1/4*log((a + b)*e^
(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^2 + 2*a*b + b^2)*d)
 - 1/4*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/
((a^2 + 2*a*b + b^2)*d) + 1/4*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b
)/sqrt(a*b))/(sqrt(a*b)*a*d)
```

3.183.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(73) = 146$.

Time $= 0.36$ (sec) , antiderivative size $= 177$, normalized size of antiderivative $= 2.08$

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{\frac{(a-b)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{(a^2+2\,ab+b^2)\sqrt{ab}} - \frac{2\,(dx+c)}{a^2+2\,ab+b^2} - \frac{2\,(ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}+a+b)}{(a^2+2\,ab+b^2)(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+}}{2\,d}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((a - b)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/s
qrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) - 2*(d*x + c)/(a^2 + 2*a*b + b^2
) - 2*(a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + a + b)/((a^2 + 2*a*b + b^2)
*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d
*x + 2*c) + a + b)))/d
```

3.183.9 Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.25

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{a\,x}{(a+b)^2} - \frac{\tanh(c+d\,x)}{2\,a\,d+2\,b\,d} + \frac{b\,x\tanh(c+d\,x)^2}{(a+b)^2}}{b\tanh\left(c+d\,x\right)^2+a} - \frac{\operatorname{atan}\left(\frac{b\tanh(c+d\,x)}{\sqrt{a\,b}}\right)\,(a-b)}{\sqrt{a\,b}\,\left(2\,d\,a^2+4\,d\,a\,b+2\,d\,b^2\right)}$$

input
```
int(tanh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
((a*x)/(a + b)^2 - tanh(c + d*x)/(2*a*d + 2*b*d) + (b*x*tanh(c + d*x)^2)/(
a + b)^2)/(a + b*tanh(c + d*x)^2) - (atan((b*tanh(c + d*x))/(a*b)^(1/2))*(
a - b))/((a*b)^(1/2)*(2*a^2*d + 2*b^2*d + 4*a*b*d))
```

3.184 $\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

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3.184.2 Mathematica [A] (verified) 1362
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3.184.1 Optimal result

Integrand size = 21, antiderivative size = 68

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)^2 d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^2 d} - \frac{1}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
ln(cosh(d*x+c))/(a+b)^2/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^2/d-1/2/(a+b)/d/
(a+b*tanh(d*x+c)^2)
```

3.184.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{-2\log(\cosh(c+dx)) - \log\left(a+b\tanh^2(c+dx)\right) + \frac{a+b}{a+b\tanh^2(c+dx)}}{2(a+b)^2 d}$$

input
```
Integrate[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-1/2*(-2*Log[Cosh[c + d*x]] - Log[a + b*Tanh[c + d*x]^2] + (a + b)/(a + b*
Tanh[c + d*x]^2))/((a + b)^2*d)
```

3.184.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 4153, 26, 353, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\tan(ic+idx)}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\downarrow 26$$

$$-i\int \frac{\tan(ic+idx)}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\downarrow 4153$$

$$-\frac{i\int \frac{i\tanh(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\tanh(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 353$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh^2(c+dx)}{2d}$$

$$\downarrow 54$$

$$\frac{\int \left(\frac{b}{(a+b)^2\left(b\tanh^2(c+dx)+a\right)}+\frac{b}{(a+b)\left(b\tanh^2(c+dx)+a\right)^2}-\frac{1}{(a+b)^2\left(\tanh^2(c+dx)-1\right)}\right)d\tanh^2(c+dx)}{2d}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\log\left(1-\tanh^2(c+dx)\right)}{(a+b)^2}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{(a+b)^2}}{2d}$$

input
```
Int[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-(Log[1 - Tanh[c + d*x]^2]/(a + b)^2) + Log[a + b*Tanh[c + d*x]^2]/(a + b
)^2 - 1/((a + b)*(a + b*Tanh[c + d*x]^2)))/(2*d)
```

3.184.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.184.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{b\left(-\frac{a+b}{b\left(a+b\tanh(dx+c)^2\right)}+\frac{\ln\left(a+b\tanh(dx+c)^2\right)}{b}\right)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}}{d}$
default	$\frac{b\left(-\frac{a+b}{b\left(a+b\tanh(dx+c)^2\right)}+\frac{\ln\left(a+b\tanh(dx+c)^2\right)}{b}\right)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}}{d}$
parallelrisch	$-\frac{-\tanh(dx+c)^2ab-a^2\ln\left(a+b\tanh(dx+c)^2\right)-b^2\tanh(dx+c)^2+2a^2dx-\ln\left(a+b\tanh(dx+c)^2\right)\tanh(dx+c)^2ab+2\ln($ $2\left(a+b\tanh(dx+c)^2\right)d(a+b)^2a$
risch	$-\frac{x}{a^2+2ab+b^2}-\frac{2c}{d(a^2+2ab+b^2)}-\frac{2b\,e^{2dx+2c}}{d(a+b)^2(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b)}+\frac{\ln\left(e^{4dx+4c}+}{2d(a^2}$

input
```
int(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*b/(a+b)^2*(-(a+b)/b/(a+b*tanh(d*x+c)^2)+1/b*ln(a+b*tanh(d*x+c)^2)
)-1/2/(a+b)^2*ln(tanh(d*x+c)+1)-1/2/(a+b)^2*ln(tanh(d*x+c)-1))
```

3.184.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. 2(64) = 128.

Time = 0.28 (sec) , antiderivative size = 623, normalized size of antiderivative = 9.16

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx =$$

$$-\frac{2\,(a+b)dx\cosh(dx+c)^4+8\,(a+b)dx\cosh(dx+c)\sinh(dx+c)^3+2\,(a+b)dx\sinh(dx+c)^4+2\,(a}{2\left((a^3+3\,a^2b+3\right.}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(a + b)*d*x*cosh(d*x + c)^4 + 8*(a + b)*d*x*cosh(d*x + c)*sinh(d*x
 + c)^3 + 2*(a + b)*d*x*sinh(d*x + c)^4 + 2*(a + b)*d*x + 4*((a - b)*d*x +
 b)*cosh(d*x + c)^2 + 4*(3*(a + b)*d*x*cosh(d*x + c)^2 + (a - b)*d*x + b)*
sinh(d*x + c)^2 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(
d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a
 + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^
3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*log(2*((a + b)*cosh(d*x
+ c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 8*((a + b)*d*x*cosh(d*x + c)^3 + ((
a - b)*d*x + b)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*d*cosh(d*x + c)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)
*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*sinh(d*x + c)^4 + 2*(
a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*cosh(d*x + c)^2 + (a^3 + a^2*b - a*b^2 - b^3)*d)*sinh(d*x + c)
^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3
)*d*cosh(d*x + c)^3 + (a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*
x + c))
```

3.184.6 Sympy [A] (verification not implemented)

Time = 58.58 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.35

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{b\left(\begin{cases}\frac{\tanh^2(c+dx)}{a^2} & \text{for } b=0\\ -\frac{1}{b\left(a+b\tanh^2(c+dx)\right)} & \text{otherwise}\end{cases}\right)}{2d\left(a+b\right)} + \frac{b\left(\begin{cases}\frac{\tanh^2(c+dx)}{a} & \text{for } b=0\\ \frac{\log\left(a+b\tanh^2(c+dx)\right)}{b} & \text{otherwise}\end{cases}\right)}{2d\left(a+b\right)^2} - \frac{\log\left(\tanh^2(c+dx)-1\right)}{2d\left(a+b\right)^2}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
b*Piecewise((tanh(c + d*x)**2/a**2, Eq(b, 0)), (-1/(b*(a + b*tanh(c + d*x)
**2)), True))/(2*d*(a + b)) + b*Piecewise((tanh(c + d*x)**2/a, Eq(b, 0)),
(log(a + b*tanh(c + d*x)**2)/b, True))/(2*d*(a + b)**2) - log(tanh(c + d*x
)**2 - 1)/(2*d*(a + b)**2)
```

3.184.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(64) = 128$.

Time = 0.22 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.50

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx =$$
$$-\frac{2\,be^{(-2\,dx-2\,c)}}{(a^3+3\,a^2b+3\,ab^2+b^3+2\,(a^3+a^2b-ab^2-b^3)e^{(-2\,dx-2\,c)}+(a^3+3\,a^2b+3\,ab^2+b^3)e^{(-4\,dx-4\,c)})d}$$
$$+\frac{dx+c}{(a^2+2\,ab+b^2)d}+\frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^2+2\,ab+b^2)d}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-2*b*e^(-2*d*x - 2*c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2*b - a
*b^2 - b^3)*e^(-2*d*x - 2*c) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-4*d*x -
 4*c))*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d) + 1/2*log(2*(a - b)*e^(-2*d*
x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d)
```

3.184.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(64) = 128$.

Time = 0.34 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.19

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$=\frac{\frac{\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{a^2+2\,ab+b^2}-\frac{e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2}{\left(a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right)}}{2\,d}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*(log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) +
 e^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^2 + 2*a*b + b^2) - (e^(2*d*x + 2*c) +
e^(-2*d*x - 2*c) + 2)/((a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d
*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2*b)*(a + b)))/d
```

3.184.9 Mupad [B] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.90

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{a\,x}{a^2+2\,a\,b+b^2}+\frac{b\,x\tanh(c+d\,x)^2}{a^2+2\,a\,b+b^2}+\frac{b\tanh(c+d\,x)^2}{2\,a\,d\,(a+b)}}{b\tanh\left(c+d\,x\right)^2+a} + \frac{\ln\left(b\tanh(c+d\,x)^2+a\right)}{2\,d\,\left(a^2+2\,a\,b+b^2\right)}-\frac{\ln\left(\tanh(c+d\,x)+1\right)}{d\,(a+b)^2}$$

input
```
int(tanh(c + d*x)/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
((a*x)/(2*a*b + a^2 + b^2) + (b*x*tanh(c + d*x)^2)/(2*a*b + a^2 + b^2) + (
b*tanh(c + d*x)^2)/(2*a*d*(a + b)))/(a + b*tanh(c + d*x)^2) + log(a + b*ta
nh(c + d*x)^2)/(2*d*(2*a*b + a^2 + b^2)) - log(tanh(c + d*x) + 1)/(d*(a +
b)^2)
```

3.185 $\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

3.185.1 Optimal result . . . 1369
3.185.2 Mathematica [A] (verified) . . . 1369
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3.185.1 Optimal result

Integrand size = 14, antiderivative size = 89

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{x}{(a+b)^2} + \frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^2 d} + \frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
x/(a+b)^2+1/2*(3*a+b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/a^(3/2)/
(a+b)^2/d+1/2*b*tanh(d*x+c)/a/(a+b)/d/(a+b*tanh(d*x+c)^2)
```

3.185.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} - \log(1-\tanh(c+dx)) + \log(1+\tanh(c+dx)) + \frac{b(a+b)\tanh(c+dx)}{a\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)^2 d}$$

input
```
Integrate[(a + b*Tanh[c + d*x]^2)^(-2),x]
```

output
```
((Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2) - Log
[1 - Tanh[c + d*x]] + Log[1 + Tanh[c + d*x]] + (b*(a + b)*Tanh[c + d*x])/(
a*(a + b*Tanh[c + d*x]^2)))/(2*(a + b)^2*d)
```

3.185.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4144, 316, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\downarrow 4144$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 316$$

$$\frac{\frac{b\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)} - \frac{\int \frac{b\tanh^2(c+dx)+b-2(a+b)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}}{d}$$

$$\downarrow 397$$

$$\frac{\frac{b\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)} - \frac{-\frac{2a\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b} - \frac{b(3a+b)\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{2a(a+b)}}{d}$$

$$\downarrow 218$$

$$\frac{\frac{b\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)} - \frac{-\frac{2a\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b} - \frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2a(a+b)}}{d}$$

$$\downarrow 219$$

$$\frac{\frac{b\tanh(c+dx)}{2a(a+b)(a+b\tanh^2(c+dx))}-\frac{-\frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}-\frac{2a\operatorname{arctanh}(\tanh(c+dx))}{a+b}}{2a(a+b)}}{d}$$

input
```
Int[(a + b*Tanh[c + d*x]^2)^(-2),x]
```

output
```
(-1/2*(-((Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt
[a]*(a + b))) - (2*a*ArcTanh[Tanh[c + d*x]])/(a + b))/(a*(a + b)) + (b*Tan
h[c + d*x])/(2*a*(a + b)*(a + b*Tanh[c + d*x]^2)))/d
```

3.185.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.185.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}+\frac{b\left(\frac{(a+b)\tanh(dx+c)}{2a\left(a+b\tanh(dx+c)^2\right)}+\frac{(3a+b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}}{d}$
default	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}+\frac{b\left(\frac{(a+b)\tanh(dx+c)}{2a\left(a+b\tanh(dx+c)^2\right)}+\frac{(3a+b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a+b)^2}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}}{d}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{b(e^{2dx+2c}a-be^{2dx+2c}+a+b)}{d(a+b)^2a(ae^{4dx+4c}+be^{4dx+4c}+2e^{2dx+2c}a-2be^{2dx+2c}+a+b)}+\frac{3\sqrt{-ab}\ln\left(e^{2dx+2c}+\frac{2\sqrt{-ab}+a-b}{a+b}\right)}{4a(a+b)^2d}$

input
```
int(1/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/(a+b)^2*ln(tanh(d*x+c)+1)+b/(a+b)^2*(1/2*(a+b)/a*tanh(d*x+c)/(a+b
*tanh(d*x+c)^2)+1/2*(3*a+b)/a/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2)
))-1/2/(a+b)^2*ln(tanh(d*x+c)-1))
```

3.185.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 810 vs. 2(77) = 154.

Time = 0.32 (sec) , antiderivative size = 1942, normalized size of antiderivative = 21.82

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*(a^2 + a*b)*d*x*cosh(d*x + c)^4 + 16*(a^2 + a*b)*d*x*cosh(d*x + c)
*sinh(d*x + c)^3 + 4*(a^2 + a*b)*d*x*sinh(d*x + c)^4 + 4*(a^2 + a*b)*d*x +
 4*(2*(a^2 - a*b)*d*x - a*b + b^2)*cosh(d*x + c)^2 + 4*(6*(a^2 + a*b)*d*x*
cosh(d*x + c)^2 + 2*(a^2 - a*b)*d*x - a*b + b^2)*sinh(d*x + c)^2 + ((3*a^2
 + 4*a*b + b^2)*cosh(d*x + c)^4 + 4*(3*a^2 + 4*a*b + b^2)*cosh(d*x + c)*si
nh(d*x + c)^3 + (3*a^2 + 4*a*b + b^2)*sinh(d*x + c)^4 + 2*(3*a^2 - 2*a*b -
 b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + 4*a*b + b^2)*cosh(d*x + c)^2 + 3*a^2
 - 2*a*b - b^2)*sinh(d*x + c)^2 + 3*a^2 + 4*a*b + b^2 + 4*((3*a^2 + 4*a*b
+ b^2)*cosh(d*x + c)^3 + (3*a^2 - 2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c
))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 +
 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2
+ a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*
cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b
)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b
)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a
 + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*
cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 +
4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)
) - 4*a*b - 4*b^2 + 8*(2*(a^2 + a*b)*d*x*cosh(d*x + c)^3 + (2*(a^2 - a*...
```

3.185.6 Sympy [F]

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(1/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**(-2), x)
```

3.185.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(77) = 154$.

Time $= 0.32$ (sec) , antiderivative size $= 206$, normalized size of antiderivative $= 2.31$

$$\int \frac{1}{\left(a + b\tanh^2(c+dx)\right)^2}\,dx = -\frac{(3\,ab+b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{2\,(a^3+2\,a^2b+ab^2)\sqrt{ab}d} + \frac{ab+b^2+(ab-b^2)e^{(-2\,dx-2\,c)}}{(a^4+3\,a^3b+3\,a^2b^2+ab^3+2\,(a^4+a^3b-a^2b^2-ab^3)e^{(-2\,dx-2\,c)}+(a^4+3\,a^3b+3\,a^2b^2+ab^3)e^{(-4\,dx-4\,c)}} + \frac{dx+c}{(a^2+2\,ab+b^2)d}$$

input
```
integrate(1/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(3*a*b + b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b)
)/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b)*d) + (a*b + b^2 + (a*b - b^2)*e^(-2*d
*x - 2*c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2
- a*b^3)*e^(-2*d*x - 2*c) + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*e^(-4*d*x
- 4*c))*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d)
```

3.185.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(77) = 154$.

Time $= 0.30$ (sec) , antiderivative size $= 195$, normalized size of antiderivative $= 2.19$

$$\int \frac{1}{\left(a + b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{(3\,ab+b^2)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{(a^3+2\,a^2b+ab^2)\sqrt{ab}} + \frac{2\,(dx+c)}{a^2+2\,ab+b^2} - \frac{2\,(abe^{(2\,dx+2\,c)}-b^2e^{(2\,dx+2\,c)}+ab+b^2)}{(a^3+2\,a^2b+ab^2)(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+}}{2\,d}$$

input
```
integrate(1/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*((3*a*b + b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a -
 b)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b)) + 2*(d*x + c)/(a^2 + 2*
a*b + b^2) - 2*(a*b*e^(2*d*x + 2*c) - b^2*e^(2*d*x + 2*c) + a*b + b^2)/((a
^3 + 2*a^2*b + a*b^2)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*
x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)))/d
```

3.185.9 Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{a\,x}{(a+b)^2}+\frac{b\,x\tanh(c+d\,x)^2}{(a+b)^2}+\frac{b\tanh(c+d\,x)}{2\,a\,d\,(a+b)}}{b\tanh\left(c+d\,x\right)^2+a} + \frac{\operatorname{atan}\left(\frac{b\tanh(c+d\,x)}{\sqrt{a\,b}}\right)\,\left(b^2+3\,a\,b\right)}{\sqrt{a\,b}\,\left(2\,a^3\,d+a\,b\,\left(4\,a\,d+2\,b\,d\right)\right)}$$

input
```
int(1/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
((a*x)/(a + b)^2 + (b*x*tanh(c + d*x)^2)/(a + b)^2 + (b*tanh(c + d*x))/(2*
a*d*(a + b)))/(a + b*tanh(c + d*x)^2) + (atan((b*tanh(c + d*x))/(a*b)^(1/2
))*(3*a*b + b^2))/((a*b)^(1/2)*(2*a^3*d + a*b*(4*a*d + 2*b*d)))
```

3.186 $\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

3.186.1 Optimal result 1376
3.186.2 Mathematica [A] (verified) 1376
3.186.3 Rubi [A] (verified) 1377
3.186.4 Maple [A] (verified) 1379
3.186.5 Fricas [B] (verification not implemented) 1379
3.186.6 Sympy [F] 1380
3.186.7 Maxima [B] (verification not implemented) 1381
3.186.8 Giac [B] (verification not implemented) 1381
3.186.9 Mupad [F(-1)] 1382

3.186.1 Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)^2 d} + \frac{\log(\tanh(c+dx))}{a^2 d} - \frac{b(2a+b)\log\left(a+b\tanh^2(c+dx)\right)}{2a^2(a+b)^2 d} + \frac{b}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
ln(cosh(d*x+c))/(a+b)^2/d+ln(tanh(d*x+c))/a^2/d-1/2*b*(2*a+b)*ln(a+b*tanh(
d*x+c)^2)/a^2/(a+b)^2/d+1/2*b/a/(a+b)/d/(a+b*tanh(d*x+c)^2)
```

3.186.2 Mathematica [A] (verified)

Time = 2.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{2\log(\cosh(c+dx))}{(a+b)^2} + \frac{2\log(\tanh(c+dx)) + \frac{b\left(-\left((2a+b)\log\left(a+b\tanh^2(c+dx)\right)\right) + \frac{a(a+b)}{a+b\tanh^2(c+dx)}\right)}{(a+b)^2}}{a^2}}{2d}$$

input
```
Integrate[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((2*Log[Cosh[c + d*x]])/(a + b)^2 + (2*Log[Tanh[c + d*x]] + (b*(-((2*a + b
)*Log[a + b*Tanh[c + d*x]^2]) + (a*(a + b))/(a + b*Tanh[c + d*x]^2)))/(a +
 b)^2)/a^2)/(2*d)
```

3.186.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 4153, 26, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{i}{\tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\downarrow 26$$

$$i\int \frac{1}{\tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\downarrow 4153$$

$$\frac{i\int -\frac{i\coth(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\coth(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 354$$

$$\frac{\int \frac{\coth(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh^2(c+dx)}{2d}$$

$$\downarrow 93$$

$$\frac{\int \left(-\frac{(2a+b)b^2}{a^2(a+b)^2(b\tanh^2(c+dx)+a)} - \frac{b^2}{a(a+b)(b\tanh^2(c+dx)+a)^2} + \frac{\coth(c+dx)}{a^2} - \frac{1}{(a+b)^2(\tanh^2(c+dx)-1)}\right) d\tanh^2(c+dx)}{2d}$$

$$\downarrow 2009$$

$$\frac{-\frac{b(2a+b)\log(a+b\tanh^2(c+dx))}{a^2(a+b)^2} + \frac{\log(\tanh^2(c+dx))}{a^2} + \frac{b}{a(a+b)(a+b\tanh^2(c+dx))} - \frac{\log(1-\tanh^2(c+dx))}{(a+b)^2}}{2d}$$

input
```
Int[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(Log[Tanh[c + d*x]^2]/a^2 - Log[1 - Tanh[c + d*x]^2]/(a + b)^2 - (b*(2*a +
 b)*Log[a + b*Tanh[c + d*x]^2])/(a^2*(a + b)^2) + b/(a*(a + b)*(a + b*Tanh
[c + d*x]^2)))/(2*d)
```

3.186.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.186.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.16

method	result
derivativedivides	$-\frac{\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c))}{a^2}+\frac{b^2\left(\frac{(2a+b)\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a(a+b)}{b\left(a+b\tanh(dx+c)^2\right)}\right)}{2(a+b)^2a^2}+\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}}{d}$
default	$-\frac{\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{\ln(\tanh(dx+c))}{a^2}+\frac{b^2\left(\frac{(2a+b)\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a(a+b)}{b\left(a+b\tanh(dx+c)^2\right)}\right)}{2(a+b)^2a^2}+\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}}{d}$
parallelrisch	$\frac{-2\left(a+b\tanh(dx+c)^2\right)b\left(a+\frac{b}{2}\right)\ln\left(a+b\tanh(dx+c)^2\right)+\left(-2\tanh(dx+c)^2a^2b-2a^3\right)\ln(1-\tanh(dx+c))+2(a+b)^2\left(a+b\right.}{2\left(a+b\tanh(dx+c)^2\right)d(a+b)^2a^2}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{2x}{a^2}-\frac{2c}{a^2d}+\frac{4bx}{a(a^2+2ab+b^2)}+\frac{4bc}{da(a^2+2ab+b^2)}+\frac{2b^2x}{a^2(a^2+2ab+b^2)}+\frac{2b^2c}{da^2(a^2+2ab+b^2)}+\frac{}{ad(a}$

input
```
int(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(1/2/(a+b)^2*ln(tanh(d*x+c)-1)-1/a^2*ln(tanh(d*x+c))+1/2*b^2/(a+b)^2/
a^2*((2*a+b)/b*ln(a+b*tanh(d*x+c)^2)-a*(a+b)/b/(a+b*tanh(d*x+c)^2))+1/2/(a
+b)^2*ln(tanh(d*x+c)+1))
```

3.186.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1148 vs. $2(91) = 182$.

Time = 0.36 (sec) , antiderivative size = 1148, normalized size of antiderivative = 12.08

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(a^3 + a^2*b)*d*x*cosh(d*x + c)^4 + 8*(a^3 + a^2*b)*d*x*cosh(d*x +
 c)*sinh(d*x + c)^3 + 2*(a^3 + a^2*b)*d*x*sinh(d*x + c)^4 + 2*(a^3 + a^2*b
)*d*x - 4*(a*b^2 - (a^3 - a^2*b)*d*x)*cosh(d*x + c)^2 + 4*(3*(a^3 + a^2*b)
*d*x*cosh(d*x + c)^2 - a*b^2 + (a^3 - a^2*b)*d*x)*sinh(d*x + c)^2 + ((2*a^
2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*
x + c)*sinh(d*x + c)^3 + (2*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^4 + 2*a^2
*b + 3*a*b^2 + b^3 + 2*(2*a^2*b - a*b^2 - b^3)*cosh(d*x + c)^2 + 2*(2*a^2*
b - a*b^2 - b^3 + 3*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x +
c)^2 + 4*((2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (2*a^2*b - a*b^2 - b
^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)
*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c)
 + sinh(d*x + c)^2)) - 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4
+ 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3
+ 2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)^2 + 2*(a^3 + a^2*b - a*b^2 -
 b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + a^2*b - a*b^
2 - b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c)
- sinh(d*x + c))) + 8*((a^3 + a^2*b)*d*x*cosh(d*x + c)^3 - (a*b^2 - (a^3 -
 a^2*b)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 3*a^4*b + 3*a^3*b^2 ...
```

3.186.6 Sympy [F]

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\coth\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(coth(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

3.186.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(91) = 182$.

Time = 0.22 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.47

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$= \frac{2\,b^2 e^{(-2\,dx-2\,c)}}{(a^4+3\,a^3b+3\,a^2b^2+ab^3+2\,(a^4+a^3b-a^2b^2-ab^3)e^{(-2\,dx-2\,c)}+(a^4+3\,a^3b+3\,a^2b^2+ab^3)e^{(-4\,dx-4\,c)})d}$$
$$- \frac{(2\,ab+b^2)\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^4+2\,a^3b+a^2b^2)d}$$
$$+ \frac{dx+c}{(a^2+2\,ab+b^2)d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{a^2d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{a^2d}$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
2*b^2*e^(-2*d*x - 2*c)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3*
b - a^2*b^2 - a*b^3)*e^(-2*d*x - 2*c) + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3
)*e^(-4*d*x - 4*c))*d) - 1/2*(2*a*b + b^2)*log(2*(a - b)*e^(-2*d*x - 2*c)
+ (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 2*a^3*b + a^2*b^2)*d) + (d*x +
 c)/((a^2 + 2*a*b + b^2)*d) + log(e^(-d*x - c) + 1)/(a^2*d) + log(e^(-d*x
- c) - 1)/(a^2*d)
```

3.186.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(91) = 182$.

Time = 0.36 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.05

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx =$$
$$- \frac{\frac{(2\,ab+b^2)\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{a^4+2\,a^3b+a^2b^2} + \frac{2\,(dx+c)}{a^2+2\,ab+b^2} - \frac{4\,b^2e^{(2\,dx+2\,c)}}{(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2}}{2\,d}$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((2*a*b + b^2)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d
*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^4 + 2*a^3*b + a^2*b^2) + 2*(d*
x + c)/(a^2 + 2*a*b + b^2) - 4*b^2*e^(2*d*x + 2*c)/((a*e^(4*d*x + 4*c) + b
*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*(a +
 b)^2*a) - 2*log(abs(e^(2*d*x + 2*c) - 1))/a^2)/d
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\coth(c+d\,x)}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(coth(c + d*x)/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
int(coth(c + d*x)/(a + b*tanh(c + d*x)^2)^2, x)
```

3.187 $\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

3.187.1 Optimal result 1383
3.187.2 Mathematica [A] (verified) 1383
3.187.3 Rubi [A] (verified) 1384
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3.187.8 Giac [B] (verification not implemented) 1389
3.187.9 Mupad [F(-1)] 1390

3.187.1 Optimal result

Integrand size = 23, antiderivative size = 119

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{x}{(a+b)^2} - \frac{b^{3/2}(5a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}(a+b)^2 d} - \frac{(2a+3b)\coth(c+dx)}{2a^2(a+b)d} + \frac{b\coth(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
x/(a+b)^2-1/2*b^(3/2)*(5*a+3*b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2
)/(a+b)^2/d-1/2*(2*a+3*b)*coth(d*x+c)/a^2/(a+b)/d+1/2*b*coth(d*x+c)/a/(a+b
)/d/(a+b*tanh(d*x+c)^2)
```

3.187.2 Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{-\frac{2(c+dx)}{(a+b)^2} + \frac{b^{3/2}(5a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a+b)^2} + \frac{2\coth(c+dx)}{a^2} + \frac{b^2\sinh(2(c+dx))}{a^2(a+b)(a-b+(a+b)\cosh(2(c+dx)))}}{2d}$$

input
```
Integrate[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-1/2*((-2*(c + d*x))/(a + b)^2 + (b^(3/2)*(5*a + 3*b)*ArcTan[(Sqrt[b]*Tanh
[c + d*x])/Sqrt[a]])/(a^(5/2)*(a + b)^2) + (2*Coth[c + d*x])/a^2 + (b^2*Si
nh[2*(c + d*x)])/(a^2*(a + b)*(a - b + (a + b)*Cosh[2*(c + d*x)])))/d
```

3.187.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 25, 4153, 25, 374, 25, 445, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\downarrow 4153$$

$$-\frac{\int -\frac{\coth^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 25$$

$$\frac{\int \frac{\coth^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 374$$

$$-\frac{\int -\frac{\coth^2(c+dx)\left(-3b\tanh^2(c+dx)+2a+3b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)} - \frac{b\coth(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 25$$

$$-\frac{-\frac{\int \frac{\coth^2(c+dx)\left(-3b\tanh^2(c+dx)+2a+3b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)} - \frac{b\coth(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 445$$

$$-\frac{-\frac{\int -\frac{2a^2-2ba-3b^2+b(2a+3b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}-\frac{(2a+3b)\coth(c+dx)}{a}}{2a(a+b)}-\frac{b\coth(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 25$$

$$-\frac{-\frac{\int \frac{2a^2-2ba-3b^2+b(2a+3b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}-\frac{(2a+3b)\coth(c+dx)}{a}}{2a(a+b)}-\frac{b\coth(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 397$$

$$-\frac{-\frac{2a^2\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}-\frac{b^2(5a+3b)\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{a}-\frac{(2a+3b)\coth(c+dx)}{a}}{2a(a+b)}-\frac{b\coth(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 218$$

$$-\frac{-\frac{2a^2\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}-\frac{b^{3/2}(5a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a}-\frac{(2a+3b)\coth(c+dx)}{a}}{2a(a+b)}-\frac{b\coth(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 219$$

$$-\frac{-\frac{2a^2\operatorname{arctanh}(\tanh(c+dx))}{a+b}-\frac{b^{3/2}(5a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a}-\frac{(2a+3b)\coth(c+dx)}{a}}{2a(a+b)}-\frac{b\coth(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

input
```
Int[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-((-1/2*((-((b^(3/2)*(5*a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/
(Sqrt[a]*(a + b))) + (2*a^2*ArcTanh[Tanh[c + d*x]])/(a + b))/a - ((2*a + 3
*b)*Coth[c + d*x])/a)/(a*(a + b)) - (b*Coth[c + d*x])/(2*a*(a + b)*(a + b*
Tanh[c + d*x]^2)))/d)
```

3.187.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.187.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01

method	result
derivativedivides	$-\frac{-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}+\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}+\frac{1}{a^2\tanh(dx+c)}+\frac{b^2\left(\frac{\left(\frac{b}{2}+\frac{a}{2}\right)\tanh(dx+c)}{a+b\tanh(dx+c)^2}+\frac{(5a+3b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a+b)^2a^2}}{d}$
default	$-\frac{-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^2}+\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}+\frac{1}{a^2\tanh(dx+c)}+\frac{b^2\left(\frac{\left(\frac{b}{2}+\frac{a}{2}\right)\tanh(dx+c)}{a+b\tanh(dx+c)^2}+\frac{(5a+3b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a+b)^2a^2}}{d}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{2a^3e^{4dx+4c}+6a^2be^{4dx+4c}+5ab^2e^{4dx+4c}+3e^{4dx+4c}b^3+4a^3e^{2dx+2c}+4a^2be^{2dx+2c}-4e^{2dx+2c}ab^2-6e^{}}{a^2d(e^{2dx+2c}-1)(a+b)^2(ae^{4dx+4c}+be^{4dx+4c}+2e^{2dx+2c}a-2be^{2dx+2c}+a}$

input
```
int(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-1/2/(a+b)^2*ln(tanh(d*x+c)+1)+1/2/(a+b)^2*ln(tanh(d*x+c)-1)+1/a^2/t
anh(d*x+c)+b^2/(a+b)^2/a^2*((1/2*b+1/2*a)*tanh(d*x+c)/(a+b*tanh(d*x+c)^2)+
1/2*(5*a+3*b)/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2))))
```

3.187.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1702 vs. 2(105) = 210.

Time = 0.34 (sec) , antiderivative size = 3725, normalized size of antiderivative = 31.30

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*(a^3 + a^2*b)*d*x*cosh(d*x + c)^6 + 24*(a^3 + a^2*b)*d*x*cosh(d*x
+ c)*sinh(d*x + c)^5 + 4*(a^3 + a^2*b)*d*x*sinh(d*x + c)^6 - 4*(2*a^3 + 6*
a^2*b + 5*a*b^2 + 3*b^3 - (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c)^4 + 4*(15*(a^
3 + a^2*b)*d*x*cosh(d*x + c)^2 - 2*a^3 - 6*a^2*b - 5*a*b^2 - 3*b^3 + (a^3
 - 3*a^2*b)*d*x)*sinh(d*x + c)^4 + 16*(5*(a^3 + a^2*b)*d*x*cosh(d*x + c)^3
 - (2*a^3 + 6*a^2*b + 5*a*b^2 + 3*b^3 - (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c))
*sinh(d*x + c)^3 - 8*a^3 - 24*a^2*b - 28*a*b^2 - 12*b^3 - 4*(a^3 + a^2*b)*
d*x - 4*(4*a^3 + 4*a^2*b - 4*a*b^2 - 6*b^3 + (a^3 - 3*a^2*b)*d*x)*cosh(d*x
 + c)^2 + 4*(15*(a^3 + a^2*b)*d*x*cosh(d*x + c)^4 - 4*a^3 - 4*a^2*b + 4*a*
b^2 + 6*b^3 - (a^3 - 3*a^2*b)*d*x - 6*(2*a^3 + 6*a^2*b + 5*a*b^2 + 3*b^3 -
 (a^3 - 3*a^2*b)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((5*a^2*b + 8*a*b
^2 + 3*b^3)*cosh(d*x + c)^6 + 6*(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)*
sinh(d*x + c)^5 + (5*a^2*b + 8*a*b^2 + 3*b^3)*sinh(d*x + c)^6 + (5*a^2*b -
 12*a*b^2 - 9*b^3)*cosh(d*x + c)^4 + (5*a^2*b - 12*a*b^2 - 9*b^3 + 15*(5*a
^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b +
 8*a*b^2 + 3*b^3)*cosh(d*x + c)^3 + (5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x
+ c))*sinh(d*x + c)^3 - 5*a^2*b - 8*a*b^2 - 3*b^3 - (5*a^2*b - 12*a*b^2 -
9*b^3)*cosh(d*x + c)^2 + (15*(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)^4 -
 5*a^2*b + 12*a*b^2 + 9*b^3 + 6*(5*a^2*b - 12*a*b^2 - 9*b^3)*cosh(d*x + c)
^2)*sinh(d*x + c)^2 + 2*(3*(5*a^2*b + 8*a*b^2 + 3*b^3)*cosh(d*x + c)^5 ...
```

3.187.6 Sympy [F]

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(coth(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(coth(c + d*x)**2/(a + b*tanh(c + d*x)**2)**2, x)
```

3.187.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 976 vs. $2(105) = 210$.

Time = 0.43 (sec) , antiderivative size = 976, normalized size of antiderivative = 8.20

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*a*b + b^2)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c)
 + a + b)/((a^4 + 2*a^3*b + a^2*b^2)*d) + 1/4*(2*a*b + b^2)*log(2*(a - b)*
e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 2*a^3*b + a^2
*b^2)*d) + 1/8*(3*a^2*b - 4*a*b^2 - 3*b^3)*arctan(1/2*((a + b)*e^(2*d*x +
2*c) + a - b)/sqrt(a*b))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(a*b)*d) - 1/8*(3*
a^2*b - 4*a*b^2 - 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqr
t(a*b))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(a*b)*d) + 1/4*(2*a^3 + 5*a^2*b + 6
*a*b^2 + 3*b^3 + (2*a^3 + 7*a^2*b + 3*b^3)*e^(4*d*x + 4*c) + 2*(2*a^3 + 2*
a^2*b + a*b^2 - 3*b^3)*e^(2*d*x + 2*c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*
b^3 - (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*e^(6*d*x + 6*c) - (a^5 - a^4*b
 - 5*a^3*b^2 - 3*a^2*b^3)*e^(4*d*x + 4*c) + (a^5 - a^4*b - 5*a^3*b^2 - 3*a
^2*b^3)*e^(2*d*x + 2*c))*d) - 1/4*(2*a^3 + 5*a^2*b + 6*a*b^2 + 3*b^3 + 2*(
2*a^3 + 2*a^2*b + a*b^2 - 3*b^3)*e^(-2*d*x - 2*c) + (2*a^3 + 7*a^2*b + 3*b
^3)*e^(-4*d*x - 4*c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + (a^5 - a^4*b
 - 5*a^3*b^2 - 3*a^2*b^3)*e^(-2*d*x - 2*c) - (a^5 - a^4*b - 5*a^3*b^2 - 3*
a^2*b^3)*e^(-4*d*x - 4*c) - (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*e^(-6*d*
x - 6*c))*d) - 1/2*(2*a^2 + 5*a*b + 3*b^2 + 2*(2*a^2 - 3*b^2)*e^(-2*d*x -
2*c) + (2*a^2 + 3*a*b + 3*b^2)*e^(-4*d*x - 4*c))/((a^4 + 2*a^3*b + a^2*b^2
 + (a^4 - 2*a^3*b - 3*a^2*b^2)*e^(-2*d*x - 2*c) - (a^4 - 2*a^3*b - 3*a^2*b
^2)*e^(-4*d*x - 4*c) - (a^4 + 2*a^3*b + a^2*b^2)*e^(-6*d*x - 6*c))*d) +...
```

3.187.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(105) = 210$.

Time = 0.37 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.82

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx =$$

$$-\frac{\frac{\left(5ab^2+3b^3\right)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{\left(a^4+2a^3b+a^2b^2\right)\sqrt{ab}} - \frac{2(dx+c)}{a^2+2ab+b^2} + \frac{2\left(2a^3e^{(4dx+4c)}+6a^2be^{(4dx+4c)}+5ab^2e^{(4dx+4c)}+3b^3e^{(4dx+4c)}\right.}{\left(a^4+2a^3b+a^2b^2\right)\left(ae^{(6dx+6c)}+be^{(6dx+}\right.}}{2d}$$

input
```
integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((5*a*b^2 + 3*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c)
+ a - b)/sqrt(a*b))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(a*b)) - 2*(d*x + c)/(a
^2 + 2*a*b + b^2) + 2*(2*a^3*e^(4*d*x + 4*c) + 6*a^2*b*e^(4*d*x + 4*c) + 5
*a*b^2*e^(4*d*x + 4*c) + 3*b^3*e^(4*d*x + 4*c) + 4*a^3*e^(2*d*x + 2*c) + 4
*a^2*b*e^(2*d*x + 2*c) - 4*a*b^2*e^(2*d*x + 2*c) - 6*b^3*e^(2*d*x + 2*c) +
 2*a^3 + 6*a^2*b + 7*a*b^2 + 3*b^3)/((a^4 + 2*a^3*b + a^2*b^2)*(a*e^(6*d*x
 + 6*c) + b*e^(6*d*x + 6*c) + a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) - a*
e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c) - a - b)))/d
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\coth(c+d\,x)^2}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2, x)
```

3.188 $\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

3.188.1 Optimal result 1391
3.188.2 Mathematica [A] (verified) 1392
3.188.3 Rubi [A] (warning: unable to verify) 1392
3.188.4 Maple [A] (verified) 1394
3.188.5 Fricas [B] (verification not implemented) 1395
3.188.6 Sympy [F] 1395
3.188.7 Maxima [B] (verification not implemented) 1396
3.188.8 Giac [B] (verification not implemented) 1396
3.188.9 Mupad [F(-1)] 1397

3.188.1 Optimal result

Integrand size = 23, antiderivative size = 124

$$\begin{aligned}\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = &-\frac{\coth^2(c+dx)}{2a^2d}+\frac{\log(\cosh(c+dx))}{(a+b)^2d}\\ &+\frac{(a-2b)\log(\tanh(c+dx))}{a^3d}\\ &+\frac{b^2(3a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{2a^3(a+b)^2d}\\ &-\frac{b^2}{2a^2(a+b)d\left(a+b\tanh^2(c+dx)\right)}\end{aligned}$$

output
```
-1/2*coth(d*x+c)^2/a^2/d+ln(cosh(d*x+c))/(a+b)^2/d+(a-2*b)*ln(tanh(d*x+c))
/a^3/d+1/2*b^2*(3*a+2*b)*ln(a+b*tanh(d*x+c)^2)/a^3/(a+b)^2/d-1/2*b^2/a^2/(
a+b)/d/(a+b*tanh(d*x+c)^2)
```

3.188.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.75

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{-\frac{\coth^2(c+dx)}{a^2}+\frac{b^3}{a^3(a+b)\left(b+a\coth^2(c+dx)\right)}+\frac{b^2(3a+2b)\log\left(b+a\coth^2(c+dx)\right)}{a^3(a+b)^2}+\frac{2\log(\sinh(c+dx))}{(a+b)^2}}{2d}$$

input
```
Integrate[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-(Coth[c + d*x]^2/a^2) + b^3/(a^3*(a + b)*(b + a*Coth[c + d*x]^2)) + (b^2
*(3*a + 2*b)*Log[b + a*Coth[c + d*x]^2])/(a^3*(a + b)^2) + (2*Log[Sinh[c +
 d*x]])/(a + b)^2)/(2*d)
```

3.188.3 Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{i}{\tan(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^2}\,dx$$
$$\downarrow 26$$
$$-i\int \frac{1}{\tan(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^2}\,dx$$
$$\downarrow 4153$$
$$-\frac{i\int \frac{i\coth^3(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}\,d\tanh(c+dx)}{d}$$
$$\downarrow 26$$

$$\frac{\int \frac{\coth^3(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2} d\tanh(c+dx)}{d}$$

$$\downarrow 354$$

$$\frac{\int \frac{\coth^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2} d\tanh^2(c+dx)}{2d}$$

$$\downarrow 99$$

$$\frac{\int \left(\frac{(3a+2b)b^3}{a^3(a+b)^2\left(b\tanh^2(c+dx)+a\right)} + \frac{b^3}{a^2(a+b)\left(b\tanh^2(c+dx)+a\right)^2} + \frac{\coth^2(c+dx)}{a^2} + \frac{(a-2b)\coth(c+dx)}{a^3} - \frac{1}{(a+b)^2\left(\tanh^2(c+dx)-1\right)}\right) d\tanh$$

$$\downarrow 2009$$

$$\frac{\frac{b^2(3a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{a^3(a+b)^2} + \frac{(a-2b)\log\left(\tanh^2(c+dx)\right)}{a^3} - \frac{b^2}{a^2(a+b)\left(a+b\tanh^2(c+dx)\right)} - \frac{\coth(c+dx)}{a^2} - \frac{\log\left(1-\tanh^2(c+dx)\right)}{(a+b)^2}}{2d}$$

input
```
Int[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-(Coth[c + d*x]/a^2) + ((a - 2*b)*Log[Tanh[c + d*x]^2])/a^3 - Log[1 - Tan
h[c + d*x]^2]/(a + b)^2 + (b^2*(3*a + 2*b)*Log[a + b*Tanh[c + d*x]^2])/(a^
3*(a + b)^2) - b^2/(a^2*(a + b)*(a + b*Tanh[c + d*x]^2)))/(2*d)
```

3.188.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.188.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{\frac{(-a+2b)\ln(\tanh(dx+c))}{a^3}+\frac{1}{2a^2\tanh(dx+c)^2}+\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{b^3\left(-\frac{a(a+b)}{b\left(a+b\tanh(dx+c)^2\right)}+\frac{(3a+2b)\ln\left(a+b\tanh(dx+c)^2\right)}{b}\right)}{2(a+b)^2a^3}}{d}$
default	$-\frac{\frac{(-a+2b)\ln(\tanh(dx+c))}{a^3}+\frac{1}{2a^2\tanh(dx+c)^2}+\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{b^3\left(-\frac{a(a+b)}{b\left(a+b\tanh(dx+c)^2\right)}+\frac{(3a+2b)\ln\left(a+b\tanh(dx+c)^2\right)}{b}\right)}{2(a+b)^2a^3}}{d}$
parallelrisch	$\frac{3\left(a+\frac{2b}{3}\right)\left(a+b\tanh(dx+c)^2\right)b^2\ln\left(a+b\tanh(dx+c)^2\right)+\left(-2\tanh(dx+c)^2a^3b-2a^4\right)\ln(1-\tanh(dx+c))+2(a-2b)(a+b$ $2\left(a+b\tanh(dx+$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{2x}{a^2}-\frac{2c}{a^2d}+\frac{4bx}{a^3}+\frac{4bc}{a^3d}-\frac{6b^2x}{a^2(a^2+2ab+b^2)}-\frac{6b^2c}{da^2(a^2+2ab+b^2)}-\frac{4b^3x}{a^3(a^2+2ab+b^2)}-\frac{4}{a^3d(a^2}$

input
```
int(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d*((-a+2*b)/a^3*ln(tanh(d*x+c))+1/2/a^2/tanh(d*x+c)^2+1/2/(a+b)^2*ln(ta
nh(d*x+c)-1)-1/2*b^3/(a+b)^2/a^3*(-a*(a+b)/b/(a+b*tanh(d*x+c)^2)+(3*a+2*b)
/b*ln(a+b*tanh(d*x+c)^2))+1/2/(a+b)^2*ln(tanh(d*x+c)+1))
```

3.188.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3468 vs. 2(118) = 236.

Time = 0.45 (sec) , antiderivative size = 3468, normalized size of antiderivative = 27.97

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(a^4 + a^3*b)*d*x*cosh(d*x + c)^8 + 16*(a^4 + a^3*b)*d*x*cosh(d*x
+ c)*sinh(d*x + c)^7 + 2*(a^4 + a^3*b)*d*x*sinh(d*x + c)^8 - 4*(2*a^3*b*d*
x - a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^6 - 4*(2*a^3*b*d*x
- 14*(a^4 + a^3*b)*d*x*cosh(d*x + c)^2 - a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b
^3)*sinh(d*x + c)^6 + 8*(14*(a^4 + a^3*b)*d*x*cosh(d*x + c)^3 - 3*(2*a^3*b
*d*x - a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5
 + 4*(2*a^4 + 2*a^3*b - 2*a^2*b^2 - 4*a*b^3 - (a^4 - 3*a^3*b)*d*x)*cosh(d*
x + c)^4 + 4*(35*(a^4 + a^3*b)*d*x*cosh(d*x + c)^4 + 2*a^4 + 2*a^3*b - 2*a
^2*b^2 - 4*a*b^3 - (a^4 - 3*a^3*b)*d*x - 15*(2*a^3*b*d*x - a^4 - 3*a^3*b -
 3*a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(7*(a^4 + a^3*
b)*d*x*cosh(d*x + c)^5 - 5*(2*a^3*b*d*x - a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*
b^3)*cosh(d*x + c)^3 + (2*a^4 + 2*a^3*b - 2*a^2*b^2 - 4*a*b^3 - (a^4 - 3*a
^3*b)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^4 + a^3*b)*d*x - 4*(2*a^3
*b*d*x - a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^2 + 4*(14*(a^4
 + a^3*b)*d*x*cosh(d*x + c)^6 - 2*a^3*b*d*x - 15*(2*a^3*b*d*x - a^4 - 3*a^
3*b - 3*a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^4 + a^4 + 3*a^3*b + 3*a^2*b^2 + 2
*a*b^3 + 6*(2*a^4 + 2*a^3*b - 2*a^2*b^2 - 4*a*b^3 - (a^4 - 3*a^3*b)*d*x)*c
osh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^2*b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x
+ c)^8 + 8*(3*a^2*b^2 + 5*a*b^3 + 2*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (
3*a^2*b^2 + 5*a*b^3 + 2*b^4)*sinh(d*x + c)^8 - 4*(3*a*b^3 + 2*b^4)*cosh...
```

3.188.6 Sympy [F]

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(coth(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(coth(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)
```

3.188.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. 2(118) = 236.

Time = 0.22 (sec) , antiderivative size = 402, normalized size of antiderivative = 3.24

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{(3\,ab^2+2\,b^3)\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^5+2\,a^4b+a^3b^2)d}+\frac{dx+c}{(a^2+2\,ab+b^2)d}$$
$$-\frac{2\left((a^3+3\,a^2b+3\,ab^2+2\,b^3)e^{(-2\,dx-2\,c)}+2\,(a^3+a^2b-ab^2-2\,b^3)e^{(-4}\right.}{(a^5+3\,a^4b+3\,a^3b^2+a^2b^3-4\,(a^4b+2\,a^3b^2+a^2b^3)e^{(-2\,dx-2\,c)}-2\,(a^5-a^4b-5\,a^3b^2-3\,a^2b^3)e^{(-4\,dx-4\,c}$$
$$+\frac{(a-2\,b)\log\left(e^{(-dx-c)}+1\right)}{a^3d}+\frac{(a-2\,b)\log\left(e^{(-dx-c)}-1\right)}{a^3d}$$

input
```
integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(3*a*b^2 + 2*b^3)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x -
 4*c) + a + b)/((a^5 + 2*a^4*b + a^3*b^2)*d) + (d*x + c)/((a^2 + 2*a*b + b
^2)*d) - 2*((a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*e^(-2*d*x - 2*c) + 2*(a^3 +
a^2*b - a*b^2 - 2*b^3)*e^(-4*d*x - 4*c) + (a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3
)*e^(-6*d*x - 6*c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 - 4*(a^4*b + 2*a
^3*b^2 + a^2*b^3)*e^(-2*d*x - 2*c) - 2*(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^
3)*e^(-4*d*x - 4*c) - 4*(a^4*b + 2*a^3*b^2 + a^2*b^3)*e^(-6*d*x - 6*c) + (
a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*e^(-8*d*x - 8*c))*d) + (a - 2*b)*log(
e^(-d*x - c) + 1)/(a^3*d) + (a - 2*b)*log(e^(-d*x - c) - 1)/(a^3*d)
```

3.188.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(118) = 236.

Time = 0.39 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.60

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{\frac{(3\,ab^2+2\,b^3)\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{a^5+2\,a^4b+a^3b^2}-\frac{2\,(dx+c)}{a^2+2\,ab+b^2}+\frac{2\,(a-2\,b)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{a^3}-\frac{4\left(\frac{\left(a^4+3\right.}{}\right.}{2\,d}$$

input
```
integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*((3*a*b^2 + 2*b^3)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(
2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^5 + 2*a^4*b + a^3*b^2) - 2*
(d*x + c)/(a^2 + 2*a*b + b^2) + 2*(a - 2*b)*log(abs(e^(2*d*x + 2*c) - 1))/
a^3 - 4*((a^4 + 3*a^3*b + 3*a^2*b^2 + 2*a*b^3)*e^(6*d*x + 6*c)/(a + b) + 2
*(a^4 + a^3*b - a^2*b^2 - 2*a*b^3)*e^(4*d*x + 4*c)/(a + b) + (a^4 + 3*a^3*
b + 3*a^2*b^2 + 2*a*b^3)*e^(2*d*x + 2*c)/(a + b))/((a*e^(4*d*x + 4*c) + b*
e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*(a +
b)*a^3*(e^(2*d*x + 2*c) - 1)^2))/d
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\coth(c+d\,x)^3}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2, x)
```

3.189 $\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

3.189.1 Optimal result . . . 1398
3.189.2 Mathematica [A] (verified) . . . 1398
3.189.3 Rubi [A] (verified) . . . 1399
3.189.4 Maple [A] (verified) . . . 1402
3.189.5 Fricas [B] (verification not implemented) . . . 1403
3.189.6 Sympy [F] . . . 1403
3.189.7 Maxima [B] (verification not implemented) . . . 1403
3.189.8 Giac [A] (verification not implemented) . . . 1404
3.189.9 Mupad [F(-1)] . . . 1405

3.189.1 Optimal result

Integrand size = 23, antiderivative size = 159

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{x}{(a+b)^2} + \frac{b^{5/2}(7a+5b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}(a+b)^2 d} - \frac{(2a^2-2ab-5b^2)\coth(c+dx)}{2a^3(a+b)d} - \frac{(2a+5b)\coth^3(c+dx)}{6a^2(a+b)d} + \frac{b\coth^3(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

output

```
x/(a+b)^2+1/2*b^(5/2)*(7*a+5*b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(7/2
)/(a+b)^2/d-1/2*(2*a^2-2*a*b-5*b^2)*coth(d*x+c)/a^3/(a+b)/d-1/6*(2*a+5*b)*
coth(d*x+c)^3/a^2/(a+b)/d+1/2*b*coth(d*x+c)^3/a/(a+b)/d/(a+b*tanh(d*x+c)^2
)
```

3.189.2 Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{6(c+dx)}{(a+b)^2} + \frac{3b^{5/2}(7a+5b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}(a+b)^2} + \frac{4(-2a+3b)\coth(c+dx)}{a^3} - \frac{2\coth(c+dx)\operatorname{csch}^2(c+dx)}{a^2} + \frac{3b^3\sinh(2(c+dx))}{a^3(a+b)(a-b+(a+b)\cosh(2($$

input
```
Integrate[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((6*(c + d*x))/(a + b)^2 + (3*b^(5/2)*(7*a + 5*b)*ArcTan[(Sqrt[b]*Tanh[c +
 d*x])/Sqrt[a]])/(a^(7/2)*(a + b)^2) + (4*(-2*a + 3*b)*Coth[c + d*x])/a^3
- (2*Coth[c + d*x]*Csch[c + d*x]^2)/a^2 + (3*b^3*Sinh[2*(c + d*x)])/(a^3*(
a + b)*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(6*d)
```

3.189.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4153, 374, 25, 445, 27, 445, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\downarrow 4153$$

$$\frac{\int \frac{\coth^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\downarrow 374$$

$$\frac{\frac{b\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)} - \frac{\int -\frac{\coth^4(c+dx)\left(-5b\tanh^2(c+dx)+2a+5b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{\coth^4(c+dx)\left(-5b\tanh^2(c+dx)+2a+5b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)} + \frac{b\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 445$$

$$\frac{-\frac{\int -\frac{3\coth^2(c+dx)\left(2a^2-2ba-5b^2+b(2a+5b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{3a}-\frac{(2a+5b)\coth^3(c+dx)}{3a}}{2a(a+b)}+\frac{b\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 27$$

$$\frac{\frac{\int \frac{\coth^2(c+dx)\left(2a^2-2ba-5b^2+b(2a+5b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}-\frac{(2a+5b)\coth^3(c+dx)}{3a}}{2a(a+b)}+\frac{b\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 445$$

$$\frac{-\frac{\int -\frac{2a^3-2ba^2+2b^2a+5b^3+b\left(2a^2-2ba-5b^2\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}-\frac{\left(2a^2-2ab-5b^2\right)\coth(c+dx)}{a}}{a}-\frac{(2a+5b)\coth^3(c+dx)}{3a}}{2a(a+b)}+\frac{b\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{2a^3-2ba^2+2b^2a+5b^3+b\left(2a^2-2ba-5b^2\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}-\frac{\left(2a^2-2ab-5b^2\right)\coth(c+dx)}{a}}{a}-\frac{(2a+5b)\coth^3(c+dx)}{3a}}{2a(a+b)}+\frac{b\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 397$$

$$\frac{\frac{\frac{2a^3\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{b^3(7a+5b)\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{a}-\frac{\left(2a^2-2ab-5b^2\right)\coth(c+dx)}{a}}{a}-\frac{(2a+5b)\coth^3(c+dx)}{3a}}{2a(a+b)}+\frac{b\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 218$$

$$\frac{\frac{\frac{2a^3\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{b^{5/2}(7a+5b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a}-\frac{\left(2a^2-2ab-5b^2\right)\coth(c+dx)}{a}}{a}-\frac{(2a+5b)\coth^3(c+dx)}{3a}}{2a(a+b)}+\frac{b\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 219$$

$$\frac{\frac{\frac{2a^3\operatorname{arctanh}(\tanh(c+dx))}{a+b}+\frac{b^{5/2}(7a+5b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a}-\frac{\left(2a^2-2ab-5b^2\right)\coth(c+dx)}{a}}{a}-\frac{(2a+5b)\coth^3(c+dx)}{3a}}{2a(a+b)}+\frac{b\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

3.189. $\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

input
```
Int[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((-1/3*((2*a + 5*b)*Coth[c + d*x]^3)/a + (((b^(5/2)*(7*a + 5*b)*ArcTan[(Sq
rt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)) + (2*a^3*ArcTanh[Tanh[c +
 d*x]])/(a + b))/a - ((2*a^2 - 2*a*b - 5*b^2)*Coth[c + d*x])/a)/a)/(2*a*(a
 + b)) + (b*Coth[c + d*x]^3)/(2*a*(a + b)*(a + b*Tanh[c + d*x]^2)))/d
```

3.189.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.189.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.89

method	result
derivativedivides	$-\frac{-\frac{-a+2b}{a^3\tanh(dx+c)}+\frac{1}{3a^2\tanh(dx+c)^3}-\frac{b^3\left(\frac{\left(\frac{b}{2}+\frac{a}{2}\right)\tanh(dx+c)}{a+b\tanh(dx+c)^2}+\frac{(7a+5b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a+b)^2a^3}+\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{\ln(\tan}{d}$
default	$-\frac{-\frac{-a+2b}{a^3\tanh(dx+c)}+\frac{1}{3a^2\tanh(dx+c)^3}-\frac{b^3\left(\frac{\left(\frac{b}{2}+\frac{a}{2}\right)\tanh(dx+c)}{a+b\tanh(dx+c)^2}+\frac{(7a+5b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a+b)^2a^3}+\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}-\frac{\ln(\tan}{d}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{12\,e^{8dx+8c}a^4+24\,e^{8dx+8c}a^3b-21a\,b^3e^{8dx+8c}-15b^4e^{8dx+8c}+12\,e^{6dx+6c}a^4-12b\,a^3e^{6dx+6c}-12a^2b^2e^{6dx}}{}$

input
```
int(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-(-a+2*b)/a^3/tanh(d*x+c)+1/3/a^2/tanh(d*x+c)^3-b^3/(a+b)^2/a^3*((1/
2*b+1/2*a)*tanh(d*x+c)/(a+b*tanh(d*x+c)^2)+1/2*(7*a+5*b)/(a*b)^(1/2)*arcta
n(b*tanh(d*x+c)/(a*b)^(1/2)))+1/2/(a+b)^2*ln(tanh(d*x+c)-1)-1/2/(a+b)^2*ln
(tanh(d*x+c)+1))
```

3.189.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4080 vs. 2(143) = 286.

Time = 0.38 (sec) , antiderivative size = 8482, normalized size of antiderivative = 53.35

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.189.6 Sympy [F]

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(coth(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(coth(c + d*x)**4/(a + b*tanh(c + d*x)**2)**2, x)
```

3.189.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2345 vs. 2(143) = 286.

Time = 0.68 (sec) , antiderivative size = 2345, normalized size of antiderivative = 14.75

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*(a^2*b - a*b^2 - b^3)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*
x + 2*c) + a + b)/((a^5 + 2*a^4*b + a^3*b^2)*d) + 1/4*(a^2*b - a*b^2 - b^3
)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^5
 + 2*a^4*b + a^3*b^2)*d) + 1/32*(3*a^3*b - 29*a^2*b^2 - 11*a*b^3 + 5*b^4)*
arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^5 + 2*a^4*b + 
a^3*b^2)*sqrt(a*b)*d) - 1/32*(3*a^3*b - 29*a^2*b^2 - 11*a*b^3 + 5*b^4)*arc
tan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^5 + 2*a^4*b + a^
3*b^2)*sqrt(a*b)*d) + 1/48*(44*a^4 + 117*a^3*b + 111*a^2*b^2 + 23*a*b^3 - 
15*b^4 + 3*(24*a^4 + 69*a^3*b + 45*a^2*b^2 + 27*a*b^3 - 5*b^4)*e^(8*d*x + 
8*c) + 6*(6*a^4 - 31*a^3*b - 50*a^2*b^2 - 51*a*b^3 + 10*b^4)*e^(6*d*x + 6*
c) - 2*(50*a^4 - 78*a^3*b - 225*a^2*b^2 - 196*a*b^3 + 45*b^4)*e^(4*d*x + 4
*c) - 2*(10*a^4 + 115*a^3*b + 182*a^2*b^2 + 95*a*b^3 - 30*b^4)*e^(2*d*x + 
2*c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 - (a^6 + 3*a^5*b + 3*a^4*b^2 + 
 a^3*b^3)*e^(10*d*x + 10*c) + (a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^3)*e^(
8*d*x + 8*c) + 2*(a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*e^(6*d*x + 6*c) - 
 2*(a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*e^(4*d*x + 4*c) - (a^6 + 7*a^5*
b + 11*a^4*b^2 + 5*a^3*b^3)*e^(2*d*x + 2*c))*d) - 1/48*(44*a^4 + 117*a^3*b
 + 111*a^2*b^2 + 23*a*b^3 - 15*b^4 - 2*(10*a^4 + 115*a^3*b + 182*a^2*b^2 + 
 95*a*b^3 - 30*b^4)*e^(-2*d*x - 2*c) - 2*(50*a^4 - 78*a^3*b - 225*a^2*b^2
- 196*a*b^3 + 45*b^4)*e^(-4*d*x - 4*c) + 6*(6*a^4 - 31*a^3*b - 50*a^2*b...
```

3.189.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.77

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$= \frac{\frac{3\,(7\,ab^3+5\,b^4)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{(a^5+2\,a^4b+a^3b^2)\sqrt{ab}} + \frac{6\,(dx+c)}{a^2+2\,ab+b^2} - \frac{6\,(ab^3e^{(2\,dx+2\,c)}-b^4e^{(2\,dx+2\,c)}+ab^3+b^4)}{(a^5+2\,a^4b+a^3b^2)(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx}}}{6\,d}$$

input
```
integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/6*(3*(7*a*b^3 + 5*b^4)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c)
 + a - b)/sqrt(a*b))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(a*b)) + 6*(d*x + c)/(
a^2 + 2*a*b + b^2) - 6*(a*b^3*e^(2*d*x + 2*c) - b^4*e^(2*d*x + 2*c) + a*b^
3 + b^4)/((a^5 + 2*a^4*b + a^3*b^2)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c)
 + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)) - 8*(3*a*e^(4*d*x + 
 4*c) - 3*b*e^(4*d*x + 4*c) - 3*a*e^(2*d*x + 2*c) + 6*b*e^(2*d*x + 2*c) + 
2*a - 3*b)/(a^3*(e^(2*d*x + 2*c) - 1)^3))/d
```

3.189.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\coth(c+d\,x)^4}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2, x)
```

3.190 $\int \frac{\tanh^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

3.190.1 Optimal result 1406
3.190.2 Mathematica [A] (verified) 1406
3.190.3 Rubi [A] (verified) 1407
3.190.4 Maple [A] (verified) 1410
3.190.5 Fricas [B] (verification not implemented) 1410
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3.190.8 Giac [B] (verification not implemented) 1412
3.190.9 Mupad [B] (verification not implemented) 1413

3.190.1 Optimal result

Integrand size = 23, antiderivative size = 144

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{x}{(a+b)^3} - \frac{\sqrt{a}(3a^2+10ab+15b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8b^{5/2}(a+b)^3 d} + \frac{a\tanh^3(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{a(3a+7b)\tanh(c+dx)}{8b^2(a+b)^2 d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
x/(a+b)^3-1/8*(3*a^2+10*a*b+15*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*a^
(1/2)/b^(5/2)/(a+b)^3/d+1/4*a*tanh(d*x+c)^3/b/(a+b)/d/(a+b*tanh(d*x+c)^2)^
2+1/8*a*(3*a+7*b)*tanh(d*x+c)/b^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

3.190.2 Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{8(c+dx) - \frac{\sqrt{a}(3a^2+10ab+15b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{5/2}} - \frac{4a^2(a+b)\sinh(2(c+dx))}{b(a-b+(a+b)\cosh(2(c+dx)))^2} + \frac{3a(a+b)(a+3b)\sinh(2(c+dx))}{b^2(a-b+(a+b)\cosh(2(c+dx)))}}{8(a+b)^3 d}$$

input
```
Integrate[Tanh[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(8*(c + d*x) - (Sqrt[a]*(3*a^2 + 10*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tanh[c +
 d*x])/Sqrt[a]])/b^(5/2) - (4*a^2*(a + b)*Sinh[2*(c + d*x)])/(b*(a - b + (
a + b)*Cosh[2*(c + d*x)])^2) + (3*a*(a + b)*(a + 3*b)*Sinh[2*(c + d*x)])/(
b^2*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(8*(a + b)^3*d)
```

3.190.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 25, 4153, 25, 372, 440, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\tan(ic+idx)^6}{\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$

$$\downarrow 25$$

$$-\int \frac{\tan(ic+idx)^6}{\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$

$$\downarrow 4153$$

$$-\frac{\int -\frac{\tanh^6(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh(c+dx)}{d}$$

$$\downarrow 25$$

$$\frac{\int \frac{\tanh^6(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh(c+dx)}{d}$$

$$\downarrow 372$$

$$-\frac{\frac{\int \frac{\tanh^2(c+dx)\left(3a-(3a+4b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}\,d\tanh(c+dx)}{4b(a+b)} - \frac{a\tanh^3(c+dx)}{4b(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 440$$

$$-\frac{\frac{\int \frac{a(3a+7b)-\left(3a^2+7ba+8b^2\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2b(a+b)}-\frac{a(3a+7b)\tanh(c+dx)}{2b(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4b(a+b)}-\frac{a\tanh^3(c+dx)}{4b(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 397$$

$$-\frac{\frac{\frac{a\left(3a^2+10ab+15b^2\right)\int\frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}-\frac{8b^2\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}}{2b(a+b)}-\frac{a(3a+7b)\tanh(c+dx)}{2b(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4b(a+b)}-\frac{a\tanh^3(c+dx)}{4b(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 218$$

$$-\frac{\frac{\frac{\sqrt{a}\left(3a^2+10ab+15b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)}-\frac{8b^2\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}}{2b(a+b)}-\frac{a(3a+7b)\tanh(c+dx)}{2b(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4b(a+b)}-\frac{a\tanh^3(c+dx)}{4b(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 219$$

$$-\frac{\frac{\frac{\sqrt{a}\left(3a^2+10ab+15b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)}-\frac{8b^2\text{arctanh}(\tanh(c+dx))}{a+b}}{2b(a+b)}-\frac{a(3a+7b)\tanh(c+dx)}{2b(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4b(a+b)}-\frac{a\tanh^3(c+dx)}{4b(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

input
```
Int[Tanh[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-((-1/4*(a*Tanh[c + d*x]^3)/(b*(a + b)*(a + b*Tanh[c + d*x]^2)^2) + (((Sqr
t[a]*(3*a^2 + 10*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(S
qrt[b]*(a + b)) - (8*b^2*ArcTanh[Tanh[c + d*x]])/(a + b))/(2*b*(a + b)) -
(a*(3*a + 7*b)*Tanh[c + d*x])/(2*b*(a + b)*(a + b*Tanh[c + d*x]^2)))/(4*b*
(a + b)))/d)
```

3.190.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.190.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{a\left(\frac{-\frac{\left(5a^2+14ab+9b^2\right)\tanh(dx+c)^3}{8b}-\frac{a\left(3a^2+10ab+7b^2\right)\tanh(dx+c)}{8b^2}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(3a^2+10ab+15b^2\right)\arctan\left(\frac{b\tanh(dx}{\sqrt{ab}}\right.\right.}{(a+b)^3}}{d}$
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{a\left(\frac{-\frac{\left(5a^2+14ab+9b^2\right)\tanh(dx+c)^3}{8b}-\frac{a\left(3a^2+10ab+7b^2\right)\tanh(dx+c)}{8b^2}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(3a^2+10ab+15b^2\right)\arctan\left(\frac{b\tanh(dx}{\sqrt{ab}}\right.\right.}{(a+b)^3}}{d}$
risch	$\frac{x}{a^3+3a^2b+3ab^2+b^3}-\frac{a\left(3a^3e^{6dx+6c}+13a^2be^{6dx+6c}+ab^2e^{6dx+6c}-9e^{6dx+6c}b^3+9a^3e^{4dx+4c}+21a^2be^{4dx+4c}-9ab^2e^{4}\right.}{4\left(ae^{4dx+4c}+be^{4dx+4c}+2e^{2dx+}\right.}$

input

```
int(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output

```
1/d*(-1/2/(a+b)^3*ln(tanh(d*x+c)-1)-a/(a+b)^3*((-1/8*(5*a^2+14*a*b+9*b^2)/
b*tanh(d*x+c)^3-1/8*a*(3*a^2+10*a*b+7*b^2)/b^2*tanh(d*x+c))/(a+b*tanh(d*x+
c)^2)^2+1/8*(3*a^2+10*a*b+15*b^2)/b^2/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*
b)^(1/2)))+1/2/(a+b)^3*ln(tanh(d*x+c)+1))
```

3.190.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3603 vs. 2(130) = 260.

Time = 0.40 (sec) , antiderivative size = 7528, normalized size of antiderivative = 52.28

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input

```
integrate(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.190.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)**6/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.190.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3354 vs. $2(130) = 260$.

Time $= 1.24$ (sec) , antiderivative size $= 3354$, normalized size of antiderivative $= 23.29$

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output

```
-1/512*(3*a^5 + 25*a^4*b + 150*a^3*b^2 - 150*a^2*b^3 - 25*a*b^4 - 3*b^5)*a
rctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^5*b^2 + 3*a^4*b
^3 + 3*a^3*b^4 + a^2*b^5)*sqrt(a*b)*d) + 1/512*(3*a^5 + 25*a^4*b + 150*a^3
*b^2 - 150*a^2*b^3 - 25*a*b^4 - 3*b^5)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c
) + a - b)/sqrt(a*b))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*sqrt(a*
b)*d) - 1/256*(3*a^6 + 30*a^5*b - 99*a^4*b^2 - 252*a^3*b^3 - 99*a^2*b^4 +
30*a*b^5 + 3*b^6 + (3*a^6 + 28*a^5*b - 465*a^4*b^2 + 465*a^2*b^4 - 28*a*b^
5 - 3*b^6)*e^(6*d*x + 6*c) + (9*a^6 + 66*a^5*b - 905*a^4*b^2 + 1148*a^3*b^
3 - 905*a^2*b^4 + 66*a*b^5 + 9*b^6)*e^(4*d*x + 4*c) + (9*a^6 + 68*a^5*b -
659*a^4*b^2 + 659*a^2*b^4 - 68*a*b^5 - 9*b^6)*e^(2*d*x + 2*c))/((a^7*b^2 +
 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6 + a^2*b^7 + (a^7*b^2 + 5*
a^6*b^3 + 10*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6 + a^2*b^7)*e^(8*d*x + 8*c) +
 4*(a^7*b^2 + 3*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - 3*a^3*b^6 - a^2*b^7)*e^(
6*d*x + 6*c) + 2*(3*a^7*b^2 + 7*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 + 7*a^3*b^
6 + 3*a^2*b^7)*e^(4*d*x + 4*c) + 4*(a^7*b^2 + 3*a^6*b^3 + 2*a^5*b^4 - 2*a^
4*b^5 - 3*a^3*b^6 - a^2*b^7)*e^(2*d*x + 2*c))*d) + 1/256*(3*a^6 + 30*a^5*b
 - 99*a^4*b^2 - 252*a^3*b^3 - 99*a^2*b^4 + 30*a*b^5 + 3*b^6 + (9*a^6 + 68*
a^5*b - 659*a^4*b^2 + 659*a^2*b^4 - 68*a*b^5 - 9*b^6)*e^(-2*d*x - 2*c) + (
9*a^6 + 66*a^5*b - 905*a^4*b^2 + 1148*a^3*b^3 - 905*a^2*b^4 + 66*a*b^5 + 9
*b^6)*e^(-4*d*x - 4*c) + (3*a^6 + 28*a^5*b - 465*a^4*b^2 + 465*a^2*b^4 ...
```

3.190.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(130) = 260$.

Time $= 0.52$ (sec) , antiderivative size $= 408$, normalized size of antiderivative $= 2.83$

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx =$$

$$-\frac{\left(3\,a^3+10\,a^2b+15\,ab^2\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{\left(a^3b^2+3\,a^2b^3+3\,ab^4+b^5\right)\sqrt{ab}} - \frac{8\,(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3} + \frac{2\left(3\,a^4e^{(6\,dx+6\,c)}+13\,a^3be^{(6\,dx+6\,c)}+a^2b^2e^{(6\,dx+6}\right.}{}$$

input

```
integrate(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((3*a^3 + 10*a^2*b + 15*a*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2
*d*x + 2*c) + a - b)/sqrt(a*b))/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*sqr
t(a*b)) - 8*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(3*a^4*e^(6*d*x
+ 6*c) + 13*a^3*b*e^(6*d*x + 6*c) + a^2*b^2*e^(6*d*x + 6*c) - 9*a*b^3*e^(6
*d*x + 6*c) + 9*a^4*e^(4*d*x + 4*c) + 21*a^3*b*e^(4*d*x + 4*c) - 9*a^2*b^2
*e^(4*d*x + 4*c) + 27*a*b^3*e^(4*d*x + 4*c) + 9*a^4*e^(2*d*x + 2*c) + 23*a
^3*b*e^(2*d*x + 2*c) - 13*a^2*b^2*e^(2*d*x + 2*c) - 27*a*b^3*e^(2*d*x + 2*
c) + 3*a^4 + 15*a^3*b + 21*a^2*b^2 + 9*a*b^3)/((a^3*b^2 + 3*a^2*b^3 + 3*a*
b^4 + b^5)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) -
2*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

3.190.9 Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 2669, normalized size of antiderivative = 18.53

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(tanh(c + d*x)^6/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
log(tanh(c + d*x) + 1)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) + ((tan
h(c + d*x)^3*(9*a*b + 5*a^2))/(8*b*(2*a*b + a^2 + b^2)) + (a*tanh(c + d*x)
*(7*a*b + 3*a^2))/(8*b^2*(2*a*b + a^2 + b^2)))/(a^2*d + b^2*d*tanh(c + d*x
)^4 + 2*a*b*d*tanh(c + d*x)^2) - log(tanh(c + d*x) - 1)/(2*d*(a + b)^3) -
(atan((((-a*b^5)^(1/2)*((tanh(c + d*x)*(60*a^5*b + 9*a^6 + 64*b^6 + 225*a^
2*b^4 + 300*a^3*b^3 + 190*a^4*b^2))/(32*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5
*d^2 + 4*a^3*b^4*d^2 + a^4*b^3*d^2)) + (((224*a*b^10*d^2 + 1440*a^2*b^9*d^
2 + 3936*a^3*b^8*d^2 + 5920*a^4*b^7*d^2 + 5280*a^5*b^6*d^2 + 2784*a^6*b^5*
d^2 + 800*a^7*b^4*d^2 + 96*a^8*b^3*d^2)/(64*(b^9*d^3 + 6*a*b^8*d^3 + 15*a^
2*b^7*d^3 + 20*a^3*b^6*d^3 + 15*a^4*b^5*d^3 + 6*a^5*b^4*d^3 + a^6*b^3*d^3)
) - (tanh(c + d*x)*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2)*(256*b^12*d^2
+ 1280*a*b^11*d^2 + 2304*a^2*b^10*d^2 + 1280*a^3*b^9*d^2 - 1280*a^4*b^8*d^
2 - 2304*a^5*b^7*d^2 - 1280*a^6*b^6*d^2 - 256*a^7*b^5*d^2))/(512*(b^8*d +
3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^
2 + 4*a^3*b^4*d^2 + a^4*b^3*d^2)))*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2
))/(16*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)))*(10*a*b + 3*a^2 + 1
5*b^2)*1i)/(16*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)) + ((-a*b^5)^
(1/2)*((tanh(c + d*x)*(60*a^5*b + 9*a^6 + 64*b^6 + 225*a^2*b^4 + 300*a^3*b
^3 + 190*a^4*b^2))/(32*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b^4*
d^2 + a^4*b^3*d^2)) - (((224*a*b^10*d^2 + 1440*a^2*b^9*d^2 + 3936*a^3*b...
```

3.191 $\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

3.191.1 Optimal result 1414
3.191.2 Mathematica [A] (verified) 1414
3.191.3 Rubi [A] (verified) 1415
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3.191.8 Giac [B] (verification not implemented) 1419
3.191.9 Mupad [B] (verification not implemented) 1420

3.191.1 Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)^3 d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^3 d} - \frac{a^2}{4b^2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{a(a+2b)}{2b^2(a+b)^2 d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
ln(cosh(d*x+c))/(a+b)^3/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^3/d-1/4*a^2/b^2/
(a+b)/d/(a+b*tanh(d*x+c)^2)^2+1/2*a*(a+2*b)/b^2/(a+b)^2/d/(a+b*tanh(d*x+c)
^2)
```

3.191.2 Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{-4\log(\cosh(c+dx)) - 2\log\left(a+b\tanh^2(c+dx)\right) + \frac{a^2(a+b)^2}{b^2\left(a+b\tanh^2(c+dx)\right)^2} - \frac{2a(a+b)(a+2b)}{b^2\left(a+b\tanh^2(c+dx)\right)}}{4(a+b)^3 d}$$

input
```
Integrate[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-1/4*(-4*Log[Cosh[c + d*x]] - 2*Log[a + b*Tanh[c + d*x]^2] + (a^2*(a + b)^
2)/(b^2*(a + b*Tanh[c + d*x]^2)^2) - (2*a*(a + b)*(a + 2*b))/(b^2*(a + b*T
anh[c + d*x]^2)))/((a + b)^3*d)
```

3.191.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\tan(ic+idx)^5}{\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$

$$\downarrow 26$$

$$-i\int \frac{\tan(ic+idx)^5}{\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$

$$\downarrow 4153$$

$$-\frac{i\int \frac{i\tanh^5(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\tanh^5(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh(c+dx)}{d}$$

$$\downarrow 354$$

$$\frac{\int \frac{\tanh^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh^2(c+dx)}{2d}$$

$$\downarrow 99$$

$$\frac{\int \left(\frac{a^2}{b(a+b)\left(b\tanh^2(c+dx)+a\right)^3} - \frac{(a+2b)a}{b(a+b)^2\left(b\tanh^2(c+dx)+a\right)^2} - \frac{1}{(a+b)^3\left(\tanh^2(c+dx)-1\right)} + \frac{b}{(a+b)^3\left(b\tanh^2(c+dx)+a\right)} \right) d\tanh^2(c+}{2d}$$

$$\downarrow 2009$$

$$\frac{-\frac{a^2}{2b^2(a+b)\left(a+b\tanh^2(c+dx)\right)^2} + \frac{a(a+2b)}{b^2(a+b)^2\left(a+b\tanh^2(c+dx)\right)} - \frac{\log\left(1-\tanh^2(c+dx)\right)}{(a+b)^3} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{(a+b)^3}}{2d}$$

input
```
Int[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-(Log[1 - Tanh[c + d*x]^2]/(a + b)^3) + Log[a + b*Tanh[c + d*x]^2]/(a + b
)^3 - a^2/(2*b^2*(a + b)*(a + b*Tanh[c + d*x]^2)^2) + (a*(a + 2*b))/(b^2*(
a + b)^2*(a + b*Tanh[c + d*x]^2)))/(2*d)
```

3.191.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.191.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{-\frac{a\left(a^2+3ab+2b^2\right)}{b^2\left(a+b\tanh(dx+c)^2\right)}+\frac{a^2\left(a^2+2ab+b^2\right)}{2b^2\left(a+b\tanh(dx+c)^2\right)^2}-\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}}{d}$
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{-\frac{a\left(a^2+3ab+2b^2\right)}{b^2\left(a+b\tanh(dx+c)^2\right)}+\frac{a^2\left(a^2+2ab+b^2\right)}{2b^2\left(a+b\tanh(dx+c)^2\right)^2}-\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}}{d}$
risch	$-\frac{x}{a^3+3a^2b+3ab^2+b^3}-\frac{2c}{d(a^3+3a^2b+3ab^2+b^3)}+\frac{4\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)a\,e^{2dx+2c}}{\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)^2d(a+b)^3}$
parallelrisch	$-\frac{4\ln(1-\tanh(dx+c))a^2b^2+4\ln(1-\tanh(dx+c))\tanh(dx+c)^4b^4-2\ln\left(a+b\tanh(dx+c)^2\right)\tanh(dx+c)^4b^4-4ab^3\tanh($

input

```
int(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output

```
1/d*(-1/2/(a+b)^3*ln(tanh(d*x+c)-1)-1/2/(a+b)^3*(-a*(a^2+3*a*b+2*b^2)/b^2/
(a+b*tanh(d*x+c)^2)+1/2*a^2*(a^2+2*a*b+b^2)/b^2/(a+b*tanh(d*x+c)^2)^2-ln(a
+b*tanh(d*x+c)^2))-1/2/(a+b)^3*ln(tanh(d*x+c)+1))
```

3.191.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2584 vs. 2(103) = 206.

Time = 0.30 (sec) , antiderivative size = 2584, normalized size of antiderivative = 23.71

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 16*(a^2 + 2*a*b + b^2)*d
*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)
^8 + 8*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b +
 b^2)*d*x*cosh(d*x + c)^2 + (a^2 - b^2)*d*x - a^2 - a*b)*sinh(d*x + c)^6 +
 16*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*((a^2 - b^2)*d*x - a^2
 - a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 - 2*a*b + 3*b^2)*d*x - 2
*a^2 + 4*a*b)*cosh(d*x + c)^4 + 4*(35*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c
)^4 + (3*a^2 - 2*a*b + 3*b^2)*d*x + 30*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(
d*x + c)^2 - 2*a^2 + 4*a*b)*sinh(d*x + c)^4 + 16*(7*(a^2 + 2*a*b + b^2)*d*
x*cosh(d*x + c)^5 + 10*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^3 + ((3
*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^3
 + 2*(a^2 + 2*a*b + b^2)*d*x + 8*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c
)^2 + 8*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 15*((a^2 - b^2)*d*x -
 a^2 - a*b)*cosh(d*x + c)^4 + (a^2 - b^2)*d*x + 3*((3*a^2 - 2*a*b + 3*b^2)
*d*x - 2*a^2 + 4*a*b)*cosh(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)^2 - ((a^2
 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh
(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 4*(a^2 - b^2)*cosh(d*x
 + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x +
 c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - b^2)*cosh(d*x
+ c))*sinh(d*x + c)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(...
```

3.191.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)**5/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.191.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(103) = 206$.

Time = 0.24 (sec) , antiderivative size = 376, normalized size of antiderivative = 3.45

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{dx+c}{(a^3+3\,a^2b+3\,ab^2+b^3)d}$$

$$+\frac{}{(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5+4\,(a^5+3\,a^4b+2\,a^3b^2-2\,a^2b^3-3\,ab^4-b^5)e^{(-2\,dx-2\,c)}+2}$$

$$+\frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)d}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
(d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 4*((a^2 + a*b)*e^(-2*d*x -
 2*c) + (a^2 - 2*a*b)*e^(-4*d*x - 4*c) + (a^2 + a*b)*e^(-6*d*x - 6*c))/((a
^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 + 4*(a^5 + 3*a^4*b
+ 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 7*a
^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*e^(-4*d*x - 4*c) + 4*(a^5
+ 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-6*d*x - 6*c) + (a^5
 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*e^(-8*d*x - 8*c))*d)
 + 1/2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/
((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

3.191.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(103) = 206$.

Time = 0.47 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.25

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$= \frac{\frac{2\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{3\,a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+3\,b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2-4}{(a^2+2\,ab+b^2)\left(a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2}\right.\right.}}{4\,d}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/4*(2*log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c)
 + e^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a*
(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 3*b*(e^(2*d*x + 2*c) + e^(-2*d*x
 - 2*c))^2 - 4*a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 12*b*(e^(2*d*x + 2*
c) + e^(-2*d*x - 2*c)) - 4*a + 12*b)/((a^2 + 2*a*b + b^2)*(a*(e^(2*d*x + 2
*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2
*b)^2))/d
```

3.191.9 Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 416, normalized size of antiderivative = 3.82

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$= \frac{a^4 + a^3\,b\left(2\tanh(c+d\,x)^2+4\right) - a\,b^3\left(-4\tanh(c+d\,x)^2 + \tanh(c+d\,x)^2\,\mathrm{atan}\left(\frac{a\tanh(c+d\,x)^2\,1\mathrm{i}+b\tanh(c}{2\,a-a\tanh(c+d\,x)^2+b\tanh}\right.\right.}{4\,d\,a^5\,b^2 + 8\,d\,a^4\,b^3\tanh\left(c+d\,x\right)^2 + 12\,d\,a^4\,b^3 + 4\,d\,a^3\,b^4\tanh\left(c+d\,x\right)^4 + 24\,d\,a^3\,b^4\tanh\left(c+d\,x\right)^2 + 12\,d}$$

input
```
int(tanh(c + d*x)^5/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
(a^4 + a^3*b*(2*tanh(c + d*x)^2 + 4) - a*b^3*(tanh(c + d*x)^2*atan((a*tanh
(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/(2*a - a*tanh(c + d*x)^2 + b*tanh(c
 + d*x)^2))*8i - 4*tanh(c + d*x)^2) + a^2*b^2*(6*tanh(c + d*x)^2 - atan((a
*tanh(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/(2*a - a*tanh(c + d*x)^2 + b*t
anh(c + d*x)^2))*4i + 3) - b^4*tanh(c + d*x)^4*atan((a*tanh(c + d*x)^2*1i
+ b*tanh(c + d*x)^2*1i)/(2*a - a*tanh(c + d*x)^2 + b*tanh(c + d*x)^2))*4i)
/(4*a^2*b^5*d + 12*a^3*b^4*d + 12*a^4*b^3*d + 4*a^5*b^2*d + 4*b^7*d*tanh(c
 + d*x)^4 + 24*a^2*b^5*d*tanh(c + d*x)^2 + 24*a^3*b^4*d*tanh(c + d*x)^2 +
8*a^4*b^3*d*tanh(c + d*x)^2 + 12*a^2*b^5*d*tanh(c + d*x)^4 + 4*a^3*b^4*d*t
anh(c + d*x)^4 + 8*a*b^6*d*tanh(c + d*x)^2 + 12*a*b^6*d*tanh(c + d*x)^4)
```

3.192 $\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

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3.192.1 Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{x}{(a+b)^3} - \frac{(a^2+6ab-3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}b^{3/2}(a+b)^3 d} + \frac{a\tanh(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a+5b)\tanh(c+dx)}{8b(a+b)^2 d\left(a+b\tanh^2(c+dx)\right)}$$

output

```
x/(a+b)^3-1/8*(a^2+6*a*b-3*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/b^(3/2
)/(a+b)^3/d/a^(1/2)+1/4*a*tanh(d*x+c)/b/(a+b)/d/(a+b*tanh(d*x+c)^2)^2-1/8*
(a+5*b)*tanh(d*x+c)/b/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

3.192.2 Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{8(c+dx) - \frac{(a^2+6ab-3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}} + \frac{4a(a+b)\sinh(2(c+dx))}{(a-b+(a+b)\cosh(2(c+dx)))^2} + \frac{(a-5b)(a+b)\sinh(2(c+dx))}{b(a-b+(a+b)\cosh(2(c+dx)))}}{8(a+b)^3 d}$$

input
```
Integrate[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(8*(c + d*x) - ((a^2 + 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[
a]])/(Sqrt[a]*b^(3/2)) + (4*a*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*
Cosh[2*(c + d*x)])^2 + ((a - 5*b)*(a + b)*Sinh[2*(c + d*x)])/(b*(a - b + (
a + b)*Cosh[2*(c + d*x)])))/(8*(a + b)^3*d)
```

3.192.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 4153, 372, 402, 25, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(ic+idx)^4}{\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$

$$\downarrow 4153$$

$$\frac{\int \frac{\tanh^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh(c+dx)}{d}$$

$$\downarrow 372$$

$$\frac{\frac{a\tanh(c+dx)}{4b(a+b)\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\int \frac{a-(a+4b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}\,d\tanh(c+dx)}{4b(a+b)}}{d}$$

$$\downarrow 402$$

$$\frac{\frac{a\tanh(c+dx)}{4b(a+b)\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\frac{(a+5b)\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)} - \frac{\int -\frac{a\left(-\left((a+5b)\tanh^2(c+dx)\right)+a-3b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}\,d\tanh(c+dx)}{2a(a+b)}}{4b(a+b)}}{d}$$

$$\downarrow 25$$

$$\frac{a\tanh(c+dx)}{4b(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\int\frac{a\left(-\left((a+5b)\tanh^2(c+dx)\right)+a-3b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}+\frac{(a+5b)\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4b(a+b)}$$
$$d$$
$$\downarrow 27$$
$$\frac{a\tanh(c+dx)}{4b(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\int\frac{-\left((a+5b)\tanh^2(c+dx)\right)+a-3b}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2(a+b)}+\frac{(a+5b)\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4b(a+b)}$$
$$d$$
$$\downarrow 397$$
$$\frac{a\tanh(c+dx)}{4b(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\left(a^2+6ab-3b^2\right)\int\frac{1}{\frac{b\tanh^2(c+dx)+a}{a+b}}d\tanh(c+dx)}{2(a+b)}-\frac{8b\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}}{4b(a+b)}+\frac{(a+5b)\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$
$$\downarrow 218$$
$$\frac{a\tanh(c+dx)}{4b(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\left(a^2+6ab-3b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(a+b)}-\frac{8b\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}}{2(a+b)}+\frac{(a+5b)\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4b(a+b)}$$
$$d$$
$$\downarrow 219$$
$$\frac{a\tanh(c+dx)}{4b(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\left(a^2+6ab-3b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(a+b)}-\frac{8b\operatorname{arctanh}(\tanh(c+dx))}{a+b}}{2(a+b)}+\frac{(a+5b)\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4b(a+b)}$$
$$d$$

input
```
Int[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((a*Tanh[c + d*x])/(4*b*(a + b)*(a + b*Tanh[c + d*x]^2)^2) - ((((a^2 + 6*a
*b - 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*(a +
 b)) - (8*b*ArcTanh[Tanh[c + d*x]])/(a + b))/(2*(a + b)) + ((a + 5*b)*Tanh
[c + d*x])/(2*(a + b)*(a + b*Tanh[c + d*x]^2)))/(4*b*(a + b)))/d
```

3.192. $\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

3.192.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.192.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}+\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}-\frac{\left(\frac{1}{8}a^2+\frac{3}{4}ab+\frac{5}{8}b^2\right)\tanh(dx+c)^3-\frac{a\left(a^2-2ab-3b^2\right)\tanh(dx+c)}{8b}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(a^2+6ab-3b^2\right)\arctan}{8b\sqrt{ab}}}{(a+b)^3}}{d}$
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}+\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}-\frac{\left(\frac{1}{8}a^2+\frac{3}{4}ab+\frac{5}{8}b^2\right)\tanh(dx+c)^3-\frac{a\left(a^2-2ab-3b^2\right)\tanh(dx+c)}{8b}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(a^2+6ab-3b^2\right)\arctan}{8b\sqrt{ab}}}{(a+b)^3}}{d}$
risch	$\frac{x}{a^3+3a^2b+3ab^2+b^3}-\frac{a^3e^{6dx+6c}-9a^2be^{6dx+6c}-5ab^2e^{6dx+6c}+5e^{6dx+6c}b^3+3a^3e^{4dx+4c}-17a^2be^{4dx+4c}+13ab^2e^{4dx}}{4b\left(ae^{4dx+4c}+be^{4dx+4c}+2e^{2dx+2c}a\right.}$

input

```
int(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output

```
1/d*(-1/2/(a+b)^3*ln(tanh(d*x+c)-1)+1/2/(a+b)^3*ln(tanh(d*x+c)+1)-1/(a+b)^
3*(((1/8*a^2+3/4*a*b+5/8*b^2)*tanh(d*x+c)^3-1/8*a*(a^2-2*a*b-3*b^2)/b*tanh
(d*x+c))/(a+b*tanh(d*x+c)^2)^2+1/8*(a^2+6*a*b-3*b^2)/b/(a*b)^(1/2)*arctan(
b*tanh(d*x+c)/(a*b)^(1/2))))
```

3.192.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3727 vs. $2(123) = 246$.

Time = 0.40 (sec) , antiderivative size = 7757, normalized size of antiderivative = 56.62

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input

```
integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.192.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.192.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2432 vs. $2(123) = 246$.

Time $= 0.95$ (sec) , antiderivative size $= 2432$, normalized size of antiderivative $= 17.75$

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output

```
-1/128*(a^4 + 24*a^3*b - 54*a^2*b^2 - 16*a*b^3 - 3*b^4)*arctan(1/2*((a + b
)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^
2*b^4)*sqrt(a*b)*d) + 1/128*(a^4 + 24*a^3*b - 54*a^2*b^2 - 16*a*b^3 - 3*b^
4)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^5*b + 3*a^
4*b^2 + 3*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) - 1/64*(a^5 - 33*a^4*b - 54*a^3*
b^2 - 2*a^2*b^3 + 21*a*b^4 + 3*b^5 + (a^5 - 71*a^4*b + 98*a^3*b^2 + 154*a^
2*b^3 - 19*a*b^4 - 3*b^5)*e^(6*d*x + 6*c) + (3*a^5 - 171*a^4*b + 310*a^3*b
^2 - 254*a^2*b^3 + 39*a*b^4 + 9*b^5)*e^(4*d*x + 4*c) + (3*a^5 - 133*a^4*b
+ 86*a^3*b^2 + 190*a^2*b^3 - 41*a*b^4 - 9*b^5)*e^(2*d*x + 2*c))/((a^7*b +
5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6 + (a^7*b + 5*a^6
*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6)*e^(8*d*x + 8*c) + 4*
(a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*e^(6*d*x
 + 6*c) + 2*(3*a^7*b + 7*a^6*b^2 + 6*a^5*b^3 + 6*a^4*b^4 + 7*a^3*b^5 + 3*a
^2*b^6)*e^(4*d*x + 4*c) + 4*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3
*a^3*b^5 - a^2*b^6)*e^(2*d*x + 2*c))*d) + 1/64*(a^5 - 33*a^4*b - 54*a^3*b^
2 - 2*a^2*b^3 + 21*a*b^4 + 3*b^5 + (3*a^5 - 133*a^4*b + 86*a^3*b^2 + 190*a
^2*b^3 - 41*a*b^4 - 9*b^5)*e^(-2*d*x - 2*c) + (3*a^5 - 171*a^4*b + 310*a^3
*b^2 - 254*a^2*b^3 + 39*a*b^4 + 9*b^5)*e^(-4*d*x - 4*c) + (a^5 - 71*a^4*b
+ 98*a^3*b^2 + 154*a^2*b^3 - 19*a*b^4 - 3*b^5)*e^(-6*d*x - 6*c))/((a^7*b +
 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6 + 4*(a^7*b +...
```

3.192.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(123) = 246$.

Time $= 0.46$ (sec) , antiderivative size $= 384$, normalized size of antiderivative $= 2.80$

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx =$$

$$-\frac{\left(a^2+6ab-3b^2\right)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{\left(a^3b+3a^2b^2+3ab^3+b^4\right)\sqrt{ab}} - \frac{8(dx+c)}{a^3+3a^2b+3ab^2+b^3} + \frac{2\left(a^3e^{(6dx+6c)}-9a^2be^{(6dx+6c)}-5ab^2e^{(6dx+6c)}+5b^3e\right.}{\left.\right.}$$

input

```
integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((a^2 + 6*a*b - 3*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2
*c) + a - b)/sqrt(a*b))/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*sqrt(a*b)) -
8*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(a^3*e^(6*d*x + 6*c) - 9*a
^2*b*e^(6*d*x + 6*c) - 5*a*b^2*e^(6*d*x + 6*c) + 5*b^3*e^(6*d*x + 6*c) + 3
*a^3*e^(4*d*x + 4*c) - 17*a^2*b*e^(4*d*x + 4*c) + 13*a*b^2*e^(4*d*x + 4*c)
 - 15*b^3*e^(4*d*x + 4*c) + 3*a^3*e^(2*d*x + 2*c) - 11*a^2*b*e^(2*d*x + 2*
c) + a*b^2*e^(2*d*x + 2*c) + 15*b^3*e^(2*d*x + 2*c) + a^3 - 3*a^2*b - 9*a*
b^2 - 5*b^3)/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*(a*e^(4*d*x + 4*c) + b*e
^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

3.192.9 Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 2574, normalized size of antiderivative = 18.79

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(tanh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
log(tanh(c + d*x) + 1)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) - ((tan
h(c + d*x)^3*(a + 5*b))/(8*(2*a*b + a^2 + b^2)) - (a*tanh(c + d*x)*(a - 3*
b))/(8*b*(2*a*b + a^2 + b^2)))/(a^2*d + b^2*d*tanh(c + d*x)^4 + 2*a*b*d*ta
nh(c + d*x)^2) - log(tanh(c + d*x) - 1)/(2*d*(a + b)^3) - (atan(((((tanh(c
 + d*x)*(12*a^3*b - 36*a*b^3 + a^4 + 73*b^4 + 30*a^2*b^2))/(32*(b^5*d^2 +
4*a*b^4*d^2 + a^4*b*d^2 + 6*a^2*b^3*d^2 + 4*a^3*b^2*d^2)) + (((96*b^9*d^2
+ 544*a*b^8*d^2 + 1248*a^2*b^7*d^2 + 1440*a^3*b^6*d^2 + 800*a^4*b^5*d^2 +
96*a^5*b^4*d^2 - 96*a^6*b^3*d^2 - 32*a^7*b^2*d^2)/(64*(b^7*d^3 + 6*a*b^6*d
^3 + a^6*b*d^3 + 15*a^2*b^5*d^3 + 20*a^3*b^4*d^3 + 15*a^4*b^3*d^3 + 6*a^5*
b^2*d^3)) - (tanh(c + d*x)*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2)*(256*b^10*
d^2 + 1280*a*b^9*d^2 + 2304*a^2*b^8*d^2 + 1280*a^3*b^7*d^2 - 1280*a^4*b^6*
d^2 - 2304*a^5*b^5*d^2 - 1280*a^6*b^4*d^2 - 256*a^7*b^3*d^2))/(512*(3*a^2*
b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)*(b^5*d^2 + 4*a*b^4*d^2 + a^4*b*
d^2 + 6*a^2*b^3*d^2 + 4*a^3*b^2*d^2)))*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2
))/(16*(3*a^2*b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)))*(-a*b^3)^(1/2)*
(6*a*b + a^2 - 3*b^2)*1i)/(16*(3*a^2*b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b
^6*d)) + (((tanh(c + d*x)*(12*a^3*b - 36*a*b^3 + a^4 + 73*b^4 + 30*a^2*b^2
))/(32*(b^5*d^2 + 4*a*b^4*d^2 + a^4*b*d^2 + 6*a^2*b^3*d^2 + 4*a^3*b^2*d^2)
) - (((96*b^9*d^2 + 544*a*b^8*d^2 + 1248*a^2*b^7*d^2 + 1440*a^3*b^6*d^2 +
800*a^4*b^5*d^2 + 96*a^5*b^4*d^2 - 96*a^6*b^3*d^2 - 32*a^7*b^2*d^2)/(64...
```

3.193 $\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

3.193.1 Optimal result 1429
3.193.2 Mathematica [A] (verified) 1429
3.193.3 Rubi [A] (verified) 1430
3.193.4 Maple [A] (verified) 1432
3.193.5 Fricas [B] (verification not implemented) 1432
3.193.6 Sympy [F(-1)] 1433
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3.193.8 Giac [B] (verification not implemented) 1434
3.193.9 Mupad [B] (verification not implemented) 1435

3.193.1 Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)^3 d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^3 d} + \frac{a}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{1}{2(a+b)^2 d\left(a+b\tanh^2(c+dx)\right)}$$

output

```
ln(cosh(d*x+c))/(a+b)^3/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^3/d+1/4*a/b/(a+b
)/d/(a+b*tanh(d*x+c)^2)^2-1/2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

3.193.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{4\log(\cosh(c+dx)) + 2\log\left(a+b\tanh^2(c+dx)\right) + \frac{a(a+b)^2}{b\left(a+b\tanh^2(c+dx)\right)^2} - \frac{2(a+b)}{a+b\tanh^2(c+dx)}}{4(a+b)^3 d}$$

input
```
Integrate[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(4*Log[Cosh[c + d*x]] + 2*Log[a + b*Tanh[c + d*x]^2] + (a*(a + b)^2)/(b*(a
 + b*Tanh[c + d*x]^2)^2) - (2*(a + b))/(a + b*Tanh[c + d*x]^2))/(4*(a + b)
^3*d)
```

3.193.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{i\tan(ic+idx)^3}{\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$
$$\downarrow 26$$
$$i\int \frac{\tan(ic+idx)^3}{\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$
$$\downarrow 4153$$
$$\frac{i\int -\frac{i\tanh^3(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh(c+dx)}{d}$$
$$\downarrow 26$$
$$\frac{\int \frac{\tanh^3(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh(c+dx)}{d}$$
$$\downarrow 354$$
$$\frac{\int \frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh^2(c+dx)}{2d}$$
$$\downarrow 86$$

$$\frac{\int \left(-\frac{a}{(a+b)\left(b\tanh^2(c+dx)+a\right)^3}-\frac{1}{(a+b)^3\left(\tanh^2(c+dx)-1\right)}+\frac{b}{(a+b)^3\left(b\tanh^2(c+dx)+a\right)}+\frac{b}{(a+b)^2\left(b\tanh^2(c+dx)+a\right)^2}\right) d\tanh^2(c+}{2d}$$

$$\downarrow 2009$$

$$\frac{\frac{a}{2b(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{1}{(a+b)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{\log\left(1-\tanh^2(c+dx)\right)}{(a+b)^3}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{(a+b)^3}}{2d}$$

input
```
Int[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-(Log[1 - Tanh[c + d*x]^2]/(a + b)^3) + Log[a + b*Tanh[c + d*x]^2]/(a + b
)^3 + a/(2*b*(a + b)*(a + b*Tanh[c + d*x]^2)^2) - 1/((a + b)^2*(a + b*Tanh
[c + d*x]^2)))/(2*d)
```

3.193.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.193.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{-\frac{-\ln\left(a+b\tanh(dx+c)^2\right)-\frac{a\left(a^2+2ab+b^2\right)}{2b\left(a+b\tanh(dx+c)^2\right)^2}-\frac{-a-b}{a+b\tanh(dx+c)^2}}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}}{d}$
default	$\frac{-\frac{-\ln\left(a+b\tanh(dx+c)^2\right)-\frac{a\left(a^2+2ab+b^2\right)}{2b\left(a+b\tanh(dx+c)^2\right)^2}-\frac{-a-b}{a+b\tanh(dx+c)^2}}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}}{d}$
risch	$-\frac{x}{a^3+3a^2b+3ab^2+b^3}-\frac{2c}{d(a^3+3a^2b+3ab^2+b^3)}+\frac{2(a^2e^{4dx+4c}-e^{4dx+4c}b^2+2a^2e^{2dx+2c}-2abe^{2dx+2c}+2e^{2dx+2c}b^2}{(ae^{4dx+4c}+be^{4dx+4c}+2e^{2dx+2c}a-2be^{2dx+2c}+a+b)^2}$
parallelrisch	$-\frac{4\ln(1-\tanh(dx+c))a^2b^2+4\ln(1-\tanh(dx+c))\tanh(dx+c)^4b^4-2\ln\left(a+b\tanh(dx+c)^2\right)\tanh(dx+c)^4b^4+2ab^3\tanh($

input

```
int(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output

```
1/d*(-1/2/(a+b)^3*(-ln(a+b*tanh(d*x+c)^2)-1/2*a*(a^2+2*a*b+b^2)/b/(a+b*tan
h(d*x+c)^2)^2-(-a-b)/(a+b*tanh(d*x+c)^2))-1/2/(a+b)^3*ln(tanh(d*x+c)-1)-1/
2/(a+b)^3*ln(tanh(d*x+c)+1))
```

3.193.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2611 vs. 2(92) = 184.

Time = 0.31 (sec) , antiderivative size = 2611, normalized size of antiderivative = 26.64

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 16*(a^2 + 2*a*b + b^2)*d
*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)
^8 + 4*(2*(a^2 - b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^6 + 4*(14*(a^2 + 2*a*
b + b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d*x - a^2 + b^2)*sinh(d*x + c
)^6 + 8*(14*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*(2*(a^2 - b^2)*d*x
 - a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 - 2*a*b + 3*b^2)*
d*x - 2*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^4 + 4*(35*(a^2 + 2*a*b + b^2)*d
*x*cosh(d*x + c)^4 + (3*a^2 - 2*a*b + 3*b^2)*d*x + 15*(2*(a^2 - b^2)*d*x -
 a^2 + b^2)*cosh(d*x + c)^2 - 2*a^2 + 2*a*b - 2*b^2)*sinh(d*x + c)^4 + 16*
(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^5 + 5*(2*(a^2 - b^2)*d*x - a^2 +
b^2)*cosh(d*x + c)^3 + ((3*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 + 2*a*b - 2*b^
2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*d*x + 4*(2*(a^2
- b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^2 + 4*(14*(a^2 + 2*a*b + b^2)*d*x*co
sh(d*x + c)^6 + 15*(2*(a^2 - b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^4 + 2*(a^
2 - b^2)*d*x + 6*((3*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 + 2*a*b - 2*b^2)*cos
h(d*x + c)^2 - a^2 + b^2)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*
b + b^2)*sinh(d*x + c)^8 + 4*(a^2 - b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a
*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b
 + b^2)*cosh(d*x + c)^3 + 3*(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 ...
```

3.193.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.193.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(92) = 184$.

Time = 0.23 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.92

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{dx+c}{(a^3+3\,a^2b+3\,ab^2+b^3)d}$$
$$+ \frac{2\,(}{(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5+4\,(a^5+3\,a^4b+2\,a^3b^2-2\,a^2b^3-3\,ab^4-b^5)e^{(-2\,dx-2\,c)}+2}$$
$$+ \frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)d}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
(d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 2*((a^2 - b^2)*e^(-2*d*x -
 2*c) + 2*(a^2 - a*b + b^2)*e^(-4*d*x - 4*c) + (a^2 - b^2)*e^(-6*d*x - 6*c
))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 + 4*(a^5 + 3*
a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-2*d*x - 2*c) + 2*(3*a^5
 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*e^(-4*d*x - 4*c) + 4
*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-6*d*x - 6*c)
+ (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*e^(-8*d*x - 8*
c))*d) + 1/2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a
 + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

3.193.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(92) = 184$.

Time = 0.44 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.50

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{\frac{2\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{3\,a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+3\,b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+4}{(a^2+2\,ab+b^2)\left(a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c}\right.\right.}}{4\,d}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/4*(2*log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c)
 + e^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a*
(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 3*b*(e^(2*d*x + 2*c) + e^(-2*d*x
 - 2*c))^2 + 4*a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 4*b*(e^(2*d*x + 2*c
) + e^(-2*d*x - 2*c)) - 4*a - 4*b)/((a^2 + 2*a*b + b^2)*(a*(e^(2*d*x + 2*c
) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2*b
)^2))/d
```

3.193.9 Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 397, normalized size of antiderivative = 4.05

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx =$$

$$-\frac{-a^3+b^3\left(2\tanh(c+dx)^2+\tanh(c+dx)^4\,\mathrm{atan}\left(\frac{a\tanh(c+dx)^2\,1\mathrm{i}+b\tanh(c+dx)^2\,1\mathrm{i}}{2\,a-a\tanh(c+dx)^2+b\tanh(c+dx)^2}\right)\,4\mathrm{i}\right)+}{4\,d\,a^5\,b+8\,d\,a^4\,b^2\tanh\left(c+dx\right)^2+12\,d\,a^4\,b^2+4\,d\,a^3\,b^3\tanh\left(c+dx\right)^4+24\,d\,a^3\,b^3\tanh\left(c+dx\right)^2+12}$$

input
```
int(tanh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
-(b^3*(tanh(c + d*x)^4*atan((a*tanh(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/
(2*a - a*tanh(c + d*x)^2 + b*tanh(c + d*x)^2))*4i + 2*tanh(c + d*x)^2) - a
^3 + a*b^2*(tanh(c + d*x)^2*atan((a*tanh(c + d*x)^2*1i + b*tanh(c + d*x)^2
*1i)/(2*a - a*tanh(c + d*x)^2 + b*tanh(c + d*x)^2))*8i + 2*tanh(c + d*x)^2
 + 1) + a^2*b*atan((a*tanh(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/(2*a - a*
tanh(c + d*x)^2 + b*tanh(c + d*x)^2))*4i)/(4*a^2*b^4*d + 12*a^3*b^3*d + 12
*a^4*b^2*d + 4*b^6*d*tanh(c + d*x)^4 + 4*a^5*b*d + 24*a^2*b^4*d*tanh(c + d
*x)^2 + 24*a^3*b^3*d*tanh(c + d*x)^2 + 8*a^4*b^2*d*tanh(c + d*x)^2 + 12*a^
2*b^4*d*tanh(c + d*x)^4 + 4*a^3*b^3*d*tanh(c + d*x)^4 + 8*a*b^5*d*tanh(c +
 d*x)^2 + 12*a*b^5*d*tanh(c + d*x)^4)
```

3.194 $\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

3.194.1 Optimal result 1436
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3.194.1 Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{x}{(a+b)^3} - \frac{(3a^2-6ab-b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}\sqrt{b}(a+b)^3 d} - \frac{\tanh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(3a-b)\tanh(c+dx)}{8a(a+b)^2 d\left(a+b\tanh^2(c+dx)\right)}$$

output

```
x/(a+b)^3-1/8*(3*a^2-6*a*b-b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2
)/(a+b)^3/d/b^(1/2)-1/4*tanh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c)^2)^2-1/8*(3*a
-b)*tanh(d*x+c)/a/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

3.194.2 Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{8(c+dx) + \frac{(-3a^2+6ab+b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}} - \frac{4b(a+b)\sinh(2(c+dx))}{(a-b+(a+b)\cosh(2(c+dx)))^2} - \frac{(5a-b)(a+b)\sinh(2(c+dx))}{a(a-b+(a+b)\cosh(2(c+dx)))}}{8(a+b)^3 d}$$

input
```
Integrate[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(8*(c + d*x) + ((-3*a^2 + 6*a*b + b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt
[a]])/(a^(3/2)*Sqrt[b]) - (4*b*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)
*Cosh[2*(c + d*x)]^2 - ((5*a - b)*(a + b)*Sinh[2*(c + d*x)])/(a*(a - b +
(a + b)*Cosh[2*(c + d*x)])))/(8*(a + b)^3*d)
```

3.194.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.18, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 25, 4153, 25, 373, 402, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\tan(ic+idx)^2}{\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$

$$\downarrow 25$$

$$-\int \frac{\tan(ic+idx)^2}{\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$

$$\downarrow 4153$$

$$-\frac{\int -\frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh(c+dx)}{d}$$

$$\downarrow 25$$

$$\frac{\int \frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh(c+dx)}{d}$$

$$\downarrow 373$$

$$-\frac{\frac{\tanh(c+dx)}{4(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\int \frac{3\tanh^2(c+dx)+1}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}\,d\tanh(c+dx)}{4(a+b)}}{d}$$

$$\downarrow 402$$

$$-\frac{\frac{\tanh(c+dx)}{4(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{-\frac{\int -\frac{(3a-b)\tanh^2(c+dx)+5a+b}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{(3a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4(a+b)}}{d}$$

$$\downarrow 25$$

$$-\frac{\frac{\tanh(c+dx)}{4(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\int \frac{(3a-b)\tanh^2(c+dx)+5a+b}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{(3a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4(a+b)}}{d}$$

$$\downarrow 397$$

$$-\frac{\frac{\tanh(c+dx)}{4(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{8a\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}-\frac{\left(3a^2-6ab-b^2\right)\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{2a(a+b)}-\frac{(3a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4(a+b)}}{d}$$

$$\downarrow 218$$

$$-\frac{\frac{\tanh(c+dx)}{4(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{8a\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}-\frac{\left(3a^2-6ab-b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(a+b)}}{2a(a+b)}-\frac{(3a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4(a+b)}}{d}$$

$$\downarrow 219$$

$$-\frac{\frac{\tanh(c+dx)}{4(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{8a\operatorname{arctanh}(\tanh(c+dx))}{a+b}-\frac{\left(3a^2-6ab-b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(a+b)}}{2a(a+b)}-\frac{(3a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4(a+b)}}{d}$$

input
```
Int[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-((Tanh[c + d*x]/(4*(a + b)*(a + b*Tanh[c + d*x]^2)^2) - ((-(((3*a^2 - 6*a
*b - b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*(a + b
))) + (8*a*ArcTanh[Tanh[c + d*x]])/(a + b))/(2*a*(a + b)) - ((3*a - b)*Tan
h[c + d*x])/(2*a*(a + b)*(a + b*Tanh[c + d*x]^2)))/(4*(a + b)))/d)
```

3.194. $\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

3.194.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.194.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{\frac{\frac{b\left(3a^2+2ab-b^2\right)\tanh(dx+c)^3}{8a}+\left(\frac{5}{8}a^2+\frac{3}{4}ab+\frac{1}{8}b^2\right)\tanh(dx+c)}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(3a^2-6ab-b^2\right)\arctan\left(\right.}{8a\sqrt{ab}}}{(a+b)^3}}{d}$
default	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{\frac{\frac{b\left(3a^2+2ab-b^2\right)\tanh(dx+c)^3}{8a}+\left(\frac{5}{8}a^2+\frac{3}{4}ab+\frac{1}{8}b^2\right)\tanh(dx+c)}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(3a^2-6ab-b^2\right)\arctan\left(\right.}{8a\sqrt{ab}}}{(a+b)^3}}{d}$
risch	$\frac{x}{a^3+3a^2b+3a\,b^2+b^3}+\frac{5a^3e^{6dx+6c}-5a^2b\,e^{6dx+6c}-9a\,b^2e^{6dx+6c}+e^{6dx+6c}b^3+15a^3e^{4dx+4c}-13a^2b\,e^{4dx+4c}+17a\,b^2e^{4d}}{4a\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a\right.}$

input
```
int(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/(a+b)^3*ln(tanh(d*x+c)+1)-1/2/(a+b)^3*ln(tanh(d*x+c)-1)-1/(a+b)^3
*((1/8*b*(3*a^2+2*a*b-b^2)/a*tanh(d*x+c)^3+(5/8*a^2+3/4*a*b+1/8*b^2)*tanh(
d*x+c))/(a+b*tanh(d*x+c)^2)^2+1/8*(3*a^2-6*a*b-b^2)/a/(a*b)^(1/2)*arctan(b
*tanh(d*x+c)/(a*b)^(1/2))))
```

3.194.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3744 vs. 2(123) = 246.

Time = 0.37 (sec) , antiderivative size = 7791, normalized size of antiderivative = 56.87

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.194.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.194.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1472 vs. $2(123) = 246$.

Time $= 0.60$ (sec) , antiderivative size $= 1472$, normalized size of antiderivative $= 10.74$

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output

```
-1/32*(3*a^3 - 21*a^2*b - 11*a*b^2 - 3*b^3)*arctan(1/2*((a + b)*e^(2*d*x +
 2*c) + a - b)/sqrt(a*b))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)
*d) + 1/32*(3*a^3 - 21*a^2*b - 11*a*b^2 - 3*b^3)*arctan(1/2*((a + b)*e^(-2
*d*x - 2*c) + a - b)/sqrt(a*b))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqr
t(a*b)*d) + 1/16*(5*a^4 - 18*a^2*b^2 - 16*a*b^3 - 3*b^4 + (5*a^4 - 46*a^3*
b - 40*a^2*b^2 + 14*a*b^3 + 3*b^4)*e^(6*d*x + 6*c) + (15*a^4 - 104*a^3*b +
 58*a^2*b^2 - 24*a*b^3 - 9*b^4)*e^(4*d*x + 4*c) + (15*a^4 - 58*a^3*b - 56*
a^2*b^2 + 26*a*b^3 + 9*b^4)*e^(2*d*x + 2*c))/((a^7 + 5*a^6*b + 10*a^5*b^2
+ 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*
b^3 + 5*a^3*b^4 + a^2*b^5)*e^(8*d*x + 8*c) + 4*(a^7 + 3*a^6*b + 2*a^5*b^2
- 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(6*d*x + 6*c) + 2*(3*a^7 + 7*a^6*b +
6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*e^(4*d*x + 4*c) + 4*(a^7 +
3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(2*d*x + 2*c))*d)
 - 1/16*(5*a^4 - 18*a^2*b^2 - 16*a*b^3 - 3*b^4 + (15*a^4 - 58*a^3*b - 56*a
^2*b^2 + 26*a*b^3 + 9*b^4)*e^(-2*d*x - 2*c) + (15*a^4 - 104*a^3*b + 58*a^2
*b^2 - 24*a*b^3 - 9*b^4)*e^(-4*d*x - 4*c) + (5*a^4 - 46*a^3*b - 40*a^2*b^2
 + 14*a*b^3 + 3*b^4)*e^(-6*d*x - 6*c))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a
^4*b^3 + 5*a^3*b^4 + a^2*b^5 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 -
3*a^3*b^4 - a^2*b^5)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6
*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*e^(-4*d*x - 4*c) + 4*(a^7 + 3*a^6*b +...
```

3.194.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(123) = 246$.

Time $= 0.42$ (sec) , antiderivative size $= 388$, normalized size of antiderivative $= 2.83$

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx =$$

$$-\frac{(3a^2-6ab-b^2)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{(a^4+3a^3b+3a^2b^2+ab^3)\sqrt{ab}} - \frac{8(dx+c)}{a^3+3a^2b+3ab^2+b^3} - \frac{2\left(5a^3e^{(6dx+6c)}-5a^2be^{(6dx+6c)}-9ab^2e^{(6dx+6c)}+b^3e\right.}{\text{...}}$$

input

```
integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((3*a^2 - 6*a*b - b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2
*c) + a - b)/sqrt(a*b))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)) -
8*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(5*a^3*e^(6*d*x + 6*c) - 5
*a^2*b*e^(6*d*x + 6*c) - 9*a*b^2*e^(6*d*x + 6*c) + b^3*e^(6*d*x + 6*c) + 1
5*a^3*e^(4*d*x + 4*c) - 13*a^2*b*e^(4*d*x + 4*c) + 17*a*b^2*e^(4*d*x + 4*c
) - 3*b^3*e^(4*d*x + 4*c) + 15*a^3*e^(2*d*x + 2*c) + a^2*b*e^(2*d*x + 2*c)
 - 11*a*b^2*e^(2*d*x + 2*c) + 3*b^3*e^(2*d*x + 2*c) + 5*a^3 + 9*a^2*b + 3*
a*b^2 - b^3)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*(a*e^(4*d*x + 4*c) + b*e
^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

3.194.9 Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.86

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{\frac{a^2\,x}{(a+b)\,(a^2+2\,a\,b+b^2)} - \frac{\tanh(c+d\,x)\,(5\,a+b)}{8\,d\,(a^2+2\,a\,b+b^2)} + \frac{b^2\,x\,\tanh(c+d\,x)^4}{a^3+3\,a^2\,b+3\,a\,b^2+b^3} + \frac{2\,a\,b\,x\,\tanh(c+d\,x)^2}{a^3+3\,a^2\,b+3\,a\,b^2+b^3} - \frac{\tanh(c+d\,x)^3\,(3\,a\,b-b^2)}{8\,a\,d\,(a^2+2\,a\,b+b^2)}}{a^2+2\,a\,b\tanh\left(c+d\,x\right)^2+b^2\tanh\left(c+d\,x\right)^4} + \frac{\operatorname{atan}\left(\frac{b\tanh(c+d\,x)}{\sqrt{a\,b}}\right)\,\left(-3\,a^2+6\,a\,b+b^2\right)}{\sqrt{a\,b}\,\left(8\,a^4\,d+a\,b\,\left(24\,d\,a^2+24\,d\,a\,b+8\,d\,b^2\right)\right)}$$

input
```
int(tanh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
((a^2*x)/((a + b)*(2*a*b + a^2 + b^2)) - (tanh(c + d*x)*(5*a + b))/(8*d*(2
*a*b + a^2 + b^2)) + (b^2*x*tanh(c + d*x)^4)/(3*a*b^2 + 3*a^2*b + a^3 + b^
3) + (2*a*b*x*tanh(c + d*x)^2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c +
 d*x)^3*(3*a*b - b^2))/(8*a*d*(2*a*b + a^2 + b^2)))/(a^2 + b^2*tanh(c + d*
x)^4 + 2*a*b*tanh(c + d*x)^2) + (atan((b*tanh(c + d*x))/(a*b)^(1/2))*(6*a*
b - 3*a^2 + b^2))/((a*b)^(1/2)*(8*a^4*d + a*b*(24*a^2*d + 8*b^2*d + 24*a*b
*d)))
```

3.195 $\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

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3.195.1 Optimal result

Integrand size = 21, antiderivative size = 94

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)^3 d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^3 d} - \frac{1}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{1}{2(a+b)^2 d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
ln(cosh(d*x+c))/(a+b)^3/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^3/d-1/4/(a+b)/d/
(a+b*tanh(d*x+c)^2)^2-1/2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

3.195.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.82

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{4\log(\cosh(c+dx)) + 2\log\left(a+b\tanh^2(c+dx)\right) - \frac{(a+b)^2}{\left(a+b\tanh^2(c+dx)\right)^2} - \frac{2(a+b)}{a+b\tanh^2(c+dx)}}{4(a+b)^3 d}$$

input
```
Integrate[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(4*Log[Cosh[c + d*x]] + 2*Log[a + b*Tanh[c + d*x]^2] - (a + b)^2/(a + b*Ta
nh[c + d*x]^2)^2 - (2*(a + b))/(a + b*Tanh[c + d*x]^2))/(4*(a + b)^3*d)
```

3.195.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 4153, 26, 353, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\tan(ic+idx)}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\downarrow 26$$

$$-i\int \frac{\tan(ic+idx)}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\downarrow 4153$$

$$-\frac{i\int \frac{i\tanh(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\tanh(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\downarrow 353$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}d\tanh^2(c+dx)}{2d}$$

$$\downarrow 54$$

$$\frac{\int \left(\frac{b}{(a+b)^3 (b \tanh^2(c+dx)+a)} + \frac{b}{(a+b)^2 (b \tanh^2(c+dx)+a)^2} + \frac{b}{(a+b)(b \tanh^2(c+dx)+a)^3} - \frac{1}{(a+b)^3 (\tanh^2(c+dx)-1)} \right) d\tanh^2(c+dx)}{2d}$$

$$\downarrow 2009$$

$$\frac{-\frac{1}{(a+b)^2 (a+b \tanh^2(c+dx))} - \frac{1}{2(a+b)(a+b \tanh^2(c+dx))^2} - \frac{\log(1-\tanh^2(c+dx))}{(a+b)^3} + \frac{\log(a+b \tanh^2(c+dx))}{(a+b)^3}}{2d}$$

input
```
Int[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-(Log[1 - Tanh[c + d*x]^2]/(a + b)^3) + Log[a + b*Tanh[c + d*x]^2]/(a + b
)^3 - 1/(2*(a + b)*(a + b*Tanh[c + d*x]^2)^2) - 1/((a + b)^2*(a + b*Tanh[c
 + d*x]^2)))/(2*d)
```

3.195.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.195.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}+\frac{b\left(\frac{\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a^2+2ab+b^2}{2b\left(a+b\tanh(dx+c)^2\right)^2}-\frac{a+b}{b\left(a+b\tanh(dx+c)^2\right)}\right)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}}{d}$
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}+\frac{b\left(\frac{\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a^2+2ab+b^2}{2b\left(a+b\tanh(dx+c)^2\right)^2}-\frac{a+b}{b\left(a+b\tanh(dx+c)^2\right)}\right)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}}{d}$
risch	$-\frac{x}{a^3+3a^2b+3ab^2+b^3}-\frac{2c}{d(a^3+3a^2b+3ab^2+b^3)}-\frac{4\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-b\,e^{2dx+2c}+a+b\right)b\,e^{2dx+2c}}{\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2b\,e^{2dx+2c}+a+b\right)^2d(a+b)^3}$
parallelrisch	$-\frac{4\ln(1-\tanh(dx+c))a^2b^2+4\ln(1-\tanh(dx+c))\tanh(dx+c)^4b^4-2\ln\left(a+b\tanh(dx+c)^2\right)\tanh(dx+c)^4b^4+2ab^3\tanh($

input

```
int(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output

```
1/d*(-1/2/(a+b)^3*ln(tanh(d*x+c)-1)+1/2*b/(a+b)^3*(1/b*ln(a+b*tanh(d*x+c)^
2)-1/2*(a^2+2*a*b+b^2)/b/(a+b*tanh(d*x+c)^2)^2-(a+b)/b/(a+b*tanh(d*x+c)^2)
)-1/2/(a+b)^3*ln(tanh(d*x+c)+1))
```

3.195.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2554 vs. $2(88) = 176$.

Time = 0.31 (sec) , antiderivative size = 2554, normalized size of antiderivative = 27.17

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 16*(a^2 + 2*a*b + b^2)*d
*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)
^8 + 8*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b +
 b^2)*d*x*cosh(d*x + c)^2 + (a^2 - b^2)*d*x + a*b + b^2)*sinh(d*x + c)^6 +
 16*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*((a^2 - b^2)*d*x + a*b
+ b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 - 2*a*b + 3*b^2)*d*x + 4
*a*b - 2*b^2)*cosh(d*x + c)^4 + 4*(35*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c
)^4 + (3*a^2 - 2*a*b + 3*b^2)*d*x + 30*((a^2 - b^2)*d*x + a*b + b^2)*cosh(
d*x + c)^2 + 4*a*b - 2*b^2)*sinh(d*x + c)^4 + 16*(7*(a^2 + 2*a*b + b^2)*d*
x*cosh(d*x + c)^5 + 10*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)^3 + ((3
*a^2 - 2*a*b + 3*b^2)*d*x + 4*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3
+ 2*(a^2 + 2*a*b + b^2)*d*x + 8*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c
)^2 + 8*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 15*((a^2 - b^2)*d*x +
 a*b + b^2)*cosh(d*x + c)^4 + (a^2 - b^2)*d*x + 3*((3*a^2 - 2*a*b + 3*b^2)
*d*x + 4*a*b - 2*b^2)*cosh(d*x + c)^2 + a*b + b^2)*sinh(d*x + c)^2 - ((a^2
 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh
(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 4*(a^2 - b^2)*cosh(d*x
 + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x +
 c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - b^2)*cosh(d*x
+ c))*sinh(d*x + c)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(...
```

3.195.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.195.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(88) = 176$.

Time = 0.24 (sec) , antiderivative size = 378, normalized size of antiderivative = 4.02

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{dx+c}{(a^3+3\,a^2b+3\,ab^2+b^3)d}$$

$$- \frac{}{(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5+4\,(a^5+3\,a^4b+2\,a^3b^2-2\,a^2b^3-3\,ab^4-b^5)e^{(-2\,dx-2\,c)}+2$$

$$+ \frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)d}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
(d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 4*((a*b + b^2)*e^(-2*d*x -
 2*c) + (2*a*b - b^2)*e^(-4*d*x - 4*c) + (a*b + b^2)*e^(-6*d*x - 6*c))/((a
^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 + 4*(a^5 + 3*a^4*b
+ 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 7*a
^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*e^(-4*d*x - 4*c) + 4*(a^5
+ 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-6*d*x - 6*c) + (a^5
 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*e^(-8*d*x - 8*c))*d)
 + 1/2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/
((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

3.195.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(88) = 176$.

Time = 0.39 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.61

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$= \frac{\frac{2\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{3\,a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+3\,b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+12}{(a^2+2\,ab+b^2)\left(a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2}\right.\right.}}{4\,d}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/4*(2*log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c)
 + e^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a*
(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 3*b*(e^(2*d*x + 2*c) + e^(-2*d*x
 - 2*c))^2 + 12*a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 4*b*(e^(2*d*x + 2*
c) + e^(-2*d*x - 2*c)) + 12*a - 4*b)/((a^2 + 2*a*b + b^2)*(a*(e^(2*d*x + 2
*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2
*b)^2))/d
```

3.195.9 Mupad [B] (verification not implemented)

Time = 2.75 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.50

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\ln\left(b\tanh(c+dx)^2+a\right)}{2da^3+6da^2b+6dab^2+2db^3} - \frac{\ln\left(1-\tanh(c+dx)\right)}{2da^3+6da^2b+6dab^2+2db^3} - \frac{\ln\left(\tanh(c+dx)+1\right)}{2da^3+6da^2b+6dab^2+2db^3} + \frac{\frac{\tanh(c+dx)^4\left(\frac{b^3}{4}+\frac{3ab^2}{4}\right)}{a^2d(a^2+2ab+b^2)} + \frac{\tanh(c+dx)^2\left(\frac{b^2}{2}+ab\right)}{ad(a^2+2ab+b^2)}}{a^2+2ab\tanh(c+dx)^2+b^2\tanh(c+dx)^4}$$

input
```
int(tanh(c + d*x)/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
log(a + b*tanh(c + d*x)^2)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) - l
og(1 - tanh(c + d*x))/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) - log(ta
nh(c + d*x) + 1)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) + ((tanh(c +
d*x)^4*((3*a*b^2)/4 + b^3/4))/(a^2*d*(2*a*b + a^2 + b^2)) + (tanh(c + d*x)
^2*(a*b + b^2/2))/(a*d*(2*a*b + a^2 + b^2)))/(a^2 + b^2*tanh(c + d*x)^4 +
2*a*b*tanh(c + d*x)^2)
```

3.196 $\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

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3.196.1 Optimal result

Integrand size = 14, antiderivative size = 142

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{x}{(a+b)^3} + \frac{\sqrt{b}(15a^2+10ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^3 d} + \frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(7a+3b)\tanh(c+dx)}{8a^2(a+b)^2 d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
x/(a+b)^3+1/8*(15*a^2+10*a*b+3*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^
(1/2)/a^(5/2)/(a+b)^3/d+1/4*b*tanh(d*x+c)/a/(a+b)/d/(a+b*tanh(d*x+c)^2)^2+
1/8*b*(7*a+3*b)*tanh(d*x+c)/a^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

3.196.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\frac{\sqrt{b}(15a^2+10ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}} - 4\log(1-\tanh(c+dx)) + 4\log(1+\tanh(c+dx)) + \frac{2b(a+b)^2\tanh(c+dx)}{a\left(a+b\tanh^2(c+dx)\right)^2}}{8(a+b)^3 d}$$

input
```
Integrate[(a + b*Tanh[c + d*x]^2)^(-3),x]
```

output
```
((Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]
])/a^(5/2) - 4*Log[1 - Tanh[c + d*x]] + 4*Log[1 + Tanh[c + d*x]] + (2*b*(a
 + b)^2*Tanh[c + d*x])/(a*(a + b*Tanh[c + d*x]^2)^2) + (b*(a + b)*(7*a + 3
*b)*Tanh[c + d*x])/(a^2*(a + b*Tanh[c + d*x]^2)))/(8*(a + b)^3*d)
```

3.196.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4144, 316, 402, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$

$$\downarrow 4144$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh(c+dx)}{d}$$

$$\downarrow 316$$

$$\frac{\frac{b\tanh(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2} - \frac{\int \frac{3b\tanh^2(c+dx)+b-4(a+b)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}\,d\tanh(c+dx)}{4a(a+b)}}{d}$$

$$\downarrow 402$$

$$\frac{\frac{b\tanh(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2} - \frac{-\frac{\int \frac{8a^2+7ba+3b^2-b(7a+3b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}\,d\tanh(c+dx)}{2a(a+b)} - \frac{b(7a+3b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4a(a+b)}}{d}$$

$$\downarrow 397$$

$$\frac{b\tanh(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{-\frac{b\left(15a^2+10ab+3b^2\right)\int\frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}+\frac{8a^2\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}}{2a(a+b)}-\frac{b(7a+3b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4a(a+b)}$$
$$d$$

$$\downarrow 218$$

$$\frac{b\tanh(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{-\frac{8a^2\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2a(a+b)}-\frac{b(7a+3b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4a(a+b)}$$
$$d$$

$$\downarrow 219$$

$$\frac{b\tanh(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{-\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}+\frac{8a^2\text{arctanh}(\tanh(c+dx))}{a+b}}{2a(a+b)}-\frac{b(7a+3b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4a(a+b)}$$
$$d$$

input
```
Int[(a + b*Tanh[c + d*x]^2)^(-3),x]
```

output
```
((b*Tanh[c + d*x])/(4*a*(a + b)*(a + b*Tanh[c + d*x]^2)^2) - (-1/2*((Sqrt[
b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqr
t[a]*(a + b)) + (8*a^2*ArcTanh[Tanh[c + d*x]])/(a + b))/(a*(a + b)) - (b*(
7*a + 3*b)*Tanh[c + d*x])/(2*a*(a + b)*(a + b*Tanh[c + d*x]^2)))/(4*a*(a +
 b)))/d
```

3.196.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.196.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}+\frac{b\left(\frac{b\left(7a^2+10ab+3b^2\right)\tanh(dx+c)^3}{8a^2}+\frac{\left(9a^2+14ab+5b^2\right)\tanh(dx+c)}{8a}+\frac{\left(15a^2+10ab+3b^2\right)a}{8a}\right.}{(a+b)^3}}{d}$
default	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}+\frac{b\left(\frac{b\left(7a^2+10ab+3b^2\right)\tanh(dx+c)^3}{8a^2}+\frac{\left(9a^2+14ab+5b^2\right)\tanh(dx+c)}{8a}+\frac{\left(15a^2+10ab+3b^2\right)a}{8a}\right.}{(a+b)^3}}{d}$
risch	$\frac{x}{a^3+3a^2b+3ab^2+b^3}-\frac{b\left(9a^3e^{6dx+6c}-a^2be^{6dx+6c}-13ab^2e^{6dx+6c}-3e^{6dx+6c}b^3+27a^3e^{4dx+4c}-9a^2be^{4dx+4c}+21ab^2e\right.}{4\left(ae^{4dx+4c}+be^{4dx+4c}+2e^{2dx+2}\right.}$

input
```
int(1/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/(a+b)^3*ln(tanh(d*x+c)+1)-1/2/(a+b)^3*ln(tanh(d*x+c)-1)+1/(a+b)^3
*b*((1/8*b*(7*a^2+10*a*b+3*b^2)/a^2*tanh(d*x+c)^3+1/8*(9*a^2+14*a*b+5*b^2)
/a*tanh(d*x+c))/(a+b*tanh(d*x+c)^2)^2+1/8*(15*a^2+10*a*b+3*b^2)/a^2/(a*b)^
(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2))))
```

3.196.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3587 vs. 2(128) = 256.

Time = 0.37 (sec) , antiderivative size = 7496, normalized size of antiderivative = 52.79

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.196.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

3.196.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(128) = 256$.

Time = 0.38 (sec) , antiderivative size = 507, normalized size of antiderivative = 3.57

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\left(15\,a^2b+10\,ab^2+3\,b^3\right)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{8\left(a^5+3\,a^4b+3\,a^3b^2+a^2b^3\right)\sqrt{ab}d}$$
$$+\frac{9\,a^3b+21\,a^2b^2+15\,ab^3+3\,b^4+(27\,a^3$$
$$+\frac{dx+c}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)d}$$

input
```
integrate(1/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*(15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) +
a - b)/sqrt(a*b))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)*d) + 1/
4*(9*a^3*b + 21*a^2*b^2 + 15*a*b^3 + 3*b^4 + (27*a^3*b + 13*a^2*b^2 - 23*a
*b^3 - 9*b^4)*e^(-2*d*x - 2*c) + 3*(9*a^3*b - 3*a^2*b^2 + 7*a*b^3 + 3*b^4)
*e^(-4*d*x - 4*c) + (9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4)*e^(-6*d*x - 6*c
))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5 + 4*(a^
7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(-2*d*x - 2*c
) + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*e^
(-4*d*x - 4*c) + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^
2*b^5)*e^(-6*d*x - 6*c) + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3
*b^4 + a^2*b^5)*e^(-8*d*x - 8*c))*d) + (d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d)
```

3.196.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(128) = 256$.

Time $= 0.33$ (sec) , antiderivative size $= 409$, normalized size of antiderivative $= 2.88$

$$\int \frac{1}{\left(a + b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{\left(15a^2b+10ab^2+3b^3\right)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{\left(a^5+3a^4b+3a^3b^2+a^2b^3\right)\sqrt{ab}} + \frac{8(dx+c)}{a^3+3a^2b+3ab^2+b^3} - \frac{2\left(9a^3be^{(6dx+6c)}-a^2b^2e^{(6dx+6c)}-13ab^3e^{(6dx+6c)}\right.}{}$$

input
```
integrate(1/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/8*((15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*
d*x + 2*c) + a - b)/sqrt(a*b))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt
(a*b)) + 8*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(9*a^3*b*e^(6*d*x
 + 6*c) - a^2*b^2*e^(6*d*x + 6*c) - 13*a*b^3*e^(6*d*x + 6*c) - 3*b^4*e^(6*
d*x + 6*c) + 27*a^3*b*e^(4*d*x + 4*c) - 9*a^2*b^2*e^(4*d*x + 4*c) + 21*a*b
^3*e^(4*d*x + 4*c) + 9*b^4*e^(4*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 13
*a^2*b^2*e^(2*d*x + 2*c) - 23*a*b^3*e^(2*d*x + 2*c) - 9*b^4*e^(2*d*x + 2*c
) + 9*a^3*b + 21*a^2*b^2 + 15*a*b^3 + 3*b^4)/((a^5 + 3*a^4*b + 3*a^3*b^2 +
 a^2*b^3)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2
*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

3.196.9 Mupad [B] (verification not implemented)

Time $= 0.81$ (sec) , antiderivative size $= 260$, normalized size of antiderivative $= 1.83$

$$\int \frac{1}{\left(a + b\tanh^2(c+dx)\right)^3}\,dx = \frac{\ln\left(\tanh(c+dx)+1\right)}{2da^3+6da^2b+6dab^2+2db^3}$$
$$- \frac{\ln\left(1-\tanh(c+dx)\right)}{2da^3+6da^2b+6dab^2+2db^3}$$
$$+ \frac{\frac{\tanh(c+dx)^3\left(\frac{3b^3}{8}+\frac{7ab^2}{8}\right)}{a^2d\left(a^2+2ab+b^2\right)} + \frac{\tanh(c+dx)\left(5b^2+9ab\right)}{8ad\left(a^2+2ab+b^2\right)}}{a^2+2ab\tanh\left(c+dx\right)^2+b^2\tanh\left(c+dx\right)^4}$$
$$+ \frac{\operatorname{atan}\left(\frac{b\tanh(c+dx)}{\sqrt{ab}}\right)\left(15a^2b+10ab^2+3b^3\right)}{\sqrt{ab}\left(8a^5d+ab\left(24a^3d+ab\left(24ad+8bd\right)\right)\right)}$$

input
```
int(1/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
log(tanh(c + d*x) + 1)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) - log(1
 - tanh(c + d*x))/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) + ((tanh(c +
 d*x)^3*((7*a*b^2)/8 + (3*b^3)/8))/(a^2*d*(2*a*b + a^2 + b^2)) + (tanh(c +
 d*x)*(9*a*b + 5*b^2))/(8*a*d*(2*a*b + a^2 + b^2)))/(a^2 + b^2*tanh(c + d*
x)^4 + 2*a*b*tanh(c + d*x)^2) + (atan((b*tanh(c + d*x))/(a*b)^(1/2))*(10*a
*b^2 + 15*a^2*b + 3*b^3))/((a*b)^(1/2)*(8*a^5*d + a*b*(24*a^3*d + a*b*(24*
a*d + 8*b*d))))
```

3.197 $\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

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3.197.1 Optimal result

Integrand size = 21, antiderivative size = 138

$$\begin{aligned}\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx &= \frac{\log(\cosh(c+dx))}{(a+b)^3 d} + \frac{\log(\tanh(c+dx))}{a^3 d} \\ &\quad - \frac{b(3a^2+3ab+b^2)\log\left(a+b\tanh^2(c+dx)\right)}{2a^3(a+b)^3 d} \\ &\quad + \frac{b}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} \\ &\quad + \frac{b(2a+b)}{2a^2(a+b)^2 d\left(a+b\tanh^2(c+dx)\right)}\end{aligned}$$

output

```
ln(cosh(d*x+c))/(a+b)^3/d+ln(tanh(d*x+c))/a^3/d-1/2*b*(3*a^2+3*a*b+b^2)*ln
(a+b*tanh(d*x+c)^2)/a^3/(a+b)^3/d+1/4*b/a/(a+b)/d/(a+b*tanh(d*x+c)^2)^2+1/
2*b*(2*a+b)/a^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

3.197.2 Mathematica [A] (verified)

Time = 1.77 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$= \frac{\frac{4\log(\cosh(c+dx))}{(a+b)^3} + \frac{4\log(\tanh(c+dx)) + \frac{b\left(-2\left(3a^2+3ab+b^2\right)\log\left(a+b\tanh^2(c+dx)\right) + \frac{a(a+b)\left(a(5a+3b)+2b(2a+b)\tanh^2(c+dx)\right)}{\left(a+b\tanh^2(c+dx)\right)^2}\right)}{(a+b)^3}}{a^3}}{4d}$$

input
```
Integrate[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((4*Log[Cosh[c + d*x]])/(a + b)^3 + (4*Log[Tanh[c + d*x]] + (b*(-2*(3*a^2
+ 3*a*b + b^2)*Log[a + b*Tanh[c + d*x]^2] + (a*(a + b)*(a*(5*a + 3*b) + 2*
b*(2*a + b)*Tanh[c + d*x]^2))/(a + b*Tanh[c + d*x]^2)^2))/(a + b)^3)/a^3)/
(4*d)
```

3.197.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 4153, 26, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{i}{\tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$

$$\downarrow 26$$

$$i\int \frac{1}{\tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$

$$\downarrow 4153$$

$$\frac{i\int -\frac{i\coth(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\coth(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\downarrow 354$$

$$\frac{\int \frac{\coth(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)^3}d\tanh^2(c+dx)}{2d}$$

$$\downarrow 93$$

$$\frac{\int \left(-\frac{(3a^2+3ba+b^2)b^2}{a^3(a+b)^3\left(b\tanh^2(c+dx)+a\right)}-\frac{(2a+b)b^2}{a^2(a+b)^2\left(b\tanh^2(c+dx)+a\right)^2}-\frac{b^2}{a(a+b)\left(b\tanh^2(c+dx)+a\right)^3}+\frac{\coth(c+dx)}{a^3}-\frac{1}{(a+b)^3(\tanh^2(c+dx)-}\right.}{2d}$$

$$\downarrow 2009$$

$$\frac{\frac{\log\left(\tanh^2(c+dx)\right)}{a^3}+\frac{b(2a+b)}{a^2(a+b)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{b\left(3a^2+3ab+b^2\right)\log\left(a+b\tanh^2(c+dx)\right)}{a^3(a+b)^3}+\frac{b}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\log(1-\tanh^2}{(a+b)}}{2d}$$

input
```
Int[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(Log[Tanh[c + d*x]^2]/a^3 - Log[1 - Tanh[c + d*x]^2]/(a + b)^3 - (b*(3*a^2
 + 3*a*b + b^2)*Log[a + b*Tanh[c + d*x]^2])/(a^3*(a + b)^3) + b/(2*a*(a +
b)*(a + b*Tanh[c + d*x]^2)^2) + (b*(2*a + b))/(a^2*(a + b)^2*(a + b*Tanh[c
 + d*x]^2)))/(2*d)
```

3.197.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.197.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.17

method	result
derivativedivides	$-\frac{-\frac{\ln(\tanh(dx+c))}{a^3}+\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}+\frac{b^2\left(-\frac{a^2\left(a^2+2ab+b^2\right)}{2b\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(3a^2+3ab+b^2\right)\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a\left(2a^2+3ab+b\right.}{b\left(a+b\tanh(dx+\right.}}{2(a+b)^3a^3}}{d}$
default	$-\frac{-\frac{\ln(\tanh(dx+c))}{a^3}+\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}+\frac{b^2\left(-\frac{a^2\left(a^2+2ab+b^2\right)}{2b\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(3a^2+3ab+b^2\right)\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a\left(2a^2+3ab+b\right.}{b\left(a+b\tanh(dx+\right.}}{2(a+b)^3a^3}}{d}$
parallelrisch	$\frac{-24\left(\frac{(a+b)^2\cosh(4dx+4c)}{4}+\left(a^2-b^2\right)\cosh(2dx+2c)+\frac{3a^2}{4}-\frac{ab}{2}+\frac{3b^2}{4}\right)\left(a^2+ab+\frac{1}{3}b^2\right)b\ln\left(a+b\tanh(dx+c)^2\right)-16\left(\frac{(a+b)^2}{4}\right.}{}$
risch	$\frac{x}{a^3+3a^2b+3ab^2+b^3}-\frac{2x}{a^3}-\frac{2c}{a^3d}+\frac{6bx}{a(a^3+3a^2b+3ab^2+b^3)}+\frac{6bc}{ad(a^3+3a^2b+3ab^2+b^3)}+\frac{6b^2x}{a^2(a^3+3a^2b+3ab^2+b^3}$

input
```
int(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-1/a^3*ln(tanh(d*x+c))+1/2/(a+b)^3*ln(tanh(d*x+c)-1)+1/2*b^2/(a+b)^3
/a^3*(-1/2*a^2*(a^2+2*a*b+b^2)/b/(a+b*tanh(d*x+c)^2)^2+(3*a^2+3*a*b+b^2)/b
*ln(a+b*tanh(d*x+c)^2)-a*(2*a^2+3*a*b+b^2)/b/(a+b*tanh(d*x+c)^2))+1/2/(a+b
)^3*ln(tanh(d*x+c)+1))
```

3.197.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4800 vs. $2(132) = 264$.

Time $= 0.52$ (sec) , antiderivative size $= 4800$, normalized size of antiderivative $= 34.78$

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^8 + 16*(a^5 + 2*a^4*b
+ a^3*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^5 + 2*a^4*b + a^3*b^2)
*d*x*sinh(d*x + c)^8 - 4*(3*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2
)*d*x)*cosh(d*x + c)^6 - 4*(3*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 14*(a^5 + 2*a^
4*b + a^3*b^2)*d*x*cosh(d*x + c)^2 - 2*(a^5 - a^3*b^2)*d*x)*sinh(d*x + c)^
6 + 8*(14*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^3 - 3*(3*a^3*b^2 + 4
*a^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 -
 4*(6*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 - (3*a^5 - 2*a^4*b + 3*a^3*b^2)*d*x)*c
osh(d*x + c)^4 + 4*(35*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^4 - 6*a
^3*b^2 + 2*a^2*b^3 + 2*a*b^4 + (3*a^5 - 2*a^4*b + 3*a^3*b^2)*d*x - 15*(3*a
^3*b^2 + 4*a^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c)^2)*sinh(
d*x + c)^4 + 16*(7*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^5 - 5*(3*a^
3*b^2 + 4*a^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c)^3 - (6*a^
3*b^2 - 2*a^2*b^3 - 2*a*b^4 - (3*a^5 - 2*a^4*b + 3*a^3*b^2)*d*x)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 2*(a^5 + 2*a^4*b + a^3*b^2)*d*x - 4*(3*a^3*b^2 + 4
*a^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c)^2 + 4*(14*(a^5 + 2
*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^6 - 3*a^3*b^2 - 4*a^2*b^3 - a*b^4 - 15
*(3*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c)^4 +
 2*(a^5 - a^3*b^2)*d*x - 6*(6*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 - (3*a^5 - 2*a
^4*b + 3*a^3*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^4*b + 9...
```

3.197.6 Sympy [F]

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\coth\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(coth(c + d*x)/(a + b*tanh(c + d*x)**2)**3, x)
```

3.197.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(132) = 264$.

Time = 0.23 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.61

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= -\frac{(3\,a^2b+3\,ab^2+b^3)\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)d}$$
$$+\frac{dx+c}{(a^3+3\,a^2b+3\,ab^2+b^3)d}$$
$$+\frac{2\left((3\,a^2b$$
$$+\frac{\log\left(e^{(-dx-c)}+1\right)}{a^3d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{a^3d}$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/2*(3*a^2*b + 3*a*b^2 + b^3)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^
(-4*d*x - 4*c) + a + b)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) + (d*x +
 c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 2*((3*a^2*b^2 + 4*a*b^3 + b^4)*e
^(-2*d*x - 2*c) + 2*(3*a^2*b^2 - a*b^3 - b^4)*e^(-4*d*x - 4*c) + (3*a^2*b^
2 + 4*a*b^3 + b^4)*e^(-6*d*x - 6*c))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4
*b^3 + 5*a^3*b^4 + a^2*b^5 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*
a^3*b^4 - a^2*b^5)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a
^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*e^(-4*d*x - 4*c) + 4*(a^7 + 3*a^6*b + 2*a^
5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(-6*d*x - 6*c) + (a^7 + 5*a^6*b
 + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*e^(-8*d*x - 8*c))*d) + l
og(e^(-d*x - c) + 1)/(a^3*d) + log(e^(-d*x - c) - 1)/(a^3*d)
```

3.197.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(132) = 264$.

Time = 0.44 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.14

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx =$$

$$-\frac{\frac{\left(3a^2b+3ab^2+b^3\right)\log\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b\right)}{a^6+3a^5b+3a^4b^2+a^3b^3}+\frac{2(dx+c)}{a^3+3a^2b+3ab^2+b^3}-\frac{2\log\left(\left|e^{(2dx+2c)}-1\right|\right)}{a^3}-\frac{4}{}}{2d}$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/2*((3*a^2*b + 3*a*b^2 + b^3)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c)
+ 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^6 + 3*a^5*b + 3*a^
4*b^2 + a^3*b^3) + 2*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*log(abs
(e^(2*d*x + 2*c) - 1))/a^3 - 4*((3*a^2*b^2 + a*b^3)*e^(6*d*x + 6*c) + (3*a
^2*b^2 + a*b^3)*e^(2*d*x + 2*c) + 2*(3*a^3*b^2 - a^2*b^3 - a*b^4)*e^(4*d*x
 + 4*c)/(a + b))/((a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x +
2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2*(a + b)^2*a^3))/d
```

3.197.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\coth(c+d\,x)}{\left(b\tanh(c+d\,x)^2+a\right)^3}\,dx$$

input
```
int(coth(c + d*x)/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
int(coth(c + d*x)/(a + b*tanh(c + d*x)^2)^3, x)
```

3.198 $\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

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3.198.1 Optimal result

Integrand size = 23, antiderivative size = 178

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{x}{(a+b)^3} - \frac{b^{3/2}(35a^2+42ab+15b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}(a+b)^3 d} - \frac{(8a^2+27ab+15b^2)\coth(c+dx)}{8a^3(a+b)^2 d} + \frac{b\coth(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(9a+5b)\coth(c+dx)}{8a^2(a+b)^2 d\left(a+b\tanh^2(c+dx)\right)}$$

output

```
x/(a+b)^3-1/8*b^(3/2)*(35*a^2+42*a*b+15*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^
(1/2))/a^(7/2)/(a+b)^3/d-1/8*(8*a^2+27*a*b+15*b^2)*coth(d*x+c)/a^3/(a+b)^2
/d+1/4*b*coth(d*x+c)/a/(a+b)/d/(a+b*tanh(d*x+c)^2)^2+1/8*b*(9*a+5*b)*coth(
d*x+c)/a^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

3.198.2 Mathematica [A] (verified)

Time = 6.80 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.93

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx =$$

$$-\frac{-\frac{8(c+dx)}{(a+b)^3}+\frac{b^{3/2}\left(35a^2+42ab+15b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}(a+b)^3}+\frac{8\coth(c+dx)}{a^3}+\frac{4b^3\sinh(2(c+dx))}{a^2(a+b)^2(a-b+(a+b)\cosh(2(c+dx)))^2}+\frac{b^2(13}{a^3(a+b)^2(a}}{8d}$$

input
```
Integrate[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-1/8*((-8*(c + d*x))/(a + b)^3 + (b^(3/2)*(35*a^2 + 42*a*b + 15*b^2)*ArcTa
n[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(7/2)*(a + b)^3) + (8*Coth[c + d*x]
)/a^3 + (4*b^3*Sinh[2*(c + d*x)])/(a^2*(a + b)^2*(a - b + (a + b)*Cosh[2*(
c + d*x)])^2) + (b^2*(13*a + 7*b)*Sinh[2*(c + d*x)])/(a^3*(a + b)^2*(a - b
 + (a + b)*Cosh[2*(c + d*x)])))/d
```

3.198.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 25, 4153, 25, 374, 25, 441, 25, 445, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\downarrow 4153$$

$$-\frac{\int -\frac{\coth^2(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\downarrow 25$$

$$\frac{\int \frac{\coth^2(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\downarrow 374$$

$$-\frac{\frac{\int -\frac{\coth^2(c+dx)\left(-5b\tanh^2(c+dx)+4a+5b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4a(a+b)}-\frac{b\coth(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 25$$

$$-\frac{\frac{\int \frac{\coth^2(c+dx)\left(-5b\tanh^2(c+dx)+4a+5b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4a(a+b)}-\frac{b\coth(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 441$$

$$-\frac{\frac{\frac{b(9a+5b)\coth(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\int -\frac{\coth^2(c+dx)\left(8a^2+27ba+15b^2-3b(9a+5b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}}{4a(a+b)}-\frac{b\coth(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 25$$

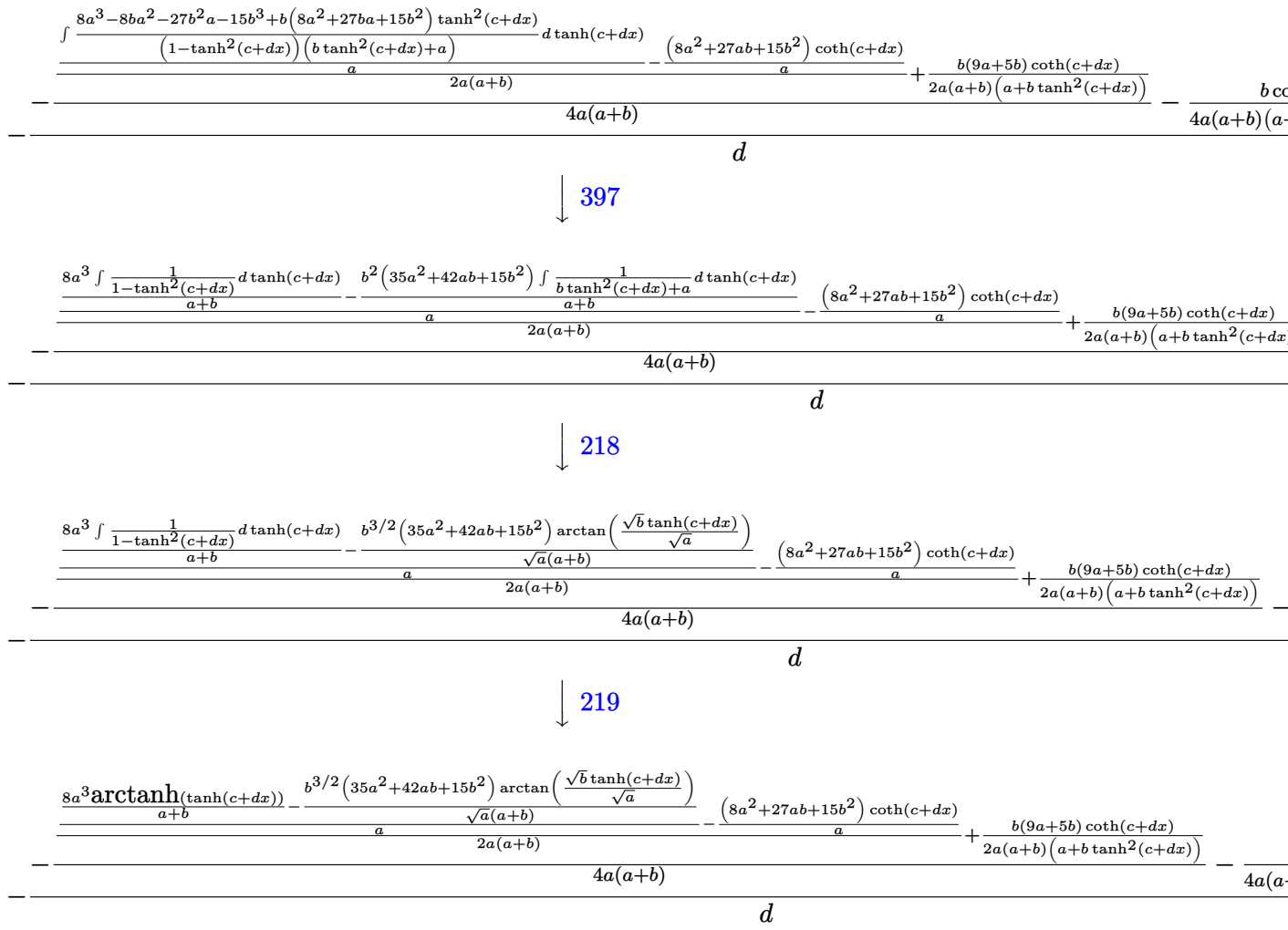
$$-\frac{\frac{\frac{b(9a+5b)\coth(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{\int \frac{\coth^2(c+dx)\left(8a^2+27ba+15b^2-3b(9a+5b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}}{4a(a+b)}-\frac{b\coth(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 445$$

$$-\frac{\frac{\frac{\left(8a^2+27ab+15b^2\right)\coth(c+dx)}{a}+\frac{b(9a+5b)\coth(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\int -\frac{8a^3-8ba^2-27b^2a-15b^3+b\left(8a^2+27ba+15b^2\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}}{2a(a+b)}}{4a(a+b)}-\frac{}{4a(a+b)}}{d}$$

$$\downarrow 25$$

3.198. $\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}dx$



input
```
Int[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-((-1/4*(b*Coth[c + d*x])/(a*(a + b)*(a + b*Tanh[c + d*x]^2)^2) - (((-((b^
(3/2)*(35*a^2 + 42*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/
(Sqrt[a]*(a + b))) + (8*a^3*ArcTanh[Tanh[c + d*x]])/(a + b))/a - ((8*a^2 +
 27*a*b + 15*b^2)*Coth[c + d*x])/a)/(2*a*(a + b)) + (b*(9*a + 5*b)*Coth[c
+ d*x])/(2*a*(a + b)*(a + b*Tanh[c + d*x]^2)))/(4*a*(a + b)))/d)
```

3.198. $\int \frac{\coth^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

3.198.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.198.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.94

method	result
derivativedivides	$-\frac{-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}+\frac{b^2\left(\frac{\left(\frac{11}{8}a^2b+\frac{9}{4}ab^2+\frac{7}{8}b^3\right)\tanh(dx+c)^3+\frac{a\left(13a^2+22ab+9b^2\right)\tanh(dx+c)}{8}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(35a^2+42ab+15b^2\right)\arctan\left(\frac{b}{8\sqrt{ab}}\right.}{(a+b)^3a^3}}{d}$
default	$-\frac{-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}+\frac{b^2\left(\frac{\left(\frac{11}{8}a^2b+\frac{9}{4}ab^2+\frac{7}{8}b^3\right)\tanh(dx+c)^3+\frac{a\left(13a^2+22ab+9b^2\right)\tanh(dx+c)}{8}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(35a^2+42ab+15b^2\right)\arctan\left(\frac{b}{8\sqrt{ab}}\right.}{(a+b)^3a^3}}{d}$
risch	$\frac{x}{a^3+3a^2b+3ab^2+b^3}-\frac{-60b^5e^{6dx+6c}+90b^5e^{4dx+4c}+8a^5+15b^5+40a^4b+93a^3b^2+113a^2b^3+67ab^4-60b^5e^{2dx+2c}+66a$

input
```
int(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-1/2/(a+b)^3*ln(tanh(d*x+c)+1)+b^2/(a+b)^3/a^3*(((11/8*a^2*b+9/4*a*b
^2+7/8*b^3)*tanh(d*x+c)^3+1/8*a*(13*a^2+22*a*b+9*b^2)*tanh(d*x+c))/(a+b*ta
nh(d*x+c)^2)^2+1/8*(35*a^2+42*a*b+15*b^2)/(a*b)^(1/2)*arctan(b*tanh(d*x+c)
/(a*b)^(1/2)))+1/a^3/tanh(d*x+c)+1/2/(a+b)^3*ln(tanh(d*x+c)-1))
```

3.198.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5772 vs. $2(162) = 324$.

Time $= 0.44$ (sec) , antiderivative size $= 11865$, normalized size of antiderivative $= 66.66$

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.198.6 Sympy [F]

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

input
```
integrate(coth(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(coth(c + d*x)**2/(a + b*tanh(c + d*x)**2)**3, x)
```

3.198.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1944 vs. $2(162) = 324$.

Time $= 0.65$ (sec) , antiderivative size $= 1944$, normalized size of antiderivative $= 10.92$

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*(3*a^2*b + 3*a*b^2 + b^3)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(
2*d*x + 2*c) + a + b)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) + 1/4*(3*a
^2*b + 3*a*b^2 + b^3)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x -
 4*c) + a + b)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) + 1/32*(15*a^3*b
- 25*a^2*b^2 - 39*a*b^3 - 15*b^4)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a
- b)/sqrt(a*b))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt(a*b)*d) - 1/32
*(15*a^3*b - 25*a^2*b^2 - 39*a*b^3 - 15*b^4)*arctan(1/2*((a + b)*e^(-2*d*x
 - 2*c) + a - b)/sqrt(a*b))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt(a*
b)*d) + 1/16*(8*a^5 + 31*a^4*b + 72*a^3*b^2 + 98*a^2*b^3 + 64*a*b^4 + 15*b
^5 + (8*a^5 + 49*a^4*b + 18*a^3*b^2 + 38*a*b^4 + 15*b^5)*e^(8*d*x + 8*c) +
 2*(16*a^5 + 57*a^4*b - 9*a^3*b^2 + 37*a^2*b^3 - 39*a*b^4 - 30*b^5)*e^(6*d
*x + 6*c) + 2*(24*a^5 + 56*a^4*b + 83*a^3*b^2 - 37*a^2*b^3 + 53*a*b^4 + 45
*b^5)*e^(4*d*x + 4*c) + 2*(16*a^5 + 39*a^4*b + 73*a^3*b^2 + 15*a^2*b^3 - 6
5*a*b^4 - 30*b^5)*e^(2*d*x + 2*c))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b
^3 + 5*a^4*b^4 + a^3*b^5 - (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^
4*b^4 + a^3*b^5)*e^(10*d*x + 10*c) - (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5
*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*e^(8*d*x + 8*c) - 2*(a^8 + a^7*b + 2*a^6*b^
2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*e^(6*d*x + 6*c) + 2*(a^8 + a^7*b
+ 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*e^(4*d*x + 4*c) + (3*a^
8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*e^(2*d*x...
```

3.198.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(162) = 324$.

Time $= 0.46$ (sec) , antiderivative size $= 437$, normalized size of antiderivative $= 2.46$

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx =$$

$$-\frac{\left(35\,a^2b^2+42\,ab^3+15\,b^4\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{\left(a^6+3\,a^5b+3\,a^4b^2+a^3b^3\right)\sqrt{ab}}-\frac{8\,(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\left(13\,a^3b^2e^{(6\,dx+6\,c)}+3\,a^2b^3e^{(6\,dx+6\,c)}-17\,ab^4e\right.}{}$$

input
```
integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((35*a^2*b^2 + 42*a*b^3 + 15*b^4)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e
^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*
sqrt(a*b)) - 8*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(13*a^3*b^2*e
^(6*d*x + 6*c) + 3*a^2*b^3*e^(6*d*x + 6*c) - 17*a*b^4*e^(6*d*x + 6*c) - 7*
b^5*e^(6*d*x + 6*c) + 39*a^3*b^2*e^(4*d*x + 4*c) - 5*a^2*b^3*e^(4*d*x + 4*
c) + 25*a*b^4*e^(4*d*x + 4*c) + 21*b^5*e^(4*d*x + 4*c) + 39*a^3*b^2*e^(2*d
*x + 2*c) + 25*a^2*b^3*e^(2*d*x + 2*c) - 35*a*b^4*e^(2*d*x + 2*c) - 21*b^5
*e^(2*d*x + 2*c) + 13*a^3*b^2 + 33*a^2*b^3 + 27*a*b^4 + 7*b^5)/((a^6 + 3*a
^5*b + 3*a^4*b^2 + a^3*b^3)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e
^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2) + 16/(a^3*(e^(2*d*x + 2*c
) - 1)))/d
```

3.198.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\coth(c+d\,x)^2}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3, x)
```

3.199 $\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

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3.199.1 Optimal result

Integrand size = 23, antiderivative size = 171

$$\begin{aligned}\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx &= -\frac{\coth^2(c+dx)}{2a^3d}+\frac{\log(\cosh(c+dx))}{(a+b)^3d}\\ &+\frac{(a-3b)\log(\tanh(c+dx))}{a^4d}\\ &+\frac{b^2(6a^2+8ab+3b^2)\log\left(a+b\tanh^2(c+dx)\right)}{2a^4(a+b)^3d}\\ &-\frac{b^2}{4a^2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2}\\ &-\frac{b^2(3a+2b)}{2a^3(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}\end{aligned}$$

output

```
-1/2*coth(d*x+c)^2/a^3/d+ln(cosh(d*x+c))/(a+b)^3/d+(a-3*b)*ln(tanh(d*x+c))
/a^4/d+1/2*b^2*(6*a^2+8*a*b+3*b^2)*ln(a+b*tanh(d*x+c)^2)/a^4/(a+b)^3/d-1/4
*b^2/a^2/(a+b)/d/(a+b*tanh(d*x+c)^2)^2-1/2*b^2*(3*a+2*b)/a^3/(a+b)^2/d/(a+
b*tanh(d*x+c)^2)
```

3.199.2 Mathematica [A] (verified)

Time = 2.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.81

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\frac{\coth^2(c+dx)}{a^3}+\frac{b^4}{2a^4(a+b)\left(b+a\coth^2(c+dx)\right)^2}-\frac{b^3(4a+3b)}{a^4(a+b)^2\left(b+a\coth^2(c+dx)\right)}-\frac{b^2\left(6a^2+8ab+3b^2\right)\log\left(b+a\coth^2(c+dx)\right)}{a^4(a+b)^3}-\frac{2\log(\sinh}{(a+}}{2d}$$

input
```
Integrate[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-1/2*(Coth[c + d*x]^2/a^3 + b^4/(2*a^4*(a + b)*(b + a*Coth[c + d*x]^2)^2)
- (b^3*(4*a + 3*b))/(a^4*(a + b)^2*(b + a*Coth[c + d*x]^2)) - (b^2*(6*a^2
+ 8*a*b + 3*b^2)*Log[b + a*Coth[c + d*x]^2])/(a^4*(a + b)^3) - (2*Log[Sinh
[c + d*x]])/(a + b)^3)/d
```

3.199.3 Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{i}{\tan(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$
$$\downarrow 26$$
$$-i\int \frac{1}{\tan(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$
$$\downarrow 4153$$
$$-\frac{i\int \frac{i\coth^3(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh(c+dx)}{d}$$

$$\downarrow 26$$

$$\frac{\int \frac{\coth^3(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)^3} d\tanh(c+dx)}{d}$$

$$\downarrow 354$$

$$\frac{\int \frac{\coth^2(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)^3} d\tanh^2(c+dx)}{2d}$$

$$\downarrow 99$$

$$\frac{\int \left(\frac{(6a^2+8ba+3b^2)b^3}{a^4(a+b)^3\left(b\tanh^2(c+dx)+a\right)} + \frac{(3a+2b)b^3}{a^3(a+b)^2\left(b\tanh^2(c+dx)+a\right)^2} + \frac{b^3}{a^2(a+b)\left(b\tanh^2(c+dx)+a\right)^3} + \frac{\coth^2(c+dx)}{a^3} + \frac{(a-3b)\coth(c+dx)}{a^4} -}{2d}$$

$$\downarrow 2009$$

$$\frac{\frac{(a-3b)\log\left(\tanh^2(c+dx)\right)}{a^4} - \frac{b^2(3a+2b)}{a^3(a+b)^2\left(a+b\tanh^2(c+dx)\right)} - \frac{\coth(c+dx)}{a^3} - \frac{b^2}{2a^2(a+b)\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b^2\left(6a^2+8ab+3b^2\right)\log\left(a+b\tanh}{a^4(a+b)^3}}{2d}$$

input
```
Int[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-(Coth[c + d*x]/a^3) + ((a - 3*b)*Log[Tanh[c + d*x]^2])/a^4 - Log[1 - Tan
h[c + d*x]^2]/(a + b)^3 + (b^2*(6*a^2 + 8*a*b + 3*b^2)*Log[a + b*Tanh[c +
d*x]^2])/(a^4*(a + b)^3) - b^2/(2*a^2*(a + b)*(a + b*Tanh[c + d*x]^2)^2) -
 (b^2*(3*a + 2*b))/(a^3*(a + b)^2*(a + b*Tanh[c + d*x]^2)))/(2*d)
```

3.199.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.199.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.08

method	result
derivativedivides	$-\frac{\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{b^3\left(\frac{\left(6a^2+8ab+3b^2\right)\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a^2\left(a^2+2ab+b^2\right)}{2b\left(a+b\tanh(dx+c)^2\right)^2}-\frac{a\left(3a^2+5ab+2b^2\right)}{b\left(a+b\tanh(dx+c)^2\right)}\right)}{2(a+b)^3a^4}+\frac{\ln(\tanh(dx+$
default	$-\frac{\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{b^3\left(\frac{\left(6a^2+8ab+3b^2\right)\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a^2\left(a^2+2ab+b^2\right)}{2b\left(a+b\tanh(dx+c)^2\right)^2}-\frac{a\left(3a^2+5ab+2b^2\right)}{b\left(a+b\tanh(dx+c)^2\right)}\right)}{2(a+b)^3a^4}+\frac{\ln(\tanh(dx+$
parallelrisch	$\frac{48\left(a^2+\frac{4}{3}ab+\frac{1}{2}b^2\right)\left(\frac{(a+b)^2\cosh(4dx+4c)}{4}+\left(a^2-b^2\right)\cosh(2dx+2c)+\frac{3a^2}{4}-\frac{ab}{2}+\frac{3b^2}{4}\right)b^2\ln\left(a+b\tanh(dx+c)^2\right)-16\left(\frac{(a+b)^2}{}\right.}{}$
risch	$\frac{x}{a^3+3a^2b+3ab^2+b^3}-\frac{2x}{a^3}-\frac{2c}{a^3d}+\frac{6bx}{a^4}+\frac{6bc}{da^4}-\frac{12b^2x}{a^2(a^3+3a^2b+3ab^2+b^3)}-\frac{12b^2c}{a^2d(a^3+3a^2b+3ab^2+b^3)}-\frac{}{a^3(}$

input
```
int(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(1/2/(a+b)^3*ln(tanh(d*x+c)-1)-1/2*b^3/(a+b)^3/a^4*((6*a^2+8*a*b+3*b^
2)/b*ln(a+b*tanh(d*x+c)^2)-1/2*a^2*(a^2+2*a*b+b^2)/b/(a+b*tanh(d*x+c)^2)^2
-a*(3*a^2+5*a*b+2*b^2)/b/(a+b*tanh(d*x+c)^2))+1/2/(a+b)^3*ln(tanh(d*x+c)+1
)+(-a+3*b)/a^4*ln(tanh(d*x+c))+1/2/a^3/tanh(d*x+c)^2)
```

3.199.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10720 vs. $2(163) = 326$.

Time $= 0.71$ (sec) , antiderivative size $= 10720$, normalized size of antiderivative $= 62.69$

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.199.6 Sympy [F]

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

input
```
integrate(coth(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(coth(c + d*x)**3/(a + b*tanh(c + d*x)**2)**3, x)
```

3.199.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(163) = 326$.

Time = 0.27 (sec) , antiderivative size = 770, normalized size of antiderivative = 4.50

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{(6\,a^2b^2+8\,ab^3+3\,b^4)\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)d}$$
$$+\frac{dx+c}{(a^3+3\,a^2b+3\,ab^2+b^3)d}$$
$$-\frac{2\left((a^5+5\,a^4b+10\,a^3b^2+14\,a^2b^3+11\,ab^4+\right.}{(a^8+5\,a^7b+10\,a^6b^2+10\,a^5b^3+5\,a^4b^4+a^3b^5+2\,(a^8+a^7b-6\,a^6b^2-14\,a^5b^3-11\,a^4b^4-3\,a^3b^5)e^{(-2\,d}$$
$$+\frac{(a-3\,b)\log\left(e^{(-dx-c)}+1\right)}{a^4d}+\frac{(a-3\,b)\log\left(e^{(-dx-c)}-1\right)}{a^4d}$$

input
```
integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/2*(6*a^2*b^2 + 8*a*b^3 + 3*b^4)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)
*e^(-4*d*x - 4*c) + a + b)/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d) + (d*
x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 2*((a^5 + 5*a^4*b + 10*a^3*b^
2 + 14*a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(-2*d*x - 2*c) + 2*(2*a^5 + 6*a^4*b +
 4*a^3*b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(-4*d*x - 4*c) + 2*(3*a^5 + 7
*a^4*b + 6*a^3*b^2 + 2*a^2*b^3 + 15*a*b^4 + 9*b^5)*e^(-6*d*x - 6*c) + 2*(2
*a^5 + 6*a^4*b + 4*a^3*b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(-8*d*x - 8*c
) + (a^5 + 5*a^4*b + 10*a^3*b^2 + 14*a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(-10*d*
x - 10*c))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5
 + 2*(a^8 + a^7*b - 6*a^6*b^2 - 14*a^5*b^3 - 11*a^4*b^4 - 3*a^3*b^5)*e^(-2
*d*x - 2*c) - (a^8 + 5*a^7*b - 6*a^6*b^2 - 38*a^5*b^3 - 43*a^4*b^4 - 15*a^
3*b^5)*e^(-4*d*x - 4*c) - 4*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4
*b^4 + 5*a^3*b^5)*e^(-6*d*x - 6*c) - (a^8 + 5*a^7*b - 6*a^6*b^2 - 38*a^5*b
^3 - 43*a^4*b^4 - 15*a^3*b^5)*e^(-8*d*x - 8*c) + 2*(a^8 + a^7*b - 6*a^6*b^
2 - 14*a^5*b^3 - 11*a^4*b^4 - 3*a^3*b^5)*e^(-10*d*x - 10*c) + (a^8 + 5*a^7
*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*e^(-12*d*x - 12*c))*d)
 + (a - 3*b)*log(e^(-d*x - c) + 1)/(a^4*d) + (a - 3*b)*log(e^(-d*x - c) -
1)/(a^4*d)
```

3.199.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(163) = 326$.

Time = 0.54 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.77

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{(6a^2b^2+8ab^3+3b^4)\log\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b\right)}{a^7+3a^6b+3a^5b^2+a^4b^3} - \frac{2(dx+c)}{a^3+3a^2b+3ab^2+b^3} + \frac{2(a-3b)\log\left(\left|e^{(2dx+2c)}-1\right|\right.}{a^4}$$

input
```
integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/2*((6*a^2*b^2 + 8*a*b^3 + 3*b^4)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*
c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^7 + 3*a^6*b + 3
*a^5*b^2 + a^4*b^3) - 2*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(a -
 3*b)*log(abs(e^(2*d*x + 2*c) - 1))/a^4 - 4*((a^5 + 5*a^4*b + 10*a^3*b^2 +
 14*a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(10*d*x + 10*c) + 2*(2*a^5 + 6*a^4*b + 4
*a^3*b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(8*d*x + 8*c) + 2*(3*a^5 + 7*a^
4*b + 6*a^3*b^2 + 2*a^2*b^3 + 15*a*b^4 + 9*b^5)*e^(6*d*x + 6*c) + 2*(2*a^5
 + 6*a^4*b + 4*a^3*b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(4*d*x + 4*c) + (
a^5 + 5*a^4*b + 10*a^3*b^2 + 14*a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(2*d*x + 2*c
))/((a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(
2*d*x + 2*c) + a + b)^2*(a + b)^3*a^3*(e^(2*d*x + 2*c) - 1)^2))/d
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\coth(c+d\,x)^3}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3, x)
```

3.200 $\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

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3.200.1 Optimal result

Integrand size = 23, antiderivative size = 228

$$\begin{aligned}\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx &= \frac{x}{(a+b)^3}+\frac{b^{5/2}(63a^2+90ab+35b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}(a+b)^3d}\\ &-\frac{(8a^3-8a^2b-55ab^2-35b^3)\coth(c+dx)}{8a^4(a+b)^2d}\\ &-\frac{(8a^2+55ab+35b^2)\coth^3(c+dx)}{24a^3(a+b)^2d}\\ &+\frac{b\coth^3(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2}\\ &+\frac{b(11a+7b)\coth^3(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}\end{aligned}$$

output

```
x/(a+b)^3+1/8*b^(5/2)*(63*a^2+90*a*b+35*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^
(1/2))/a^(9/2)/(a+b)^3/d-1/8*(8*a^3-8*a^2*b-55*a*b^2-35*b^3)*coth(d*x+c)/a
^4/(a+b)^2/d-1/24*(8*a^2+55*a*b+35*b^2)*coth(d*x+c)^3/a^3/(a+b)^2/d+1/4*b*
coth(d*x+c)^3/a/(a+b)/d/(a+b*tanh(d*x+c)^2)^2+1/8*b*(11*a+7*b)*coth(d*x+c)
^3/a^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

3.200.2 Mathematica [A] (verified)

Time = 3.91 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.85

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$= \frac{\frac{24(c+dx)}{(a+b)^3} + \frac{3b^{5/2}(63a^2+90ab+35b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{9/2}(a+b)^3} + \frac{8(-4a+9b)\coth(c+dx)}{a^4} - \frac{8\coth(c+dx)\text{csch}^2(c+dx)}{a^3} + \frac{12b^4 \text{ si}}{a^3(a+b)^2(a-b+}}{24d}$$

input
```
Integrate[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((24*(c + d*x))/(a + b)^3 + (3*b^(5/2)*(63*a^2 + 90*a*b + 35*b^2)*ArcTan[(
Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(9/2)*(a + b)^3) + (8*(-4*a + 9*b)*Cot
h[c + d*x])/a^4 - (8*Coth[c + d*x]*Csch[c + d*x]^2)/a^3 + (12*b^4*Sinh[2*(
c + d*x)])/(a^3*(a + b)^2*(a - b + (a + b)*Cosh[2*(c + d*x)])^2) + (3*b^3*
(17*a + 11*b)*Sinh[2*(c + d*x)])/(a^4*(a + b)^2*(a - b + (a + b)*Cosh[2*(c
 + d*x)])))/(24*d)
```

3.200.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 4153, 374, 25, 441, 25, 445, 27, 445, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\tan(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)^3}\,dx$$

$$\downarrow 4153$$

$$\frac{\int \frac{\coth^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}\,d\tanh(c+dx)}{d}$$

$$\downarrow 374$$

$$\frac{\frac{b\coth^3(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\int-\frac{\coth^4(c+dx)\left(-7b\tanh^2(c+dx)+4a+7b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4a(a+b)}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{\int\frac{\coth^4(c+dx)\left(-7b\tanh^2(c+dx)+4a+7b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4a(a+b)}+\frac{b\coth^3(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 441$$

$$\frac{\frac{b(11a+7b)\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\int-\frac{\coth^4(c+dx)\left(8a^2+55ba+35b^2-5b(11a+7b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}}{4a(a+b)}+\frac{b\coth^3(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{\int\frac{\coth^4(c+dx)\left(8a^2+55ba+35b^2-5b(11a+7b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}+\frac{b(11a+7b)\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4a(a+b)}+\frac{b\coth^3(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\downarrow 445$$

$$\frac{-\frac{\int-\frac{3\coth^2(c+dx)\left(8a^3-8ba^2-55b^2a-35b^3+b\left(8a^2+55ba+35b^2\right)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{3a}}{2a(a+b)}-\frac{\left(8a^2+55ab+35b^2\right)\coth^3(c+dx)}{3a}}{4a(a+b)}+\frac{b(11a+7b)\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\downarrow 27$$

$$\frac{\frac{\int\frac{\coth^2(c+dx)\left(8a^3-8ba^2-55b^2a-35b^3+b\left(8a^2+55ba+35b^2\right)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}}{2a(a+b)}-\frac{\left(8a^2+55ab+35b^2\right)\coth^3(c+dx)}{3a}}{4a(a+b)}+\frac{b(11a+7b)\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}+\frac{}{4}$$

$$\downarrow 445$$

3.200. $\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

$$\frac{\frac{\int -\frac{8a^4-8ba^3+8b^2a^2+55b^3a+35b^4+b\left(8a^3-8ba^2-55b^2a-35b^3\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}-\frac{\left(8a^3-8a^2b-55ab^2-35b^3\right)\coth(c+dx)}{a}}{2a(a+b)}-\frac{\left(8a^2+55ab+35b^2\right)\coth^3}{3a}}{4a(a+b)}$$
$$d$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{8a^4-8ba^3+8b^2a^2+55b^3a+35b^4+b\left(8a^3-8ba^2-55b^2a-35b^3\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}-\frac{\left(8a^3-8a^2b-55ab^2-35b^3\right)\coth(c+dx)}{a}}{2a(a+b)}-\frac{\left(8a^2+55ab+35b^2\right)\coth^3(c+}{3a}}{4a(a+b)}$$
$$d$$

$$\downarrow 397$$

$$\frac{\frac{8a^4\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{b^3\left(63a^2+90ab+35b^2\right)\int\frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{a}-\frac{\left(8a^3-8a^2b-55ab^2-35b^3\right)\coth(c+dx)}{a}}{2a(a+b)}-\frac{\left(8a^2+55ab+35b^2\right)\coth}{3a}}{4a(a+b)}$$
$$d$$

$$\downarrow 218$$

$$\frac{\frac{8a^4\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{b^{5/2}\left(63a^2+90ab+35b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a}-\frac{\left(8a^3-8a^2b-55ab^2-35b^3\right)\coth(c+dx)}{a}}{2a(a+b)}-\frac{\left(8a^2+55ab+35b^2\right)\coth^3(c+}{3a}}{4a(a+b)}$$
$$d$$

$$\downarrow 219$$

$$\frac{\frac{8a^4\text{arctanh}(\tanh(c+dx))}{a+b}+\frac{b^{5/2}\left(63a^2+90ab+35b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a}-\frac{\left(8a^3-8a^2b-55ab^2-35b^3\right)\coth(c+dx)}{a}}{2a(a+b)}-\frac{\left(8a^2+55ab+35b^2\right)\coth^3(c+dx)}{3a}+$$
$$4a(a+b)$$
$$d$$

input
```
Int[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

3.200. $\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

output
```
((b*Coth[c + d*x]^3)/(4*a*(a + b)*(a + b*Tanh[c + d*x]^2)^2) + ((-1/3*((8*
a^2 + 55*a*b + 35*b^2)*Coth[c + d*x]^3)/a + (((b^(5/2)*(63*a^2 + 90*a*b +
35*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)) + (8*a^
4*ArcTanh[Tanh[c + d*x]])/(a + b))/a - ((8*a^3 - 8*a^2*b - 55*a*b^2 - 35*b
^3)*Coth[c + d*x])/a)/a)/(2*a*(a + b)) + (b*(11*a + 7*b)*Coth[c + d*x]^3)/
(2*a*(a + b)*(a + b*Tanh[c + d*x]^2)))/(4*a*(a + b)))/d
```

3.200.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.200.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.83

method	result
derivativedivides	$-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}-\frac{b^3\left(\frac{\left(\frac{15}{8}a^2b+\frac{13}{4}ab^2+\frac{11}{8}b^3\right)\tanh(dx+c)^3+\frac{a\left(17a^2+30ab+13b^2\right)\tanh(dx+c)}{8}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(63a^2-\right.}{(a+b)^3a^4}}{d}$
default	$-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}-\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^3}-\frac{b^3\left(\frac{\left(\frac{15}{8}a^2b+\frac{13}{4}ab^2+\frac{11}{8}b^3\right)\tanh(dx+c)^3+\frac{a\left(17a^2+30ab+13b^2\right)\tanh(dx+c)}{8}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(63a^2-\right.}{(a+b)^3a^4}}{d}$
risch	Expression too large to display

input
```
int(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(1/2/(a+b)^3*ln(tanh(d*x+c)-1)-1/2/(a+b)^3*ln(tanh(d*x+c)+1)-b^3/(a+b
)^3/a^4*(((15/8*a^2*b+13/4*a*b^2+11/8*b^3)*tanh(d*x+c)^3+1/8*a*(17*a^2+30*
a*b+13*b^2)*tanh(d*x+c))/(a+b*tanh(d*x+c)^2)^2+1/8*(63*a^2+90*a*b+35*b^2)/
(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2)))-(-a+3*b)/a^4/tanh(d*x+c)+1/
3/a^3/tanh(d*x+c)^3)
```

3.200.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10858 vs. $2(210) = 420$.

Time $= 0.55$ (sec) , antiderivative size $= 22038$, normalized size of antiderivative $= 96.66$

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.200.6 Sympy [F]

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

input
```
integrate(coth(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(coth(c + d*x)**4/(a + b*tanh(c + d*x)**2)**3, x)
```

3.200.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4285 vs. 2(210) = 420.

Time = 1.08 (sec) , antiderivative size = 4285, normalized size of antiderivative = 18.79

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*(3*a^3*b - 3*a^2*b^2 - 7*a*b^3 - 3*b^4)*log((a + b)*e^(4*d*x + 4*c) +
 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)
*d) + 1/8*(3*a^3*b - 3*a^2*b^2 - 7*a*b^3 - 3*b^4)*log(2*(a - b)*e^(-2*d*x
- 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^7 + 3*a^6*b + 3*a^5*b^2 + a
^4*b^3)*d) + 1/128*(15*a^4*b - 200*a^3*b^2 - 186*a^2*b^3 + 35*b^5)*arctan(
1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^7 + 3*a^6*b + 3*a^5*b
^2 + a^4*b^3)*sqrt(a*b)*d) - 1/128*(15*a^4*b - 200*a^3*b^2 - 186*a^2*b^3 +
 35*b^5)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^7 +
3*a^6*b + 3*a^5*b^2 + a^4*b^3)*sqrt(a*b)*d) + 1/192*(176*a^6 + 781*a^5*b +
 1571*a^4*b^2 + 1538*a^3*b^3 + 502*a^2*b^4 - 175*a*b^5 - 105*b^6 + 3*(96*a
^6 + 465*a^5*b + 665*a^4*b^2 + 706*a^3*b^3 + 506*a^2*b^4 + 61*a*b^5 - 35*b
^6)*e^(12*d*x + 12*c) + 6*(120*a^6 + 192*a^5*b - 315*a^4*b^2 - 728*a^3*b^3
 - 1070*a^2*b^4 - 240*a*b^5 + 105*b^6)*e^(10*d*x + 10*c) + (176*a^6 - 281*
a^5*b + 3509*a^4*b^2 + 3950*a^3*b^3 + 12226*a^2*b^4 + 3755*a*b^5 - 1575*b^
6)*e^(8*d*x + 8*c) - 4*(184*a^6 + 48*a^5*b + 473*a^4*b^2 + 970*a^3*b^3 + 3
684*a^2*b^4 + 1070*a*b^5 - 525*b^6)*e^(6*d*x + 6*c) - (384*a^6 + 1127*a^5*
b - 861*a^4*b^2 - 7146*a^3*b^3 - 11386*a^2*b^4 - 1965*a*b^5 + 1575*b^6)*e^
(4*d*x + 4*c) + 2*(136*a^6 - 96*a^5*b - 1309*a^4*b^2 - 2996*a^3*b^3 - 2238
*a^2*b^4 - 4*a*b^5 + 315*b^6)*e^(2*d*x + 2*c))/((a^9 + 5*a^8*b + 10*a^7*b^
2 + 10*a^6*b^3 + 5*a^5*b^4 + a^4*b^5 - (a^9 + 5*a^8*b + 10*a^7*b^2 + 10...
```

3.200.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(210) = 420.

Time = 0.56 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.16

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$= \frac{3\left(63\,a^2b^3+90\,ab^4+35\,b^5\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{\left(a^7+3\,a^6b+3\,a^5b^2+a^4b^3\right)\sqrt{ab}} + \frac{24\,(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{6\left(17\,a^3b^3e^{(6\,dx+6\,c)}+7\,a^2b^4e^{(6\,dx+6\,c)}-21\,ab^5e^{(6}\right.}{}$$

input
```
integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/24*(3*(63*a^2*b^3 + 90*a*b^4 + 35*b^5)*arctan(1/2*(a*e^(2*d*x + 2*c) + b
*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3
)*sqrt(a*b)) + 24*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 6*(17*a^3*b^
3*e^(6*d*x + 6*c) + 7*a^2*b^4*e^(6*d*x + 6*c) - 21*a*b^5*e^(6*d*x + 6*c) -
 11*b^6*e^(6*d*x + 6*c) + 51*a^3*b^3*e^(4*d*x + 4*c) - a^2*b^4*e^(4*d*x +
4*c) + 29*a*b^5*e^(4*d*x + 4*c) + 33*b^6*e^(4*d*x + 4*c) + 51*a^3*b^3*e^(2
*d*x + 2*c) + 37*a^2*b^4*e^(2*d*x + 2*c) - 47*a*b^5*e^(2*d*x + 2*c) - 33*b
^6*e^(2*d*x + 2*c) + 17*a^3*b^3 + 45*a^2*b^4 + 39*a*b^5 + 11*b^6)/((a^7 +
3*a^6*b + 3*a^5*b^2 + a^4*b^3)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*
a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2) - 16*(6*a*e^(4*d*x + 4
*c) - 9*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) + 18*b*e^(2*d*x + 2*c) + 4
*a - 9*b)/(a^4*(e^(2*d*x + 2*c) - 1)^3))/d
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\coth(c+d\,x)^4}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3, x)
```

3.201 $\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx$

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3.201.1 Optimal result

Integrand size = 14, antiderivative size = 201

$$\begin{aligned}\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx &= \frac{x}{(a+b)^4} \\ &\quad + \frac{\sqrt{b}(35a^3+35a^2b+21ab^2+5b^3)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{16a^{7/2}(a+b)^4d} \\ &\quad + \frac{b\tanh(c+dx)}{6a(a+b)d\left(a+b\tanh^2(c+dx)\right)^3} \\ &\quad + \frac{b(11a+5b)\tanh(c+dx)}{24a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} \\ &\quad + \frac{b(19a^2+16ab+5b^2)\tanh(c+dx)}{16a^3(a+b)^3d\left(a+b\tanh^2(c+dx)\right)}\end{aligned}$$

output

```
x/(a+b)^4+1/16*(35*a^3+35*a^2*b+21*a*b^2+5*b^3)*arctan(b^(1/2)*tanh(d*x+c)
/a^(1/2))*b^(1/2)/a^(7/2)/(a+b)^4/d+1/6*b*tanh(d*x+c)/a/(a+b)/d/(a+b*tanh(
d*x+c)^2)^3+1/24*b*(11*a+5*b)*tanh(d*x+c)/a^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2
)^2+1/16*b*(19*a^2+16*a*b+5*b^2)*tanh(d*x+c)/a^3/(a+b)^3/d/(a+b*tanh(d*x+c
)^2)
```

3.201.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.01

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx$$
$$= \frac{\frac{3\sqrt{b}(35a^3+35a^2b+21ab^2+5b^3)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}} - 24\log(1-\tanh(c+dx)) + 24\log(1+\tanh(c+dx)) + \frac{8b(a+b)}{a(a+b\tanh}}{48(a+b)^4 d}$$

input
```
Integrate[(a + b*Tanh[c + d*x]^2)^(-4),x]
```

output
```
((3*Sqrt[b]*(35*a^3 + 35*a^2*b + 21*a*b^2 + 5*b^3)*ArcTan[(Sqrt[b]*Tanh[c
+ d*x])/Sqrt[a]])/a^(7/2) - 24*Log[1 - Tanh[c + d*x]] + 24*Log[1 + Tanh[c
+ d*x]] + (8*b*(a + b)^3*Tanh[c + d*x])/(a*(a + b*Tanh[c + d*x]^2)^3) + (2
*b*(a + b)^2*(11*a + 5*b)*Tanh[c + d*x])/(a^2*(a + b*Tanh[c + d*x]^2)^2) +
 (3*b*(a + b)*(19*a^2 + 16*a*b + 5*b^2)*Tanh[c + d*x])/(a^3*(a + b*Tanh[c
+ d*x]^2)))/(48*(a + b)^4*d)
```

3.201.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4144, 316, 402, 27, 402, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\left(a-b\tan(ic+idx)^2\right)^4}dx$$
$$\downarrow 4144$$
$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^4}d\tanh(c+dx)}{d}$$
$$\downarrow 316$$

$$\frac{\frac{b\tanh(c+dx)}{6a(a+b)\left(a+b\tanh^2(c+dx)\right)^3}-\frac{\int\frac{5b\tanh^2(c+dx)+b-6(a+b)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{6a(a+b)}}{d}$$

$$\downarrow 402$$

$$\frac{\frac{b\tanh(c+dx)}{6a(a+b)\left(a+b\tanh^2(c+dx)\right)^3}-\frac{-\frac{\int\frac{3\left(8a^2+11ba+5b^2-b(11a+5b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4a(a+b)}-\frac{b(11a+5b)\tanh(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{6a(a+b)}}{d}$$

$$\downarrow 27$$

$$\frac{\frac{b\tanh(c+dx)}{6a(a+b)\left(a+b\tanh^2(c+dx)\right)^3}-\frac{-\frac{3\int\frac{8a^2+11ba+5b^2-b(11a+5b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4a(a+b)}-\frac{b(11a+5b)\tanh(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{6a(a+b)}}{d}$$

$$\downarrow 402$$

$$\frac{\frac{b\tanh(c+dx)}{6a(a+b)\left(a+b\tanh^2(c+dx)\right)^3}-\frac{-\frac{3\left(\frac{b\left(19a^2+16ab+5b^2\right)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\int-\frac{16a^3+19ba^2+16b^2a+5b^3-b\left(19a^2+16ba+5b^2\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}\right)}{4a(a+b)}-\frac{}{4a}}{6a(a+b)}}{d}$$

$$\downarrow 25$$

$$\frac{\frac{b\tanh(c+dx)}{6a(a+b)\left(a+b\tanh^2(c+dx)\right)^3}-\frac{-\frac{3\left(\frac{\int\frac{16a^3+19ba^2+16b^2a+5b^3-b\left(19a^2+16ba+5b^2\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}+\frac{b\left(19a^2+16ab+5b^2\right)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}\right)}{4a(a+b)}-\frac{b}{4a(a}}{6a(a+b)}}{d}$$

$$\downarrow 397$$

$$\frac{\frac{b\tanh(c+dx)}{6a(a+b)\left(a+b\tanh^2(c+dx)\right)^3}-\frac{-\frac{3\left(\frac{16a^3\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{b\left(35a^3+35a^2b+21ab^2+5b^3\right)\int\frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{2a(a+b)}+\frac{b\left(19a^2+16ab+\right.}{2a(a+b)\left(a+b\right.}\right)}{4a(a+b)}}{6a(a+b)}}{d}$$

$$\downarrow 218$$

3.201. $\int\frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}dx$

$$\frac{b\tanh(c+dx)}{6a(a+b)\left(a+b\tanh^2(c+dx)\right)^3}-\frac{3\left(\frac{16a^3\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{\sqrt{b}\left(35a^3+35a^2b+21ab^2+5b^3\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2a(a+b)}+\frac{b\left(19a^2+16ab+5b^2\right)t}{2a(a+b)\left(a+b\tanh^2\right.}\right)}{4a(a+b)}}{6a(a+b)}$$

$$\Big\downarrow 219$$

$$\frac{b\tanh(c+dx)}{6a(a+b)\left(a+b\tanh^2(c+dx)\right)^3}-\frac{3\left(\frac{b\left(19a^2+16ab+5b^2\right)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{16a^3\operatorname{arctanh}(\tanh(c+dx))}{a+b}+\frac{\sqrt{b}\left(35a^3+35a^2b+21ab^2+5b^3\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right.}{\sqrt{a}(a+b)}}{2a(a+b)}\right)}{4a(a+b)}}{6a(a+b)}$$

input
```
Int[(a + b*Tanh[c + d*x]^2)^(-4),x]
```

output
```
((b*Tanh[c + d*x])/(6*a*(a + b)*(a + b*Tanh[c + d*x]^2)^3) - (-1/4*(b*(11*
a + 5*b)*Tanh[c + d*x])/(a*(a + b)*(a + b*Tanh[c + d*x]^2)^2) - (3*(((Sqrt
[b]*(35*a^3 + 35*a^2*b + 21*a*b^2 + 5*b^3)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/
Sqrt[a]])/(Sqrt[a]*(a + b)) + (16*a^3*ArcTanh[Tanh[c + d*x]])/(a + b))/(2*
a*(a + b)) + (b*(19*a^2 + 16*a*b + 5*b^2)*Tanh[c + d*x])/(2*a*(a + b)*(a +
 b*Tanh[c + d*x]^2))))/(4*a*(a + b)))/(6*a*(a + b)))/d
```

3.201.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.201.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^4}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^4}+\frac{b\left(\frac{\frac{b^2\left(19a^3+35a^2b+21a\,b^2+5b^3\right)\tanh(dx+c)^5}{16a^3}+\frac{b\left(17a^3+33a^2b+21a\,b^2+5b^3\right)\tanh(dx+c)}{6a^2}}{\left(a+b\tanh(dx+c)^2\right)^3}\right.}{}}{d}$
default	$\frac{\frac{\ln(\tanh(dx+c)+1)}{2(a+b)^4}-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^4}+\frac{b\left(\frac{\frac{b^2\left(19a^3+35a^2b+21a\,b^2+5b^3\right)\tanh(dx+c)^5}{16a^3}+\frac{b\left(17a^3+33a^2b+21a\,b^2+5b^3\right)\tanh(dx+c)}{6a^2}}{\left(a+b\tanh(dx+c)^2\right)^3}\right.}{}}{d}$
risch	Expression too large to display

input
```
int(1/(a+b*tanh(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/(a+b)^4*ln(tanh(d*x+c)+1)-1/2/(a+b)^4*ln(tanh(d*x+c)-1)+1/(a+b)^4
*b*((1/16*b^2*(19*a^3+35*a^2*b+21*a*b^2+5*b^3)/a^3*tanh(d*x+c)^5+1/6*b*(17
*a^3+33*a^2*b+21*a*b^2+5*b^3)/a^2*tanh(d*x+c)^3+1/16*(29*a^3+61*a^2*b+43*a
*b^2+11*b^3)/a*tanh(d*x+c))/(a+b*tanh(d*x+c)^2)^3+1/16*(35*a^3+35*a^2*b+21
*a*b^2+5*b^3)/a^3/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2))))
```

3.201.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9849 vs. 2(185) = 370.

Time = 0.51 (sec) , antiderivative size = 20020, normalized size of antiderivative = 99.60

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tanh(d*x+c)^2)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

3.201.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*tanh(d*x+c)**2)**4,x)
```

output
```
Timed out
```

3.201.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(185) = 370$.

Time = 0.50 (sec) , antiderivative size = 925, normalized size of antiderivative = 4.60

$$\begin{aligned}
&\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx \\
&= -\frac{\left(35\,a^3b+35\,a^2b^2+21\,ab^3+5\,b^4\right)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{16\left(a^7+4\,a^6b+6\,a^5b^2+4\,a^4b^3+a^3b^4\right)\sqrt{ab}d} \\
&\quad + \frac{87\,a^5b+319\,a^4b^2+450}{24\left(a^{10}+7\,a^9b+21\,a^8b^2+35\,a^7b^3+35\,a^6b^4+21\,a^5b^5+7\,a^4b^6+a^3b^7+6\left(a^{10}+5\,a^9b+9\,a^8b^2+5\,a^7b^3\right.\right.} \\
&\quad + \frac{dx+c}{\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)d}
\end{aligned}$$

input
```
integrate(1/(a+b*tanh(d*x+c)^2)^4,x, algorithm="maxima")
```

output

```
-1/16*(35*a^3*b + 35*a^2*b^2 + 21*a*b^3 + 5*b^4)*arctan(1/2*((a + b)*e^(-2
*d*x - 2*c) + a - b)/sqrt(a*b))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 +
a^3*b^4)*sqrt(a*b)*d) + 1/24*(87*a^5*b + 319*a^4*b^2 + 450*a^3*b^3 + 306*a
^2*b^4 + 103*a*b^5 + 15*b^6 + 3*(145*a^5*b + 267*a^4*b^2 + 34*a^3*b^3 - 17
8*a^2*b^4 - 115*a*b^5 - 25*b^6)*e^(-2*d*x - 2*c) + 6*(145*a^5*b + 93*a^4*b
^2 - 6*a^3*b^3 + 106*a^2*b^4 + 85*a*b^5 + 25*b^6)*e^(-4*d*x - 4*c) + 2*(43
5*a^5*b + 29*a^4*b^2 + 162*a^3*b^3 - 306*a^2*b^4 - 245*a*b^5 - 75*b^6)*e^(
-6*d*x - 6*c) + 3*(145*a^5*b + 17*a^4*b^2 - 58*a^3*b^3 + 150*a^2*b^4 + 105
*a*b^5 + 25*b^6)*e^(-8*d*x - 8*c) + 3*(29*a^5*b + 23*a^4*b^2 - 62*a^3*b^3
- 82*a^2*b^4 - 31*a*b^5 - 5*b^6)*e^(-10*d*x - 10*c))/((a^10 + 7*a^9*b + 21
*a^8*b^2 + 35*a^7*b^3 + 35*a^6*b^4 + 21*a^5*b^5 + 7*a^4*b^6 + a^3*b^7 + 6*
(a^10 + 5*a^9*b + 9*a^8*b^2 + 5*a^7*b^3 - 5*a^6*b^4 - 9*a^5*b^5 - 5*a^4*b^
6 - a^3*b^7)*e^(-2*d*x - 2*c) + 3*(5*a^10 + 19*a^9*b + 25*a^8*b^2 + 15*a^7
*b^3 + 15*a^6*b^4 + 25*a^5*b^5 + 19*a^4*b^6 + 5*a^3*b^7)*e^(-4*d*x - 4*c)
+ 4*(5*a^10 + 17*a^9*b + 21*a^8*b^2 + 9*a^7*b^3 - 9*a^6*b^4 - 21*a^5*b^5 -
 17*a^4*b^6 - 5*a^3*b^7)*e^(-6*d*x - 6*c) + 3*(5*a^10 + 19*a^9*b + 25*a^8*
b^2 + 15*a^7*b^3 + 15*a^6*b^4 + 25*a^5*b^5 + 19*a^4*b^6 + 5*a^3*b^7)*e^(-8
*d*x - 8*c) + 6*(a^10 + 5*a^9*b + 9*a^8*b^2 + 5*a^7*b^3 - 5*a^6*b^4 - 9*a^
5*b^5 - 5*a^4*b^6 - a^3*b^7)*e^(-10*d*x - 10*c) + (a^10 + 7*a^9*b + 21*a^8
*b^2 + 35*a^7*b^3 + 35*a^6*b^4 + 21*a^5*b^5 + 7*a^4*b^6 + a^3*b^7)*e^(-...
```

3.201.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 750 vs. $2(185) = 370$.

Time $= 0.36$ (sec) , antiderivative size $= 750$, normalized size of antiderivative $= 3.73$

$$\int \frac{1}{\left(a + b\tanh^2(c + dx)\right)^4}\,dx$$

$$= \frac{3\left(35a^3b+35a^2b^2+21ab^3+5b^4\right)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{\left(a^7+4a^6b+6a^5b^2+4a^4b^3+a^3b^4\right)\sqrt{ab}} + \frac{48(dx+c)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4} - \frac{2\left(87a^5be^{(10dx+10c)}+69a^4b^2e^{(10}\right.}{}$$

input

```
integrate(1/(a+b*tanh(d*x+c)^2)^4,x, algorithm="giac")
```

output
```
1/48*(3*(35*a^3*b + 35*a^2*b^2 + 21*a*b^3 + 5*b^4)*arctan(1/2*(a*e^(2*d*x
+ 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^7 + 4*a^6*b + 6*a^5*b^2
 + 4*a^4*b^3 + a^3*b^4)*sqrt(a*b)) + 48*(d*x + c)/(a^4 + 4*a^3*b + 6*a^2*b
^2 + 4*a*b^3 + b^4) - 2*(87*a^5*b*e^(10*d*x + 10*c) + 69*a^4*b^2*e^(10*d*x
 + 10*c) - 186*a^3*b^3*e^(10*d*x + 10*c) - 246*a^2*b^4*e^(10*d*x + 10*c) -
 93*a*b^5*e^(10*d*x + 10*c) - 15*b^6*e^(10*d*x + 10*c) + 435*a^5*b*e^(8*d*
x + 8*c) + 51*a^4*b^2*e^(8*d*x + 8*c) - 174*a^3*b^3*e^(8*d*x + 8*c) + 450*
a^2*b^4*e^(8*d*x + 8*c) + 315*a*b^5*e^(8*d*x + 8*c) + 75*b^6*e^(8*d*x + 8*
c) + 870*a^5*b*e^(6*d*x + 6*c) + 58*a^4*b^2*e^(6*d*x + 6*c) + 324*a^3*b^3*
e^(6*d*x + 6*c) - 612*a^2*b^4*e^(6*d*x + 6*c) - 490*a*b^5*e^(6*d*x + 6*c)
- 150*b^6*e^(6*d*x + 6*c) + 870*a^5*b*e^(4*d*x + 4*c) + 558*a^4*b^2*e^(4*d
*x + 4*c) - 36*a^3*b^3*e^(4*d*x + 4*c) + 636*a^2*b^4*e^(4*d*x + 4*c) + 510
*a*b^5*e^(4*d*x + 4*c) + 150*b^6*e^(4*d*x + 4*c) + 435*a^5*b*e^(2*d*x + 2*
c) + 801*a^4*b^2*e^(2*d*x + 2*c) + 102*a^3*b^3*e^(2*d*x + 2*c) - 534*a^2*b
^4*e^(2*d*x + 2*c) - 345*a*b^5*e^(2*d*x + 2*c) - 75*b^6*e^(2*d*x + 2*c) +
87*a^5*b + 319*a^4*b^2 + 450*a^3*b^3 + 306*a^2*b^4 + 103*a*b^5 + 15*b^6)/(
(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*(a*e^(4*d*x + 4*c) + b*e
^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^3))/d
```

3.201.9 Mupad [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 3685, normalized size of antiderivative = 18.33

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*tanh(c + d*x)^2)^4,x)
```

output

```
log(tanh(c + d*x) + 1)/(2*a^4*d + 2*b^4*d + 12*a^2*b^2*d + 8*a*b^3*d + 8*a
^3*b*d) + ((tanh(c + d*x)^3*(16*a*b^3 + 5*b^4 + 17*a^2*b^2))/(6*a^2*(3*a*b
^2 + 3*a^2*b + a^3 + b^3)) + (tanh(c + d*x)*(32*a*b^2 + 29*a^2*b + 11*b^3)
)/(16*a*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + (b^2*tanh(c + d*x)^5*(16*a*b^2
+ 19*a^2*b + 5*b^3))/(16*a^2*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)))/(a^3*d
+ b^3*d*tanh(c + d*x)^6 + 3*a^2*b*d*tanh(c + d*x)^2 + 3*a*b^2*d*tanh(c + d
*x)^4) - log(tanh(c + d*x) - 1)/(2*d*(a + b)^4) - (atan((((-a^7*b)^(1/2)*(
(tanh(c + d*x)*(210*a*b^8 + 25*b^9 + 791*a^2*b^7 + 1820*a^3*b^6 + 2695*a^4
*b^5 + 2450*a^5*b^4 + 1481*a^6*b^3))/(128*(a^12*d^2 + 6*a^11*b*d^2 + a^6*b
^6*d^2 + 6*a^7*b^5*d^2 + 15*a^8*b^4*d^2 + 20*a^9*b^3*d^2 + 15*a^10*b^2*d^2
)) + ((((5*a^3*b^13*d^2)/4 + 14*a^4*b^12*d^2 + (287*a^5*b^11*d^2)/4 + 224*
a^6*b^10*d^2 + (953*a^7*b^9*d^2)/2 + 728*a^8*b^8*d^2 + (1631*a^9*b^7*d^2)/
2 + 668*a^10*b^6*d^2 + (1561*a^11*b^5*d^2)/4 + 154*a^12*b^4*d^2 + (147*a^1
3*b^3*d^2)/4 + 4*a^14*b^2*d^2)/(a^15*d^3 + 9*a^14*b*d^3 + a^6*b^9*d^3 + 9*
a^7*b^8*d^3 + 36*a^8*b^7*d^3 + 84*a^9*b^6*d^3 + 126*a^10*b^5*d^3 + 126*a^1
1*b^4*d^3 + 84*a^12*b^3*d^3 + 36*a^13*b^2*d^3) - (tanh(c + d*x)*(-a^7*b)^(
1/2)*(21*a*b^2 + 35*a^2*b + 35*a^3 + 5*b^3)*(1024*a^6*b^11*d^2 + 7168*a^7*
b^10*d^2 + 20480*a^8*b^9*d^2 + 28672*a^9*b^8*d^2 + 14336*a^10*b^7*d^2 - 14
336*a^11*b^6*d^2 - 28672*a^12*b^5*d^2 - 20480*a^13*b^4*d^2 - 7168*a^14*b^3
*d^2 - 1024*a^15*b^2*d^2))/(4096*(a^11*d + a^7*b^4*d + 4*a^8*b^3*d + 6*...
```

3.202 $\int \sqrt{1-\tanh^2(x)}\,dx$

3.202.1 Optimal result 1501
3.202.2 Mathematica [B] (verified) 1501
3.202.3 Rubi [A] (verified) 1502
3.202.4 Maple [A] (verified) 1503
3.202.5 Fricas [B] (verification not implemented) 1504
3.202.6 Sympy [F] 1504
3.202.7 Maxima [A] (verification not implemented) 1504
3.202.8 Giac [A] (verification not implemented) 1505
3.202.9 Mupad [B] (verification not implemented) 1505

3.202.1 Optimal result

Integrand size = 12, antiderivative size = 3

$$\int \sqrt{1-\tanh^2(x)}\,dx = \arcsin(\tanh(x))$$

output
```
arcsin(tanh(x))
```

3.202.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 14 vs. $2(3) = 6$.

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 4.67

$$\int \sqrt{1-\tanh^2(x)}\,dx = \arctan(\sinh(x))\cosh(x)\sqrt{\operatorname{sech}^2(x)}$$

input
```
Integrate[Sqrt[1 - Tanh[x]^2],x]
```

output
```
ArcTan[Sinh[x]]*Cosh[x]*Sqrt[Sech[x]^2]
```

3.202.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4140, 3042, 4610, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-\tanh^2(x)}\,dx$$
$$\downarrow 3042$$
$$\int \sqrt{1+\tan(ix)^2}dx$$
$$\downarrow 4140$$
$$\int \sqrt{\operatorname{sech}^2(x)}dx$$
$$\downarrow 3042$$
$$\int \sqrt{\sec(ix)^2}dx$$
$$\downarrow 4610$$
$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}d\tanh(x)$$
$$\downarrow 223$$
$$\arcsin(\tanh(x))$$

input
```
Int[Sqrt[1 - Tanh[x]^2],x]
```

output
```
ArcSin[Tanh[x]]
```

3.202.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

3.202.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\arcsin(\tanh(x))$	4
default	$\arcsin(\tanh(x))$	4
risch	$i\sqrt{\frac{e^{2x}}{(1+e^{2x})^2}}\,e^{-x}(1+e^{2x})\ln(e^x+i) - i\sqrt{\frac{e^{2x}}{(1+e^{2x})^2}}\,e^{-x}(1+e^{2x})\ln(e^x-i)$	70

input
```
int((1-tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(tanh(x))
```

3.202.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8 vs. $2(3) = 6$.

Time $= 0.24$ (sec) , antiderivative size $= 8$, normalized size of antiderivative $= 2.67$

$$\int \sqrt{1 - \tanh^2(x)}\, dx = 2 \arctan\left(\cosh\left(x\right) + \sinh\left(x\right)\right)$$

input
```
integrate((1-tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
2*arctan(cosh(x) + sinh(x))
```

3.202.6 Sympy [F]

$$\int \sqrt{1 - \tanh^2(x)}\, dx = \int \sqrt{1 - \tanh^2\left(x\right)}\, dx$$

input
```
integrate((1-tanh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(1 - tanh(x)**2), x)
```

3.202.7 Maxima [A] (verification not implemented)

Time $= 0.34$ (sec) , antiderivative size $= 5$, normalized size of antiderivative $= 1.67$

$$\int \sqrt{1 - \tanh^2(x)}\, dx = 2 \arctan\left(e^{x}\right)$$

input
```
integrate((1-tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
2*arctan(e^x)
```

3.202.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \sqrt{1-\tanh^2(x)}\,dx = 2\arctan\left(e^x\right)$$

input
```
integrate((1-tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(e^x)
```

3.202.9 Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \sqrt{1-\tanh^2(x)}\,dx = \operatorname{asin}(\tanh(x))$$

input
```
int((1 - tanh(x)^2)^(1/2),x)
```

output
```
asin(tanh(x))
```

3.203 $\int \sqrt{-1+\tanh^2(x)}\,dx$

3.203.1 Optimal result 1506
3.203.2 Mathematica [A] (verified) 1506
3.203.3 Rubi [A] (verified) 1507
3.203.4 Maple [A] (verified) 1508
3.203.5 Fricas [B] (verification not implemented) 1509
3.203.6 Sympy [F] 1509
3.203.7 Maxima [C] (verification not implemented) 1510
3.203.8 Giac [A] (verification not implemented) 1510
3.203.9 Mupad [B] (verification not implemented) 1510

3.203.1 Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \sqrt{-1+\tanh^2(x)}\,dx = -\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}\right)$$

output
```
-arctanh(tanh(x)/(-sech(x)^2)^(1/2))
```

3.203.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{-1+\tanh^2(x)}\,dx = \arctan(\sinh(x))\cosh(x)\sqrt{-\operatorname{sech}^2(x)}$$

input
```
Integrate[Sqrt[-1 + Tanh[x]^2],x]
```

output
```
ArcTan[Sinh[x]]*Cosh[x]*Sqrt[-Sech[x]^2]
```

3.203.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4140, 3042, 4610, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tanh^2(x) - 1}\, dx$$

$$\downarrow 3042$$

$$\int \sqrt{-1 - \tan(ix)^2} dx$$

$$\downarrow 4140$$

$$\int \sqrt{-\operatorname{sech}^2(x)} dx$$

$$\downarrow 3042$$

$$\int \sqrt{-\sec(ix)^2} dx$$

$$\downarrow 4610$$

$$-\int \frac{1}{\sqrt{\tanh^2(x) - 1}} d\tanh(x)$$

$$\downarrow 224$$

$$-\int \frac{1}{1 - \frac{\tanh^2(x)}{\tanh^2(x)-1}} d\frac{\tanh(x)}{\sqrt{\tanh^2(x) - 1}}$$

$$\downarrow 219$$

$$-\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{\tanh^2(x) - 1}}\right)$$

input
```
Int[Sqrt[-1 + Tanh[x]^2],x]
```

output
```
-ArcTanh[Tanh[x]/Sqrt[-1 + Tanh[x]^2]]
```

3.203.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

3.203.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$-\ln\left(\tanh(x)+\sqrt{\tanh(x)^2-1}\right)$	15
default	$-\ln\left(\tanh(x)+\sqrt{\tanh(x)^2-1}\right)$	15
risch	$i\sqrt{-\frac{e^{2x}}{(1+e^{2x})^2}}\,e^{-x}(1+e^{2x})\ln(e^x+i)-i\sqrt{-\frac{e^{2x}}{(1+e^{2x})^2}}\,e^{-x}(1+e^{2x})\ln(e^x-i)$	72

input
```
int((tanh(x)^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-ln(tanh(x)+(tanh(x)^2-1)^(1/2))
```

3.203.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(14) = 28$.

Time $= 0.25$ (sec) , antiderivative size $= 96$, normalized size of antiderivative $= 6.00$

$$\int \sqrt{-1+\tanh^2(x)}\,dx$$
$$= -\log\left(\left(\cosh(x)\,e^x + e^x\sinh(x) + \sqrt{-\frac{e^{(2\,x)}}{e^{(4\,x)}+2\,e^{(2\,x)}+1}}\left(e^{(2\,x)}+1\right)\right)e^{(-x)}\right)$$
$$+\log\left(\left(\cosh(x)\,e^x + e^x\sinh(x) - \sqrt{-\frac{e^{(2\,x)}}{e^{(4\,x)}+2\,e^{(2\,x)}+1}}\left(e^{(2\,x)}+1\right)\right)e^{(-x)}\right)$$

input
```
integrate((-1+tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-log((cosh(x)*e^x + e^x*sinh(x) + sqrt(-e^(2*x)/(e^(4*x) + 2*e^(2*x) + 1))
*(e^(2*x) + 1))*e^(-x)) + log((cosh(x)*e^x + e^x*sinh(x) - sqrt(-e^(2*x)/(
e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1))*e^(-x))
```

3.203.6 Sympy [F]

$$\int \sqrt{-1+\tanh^2(x)}\,dx = \int \sqrt{\tanh^2(x)-1}\,dx$$

input
```
integrate((-1+tanh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(tanh(x)**2 - 1), x)
```

3.203.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.31

$$\int \sqrt{-1+\tanh^2(x)}\,dx = 2i\,\arctan\left(e^x\right)$$

input
```
integrate((-1+tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
2*I*arctan(e^x)
```

3.203.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.06

$$\int \sqrt{-1+\tanh^2(x)}\,dx = 0$$

input
```
integrate((-1+tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
0
```

3.203.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \sqrt{-1+\tanh^2(x)}\,dx = -\ln\left(\tanh(x)+\sqrt{\tanh\left(x\right)^2-1}\right)$$

input
```
int((tanh(x)^2 - 1)^(1/2),x)
```

output
```
-log(tanh(x) + (tanh(x)^2 - 1)^(1/2))
```

3.204 $\int \left(1-\tanh^2(x)\right)^{3/2}\,dx$

3.204.1 Optimal result 1511
3.204.2 Mathematica [A] (verified) 1511
3.204.3 Rubi [A] (verified) 1512
3.204.4 Maple [A] (verified) 1513
3.204.5 Fricas [B] (verification not implemented) 1514
3.204.6 Sympy [F] 1514
3.204.7 Maxima [A] (verification not implemented) 1514
3.204.8 Giac [B] (verification not implemented) 1515
3.204.9 Mupad [B] (verification not implemented) 1515

3.204.1 Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \left(1-\tanh^2(x)\right)^{3/2}\,dx = \frac{1}{2}\arcsin(\tanh(x)) + \frac{1}{2}\sqrt{\operatorname{sech}^2(x)}\tanh(x)$$

output
```
1/2*arcsin(tanh(x))+1/2*(sech(x)^2)^(1/2)*tanh(x)
```

3.204.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \left(1-\tanh^2(x)\right)^{3/2}\,dx = \frac{\operatorname{sech}(x)(\arctan(\sinh(x)) + \operatorname{sech}(x)\tanh(x))}{2\sqrt{\operatorname{sech}^2(x)}}$$

input
```
Integrate[(1 - Tanh[x]^2)^(3/2),x]
```

output
```
(Sech[x]*(ArcTan[Sinh[x]] + Sech[x]*Tanh[x]))/(2*Sqrt[Sech[x]^2])
```

3.204.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4140, 3042, 4610, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-\tanh^2(x)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(1+\tan(ix)^2\right)^{3/2}\,dx$$

$$\downarrow 4140$$

$$\int \operatorname{sech}^2(x)^{3/2}dx$$

$$\downarrow 3042$$

$$\int \left(\sec(ix)^2\right)^{3/2}\,dx$$

$$\downarrow 4610$$

$$\int \sqrt{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 211$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-\tanh^2(x)}}d\tanh(x)+\frac{1}{2}\sqrt{1-\tanh^2(x)}\tanh(x)$$

$$\downarrow 223$$

$$\frac{1}{2}\arcsin(\tanh(x))+\frac{1}{2}\tanh(x)\sqrt{1-\tanh^2(x)}$$

input
```
Int[(1 - Tanh[x]^2)^(3/2),x]
```

output
```
ArcSin[Tanh[x]]/2 + (Tanh[x]*Sqrt[1 - Tanh[x]^2])/2
```

3.204.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

3.204.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\tanh(x)\sqrt{1-\tanh(x)^2}}{2}+\frac{\arcsin(\tanh(x))}{2}$	21
default	$\frac{\tanh(x)\sqrt{1-\tanh(x)^2}}{2}+\frac{\arcsin(\tanh(x))}{2}$	21
risch	$\frac{\sqrt{\frac{e^{2x}}{\left(1+e^{2x}\right)^2}}\left(e^{2x}-1\right)}{1+e^{2x}}+\frac{ie^{-x}\left(1+e^{2x}\right)\sqrt{\frac{e^{2x}}{\left(1+e^{2x}\right)^2}}\ln(e^x+i)}{2}-\frac{i\sqrt{\frac{e^{2x}}{\left(1+e^{2x}\right)^2}}e^{-x}\left(1+e^{2x}\right)\ln(e^x-i)}{2}$	100

input
```
int((1-tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*tanh(x)*(1-tanh(x)^2)^(1/2)+1/2*arcsin(tanh(x))
```

3.204.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(16) = 32$.

Time $= 0.26$ (sec) , antiderivative size $= 140$, normalized size of antiderivative $= 6.36$

$$\int (1 - \tanh^2(x))^{3/2}\, dx = \frac{\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + (\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x$$
$$\cosh(x)^4 + 4\cosh(x)\sinh(x$$

input
```
integrate((1-tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (cosh(x)^4 + 4*cosh(x)*sinh
(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(
x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + (3*cosh(x)^2 - 1)
*sinh(x) - cosh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*co
sh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1
)
```

3.204.6 Sympy [F]

$$\int (1 - \tanh^2(x))^{3/2}\, dx = \int (1 - \tanh^2(x))^{\frac{3}{2}}\, dx$$

input
```
integrate((1-tanh(x)**2)**(3/2),x)
```

output
```
Integral((1 - tanh(x)**2)**(3/2), x)
```

3.204.7 Maxima [A] (verification not implemented)

Time $= 0.31$ (sec) , antiderivative size $= 28$, normalized size of antiderivative $= 1.27$

$$\int (1 - \tanh^2(x))^{3/2}\, dx = \frac{e^{(3x)} - e^x}{e^{(4x)} + 2e^{(2x)} + 1} + \arctan(e^x)$$

input
```
integrate((1-tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
(e^(3*x) - e^x)/(e^(4*x) + 2*e^(2*x) + 1) + arctan(e^x)
```

3.204.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(16) = 32$.

Time $= 0.27$ (sec) , antiderivative size $= 45$, normalized size of antiderivative $= 2.05$

$$\int \left(1 - \tanh^2(x)\right)^{3/2}\, dx = \frac{1}{4}\pi - \frac{e^{(-x)} - e^{x}}{\left(e^{(-x)} - e^{x}\right)^2 + 4} + \frac{1}{2}\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)$$

input
```
integrate((1-tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/4*pi - (e^(-x) - e^x)/((e^(-x) - e^x)^2 + 4) + 1/2*arctan(1/2*(e^(2*x) -
 1)*e^(-x))
```

3.204.9 Mupad [B] (verification not implemented)

Time $= 0.09$ (sec) , antiderivative size $= 20$, normalized size of antiderivative $= 0.91$

$$\int \left(1 - \tanh^2(x)\right)^{3/2}\, dx = \frac{\operatorname{asin}(\tanh(x))}{2} + \frac{\tanh(x)\,\sqrt{1 - \tanh\left(x\right)^2}}{2}$$

input
```
int((1 - tanh(x)^2)^(3/2),x)
```

output
```
asin(tanh(x))/2 + (tanh(x)*(1 - tanh(x)^2)^(1/2))/2
```

3.205 $\int \left(-1+\tanh^2(x)\right)^{3/2}\,dx$

3.205.1 Optimal result 1516
3.205.2 Mathematica [A] (verified) 1516
3.205.3 Rubi [A] (verified) 1517
3.205.4 Maple [A] (verified) 1519
3.205.5 Fricas [B] (verification not implemented) 1519
3.205.6 Sympy [F] 1520
3.205.7 Maxima [C] (verification not implemented) 1520
3.205.8 Giac [A] (verification not implemented) 1521
3.205.9 Mupad [B] (verification not implemented) 1521

3.205.1 Optimal result

Integrand size = 10, antiderivative size = 35

$$\int \left(-1+\tanh^2(x)\right)^{3/2}\,dx = \frac{1}{2}\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}\right) - \frac{1}{2}\sqrt{-\operatorname{sech}^2(x)}\tanh(x)$$

output
```
1/2*arctanh(tanh(x)/(-sech(x)^2)^(1/2))-1/2*(-sech(x)^2)^(1/2)*tanh(x)
```

3.205.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \left(-1+\tanh^2(x)\right)^{3/2}\,dx = -\frac{1}{2}\sqrt{-\operatorname{sech}^2(x)}\left(\arctan(\sinh(x))\cosh(x)+\tanh(x)\right)$$

input
```
Integrate[(-1 + Tanh[x]^2)^(3/2),x]
```

output
```
-1/2*(Sqrt[-Sech[x]^2]*(ArcTan[Sinh[x]]*Cosh[x] + Tanh[x]))
```

3.205.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3042, 4140, 3042, 4610, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\tanh^2(x)-1\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(-1-\tan(ix)^2\right)^{3/2}\,dx$$

$$\downarrow 4140$$

$$\int \left(-\operatorname{sech}^2(x)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(-\sec(ix)^2\right)^{3/2}\,dx$$

$$\downarrow 4610$$

$$-\int \sqrt{\tanh^2(x)-1}d\tanh(x)$$

$$\downarrow 211$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\tanh^2(x)-1}}d\tanh(x)-\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)-1}$$

$$\downarrow 224$$

$$\frac{1}{2}\int \frac{1}{1-\frac{\tanh^2(x)}{\tanh^2(x)-1}}d\frac{\tanh(x)}{\sqrt{\tanh^2(x)-1}}-\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)-1}$$

$$\downarrow 219$$

$$\frac{1}{2}\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{\tanh^2(x)-1}}\right)-\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)-1}$$

input
```
Int[(-1 + Tanh[x]^2)^(3/2),x]
```

output
```
ArcTanh[Tanh[x]/Sqrt[-1 + Tanh[x]^2]]/2 - (Tanh[x]*Sqrt[-1 + Tanh[x]^2])/2
```

3.205.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

3.205.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{\tanh(x)\sqrt{\tanh(x)^2-1}}{2}+\frac{\ln\left(\tanh(x)+\sqrt{\tanh(x)^2-1}\right)}{2}$	28
default	$-\frac{\tanh(x)\sqrt{\tanh(x)^2-1}}{2}+\frac{\ln\left(\tanh(x)+\sqrt{\tanh(x)^2-1}\right)}{2}$	28
risch	$-\frac{\sqrt{-\frac{e^{2x}}{\left(1+e^{2x}\right)^2}}\left(e^{2x}-1\right)}{1+e^{2x}}-\frac{i\sqrt{-\frac{e^{2x}}{\left(1+e^{2x}\right)^2}}e^{-x}\left(1+e^{2x}\right)\ln(e^x+i)}{2}+\frac{i\sqrt{-\frac{e^{2x}}{\left(1+e^{2x}\right)^2}}e^{-x}\left(1+e^{2x}\right)\ln(e^x-i)}{2}$	104

input
```
int((tanh(x)^2-1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*tanh(x)*(tanh(x)^2-1)^(1/2)+1/2*ln(tanh(x)+(tanh(x)^2-1)^(1/2))
```

3.205.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(27) = 54.

Time = 0.30 (sec) , antiderivative size = 385, normalized size of antiderivative = 11.00

$$\int (-1+\tanh^2(x))^{3/2}\,dx = \frac{\left(4\cosh(x)e^x\sinh(x)^3+e^x\sinh(x)^4+2\left(3\cosh(x)^2+1\right)e^x\sinh(x)^2+4\left(\cosh(x)^3+\right.\right.}{}$$

input
```
integrate((-1+tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((4*cosh(x)*e^x*sinh(x)^3 + e^x*sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*e^x*si
nh(x)^2 + 4*(cosh(x)^3 + cosh(x))*e^x*sinh(x) + (cosh(x)^4 + 2*cosh(x)^2 +
 1)*e^x)*log((cosh(x)*e^x + e^x*sinh(x) + sqrt(-e^(2*x)/(e^(4*x) + 2*e^(2*
x) + 1))*(e^(2*x) + 1))*e^(-x)) - (4*cosh(x)*e^x*sinh(x)^3 + e^x*sinh(x)^4
 + 2*(3*cosh(x)^2 + 1)*e^x*sinh(x)^2 + 4*(cosh(x)^3 + cosh(x))*e^x*sinh(x)
 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^x)*log((cosh(x)*e^x + e^x*sinh(x) - sqr
t(-e^(2*x)/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1))*e^(-x)) - 2*((e^(2*x)
 + 1)*sinh(x)^3 + cosh(x)^3 + 3*(cosh(x)*e^(2*x) + cosh(x))*sinh(x)^2 + (c
osh(x)^3 - cosh(x))*e^(2*x) + (3*cosh(x)^2 + (3*cosh(x)^2 - 1)*e^(2*x) - 1
)*sinh(x) - cosh(x))*sqrt(-e^(2*x)/(e^(4*x) + 2*e^(2*x) + 1)))/(4*cosh(x)*
e^x*sinh(x)^3 + e^x*sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*e^x*sinh(x)^2 + 4*(cos
h(x)^3 + cosh(x))*e^x*sinh(x) + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^x)
```

3.205.6 Sympy [F]

$$\int \left(-1+\tanh^2(x)\right)^{3/2}\,dx = \int \left(\tanh^2(x)-1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-1+tanh(x)**2)**(3/2),x)
```

output
```
Integral((tanh(x)**2 - 1)**(3/2), x)
```

3.205.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \left(-1+\tanh^2(x)\right)^{3/2}\,dx = \frac{-i\,e^{(3\,x)}+i\,e^{x}}{e^{(4\,x)}+2\,e^{(2\,x)}+1} - i\,\arctan\left(e^{x}\right)$$

input
```
integrate((-1+tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
(-I*e^(3*x) + I*e^x)/(e^(4*x) + 2*e^(2*x) + 1) - I*arctan(e^x)
```

3.205.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \left(-1+\tanh^2(x)\right)^{3/2}\,dx = \frac{\sqrt{-e^{(2\,x)}}+\frac{1}{\sqrt{-e^{(2\,x)}}}}{\left(\sqrt{-e^{(2\,x)}}+\frac{1}{\sqrt{-e^{(2\,x)}}}\right)^2-4}$$

input
```
integrate((-1+tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
(sqrt(-e^(2*x)) + 1/sqrt(-e^(2*x)))/((sqrt(-e^(2*x)) + 1/sqrt(-e^(2*x)))^2
 - 4)
```

3.205.9 Mupad [B] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \left(-1+\tanh^2(x)\right)^{3/2}\,dx = \frac{\ln\left(\tanh(x)+\sqrt{\tanh(x)^2-1}\right)}{2}-\frac{\tanh(x)\,\sqrt{\tanh(x)^2-1}}{2}$$

input
```
int((tanh(x)^2 - 1)^(3/2),x)
```

output
```
log(tanh(x) + (tanh(x)^2 - 1)^(1/2))/2 - (tanh(x)*(tanh(x)^2 - 1)^(1/2))/2
```

3.206 $\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx$

3.206.1 Optimal result 1522
3.206.2 Mathematica [A] (verified) 1522
3.206.3 Rubi [A] (verified) 1523
3.206.4 Maple [A] (verified) 1524
3.206.5 Fricas [A] (verification not implemented) 1525
3.206.6 Sympy [F] 1525
3.206.7 Maxima [A] (verification not implemented) 1525
3.206.8 Giac [A] (verification not implemented) 1526
3.206.9 Mupad [B] (verification not implemented) 1526

3.206.1 Optimal result

Integrand size = 12, antiderivative size = 11

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx = \frac{\tanh(x)}{\sqrt{\operatorname{sech}^2(x)}}$$

output
```
tanh(x)/(sech(x)^2)^(1/2)
```

3.206.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx = \frac{\tanh(x)}{\sqrt{\operatorname{sech}^2(x)}}$$

input
```
Integrate[1/Sqrt[1 - Tanh[x]^2],x]
```

output
```
Tanh[x]/Sqrt[Sech[x]^2]
```

3.206.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4140, 3042, 4610, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\sqrt{1+\tan(ix)^2}}dx$$
$$\downarrow 4140$$
$$\int \frac{1}{\sqrt{\operatorname{sech}^2(x)}}dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\sqrt{\sec(ix)^2}}dx$$
$$\downarrow 4610$$
$$\int \frac{1}{\left(1-\tanh^2(x)\right)^{3/2}}d\tanh(x)$$
$$\downarrow 208$$
$$\frac{\tanh(x)}{\sqrt{1-\tanh^2(x)}}$$

input
```
Int[1/Sqrt[1 - Tanh[x]^2],x]
```

output
```
Tanh[x]/Sqrt[1 - Tanh[x]^2]
```

3.206.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

3.206.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$\frac{\tanh(x)}{\sqrt{1-\tanh(x)^2}}$	14
default	$\frac{\tanh(x)}{\sqrt{1-\tanh(x)^2}}$	14
parallelrisch	$\frac{\tanh(x)}{\sqrt{1-\tanh(x)^2}}$	14
risch	$\frac{e^{2x}}{2\sqrt{\frac{e^{2x}}{(1+e^{2x})^2}}\,(1+e^{2x})} - \frac{1}{2(1+e^{2x})\sqrt{\frac{e^{2x}}{(1+e^{2x})^2}}}$	56

input
```
int(1/(1-tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(1-tanh(x)^2)^(1/2)*tanh(x)
```

3.206.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.18

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx = \sinh(x)$$

input
```
integrate(1/(1-tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
sinh(x)
```

3.206.6 Sympy [F]

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx = \int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx$$

input
```
integrate(1/(1-tanh(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(1 - tanh(x)**2), x)
```

3.206.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx = -\frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

input
```
integrate(1/(1-tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*e^(-x) + 1/2*e^x
```

3.206.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx = -\frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

input
```
integrate(1/(1-tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*e^(-x) + 1/2*e^x
```

3.206.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.18

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx = \sinh(x)$$

input
```
int(1/(1 - tanh(x)^2)^(1/2),x)
```

output
```
sinh(x)
```

3.207 $\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx$

3.207.1 Optimal result 1527
3.207.2 Mathematica [A] (verified) 1527
3.207.3 Rubi [A] (verified) 1528
3.207.4 Maple [A] (verified) 1529
3.207.5 Fricas [B] (verification not implemented) 1530
3.207.6 Sympy [A] (verification not implemented) 1530
3.207.7 Maxima [B] (verification not implemented) 1530
3.207.8 Giac [A] (verification not implemented) 1531
3.207.9 Mupad [B] (verification not implemented) 1531

3.207.1 Optimal result

Integrand size = 10, antiderivative size = 13

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = \frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}$$

output
```
tanh(x)/(-sech(x)^2)^(1/2)
```

3.207.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = \frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}$$

input
```
Integrate[1/Sqrt[-1 + Tanh[x]^2],x]
```

output
```
Tanh[x]/Sqrt[-Sech[x]^2]
```

3.207.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4140, 3042, 4610, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\tanh^2(x)-1}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\sqrt{-1-\tan(ix)^2}}dx$$
$$\downarrow 4140$$
$$\int \frac{1}{\sqrt{-\operatorname{sech}^2(x)}}dx$$
$$\downarrow 3042$$
$$\int \frac{1}{\sqrt{-\sec(ix)^2}}dx$$
$$\downarrow 4610$$
$$-\int \frac{1}{\left(\tanh^2(x)-1\right)^{3/2}}d\tanh(x)$$
$$\downarrow 208$$
$$\frac{\tanh(x)}{\sqrt{\tanh^2(x)-1}}$$

input
```
Int[1/Sqrt[-1 + Tanh[x]^2],x]
```

output
```
Tanh[x]/Sqrt[-1 + Tanh[x]^2]
```

3.207.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

3.207.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\tanh(x)}{\sqrt{\tanh(x)^2-1}}$	12
default	$\frac{\tanh(x)}{\sqrt{\tanh(x)^2-1}}$	12
risch	$\frac{e^{2x}}{2\sqrt{-\frac{e^{2x}}{\left(1+e^{2x}\right)^2}}\left(1+e^{2x}\right)}-\frac{1}{2\left(1+e^{2x}\right)\sqrt{-\frac{e^{2x}}{\left(1+e^{2x}\right)^2}}}$	58

input
```
int(1/(tanh(x)^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
tanh(x)/(tanh(x)^2-1)^(1/2)
```

3.207.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.77

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = -\sqrt{-\frac{e^{(2\,x)}}{e^{(4\,x)}+2\,e^{(2\,x)}+1}}\left(e^{(2\,x)}+1\right)e^{(-x)}\sinh(x)$$

input
```
integrate(1/(-1+tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-e^(2*x)/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1)*e^(-x)*sinh(x)
```

3.207.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = \frac{\tanh(x)}{\sqrt{\tanh^2(x)-1}}$$

input
```
integrate(1/(-1+tanh(x)**2)**(1/2),x)
```

output
```
tanh(x)/sqrt(tanh(x)**2 - 1)
```

3.207.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = -\frac{e^{(-2\,x)}}{2\sqrt{-e^{(-2\,x)}}}+\frac{1}{2\sqrt{-e^{(-2\,x)}}}$$

input
```
integrate(1/(-1+tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*e^(-2*x)/sqrt(-e^(-2*x)) + 1/2/sqrt(-e^(-2*x))
```

3.207.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = -\frac{1}{2}\sqrt{-e^{(2\,x)}} - \frac{1}{2\sqrt{-e^{(2\,x)}}}$$

input
```
integrate(1/(-1+tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-e^(2*x)) - 1/2/sqrt(-e^(2*x))
```

3.207.9 Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = -\frac{\sinh(2\,x)\,\sqrt{-\frac{1}{\cosh(x)^2}}}{2}$$

input
```
int(1/(tanh(x)^2 - 1)^(1/2),x)
```

output
```
-(sinh(2*x)*(-1/cosh(x)^2)^(1/2))/2
```

3.208 $\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx$

3.208.1 Optimal result 1532
3.208.2 Mathematica [A] (verified) 1532
3.208.3 Rubi [A] (verified) 1533
3.208.4 Maple [B] (verified) 1535
3.208.5 Fricas [B] (verification not implemented) 1536
3.208.6 Sympy [A] (verification not implemented) 1536
3.208.7 Maxima [F] 1537
3.208.8 Giac [B] (verification not implemented) 1537
3.208.9 Mupad [B] (verification not implemented) 1538

3.208.1 Optimal result

Integrand size = 17, antiderivative size = 87

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \sqrt{a+b\tanh^2(x)} + \frac{(a-b)\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2} - \frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b^2}$$

output
```
arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)-(a+b*tanh(x)^2)^(1/
2)+1/3*(a-b)*(a+b*tanh(x)^2)^(3/2)/b^2-1/5*(a+b*tanh(x)^2)^(5/2)/b^2
```

3.208.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) + \frac{\sqrt{a+b\tanh^2(x)}\left(2a^2-5ab-15b^2-b(a+5b)\tanh^2(x)-3b^2\tanh^4(x)\right)}{15b^2}$$

input
```
Integrate[Tanh[x]^5*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] + (Sqrt[a + b*Tanh[
x]^2]*(2*a^2 - 5*a*b - 15*b^2 - b*(a + 5*b)*Tanh[x]^2 - 3*b^2*Tanh[x]^4))/
(15*b^2)
```

3.208.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx$$

$$\downarrow 3042$$

$$\int -i\tan(ix)^5\sqrt{a-b\tan(ix)^2}dx$$

$$\downarrow 26$$

$$-i\int \tan(ix)^5\sqrt{a-b\tan(ix)^2}dx$$

$$\downarrow 4153$$

$$-i\int \frac{i\tanh^5(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\tanh^5(x)\sqrt{a+b\tanh^2(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 354$$

$$\frac{1}{2}\int \frac{\tanh^4(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh^2(x)$$

$$\downarrow 99$$

$$\frac{1}{2}\int\left(-\frac{\left(b\tanh^2(x)+a\right)^{3/2}}{b}+\frac{(a-b)\sqrt{b\tanh^2(x)+a}}{b}+\frac{\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}\right)d\tanh^2(x)$$

$$\downarrow \text{2009}$$

$$\frac{1}{2}\left(2\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\frac{2\left(a+b\tanh^2(x)\right)^{5/2}}{5b^2}+\frac{2(a-b)\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}-2\sqrt{a+b\tanh^2}\right.$$

input
```
Int[Tanh[x]^5*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
(2*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - 2*Sqrt[a + b*T
anh[x]^2] + (2*(a - b)*(a + b*Tanh[x]^2)^(3/2))/(3*b^2) - (2*(a + b*Tanh[x
]^2)^(5/2))/(5*b^2))/2
```

3.208.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.208.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(71) = 142$.

Time = 0.18 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.31

method	result
derivativedivides	$-\frac{\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{3b}-\frac{\tanh(x)^2\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{5b}+\frac{2a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{15b^2}-\frac{\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a}}{2}$
default	$-\frac{\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{3b}-\frac{\tanh(x)^2\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{5b}+\frac{2a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{15b^2}-\frac{\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a}}{2}$

input

```
int((a+b*tanh(x)^2)^(1/2)*tanh(x)^5,x,method=_RETURNVERBOSE)
```

output

```
-1/3*(a+b*tanh(x)^2)^(3/2)/b-1/5*tanh(x)^2*(a+b*tanh(x)^2)^(3/2)/b+2/15*a/
b^2*(a+b*tanh(x)^2)^(3/2)-1/2*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)+
1/2*b^(1/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2*b*(1+tanh(x))+
a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(
1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))-1/2*(b*(tanh(x)-1)^2
+2*b*(tanh(x)-1)+a+b)^(1/2)-1/2*b^(1/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(t
anh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*b*(t
anh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh
(x)-1))
```

3.208.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1982 vs. 2(71) = 142.

Time = 0.49 (sec) , antiderivative size = 4529, normalized size of antiderivative = 52.06

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^5,x, algorithm="fricas")
```

output
```
[1/60*(15*(b^2*cosh(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*
b^2*cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 4
0*(3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^
2*cosh(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70
*b^2*cosh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2
*cosh(x)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b
^2*cosh(x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3
+ 5*(9*b^2*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x
)^2 + b^2)*sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*c
osh(x)^5 + 4*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 +
 a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh
(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*
cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*c
osh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 +
 a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cos
h(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cos
h(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*
b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2
*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(
6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh...
```

3.208.6 Sympy [A] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.45

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx =$$

$$-\begin{cases} \dfrac{2\left(\frac{b^3\sqrt{a+b\tanh^2(x)}}{2}+\frac{b^3(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}+\frac{b\left(a+b\tanh^2(x)\right)^{\frac{5}{2}}}{10}+\frac{\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}\left(-\frac{ab}{2}+\frac{b^2}{2}\right)}{3}\right)}{b^3} & \text{for } b\neq 0 \\ \sqrt{a}\left(\frac{\log\left(\tanh^2(x)-1\right)}{2}+\frac{\tanh^4(x)}{4}+\frac{\tanh^2(x)}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*tanh(x)**2)**(1/2)*tanh(x)**5,x)
```

output
```
-Piecewise((2*(b**3*sqrt(a + b*tanh(x)**2)/2 + b**3*(a + b)*atan(sqrt(a +
b*tanh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a + b*tanh(x)**2)**(5/2)
/10 + (a + b*tanh(x)**2)**(3/2)*(-a*b/2 + b**2/2)/3)/b**3, Ne(b, 0)), (sqr
t(a)*(log(tanh(x)**2 - 1)/2 + tanh(x)**4/4 + tanh(x)**2/2), True))
```

3.208.7 Maxima [F]

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh(x)^2+a}\tanh(x)^5\,dx$$

input
```
integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^5, x)
```

3.208.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 980 vs. $2(71) = 142$.

Time $= 1.13$ (sec) , antiderivative size $= 980$, normalized size of antiderivative $= 11.26$

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^5,x, algorithm="giac")
```

output
```
-1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) +
1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log
(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b
*e^(2*x) + a + b) - sqrt(a + b))) - 4/15*(15*(sqrt(a + b)*e^(2*x) - sqrt(a
*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^9*(2*a + 3*b) +
 15*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*
e^(2*x) + a + b))^8*(10*a + 9*b)*sqrt(a + b) + 20*(18*a^2 + 23*a*b + b^2)*
(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b))^7 + 20*(30*a^2 - 7*a*b - 65*b^2)*(sqrt(a + b)*e^(2*x) - sqrt
(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^6*sqrt(a + b)
 + 2*(330*a^3 - 705*a^2*b - 1480*a*b^2 + 19*b^3)*(sqrt(a + b)*e^(2*x) - sq
rt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5 + 10*(18*
a^3 - 279*a^2*b + 68*a*b^2 + 349*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x
) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a + b) - 20*(30
*a^4 + 81*a^3*b - 149*a^2*b^2 - 245*a*b^3 + 19*b^4)*(sqrt(a + b)*e^(2*x) -
 sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3 - 20*(
42*a^4 - 33*a^3*b - 139*a^2*b^2 + 69*a*b^3 + 325*b^4)*(sqrt(a + b)*e^(2*x)
 - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*s...
```

3.208.9 Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.37

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx = -\frac{\left(b\tanh(x)^2+a\right)^{5/2}}{5\,b^2} - 2\,\mathrm{atan}\left(\frac{2\sqrt{b\tanh(x)^2+a}\sqrt{-\frac{a}{4}-\frac{b}{4}}}{a+b}\right)\sqrt{-\frac{a}{4}-\frac{b}{4}} - \sqrt{b\tanh(x)^2+a}\left((a+b)\left(\frac{a+b}{b^2}-\frac{2\,a}{b^2}\right)+\frac{a^2}{b^2}\right) - \left(\frac{a+b}{3\,b^2}-\frac{2\,a}{3\,b^2}\right)\left(b\tanh(x)^2+a\right)^{3/2}$$

input
```
int(tanh(x)^5*(a + b*tanh(x)^2)^(1/2),x)
```

output
```
- (a + b*tanh(x)^2)^(5/2)/(5*b^2) - 2*atan((2*(a + b*tanh(x)^2)^(1/2)*(- a
/4 - b/4)^(1/2))/(a + b))*(- a/4 - b/4)^(1/2) - (a + b*tanh(x)^2)^(1/2)*((
a + b)*((a + b)/b^2 - (2*a)/b^2) + a^2/b^2) - ((a + b)/(3*b^2) - (2*a)/(3*
b^2))*(a + b*tanh(x)^2)^(3/2)
```

3.209 $\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx$

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3.209.1 Optimal result

Integrand size = 17, antiderivative size = 121

$$\begin{aligned}\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx = &\frac{(a^2-4ab-8b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{8b^{3/2}}\\ &+\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\\ &-\frac{(a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}}{8b}\\ &-\frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}\end{aligned}$$

output

```
1/8*(a^2-4*a*b-8*b^2)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(3/
2)+arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))*(a+b)^(1/2)-1/8*(a+4
*b)*(a+b*tanh(x)^2)^(1/2)*tanh(x)/b-1/4*(a+b*tanh(x)^2)^(1/2)*tanh(x)^3
```

3.209.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 6.21 (sec) , antiderivative size = 580, normalized size of antiderivative = 4.79

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx$$

$$= \frac{-\frac{b(a^2-4b^2)\sqrt{\frac{a-b+(a+b)\cosh(2x)}{1+\cosh(2x)}}\sqrt{-\frac{a\coth^2(x)}{b}}\sqrt{-\frac{a(1+\cosh(2x))\operatorname{csch}^2(x)}{b}}\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}\operatorname{csch}(2x)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{(a-}\right.\right.}{a(a-b+(a+b)\cosh(2x))}}{}$$

$$+\sqrt{\frac{a-b+a\cosh(2x)+b\cosh(2x)}{1+\cosh(2x)}}\left(\frac{\operatorname{sech}(x)(-a\sinh(x)-6b\sinh(x))}{8b}\right.$$

$$\left.+\frac{1}{4}\operatorname{sech}^2(x)\tanh(x)\right)$$

input
```
Integrate[Tanh[x]^4*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
(-((b*(a^2 - 4*b^2)*Sqrt[(a - b + (a + b)*Cosh[2*x])/(1 + Cosh[2*x])]*Sqrt
[-((a*Coth[x]^2)/b)]*Sqrt[-((a*(1 + Cosh[2*x])*Csch[x]^2)/b)]*Sqrt[((a - b
 + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Csch[2*x]*EllipticF[ArcSin[Sqrt[((a -
b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]*Sinh[x]^4)/(a*(a - b + (
a + b)*Cosh[2*x]))) - ((4*I)*b*(4*a*b + 4*b^2)*Sqrt[1 + Cosh[2*x]]*Sqrt[(a
 - b + (a + b)*Cosh[2*x])/(1 + Cosh[2*x])]*(((-1/4*I)*Sqrt[-((a*Coth[x]^2)
/b)]*Sqrt[-((a*(1 + Cosh[2*x])*Csch[x]^2)/b)]*Sqrt[((a - b + (a + b)*Cosh[
2*x])*Csch[x]^2)/b]*Csch[2*x]*EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh
[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]*Sinh[x]^4)/(a*Sqrt[1 + Cosh[2*x]]*Sqrt[a
 - b + (a + b)*Cosh[2*x]]) + ((I/2)*Sqrt[-((a*Coth[x]^2)/b)]*Sqrt[-((a*(1
+ Cosh[2*x])*Csch[x]^2)/b)]*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b
]*Csch[2*x]*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])
*Csch[x]^2)/b]/Sqrt[2]], 1]*Sinh[x]^4)/((a + b)*Sqrt[1 + Cosh[2*x]]*Sqrt[a
 - b + (a + b)*Cosh[2*x]])))/Sqrt[a - b + (a + b)*Cosh[2*x]])/(4*b) + Sqrt
[(a - b + a*Cosh[2*x] + b*Cosh[2*x])/(1 + Cosh[2*x])]*((Sech[x]*(-(a*Sinh[
x]) - 6*b*Sinh[x]))/(8*b) + (Sech[x]^2*Tanh[x])/4)
```

3.209.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 4153, 380, 444, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx$$

$$\downarrow 3042$$

$$\int \tan(ix)^4\sqrt{a-b\tan(ix)^2}dx$$

$$\downarrow 4153$$

$$\int \frac{\tanh^4(x)\sqrt{a+b\tanh^2(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 380$$

$$\frac{1}{4}\int \frac{\tanh^2(x)\left((a+4b)\tanh^2(x)+3a\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x) - \frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 444$$

$$\frac{1}{4}\left(\frac{\int \frac{a(a+4b)-\left(a^2-4ba-8b^2\right)\tanh^2(x)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{2b} - \frac{(a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}\right) - \frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 398$$

$$\frac{1}{4}\left(\frac{\left(a^2-4ab-8b^2\right)\int \frac{1}{\sqrt{b\tanh^2(x)+a}}d\tanh(x) + 8b(a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{2b} - \frac{(a+4b)\tanh(x)}{2b}\right) - \frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 224$$

$$\frac{1}{4}\left(\frac{\left(a^2-4ab-8b^2\right)\int\frac{1}{1-\frac{b\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}+8b(a+b)\int\frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{2b}-\frac{(a+4b)\tanh}{}\right.$$

$$\frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 219$$

$$\frac{1}{4}\left(\frac{8b(a+b)\int\frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)+\frac{\left(a^2-4ab-8b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}}{2b}-\frac{(a+4b)\tanh(x)\sqrt{a+b}}{2b}\right.$$

$$\frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 291$$

$$\frac{1}{4}\left(\frac{8b(a+b)\int\frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}+\frac{\left(a^2-4ab-8b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}}{2b}-\frac{(a+4b)\tanh(x)\sqrt{a+b\tanh^2}}{2b}\right.$$

$$\frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 219$$

$$\frac{1}{4}\left(\frac{\frac{\left(a^2-4ab-8b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}+8b\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2b}-\frac{(a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}\right)$$

$$\frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

input
```
Int[Tanh[x]^4*Sqrt[a + b*Tanh[x]^2],x]
```

3.209. $\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx$

output
```
-1/4*(Tanh[x]^3*Sqrt[a + b*Tanh[x]^2]) + ((((a^2 - 4*a*b - 8*b^2)*ArcTanh[
(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[b] + 8*b*Sqrt[a + b]*ArcTan
h[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/(2*b) - ((a + 4*b)*Tanh[x]
*Sqrt[a + b*Tanh[x]^2])/(2*b))/4
```

3.209.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.209.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(99) = 198$.

Time $= 0.10$ (sec) , antiderivative size $= 337$, normalized size of antiderivative $= 2.79$

method	result
derivativedivides	$-\frac{\sqrt{a+b\tanh(x)^2}\,\tanh(x)}{2} - \frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{2\sqrt{b}} - \frac{\tanh(x)\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{4b} + \frac{a\tanh(x)\sqrt{a+b}}{8b}$
default	$-\frac{\sqrt{a+b\tanh(x)^2}\,\tanh(x)}{2} - \frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{2\sqrt{b}} - \frac{\tanh(x)\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{4b} + \frac{a\tanh(x)\sqrt{a+b}}{8b}$

input
```
int((a+b*tanh(x)^2)^(1/2)*tanh(x)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a+b*tanh(x)^2)^(1/2)*tanh(x)-1/2*a/b^(1/2)*ln(b^(1/2)*tanh(x)+(a+b*t
anh(x)^2)^(1/2))-1/4*tanh(x)*(a+b*tanh(x)^2)^(3/2)/b+1/8*a/b*tanh(x)*(a+b*
tanh(x)^2)^(1/2)+1/8*a^2/b^(3/2)*ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2))
-1/2*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-1/2*b^(1/2)*ln((b*(tanh(x
)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))+1/2*(a+b)^(1/
2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)
-1)+a+b)^(1/2))/(tanh(x)-1))+1/2*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/
2)-1/2*b^(1/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2*b*(1+tanh(x
))+a+b)^(1/2))-1/2*(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(
b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))
```

3.209.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2030 vs. $2(99) = 198$.

Time $= 0.58$ (sec) , antiderivative size $= 9360$, normalized size of antiderivative $= 77.36$

$$\int \tanh^4(x)\sqrt{a + b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

3.209.6 Sympy [F]

$$\int \tanh^4(x)\sqrt{a + b\tanh^2(x)}\,dx = \int \sqrt{a + b\tanh^2(x)}\tanh^4(x)\,dx$$

input
```
integrate((a+b*tanh(x)**2)**(1/2)*tanh(x)**4,x)
```

output
```
Integral(sqrt(a + b*tanh(x)**2)*tanh(x)**4, x)
```

3.209.7 Maxima [F]

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh(x)^2+a}\tanh(x)^4\,dx$$

input
```
integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^4, x)
```

3.209.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 938 vs. $2(99) = 198$.

Time = 0.99 (sec) , antiderivative size = 938, normalized size of antiderivative = 7.75

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^4,x, algorithm="giac")
```

output
```
-1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) -
1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*sqrt(a + b)*log
(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b
*e^(2*x) + a + b) - sqrt(a + b))) + 1/4*(a^2 - 4*a*b - 8*b^2)*arctan(-1/2*
(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b) + sqrt(a + b))/sqrt(-b))/(sqrt(-b)*b) - 1/2*((a^2 + 12*a*b +
16*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
2*b*e^(2*x) + a + b))^7 + (7*a^2 + 52*a*b + 16*b^2)*(sqrt(a + b)*e^(2*x) -
 sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^6*sqrt(a
 + b) + (21*a^3 + 109*a^2*b + 28*a*b^2 - 48*b^3)*(sqrt(a + b)*e^(2*x) - sq
rt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5 + (35*a^3
 + 115*a^2*b - 156*a*b^2 - 176*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a + b) + (35*a^4
+ 130*a^3*b - 317*a^2*b^2 - 156*a*b^3 + 304*b^4)*(sqrt(a + b)*e^(2*x) - sq
rt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3 + (21*a^4
 + 94*a^3*b - 379*a^2*b^2 + 476*a*b^3 + 48*b^4)*(sqrt(a + b)*e^(2*x) - sqr
t(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b
) + (7*a^5 + 53*a^4*b - 135*a^3*b^2 + 271*a^2*b^3 - 140*a*b^4 - 272*b^5...
```

3.209.9 Mupad [F(-1)]

Timed out.

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \tanh(x)^4\sqrt{b\tanh(x)^2+a}\,dx$$

input
```
int(tanh(x)^4*(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(tanh(x)^4*(a + b*tanh(x)^2)^(1/2), x)
```

3.210 $\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx$

3.210.1 Optimal result . . . 1549
3.210.2 Mathematica [A] (verified) . . . 1549
3.210.3 Rubi [A] (verified) . . . 1550
3.210.4 Maple [B] (verified) . . . 1552
3.210.5 Fricas [B] (verification not implemented) . . . 1553
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3.210.9 Mupad [B] (verification not implemented) . . . 1556

3.210.1 Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \sqrt{a+b\tanh^2(x)} - \frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b}$$

output
```
arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)-(a+b*tanh(x)^2)^(1/
2)-1/3*(a+b*tanh(x)^2)^(3/2)/b
```

3.210.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{\sqrt{a+b\tanh^2(x)}\left(a+3b+b\tanh^2(x)\right)}{3b}$$

input
```
Integrate[Tanh[x]^3*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Sqrt[a + b*Tanh[
x]^2]*(a + 3*b + b*Tanh[x]^2))/(3*b)
```

3.210.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 26, 4153, 26, 354, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx$$

$$\downarrow 3042$$

$$\int i\tan(ix)^3\sqrt{a-b\tan(ix)^2}dx$$

$$\downarrow 26$$

$$i\int \tan(ix)^3\sqrt{a-b\tan(ix)^2}dx$$

$$\downarrow 4153$$

$$i\int -\frac{i\tanh^3(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\tanh^3(x)\sqrt{a+b\tanh^2(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 354$$

$$\frac{1}{2}\int \frac{\tanh^2(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh^2(x)$$

$$\downarrow 90$$

$$\frac{1}{2}\left(\int \frac{\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh^2(x)-\frac{2\left(a+b\tanh^2(x)\right)^{3/2}}{3b}\right)$$

$$\downarrow 60$$

$$\frac{1}{2}\left((a+b)\int\frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)-\frac{2\left(a+b\tanh^2(x)\right)^{3/2}}{3b}-2\sqrt{a+b\tanh^2(x)}\right)$$

$$\downarrow 73$$

$$\frac{1}{2}\left(\frac{2(a+b)\int\frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}-\frac{2\left(a+b\tanh^2(x)\right)^{3/2}}{3b}-2\sqrt{a+b\tanh^2(x)}\right)$$

$$\downarrow 221$$

$$\frac{1}{2}\left(2\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\frac{2\left(a+b\tanh^2(x)\right)^{3/2}}{3b}-2\sqrt{a+b\tanh^2(x)}\right)$$

input
```
Int[Tanh[x]^3*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
(2*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - 2*Sqrt[a + b*T
anh[x]^2] - (2*(a + b*Tanh[x]^2)^(3/2))/(3*b))/2
```

3.210.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.210.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(51) = 102$.

Time $= 0.09$ (sec) , antiderivative size $= 253$, normalized size of antiderivative $= 4.02$

method	result
derivativedivides	$-\frac{\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{3b} - \frac{\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}{2} + \frac{\sqrt{b}\ln\left(\frac{b(1+\tanh(x))-b}{\sqrt{b}}+\sqrt{b(1+\tanh(x))^2-2b(1+}\right.$
default	$-\frac{\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{3b} - \frac{\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}{2} + \frac{\sqrt{b}\ln\left(\frac{b(1+\tanh(x))-b}{\sqrt{b}}+\sqrt{b(1+\tanh(x))^2-2b(1+}\right.$

input
```
int((a+b*tanh(x)^2)^(1/2)*tanh(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a+b*tanh(x)^2)^(3/2)/b-1/2*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/
2)+1/2*b^(1/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2*b*(1+tanh(x
))+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(
b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))-1/2*(b*(tanh(x)-1
)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-1/2*b^(1/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b
*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*b
*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(t
anh(x)-1))
```

3.210.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(51) = 102$.

Time $= 0.34$ (sec) , antiderivative size $= 2329$, normalized size of antiderivative $= 36.97$

$$\int \tanh^3(x)\sqrt{a + b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^3,x, algorithm="fricas")
```

output

```
[1/12*(3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^
4 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh
(x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 +
6*(b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a + b)*log((
(a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b
)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a
^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^
2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*
(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*
b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*
b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 +
3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3
 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3
 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*c
osh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(
5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sin
h(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh
(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh
(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a +
 b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2...
```

3.210.6 Sympy [A] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.49

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx$$

$$= -\begin{cases} \dfrac{2\left(\frac{b^2\sqrt{a+b\tanh^2(x)}}{2}+\frac{b^2(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}+\frac{b\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}{6}\right)}{b^2} & \text{for } b\neq 0 \\ \sqrt{a}\left(\frac{\log\left(\tanh^2(x)-1\right)}{2}+\frac{\tanh^2(x)}{2}\right) & \text{otherwise} \end{cases}$$

input

```
integrate((a+b*tanh(x)**2)**(1/2)*tanh(x)**3,x)
```

output

```
-Piecewise((2*(b**2*sqrt(a + b*tanh(x)**2)/2 + b**2*(a + b)*atan(sqrt(a +
b*tanh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a + b*tanh(x)**2)**(3/2)
/6)/b**2, Ne(b, 0)), (sqrt(a)*(log(tanh(x)**2 - 1)/2 + tanh(x)**2/2), True
))
```

3.210.7 Maxima [F]

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh(x)^2+a}\tanh(x)^3\,dx$$

input
```
integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^3, x)
```

3.210.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(51) = 102$.

Time $= 0.77$ (sec) , antiderivative size $= 630$, normalized size of antiderivative $= 10.00$

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx =$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b\right.\right.$$
$$+\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)$$
$$-\frac{4\left(3\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^5(a+2b)+3\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(}\right.\right.}{}$$

input
```
integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^3,x, algorithm="giac")
```

output
```
-1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) +
1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log
(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b
*e^(2*x) + a + b) - sqrt(a + b))) - 4/3*(3*(sqrt(a + b)*e^(2*x) - sqrt(a*e
^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5*(a + 2*b) + 3*(
sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b))^4*(3*a + 2*b)*sqrt(a + b) + 2*(3*a^2 - 3*a*b - 10*b^2)*(sqrt(
a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
a + b))^3 - 6*(a^2 + 3*a*b + 6*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) - 3*(3*a^3
 + 4*a^2*b - 9*a*b^2 - 26*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e
^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) - (3*a^3 - 17*a*b^2 + 34*b^3)
*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(
2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) +
 b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)^3
```

3.210.9 Mupad [B] (verification not implemented)

Time = 3.68 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx = -\sqrt{b\tanh(x)^2+a} - \frac{\left(b\tanh(x)^2+a\right)^{3/2}}{3\,b} - 2\,\mathrm{atan}\left(\frac{2\sqrt{b\tanh(x)^2+a}\,\sqrt{-\frac{a}{4}-\frac{b}{4}}}{a+b}\right)\sqrt{-\frac{a}{4}-\frac{b}{4}}$$

input
```
int(tanh(x)^3*(a + b*tanh(x)^2)^(1/2),x)
```

output
```
- (a + b*tanh(x)^2)^(1/2) - (a + b*tanh(x)^2)^(3/2)/(3*b) - 2*atan((2*(a +
 b*tanh(x)^2)^(1/2)*(- a/4 - b/4)^(1/2))/(a + b))*(- a/4 - b/4)^(1/2)
```

3.211 $\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx$

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3.211.1 Optimal result

Integrand size = 17, antiderivative size = 85

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx = -\frac{(a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2\sqrt{b}} + \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}$$

output
```
-1/2*(a+2*b)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(1/2)+arctan
h((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))*(a+b)^(1/2)-1/2*(a+b*tanh(x)^
2)^(1/2)*tanh(x)
```

3.211.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 3.74 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.27

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \frac{\left(\sqrt{2}a\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}}{\sqrt{2}}\right),1\right) - 2\sqrt{2}a\sqrt{\frac{(a-b+(a+b}\right.}{2\sqrt{2}\sqrt{(a-b}}$$

input
```
Integrate[Tanh[x]^2*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
((Sqrt[2]*a*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticF[ArcS
in[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] - 2*Sqrt[2
]*a*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticPi[b/(a + b),
ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] - (a -
 b + (a + b)*Cosh[2*x])*Sech[x]^2)*Tanh[x])/(2*Sqrt[2]*Sqrt[(a - b + (a +
b)*Cosh[2*x])*Sech[x]^2])
```

3.211.3 Rubi [A] (verified)

Time $= 0.32$ (sec) , antiderivative size $= 89$, normalized size of antiderivative $= 1.05$, number of steps used $= 11$, number of rules used $= 10$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used $= \{3042, 25, 4153, 25, 380, 398, 224, 219, 291, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx$$

$$\downarrow 3042$$

$$\int -\tan(ix)^2\sqrt{a-b\tan(ix)^2}dx$$

$$\downarrow 25$$

$$-\int \tan(ix)^2\sqrt{a-b\tan(ix)^2}dx$$

$$\downarrow 4153$$

$$-\int -\frac{\tanh^2(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 25$$

$$\int \frac{\tanh^2(x)\sqrt{a+b\tanh^2(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 380$$

$$\frac{1}{2}\int \frac{(a+2b)\tanh^2(x)+a}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x) - \frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 398$$

$$\frac{1}{2}\left(2(a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-(a+2b)\int \frac{1}{\sqrt{b\tanh^2(x)+a}}d\tanh(x)\right)-\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 224$$

$$\frac{1}{2}\left(2(a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-(a+2b)\int \frac{1}{1-\frac{b\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}\right)-\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 219$$

$$\frac{1}{2}\left(2(a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\frac{(a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}\right)-\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 291$$

$$\frac{1}{2}\left(2(a+b)\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}-\frac{(a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}\right)-\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 219$$

$$\frac{1}{2}\left(2\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{(a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}\right)-\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}$$

input
```
Int[Tanh[x]^2*Sqrt[a + b*Tanh[x]^2],x]
```

3.211. $\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx$

output
```
(-(((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[b]) +
 2*Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/2 - (
Tanh[x]*Sqrt[a + b*Tanh[x]^2])/2
```

3.211.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.211.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(67) = 134$.

Time = 0.09 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.25

method	result
derivativedivides	$-\frac{\sqrt{a+b\tanh(x)^2}\,\tanh(x)}{2} - \frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{2\sqrt{b}} + \frac{\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}{2} - \sqrt{}$
default	$-\frac{\sqrt{a+b\tanh(x)^2}\,\tanh(x)}{2} - \frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{2\sqrt{b}} + \frac{\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}{2} - \sqrt{}$

input

```
int((a+b*tanh(x)^2)^(1/2)*tanh(x)^2,x,method=_RETURNVERBOSE)
```

output

```
-1/2*(a+b*tanh(x)^2)^(1/2)*tanh(x)-1/2*a/b^(1/2)*ln(b^(1/2)*tanh(x)+(a+b*t
anh(x)^2)^(1/2))+1/2*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-1/2*b^(1/
2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2
))-1/2*(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x)
)^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))-1/2*(b*(tanh(x)-1)^2+2*b*(tan
h(x)-1)+a+b)^(1/2)-1/2*b^(1/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)
^2+2*b*(tanh(x)-1)+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)
+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))
```

3.211.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 896 vs. 2(67) = 134.

Time = 0.40 (sec) , antiderivative size = 4825, normalized size of antiderivative = 56.76

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^2,x, algorithm="fricas")
```

output
```
[1/4*((b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 +
 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b
)*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh
(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 +
 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(
x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b
^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^
3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^
5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)
)*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*co
sh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a
^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x
)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3
*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3
 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cos
h(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b -
 b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x
))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(
x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*co...
```

3.211.6 Sympy [F]

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{a+b\tanh^2(x)}\tanh^2(x)\,dx$$

input
```
integrate((a+b*tanh(x)**2)**(1/2)*tanh(x)**2,x)
```

output
```
Integral(sqrt(a + b*tanh(x)**2)*tanh(x)**2, x)
```

3.211.7 Maxima [F]

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh(x)^2+a}\tanh(x)^2\,dx$$

input
```
integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^2, x)
```

3.211.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. 2(67) = 134.

Time = 0.74 (sec) , antiderivative size = 554, normalized size of antiderivative = 6.52

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx$$
$$= -\frac{(a+2b)\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}}$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b\right.\right.$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)$$
$$+\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)$$
$$-\frac{2\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^3(a+2b)+\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}}\right.\right.}{\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2}\right.\right.}$$

input
```
integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^2,x, algorithm="giac")
```

output
```
-(a + 2*b)*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/
2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) - 1/2
*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*sqrt(a + b)*log(ab
s(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^
(2*x) + a + b) - sqrt(a + b))) - 2*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3*(a + 2*b) + (sqrt(a +
b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
b))^2*(3*a - 2*b)*sqrt(a + b) + (3*a^2 - 3*a*b - 2*b^2)*(sqrt(a + b)*e^(2*
x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) + (a
^2 - a*b + 2*b^2)*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*
e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) -
 sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a +
 b) + a - 3*b)^2
```

3.211.9 Mupad [F(-1)]

Timed out.

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \tanh(x)^2\sqrt{b\tanh(x)^2+a}\,dx$$

input
```
int(tanh(x)^2*(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(tanh(x)^2*(a + b*tanh(x)^2)^(1/2), x)
```

3.212 $\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx$

3.212.1 Optimal result 1565
3.212.2 Mathematica [A] (verified) 1565
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3.212.9 Mupad [B] (verification not implemented) 1571

3.212.1 Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \sqrt{a+b\tanh^2(x)}$$

output
```
arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)-(a+b*tanh(x)^2)^(1/
2)
```

3.212.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \sqrt{a+b\tanh^2(x)}$$

input
```
Integrate[Tanh[x]*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Sqrt[a + b*Tanh[x
]^2]
```

3.212.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 26, 4153, 26, 353, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx$$

$$\downarrow 3042$$

$$\int -i\tan(ix)\sqrt{a-b\tan(ix)^2}dx$$

$$\downarrow 26$$

$$-i\int \tan(ix)\sqrt{a-b\tan(ix)^2}dx$$

$$\downarrow 4153$$

$$-i\int \frac{i\tanh(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 353$$

$$\frac{1}{2}\int \frac{\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh^2(x)$$

$$\downarrow 60$$

$$\frac{1}{2}\left((a+b)\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh^2(x) - 2\sqrt{a+b\tanh^2(x)}\right)$$

$$\downarrow 73$$

$$\frac{1}{2}\left(\frac{2(a+b)\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b} - 2\sqrt{a+b\tanh^2(x)}\right)$$

$$\downarrow 221$$

$$\frac{1}{2}\left(2\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-2\sqrt{a+b\tanh^2(x)}\right)$$

input
```
Int[Tanh[x]*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
(2*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - 2*Sqrt[a + b*T
anh[x]^2])/2
```

3.212.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.212.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(36) = 72$.

Time $= 0.07$ (sec) , antiderivative size $= 238$, normalized size of antiderivative $= 5.41$

method	result
derivativedivides	$-\frac{\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{2}-\frac{\sqrt{b}\,\ln\left(\frac{b(\tanh(x)-1)+b}{\sqrt{b}}+\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}\right)}{2}+$
default	$-\frac{\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{2}-\frac{\sqrt{b}\,\ln\left(\frac{b(\tanh(x)-1)+b}{\sqrt{b}}+\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}\right)}{2}+$

input
```
int((a+b*tanh(x)^2)^(1/2)*tanh(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-1/2*b^(1/2)*ln((b*(tanh(x
)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))+1/2*(a+b)^(1/
2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)
-1)+a+b)^(1/2))/(tanh(x)-1))-1/2*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/
2)+1/2*b^(1/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2*b*(1+tanh(x
))+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(
b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))
```

3.212.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. 2(36) = 72.

Time = 0.30 (sec) , antiderivative size = 1543, normalized size of antiderivative = 35.07

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tanh(x)^2)^(1/2)*tanh(x),x, algorithm="fricas")
```

output
```
[1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*log(((a^
3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*s
inh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*
b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b
)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^
3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*
cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*
cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a
^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 +
a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 +
3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh
(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a
^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x
)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)
^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)
^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)
*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(
x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b
)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b
 - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)...
```

3.212.6 Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.55

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx = -\begin{cases} \dfrac{2\left(\frac{b\sqrt{a+b\tanh^2(x)}}{2} + \frac{b(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}\right)}{b} & \text{for } b \neq 0 \\ \dfrac{\sqrt{a}\log\left(2\tanh^2(x)-2\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*tanh(x)**2)**(1/2)*tanh(x),x)
```

output
```
-Piecewise((2*(b*sqrt(a + b*tanh(x)**2)/2 + b*(a + b)*atan(sqrt(a + b*tanh
(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)))/b, Ne(b, 0)), (sqrt(a)*log(2*tanh(
x)**2 - 2)/2, True))
```

3.212.7 Maxima [F]

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh(x)^2+a}\tanh(x)\,dx$$

input
```
integrate((a+b*tanh(x)^2)^(1/2)*tanh(x),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^2 + a)*tanh(x), x)
```

3.212.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(36) = 72$.

Time $= 0.56$ (sec) , antiderivative size $= 349$, normalized size of antiderivative $= 7.93$

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx =$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b\right.\right.$$
$$+\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)$$
$$-\frac{4\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)b-\right.}{\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2+2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+\right.}$$

input
```
integrate((a+b*tanh(x)^2)^(1/2)*tanh(x),x, algorithm="giac")
```

output
```
-1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) +
1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log
(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b
*e^(2*x) + a + b) - sqrt(a + b))) - 4*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*
x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*b - sqrt(a + b)*b)/((
sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a
*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)
```

3.212.9 Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx = -\sqrt{b\tanh(x)^2+a} - 2\operatorname{atan}\left(\frac{2\sqrt{b\tanh(x)^2+a}\sqrt{-\frac{a}{4}-\frac{b}{4}}}{a+b}\right)\sqrt{-\frac{a}{4}-\frac{b}{4}}$$

input
```
int(tanh(x)*(a + b*tanh(x)^2)^(1/2),x)
```

output
```
- (a + b*tanh(x)^2)^(1/2) - 2*atan((2*(a + b*tanh(x)^2)^(1/2)*(- a/4 - b/4
)^(1/2))/(a + b))*(- a/4 - b/4)^(1/2)
```

3.213 $\int \sqrt{a+b\tanh^2(x)}\,dx$

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3.213.2 Mathematica [A] (verified) . . . 1572
3.213.3 Rubi [A] (verified) . . . 1573
3.213.4 Maple [B] (verified) . . . 1575
3.213.5 Fricas [B] (verification not implemented) . . . 1575
3.213.6 Sympy [F] . . . 1576
3.213.7 Maxima [F] . . . 1577
3.213.8 Giac [B] (verification not implemented) . . . 1577
3.213.9 Mupad [F(-1)] . . . 1578

3.213.1 Optimal result

Integrand size = 12, antiderivative size = 60

$$\int \sqrt{a+b\tanh^2(x)}\,dx = -\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) + \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$

output
```
-arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))*b^(1/2)+arctanh((a+b)^(1/2
)*tanh(x)/(a+b*tanh(x)^2)^(1/2))*(a+b)^(1/2)
```

3.213.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.35

$$\int \sqrt{a+b\tanh^2(x)}\,dx = \sqrt{-a-b}\arctan\left(\frac{\sqrt{b}\operatorname{sech}^2(x)+\tanh(x)\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right) + \sqrt{b}\log\left(-\sqrt{b}\tanh(x)+\sqrt{a+b\tanh^2(x)}\right)$$

input
```
Integrate[Sqrt[a + b*Tanh[x]^2],x]
```

output
```
Sqrt[-a - b]*ArcTan[(Sqrt[b]*Sech[x]^2 + Tanh[x]*Sqrt[a + b*Tanh[x]^2])/Sq
rt[-a - b]] + Sqrt[b]*Log[-(Sqrt[b]*Tanh[x]) + Sqrt[a + b*Tanh[x]^2]]
```

3.213.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 4144, 301, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\tanh^2(x)}\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{a-b\tan(ix)^2}dx$$

$$\downarrow 4144$$

$$\int \frac{\sqrt{a+b\tanh^2(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 301$$

$$(a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x) - b\int \frac{1}{\sqrt{b\tanh^2(x)+a}}d\tanh(x)$$

$$\downarrow 224$$

$$(a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x) - b\int \frac{1}{1-\frac{b\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}$$

$$\downarrow 219$$

$$(a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x) - \sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$

$$\downarrow 291$$

$$(a+b)\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}} - \sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$

$$\downarrow 219$$

$$\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$

input
```
Int[Sqrt[a + b*Tanh[x]^2],x]
```

output
```
-(Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]) + Sqrt[a + b]*
ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]
```

3.213.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.213.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(48) = 96$.

Time $= 0.11$ (sec) , antiderivative size $= 238$, normalized size of antiderivative $= 3.97$

method	result
derivativedivides	$\frac{\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}{2} - \frac{\sqrt{b}\,\ln\left(\frac{b(1+\tanh(x))-b}{\sqrt{b}}+\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}\right)}{2} - \frac{\sqrt{a-}}{}$
default	$\frac{\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}{2} - \frac{\sqrt{b}\,\ln\left(\frac{b(1+\tanh(x))-b}{\sqrt{b}}+\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}\right)}{2} - \frac{\sqrt{a-}}{}$

input
```
int((a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-1/2*b^(1/2)*ln((b*(1+tanh(
x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))-1/2*(a+b)^(1/2
)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x
))+a+b)^(1/2))/(1+tanh(x)))-1/2*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2
)-1/2*b^(1/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1
)+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b
*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))
```

3.213.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(48) = 96$.

Time $= 0.36$ (sec) , antiderivative size $= 3443$, normalized size of antiderivative $= 57.38$

$$\int \sqrt{a + b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*s
inh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^
2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*co
sh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 +
6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6
*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(
x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh
(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)
*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4
+ a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sin
h(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6
- 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)
^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*
cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*
b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cos
h(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*co
sh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*co
sh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)
^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(...
```

3.213.6 Sympy [F]

$$\int \sqrt{a + b \tanh^2(x)}\, dx = \int \sqrt{a + b \tanh^2(x)}\, dx$$

input
```
integrate((a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tanh(x)**2), x)
```

3.213.7 Maxima [F]

$$\int \sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh(x)^2+a}\,dx$$

input
```
integrate((a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^2 + a), x)
```

3.213.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(48) = 96$.

Time $= 0.49$ (sec) , antiderivative size $= 253$, normalized size of antiderivative $= 4.22$

$$\int \sqrt{a+b\tanh^2(x)}\,dx = -\frac{2\,b\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}+\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}}$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b\right.\right.$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)$$
$$+\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)$$

input
```
integrate((a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-2*b*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e
^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*sqrt
(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e
^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) - 1/2*sqrt(
a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(
2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*sqrt(a + b)*log(abs(-sqr
t(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
+ a + b) - sqrt(a + b)))
```

3.213.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + b \tanh^2(x)}\, dx = \int \sqrt{b \tanh(x)^2 + a}\, dx$$

input
```
int((a + b*tanh(x)^2)^(1/2),x)
```

output
```
int((a + b*tanh(x)^2)^(1/2), x)
```

3.214 $\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx$

3.214.1 Optimal result . . . 1579
3.214.2 Mathematica [A] (verified) . . . 1579
3.214.3 Rubi [A] (verified) . . . 1580
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3.214.5 Fricas [B] (verification not implemented) . . . 1582
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3.214.8 Giac [F] . . . 1584
3.214.9 Mupad [F(-1)] . . . 1584

3.214.1 Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx = -\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right) + \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)$$

output
```
-arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))*a^(1/2)+arctanh((a+b*tanh(x)^2)^(1
/2)/(a+b)^(1/2))*(a+b)^(1/2)
```

3.214.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx = -\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right) + \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)$$

input
```
Integrate[Coth[x]*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
-(Sqrt[a]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]) + Sqrt[a + b]*ArcTanh[Sq
rt[a + b*Tanh[x]^2]/Sqrt[a + b]]
```

3.214.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 26, 4153, 26, 354, 94, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{i\sqrt{a-b\tan(ix)^2}}{\tan(ix)}dx$$

$$\downarrow 26$$

$$i\int \frac{\sqrt{a-b\tan(ix)^2}}{\tan(ix)}dx$$

$$\downarrow 4153$$

$$i\int -\frac{i\coth(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 354$$

$$\frac{1}{2}\int \frac{\coth(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh^2(x)$$

$$\downarrow 94$$

$$\frac{1}{2}\left((a+b)\int\frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)+a\int\frac{\coth(x)}{\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)\right)$$

$$\downarrow 73$$

$$\frac{1}{2}\left(\frac{2(a+b)\int\frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}+\frac{2a\int\frac{1}{\frac{\tanh^4(x)}{b}-\frac{a}{b}}d\sqrt{b\tanh^2(x)+a}}{b}\right)$$

$$\downarrow 221$$

$$\frac{1}{2}\left(2\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)\right)$$

input
```
Int[Coth[x]*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]] + 2*Sqrt[a + b]*ArcTanh
[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]])/2
```

3.214.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.214.4 Maple [F]

$$\int \coth(x)\sqrt{a+b\tanh(x)^2}dx$$

input
```
int(coth(x)*(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)*(a+b*tanh(x)^2)^(1/2),x)
```

3.214.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(44) = 88$.

Time $= 0.36$ (sec) , antiderivative size $= 3467$, normalized size of antiderivative $= 61.91$

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*si
nh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3
 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cos
h(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 +
 b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 +
b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x
)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(
x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*
cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 +
 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh
(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 +
 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^
3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*c
osh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b
 + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh
(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
- b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cos
h(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cos
h(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^
5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x...
```

3.214.6 Sympy [F]

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{a+b\tanh^2(x)}\coth(x)\,dx$$

input
```
integrate(coth(x)*(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tanh(x)**2)*coth(x), x)
```

3.214.7 Maxima [F]

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh(x)^2+a}\coth(x)\,dx$$

input
```
integrate(coth(x)*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^2 + a)*coth(x), x)
```

3.214.8 Giac [F]

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh(x)^2+a}\coth(x)\,dx$$

input
```
integrate(coth(x)*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.214.9 Mupad [F(-1)]

Timed out.

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \coth(x)\,\sqrt{b\tanh(x)^2+a}\,dx$$

input
```
int(coth(x)*(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)*(a + b*tanh(x)^2)^(1/2), x)
```

3.215 $\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx$

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3.215.1 Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \coth(x)\sqrt{a+b\tanh^2(x)}$$

output
```
arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))*(a+b)^(1/2)-coth(x)*(a+
b*tanh(x)^2)^(1/2)
```

3.215.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx = -\coth(x)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{(a+b)\tanh^2(x)}{a+b\tanh^2(x)}\right)\sqrt{a+b\tanh^2(x)}$$

input
```
Integrate[Coth[x]^2*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
-(Coth[x]*Hypergeometric2F1[-1/2, 1, 1/2, ((a + b)*Tanh[x]^2)/(a + b*Tanh[
x]^2)]*Sqrt[a + b*Tanh[x]^2])
```

3.215.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 25, 4153, 25, 377, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\sqrt{a-b\tan(ix)^2}}{\tan(ix)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\sqrt{a-b\tan(ix)^2}}{\tan(ix)^2}dx$$

$$\downarrow 4153$$

$$-\int -\frac{\coth^2(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 25$$

$$\int \frac{\coth^2(x)\sqrt{a+b\tanh^2(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 377$$

$$\int \frac{a+b}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x) - \coth(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 27$$

$$(a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x) - \coth(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 291$$

$$(a+b)\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}} - \coth(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 219$$

$$\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\coth(x)\sqrt{a+b\tanh^2(x)}$$

input
```
Int[Coth[x]^2*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - Coth[x]
*Sqrt[a + b*Tanh[x]^2]
```

3.215.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.215.4 Maple [F]

$$\int \coth(x)^2 \sqrt{a + b \tanh(x)^2}\, dx$$

input
```
int(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x)
```

3.215.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(40) = 80$.

Time $= 0.30$ (sec) , antiderivative size $= 1539$, normalized size of antiderivative $= 32.06$

$$\int \coth^2(x) \sqrt{a + b \tanh^2(x)}\, dx = \text{Too large to display}$$

input
```
integrate(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*log(-((a
*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*
sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b
^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^
3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a
*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)
*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)
*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*
a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2
 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 +
 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cos
h(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*
b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(
x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x
)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x
)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b
)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh
(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^
3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2
- 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x...
```

3.215.6 Sympy [F]

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{a+b\tanh^2(x)}\coth^2(x)\,dx$$

input
```
integrate(coth(x)**2*(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tanh(x)**2)*coth(x)**2, x)
```

3.215.7 Maxima [F]

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh(x)^2+a}\coth(x)^2\,dx$$

input
```
integrate(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^2, x)
```

3.215.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(40) = 80$.

Time = 0.56 (sec) , antiderivative size = 348, normalized size of antiderivative = 7.25

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx =$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b\right.\right.$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)$$
$$+\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)$$
$$+\frac{4\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)a+\right.}{\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2-2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+\right.}$$

input
```
integrate(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) -
1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*sqrt(a + b)*log
(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b
*e^(2*x) + a + b) - sqrt(a + b))) + 4*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*
x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*a + sqrt(a + b)*a)/((
sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a
*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a + b)
```

3.215.9 Mupad [F(-1)]

Timed out.

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \coth(x)^2\sqrt{b\tanh(x)^2+a}\,dx$$

input
```
int(coth(x)^2*(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^2*(a + b*tanh(x)^2)^(1/2), x)
```

3.216 $\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx$

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3.216.8 Giac [B] (verification not implemented) . . . 1598
3.216.9 Mupad [F(-1)] . . . 1599

3.216.1 Optimal result

Integrand size = 17, antiderivative size = 83

$$\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx = -\frac{(2a+b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2\sqrt{a}} + \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)}$$

output

```
-1/2*(2*a+b)*arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(1/2)+arctanh((a+b*t
anh(x)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)-1/2*coth(x)^2*(a+b*tanh(x)^2)^(1/
2)
```

3.216.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx = -\frac{(2a+b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2\sqrt{a}} + \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)}$$

input
```
Integrate[Coth[x]^3*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
-1/2*((2*a + b)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/Sqrt[a] + Sqrt[a +
 b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Coth[x]^2*Sqrt[a + b*Tan
h[x]^2])/2
```

3.216.3 Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {3042, 26, 4153, 26, 354, 110, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{i\sqrt{a-b\tan(ix)^2}}{\tan(ix)^3}dx$$
$$\downarrow 26$$
$$-i\int \frac{\sqrt{a-b\tan(ix)^2}}{\tan(ix)^3}dx$$
$$\downarrow 4153$$

$$-i\int \frac{i\coth^3(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\coth^3(x)\sqrt{a+b\tanh^2(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 354$$

$$\frac{1}{2}\int \frac{\coth^2(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh^2(x)$$

$$\downarrow 110$$

$$\frac{1}{2}\left(\int \frac{\coth(x)\left(b\tanh^2(x)+2a+b\right)}{2\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)-\coth(x)\sqrt{a+b\tanh^2(x)}\right)$$

$$\downarrow 27$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{\coth(x)\left(b\tanh^2(x)+2a+b\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)-\coth(x)\sqrt{a+b\tanh^2(x)}\right)$$

$$\downarrow 174$$

$$\frac{1}{2}\left(\frac{1}{2}\left(2(a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)+(2a+b)\int \frac{\coth(x)}{\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)\right)-\coth$$

$$\downarrow 73$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{4(a+b)\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}+\frac{2(2a+b)\int \frac{1}{\frac{\tanh^4(x)}{b}-\frac{a}{b}}d\sqrt{b\tanh^2(x)+a}}{b}\right)-\coth(x)\sqrt{a+b\tanh^2(x)}\right)$$

$$\downarrow 221$$

$$\frac{1}{2}\left(\frac{1}{2}\left(4\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\frac{2(2a+b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}\right)-\coth(x)\sqrt{a+b\tanh^2(x)}\right)$$

input
```
Int[Coth[x]^3*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
(((-2*(2*a + b)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/Sqrt[a] + 4*Sqrt[a
 + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]])/2 - Coth[x]*Sqrt[a + b*T
anh[x]^2])/2
```

3.216.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.216.4 Maple [F]

$$\int \coth(x)^3 \sqrt{a + b \tanh(x)^2}\, dx$$

input
```
int(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x)
```

3.216.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 907 vs. $2(65) = 130$.

Time $= 0.41$ (sec) , antiderivative size $= 4891$, normalized size of antiderivative $= 58.93$

$$\int \coth^3(x)\sqrt{a + b\tanh^2(x)}\, dx = \text{Too large to display}$$

input
```
integrate(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output

```
[1/4*((a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 - 2*a*cosh(x)^2 +
 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh(x)^3 - a*cosh(x))*sinh(x) + a
)*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(
x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 +
a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x
)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^
3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3
 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5
 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))
*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cos
h(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*
a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)
^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*
a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 +
 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh
(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b +
b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)
)*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x
)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cos...
```

3.216.6 Sympy [F]

$$\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{a+b\tanh^2(x)}\coth^3(x)\,dx$$

input

```
integrate(coth(x)**3*(a+b*tanh(x)**2)**(1/2),x)
```

output

```
Integral(sqrt(a + b*tanh(x)**2)*coth(x)**3, x)
```

3.216.7 Maxima [F]

$$\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh(x)^2+a}\coth(x)^3\,dx$$

input
```
integrate(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^3, x)
```

3.216.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. 2(65) = 130.

Time = 0.72 (sec) , antiderivative size = 557, normalized size of antiderivative = 6.71

$$\begin{aligned}
&\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx \\
&= \frac{(2a+b)\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}} \\
&\quad -\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b\right.\right. \\
&\quad +\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right) \\
&\quad -\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right) \\
&\quad +\frac{2\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^3(2a+b)+\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}\right.\right.}{\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2}\right.\right.}
\end{aligned}$$

input
```
integrate(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
(2*a + b)*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/sqrt(-a))/sqrt(-a) - 1/2
*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) + 1/2*
sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*
a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log(abs
(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(
2*x) + a + b) - sqrt(a + b))) + 2*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) +
 b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3*(2*a + b) + (sqrt(a + b
)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b
))^2*(2*a - 3*b)*sqrt(a + b) - (2*a^2 + 3*a*b - 3*b^2)*(sqrt(a + b)*e^(2*x
) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) - (2*
a^2 - a*b + b^2)*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e
^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) -
sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a +
b) - 3*a + b)^2
```

3.216.9 Mupad [F(-1)]

Timed out.

$$\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \coth(x)^3\sqrt{b\tanh(x)^2+a}\,dx$$

input
```
int(coth(x)^3*(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^3*(a + b*tanh(x)^2)^(1/2), x)
```

3.217 $\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx$

3.217.1 Optimal result 1600
3.217.2 Mathematica [C] (warning: unable to verify) 1600
3.217.3 Rubi [A] (verified) 1601
3.217.4 Maple [F] 1603
3.217.5 Fricas [B] (verification not implemented) 1604
3.217.6 Sympy [F] 1604
3.217.7 Maxima [F] 1605
3.217.8 Giac [B] (verification not implemented) 1605
3.217.9 Mupad [F(-1)] 1606

3.217.1 Optimal result

Integrand size = 17, antiderivative size = 78

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a} - \frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)}$$

output
```
arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))*(a+b)^(1/2)-1/3*(3*a+b)
*coth(x)*(a+b*tanh(x)^2)^(1/2)/a-1/3*coth(x)^3*(a+b*tanh(x)^2)^(1/2)
```

3.217.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.06

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \frac{\cosh^4(x)\coth^3(x)\left(1+\frac{b\tanh^2(x)}{a}\right)\left(-\frac{\operatorname{sech}^4(x)\left(\arcsin\left(\sqrt{-\frac{(a+b)\sinh^2(x)}{a}}\right)\sqrt{-\frac{(a+b)\sinh^2(x)}{a}}+\sqrt{\cosh^2(x)+\frac{b\sinh^2(x)}{a}}\right)(a-2b\tan}{\sqrt{\cosh^2(x)+\frac{b\sinh^2(x)}{a}}}\right.}{3\sqrt{a+b\tanh^2(x)}}$$

input
```
Integrate[Coth[x]^4*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
(Cosh[x]^4*Coth[x]^3*(1 + (b*Tanh[x]^2)/a)*(-((Sech[x]^4*(ArcSin[Sqrt[-(((
a + b)*Sinh[x]^2)/a)]]*Sqrt[-(((a + b)*Sinh[x]^2)/a)] + Sqrt[Cosh[x]^2 + (
b*Sinh[x]^2)/a])*(a - 2*b*Tanh[x]^2))/Sqrt[Cosh[x]^2 + (b*Sinh[x]^2)/a]) -
 (4*(a + b)*Hypergeometric2F1[2, 2, 3/2, -(((a + b)*Sinh[x]^2)/a)]*(a*Tanh
[x] + b*Tanh[x]^3)^2)/a^2))/(3*Sqrt[a + b*Tanh[x]^2])
```

3.217.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 4153, 377, 445, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a-b\tan(ix)^2}}{\tan(ix)^4}dx$$

$$\downarrow 4153$$

$$\int \frac{\coth^4(x)\sqrt{a+b\tanh^2(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 377$$

$$\frac{1}{3}\int \frac{\coth^2(x)\left(2b\tanh^2(x)+3a+b\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x) - \frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 445$$

$$\frac{1}{3}\left(-\frac{\int -\frac{3a(a+b)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{a} - \frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right) - \frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 27$$

$$\frac{1}{3}\left(3(a+b)\int\frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right)-\frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 291$$

$$\frac{1}{3}\left(3(a+b)\int\frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}-\frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right)-\frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 219$$

$$\frac{1}{3}\left(3\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right)-\frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)}$$

input
```
Int[Coth[x]^4*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
-1/3*(Coth[x]^3*Sqrt[a + b*Tanh[x]^2]) + (3*Sqrt[a + b]*ArcTanh[(Sqrt[a +
b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - ((3*a + b)*Coth[x]*Sqrt[a + b*Tanh[x]
^2])/a)/3
```

3.217.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.217.4 Maple [F]

$$\int \coth(x)^4 \sqrt{a + b\tanh(x)^2}\, dx$$

input
```
int(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x)
```

3.217.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. 2(64) = 128.

Time = 0.35 (sec) , antiderivative size = 2355, normalized size of antiderivative = 30.19

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 - 3*a*cosh(x)^
4 + 3*(5*a*cosh(x)^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 3*a*cosh(x))*sinh
(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^2 + a)*sinh(x)^2 +
6*(a*cosh(x)^5 - 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) - a)*sqrt(a + b)*log(-
((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^
3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2
+ b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2
*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70
*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b
^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b
^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 +
 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*
b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^
3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*
cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*
(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*si
nh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cos
h(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cos
h(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a
+ b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - ...
```

3.217.6 Sympy [F]

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{a+b\tanh^2(x)}\coth^4(x)\,dx$$

input
```
integrate(coth(x)**4*(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tanh(x)**2)*coth(x)**4, x)
```

3.217.7 Maxima [F]

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh(x)^2+a}\coth(x)^4\,dx$$

input
```
integrate(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^4, x)
```

3.217.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(64) = 128$.

Time $= 0.83$ (sec) , antiderivative size $= 629$, normalized size of antiderivative $= 8.06$

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx =$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b\right.\right.$$
$$\left.\left.-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)\right.\right.$$
$$\left.\left.+\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)\right.\right.$$
$$\left.\left.+\frac{4\left(3\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^5(2a+b)-3\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(}\right.\right.\right.\right.$$

input
```
integrate(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) -
1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*sqrt(a + b)*log
(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b
*e^(2*x) + a + b) - sqrt(a + b))) + 4/3*(3*(sqrt(a + b)*e^(2*x) - sqrt(a*e
^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5*(2*a + b) - 3*(
sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b))^4*(2*a + 3*b)*sqrt(a + b) - 2*(10*a^2 + 3*a*b - 3*b^2)*(sqrt(
a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
a + b))^3 + 6*(6*a^2 + 3*a*b + b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) + 3*(26*a^
3 + 9*a^2*b - 4*a*b^2 - 3*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e
^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) + (34*a^3 - 17*a^2*b + 3*b^3)
*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(
2*x) - 2*b*e^(2*x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) +
 b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a + b)^3
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \coth(x)^4\sqrt{b\tanh(x)^2+a}\,dx$$

input
```
int(coth(x)^4*(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^4*(a + b*tanh(x)^2)^(1/2), x)
```

3.218 $\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx$

3.218.1 Optimal result 1607
3.218.2 Mathematica [A] (verified) 1608
3.218.3 Rubi [A] (warning: unable to verify) 1608
3.218.4 Maple [F] 1612
3.218.5 Fricas [B] (verification not implemented) 1612
3.218.6 Sympy [F] 1612
3.218.7 Maxima [F] 1613
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3.218.9 Mupad [F(-1)] 1614

3.218.1 Optimal result

Integrand size = 17, antiderivative size = 121

$$\begin{aligned}\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx = &-\frac{(8a^2+4ab-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{8a^{3/2}}\\ &+\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)\\ &-\frac{(4a+b)\coth^2(x)\sqrt{a+b\tanh^2(x)}}{8a}\\ &-\frac{1}{4}\coth^4(x)\sqrt{a+b\tanh^2(x)}\end{aligned}$$

output

```
-1/8*(8*a^2+4*a*b-b^2)*arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(3/2)+arct
anh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)-1/8*(4*a+b)*coth(x)^2*(
a+b*tanh(x)^2)^(1/2)/a-1/4*coth(x)^4*(a+b*tanh(x)^2)^(1/2)
```

3.218.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

$$\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx$$

$$= \frac{(-8a^2-4ab+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)+\sqrt{a}\left(8a\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\coth^2(x)\,(4a+b+$$

$$8a^{3/2}$$

input
```
Integrate[Coth[x]^5*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
((-8*a^2 - 4*a*b + b^2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]] + Sqrt[a]*(
8*a*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Coth[x]^2*(4*
a + b + 2*a*Coth[x]^2)*Sqrt[a + b*Tanh[x]^2]))/(8*a^(3/2))
```

3.218.3 Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {3042, 26, 4153, 26, 354, 110, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{i\sqrt{a-b\tan(ix)^2}}{\tan(ix)^5}dx$$

$$\downarrow 26$$

$$i\int \frac{\sqrt{a-b\tan(ix)^2}}{\tan(ix)^5}dx$$

$$\downarrow 4153$$

$$i\int -\frac{i\coth^5(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\coth^5(x)\sqrt{a+b\tanh^2(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 354$$

$$\frac{1}{2}\int \frac{\coth^3(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh^2(x)$$

$$\downarrow 110$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{\coth^2(x)\left(3b\tanh^2(x)+4a+b\right)}{2\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)-\frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)}\right)$$

$$\downarrow 27$$

$$\frac{1}{2}\left(\frac{1}{4}\int \frac{\coth^2(x)\left(3b\tanh^2(x)+4a+b\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)-\frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)}\right)$$

$$\downarrow 168$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{\int -\frac{\coth(x)\left(8a^2+4ba-b^2+b(4a+b)\tanh^2(x)\right)}{2\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a}-\frac{(4a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right)-\frac{1}{2}\coth^2(x)\sqrt{a+\right.$$

$$\downarrow 27$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int \frac{\coth(x)\left(8a^2+4ba-b^2+b(4a+b)\tanh^2(x)\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{2a}-\frac{(4a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right)-\frac{1}{2}\coth^2(x)\sqrt{a+b}\right.$$

$$\downarrow 174$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\left(8a^2+4ab-b^2\right)\int \frac{\coth(x)}{\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)+8a(a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{2a}-\frac{(4a+b)\coth\right.\right.$$

$$\downarrow 73$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\frac{2\left(8a^2+4ab-b^2\right)\int \frac{1}{\frac{\tanh^4(x)}{b}-\frac{a}{b}}d\sqrt{b\tanh^2(x)+a}}{b}+\frac{16a(a+b)\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}}{2a}-\frac{(4a+b)\coth(x)\sqrt{a+b\tanh\right.\right.$$

3.218. $\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx$

$$\downarrow 221$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{16a\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\frac{2\left(8a^2+4ab-b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}}{2a}-\frac{(4a+b)\coth(x)\sqrt{a+b\tanh^2}}{a}\right.\right.$$

input
```
Int[Coth[x]^5*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
(-1/2*(Coth[x]^2*Sqrt[a + b*Tanh[x]^2]) + (((-2*(8*a^2 + 4*a*b - b^2)*ArcT
anh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/Sqrt[a] + 16*a*Sqrt[a + b]*ArcTanh[Sqr
t[a + b*Tanh[x]^2]/Sqrt[a + b]])/(2*a) - ((4*a + b)*Coth[x]*Sqrt[a + b*Tan
h[x]^2])/a)/4)/2
```

3.218.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.218.4 Maple [F]

$$\int \coth(x)^5 \sqrt{a + b \tanh(x)^2}\, dx$$

input
```
int(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x)
```

3.218.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2094 vs. $2(99) = 198$.

Time $= 0.57$ (sec) , antiderivative size $= 9642$, normalized size of antiderivative $= 79.69$

$$\int \coth^5(x) \sqrt{a + b \tanh^2(x)}\, dx = \text{Too large to display}$$

input
```
integrate(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.218.6 Sympy [F]

$$\int \coth^5(x) \sqrt{a + b \tanh^2(x)}\, dx = \int \sqrt{a + b \tanh^2(x)} \coth^5(x)\, dx$$

input
```
integrate(coth(x)**5*(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tanh(x)**2)*coth(x)**5, x)
```

3.218.7 Maxima [F]

$$\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh(x)^2+a}\coth(x)^5\,dx$$

input
```
integrate(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^5, x)
```

3.218.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 947 vs. $2(99) = 198$.

Time = 1.01 (sec) , antiderivative size = 947, normalized size of antiderivative = 7.83

$$\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) +
1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log
(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b
*e^(2*x) + a + b) - sqrt(a + b))) + 1/4*(8*a^2 + 4*a*b - b^2)*arctan(-1/2*
(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b) - sqrt(a + b))/sqrt(-a))/(sqrt(-a)*a) + 1/2*((16*a^2 + 12*a*b
 + b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
2*b*e^(2*x) + a + b))^7 - (16*a^2 + 52*a*b + 7*b^2)*(sqrt(a + b)*e^(2*x) -
 sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^6*sqrt(a
 + b) - (48*a^3 - 28*a^2*b - 109*a*b^2 - 21*b^3)*(sqrt(a + b)*e^(2*x) - sq
rt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5 + (176*a^
3 + 156*a^2*b - 115*a*b^2 - 35*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a + b) + (304*a^4
 - 156*a^3*b - 317*a^2*b^2 + 130*a*b^3 + 35*b^4)*(sqrt(a + b)*e^(2*x) - sq
rt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3 - (48*a^4
 + 476*a^3*b - 379*a^2*b^2 + 94*a*b^3 + 21*b^4)*(sqrt(a + b)*e^(2*x) - sqr
t(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b
) - (272*a^5 + 140*a^4*b - 271*a^3*b^2 + 135*a^2*b^3 - 53*a*b^4 - 7*b^5...
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \coth(x)^5\sqrt{b\tanh(x)^2+a}\,dx$$

input
```
int(coth(x)^5*(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^5*(a + b*tanh(x)^2)^(1/2), x)
```

3.219 $\int \tanh^3(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx$

3.219.1 Optimal result 1615
3.219.2 Mathematica [A] (verified) 1615
3.219.3 Rubi [A] (verified) 1616
3.219.4 Maple [B] (verified) 1619
3.219.5 Fricas [B] (verification not implemented) 1619
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3.219.7 Maxima [F] 1621
3.219.8 Giac [B] (verification not implemented) 1621
3.219.9 Mupad [B] (verification not implemented) 1622

3.219.1 Optimal result

Integrand size = 17, antiderivative size = 82

$$\int \tanh^3(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = (a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - (a+b)\sqrt{a+b\tanh^2(x)} - \frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2} - \frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b}$$

output
```
(a+b)^(3/2)*arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))-(a+b)*(a+b*tanh(x)^
2)^(1/2)-1/3*(a+b*tanh(x)^2)^(3/2)-1/5*(a+b*tanh(x)^2)^(5/2)/b
```

3.219.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int \tanh^3(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = (a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{\sqrt{a+b\tanh^2(x)}\left(3a^2+20ab+15b^2+b(6a+5b)\tanh^2(x)+3b^2\tanh^4(x)\right)}{15b}$$

input
```
Integrate[Tanh[x]^3*(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
(a + b)^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Sqrt[a + b*Tan
h[x]^2]*(3*a^2 + 20*a*b + 15*b^2 + b*(6*a + 5*b)*Tanh[x]^2 + 3*b^2*Tanh[x]
^4))/(15*b)
```

3.219.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {3042, 26, 4153, 26, 354, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int i\tan(ix)^3\left(a-b\tan(ix)^2\right)^{3/2}dx$$

$$\downarrow 26$$

$$i\int \tan(ix)^3\left(a-b\tan(ix)^2\right)^{3/2}dx$$

$$\downarrow 4153$$

$$i\int -\frac{i\tanh^3(x)\left(b\tanh^2(x)+a\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\tanh^3(x)\left(a+b\tanh^2(x)\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 354$$

$$\frac{1}{2}\int \frac{\tanh^2(x)\left(b\tanh^2(x)+a\right)^{3/2}}{1-\tanh^2(x)}d\tanh^2(x)$$

$$\downarrow 90$$

$$\frac{1}{2}\left(\int \frac{\left(b\tanh^2(x)+a\right)^{3/2}}{1-\tanh^2(x)}d\tanh^2(x)-\frac{2\left(a+b\tanh^2(x)\right)^{5/2}}{5b}\right)$$

$$\downarrow 60$$

$$\frac{1}{2}\left((a+b)\int\frac{\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh^2(x)-\frac{2\left(a+b\tanh^2(x)\right)^{5/2}}{5b}-\frac{2}{3}\left(a+b\tanh^2(x)\right)^{3/2}\right)$$

$$\downarrow 60$$

$$\frac{1}{2}\left((a+b)\left((a+b)\int\frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)-2\sqrt{a+b\tanh^2(x)}\right)-\frac{2\left(a+b\tanh^2(x)\right)^{5/2}}{5b}\right.$$

$$\downarrow 73$$

$$\frac{1}{2}\left((a+b)\left(\frac{2(a+b)\int\frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}-2\sqrt{a+b\tanh^2(x)}\right)-\frac{2\left(a+b\tanh^2(x)\right)^{5/2}}{5b}-\frac{2}{3}(a+b\tanh\right.$$

$$\downarrow 221$$

$$\frac{1}{2}\left((a+b)\left(2\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-2\sqrt{a+b\tanh^2(x)}\right)-\frac{2\left(a+b\tanh^2(x)\right)^{5/2}}{5b}-\frac{2}{3}(a+b\tanh\right.$$

input
```
Int[Tanh[x]^3*(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
((-2*(a + b*Tanh[x]^2)^(3/2))/3 - (2*(a + b*Tanh[x]^2)^(5/2))/(5*b) + (a +
 b)*(2*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - 2*Sqrt[a +
 b*Tanh[x]^2]))/2
```

3.219.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.219.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(66) = 132$.

Time = 0.08 (sec) , antiderivative size = 488, normalized size of antiderivative = 5.95

method	result
derivativedivides	$-\frac{\left(a+b\tanh(x)^2\right)^{\frac{5}{2}}}{5b} - \frac{\left(b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b\right)^{\frac{3}{2}}}{6} + b\left(\frac{(2b(1+\tanh(x))-2b)\sqrt{b(1+\tanh(x))^2-2b(1+\tanh}}{4b}\right.$
default	$-\frac{\left(a+b\tanh(x)^2\right)^{\frac{5}{2}}}{5b} - \frac{\left(b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b\right)^{\frac{3}{2}}}{6} + b\left(\frac{(2b(1+\tanh(x))-2b)\sqrt{b(1+\tanh(x))^2-2b(1+\tanh}}{4b}\right.$

input
```
int(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(a+b*tanh(x)^2)^(5/2)/b-1/6*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/
2)+1/2*b*(1/4*(2*b*(1+tanh(x))-2*b)/b*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b
)^(1/2)+1/8*(4*(a+b)*b-4*b^2)/b^(3/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+t
anh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)))-1/2*(a+b)*((b*(1+tanh(x))^2-2*b*(1+
tanh(x))+a+b)^(1/2)-b^(1/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-
2*b*(1+tanh(x))+a+b)^(1/2))-(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b
)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x))))-1/6*(b*
(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)-1/2*b*(1/4*(2*b*(tanh(x)-1)+2*b)/
b*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/8*(4*(a+b)*b-4*b^2)/b^(3/2
)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)
))-1/2*(a+b)*((b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b^(1/2)*ln((b*(t
anh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))-(a+b)^(1
/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x
)-1)+a+b)^(1/2))/(tanh(x)-1)))
```

3.219.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2188 vs. $2(66) = 132$.

Time = 0.49 (sec) , antiderivative size = 4941, normalized size of antiderivative = 60.26

$$\int \tanh^3(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output

```
[1/60*(15*((a*b + b^2)*cosh(x)^10 + 10*(a*b + b^2)*cosh(x)*sinh(x)^9 + (a*
b + b^2)*sinh(x)^10 + 5*(a*b + b^2)*cosh(x)^8 + 5*(9*(a*b + b^2)*cosh(x)^2
 + a*b + b^2)*sinh(x)^8 + 40*(3*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x
))*sinh(x)^7 + 10*(a*b + b^2)*cosh(x)^6 + 10*(21*(a*b + b^2)*cosh(x)^4 + 1
4*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^6 + 4*(63*(a*b + b^2)*cosh(x)
^5 + 70*(a*b + b^2)*cosh(x)^3 + 15*(a*b + b^2)*cosh(x))*sinh(x)^5 + 10*(a*
b + b^2)*cosh(x)^4 + 10*(21*(a*b + b^2)*cosh(x)^6 + 35*(a*b + b^2)*cosh(x)
^4 + 15*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^4 + 40*(3*(a*b + b^2)*c
osh(x)^7 + 7*(a*b + b^2)*cosh(x)^5 + 5*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)
*cosh(x))*sinh(x)^3 + 5*(a*b + b^2)*cosh(x)^2 + 5*(9*(a*b + b^2)*cosh(x)^8
 + 28*(a*b + b^2)*cosh(x)^6 + 30*(a*b + b^2)*cosh(x)^4 + 12*(a*b + b^2)*co
sh(x)^2 + a*b + b^2)*sinh(x)^2 + a*b + b^2 + 10*((a*b + b^2)*cosh(x)^9 + 4
*(a*b + b^2)*cosh(x)^7 + 6*(a*b + b^2)*cosh(x)^5 + 4*(a*b + b^2)*cosh(x)^3
 + (a*b + b^2)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8
+ 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 +
 a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)
^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5
+ (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4
+ 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4
+ 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3...
```

3.219.6 Sympy [A] (verification not implemented)

Time = 10.26 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.72

$$\int \tanh^3(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx =$$

$$-a\left(\begin{cases} \dfrac{2\left(\frac{b^2\sqrt{a+b\tanh^2(x)}}{2}+\frac{b^2(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}+\frac{b\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}{6}\right)}{b^2} & \text{for } b\neq 0 \\ \sqrt{a}\left(\frac{\log\left(\tanh^2(x)-1\right)}{2}+\frac{\tanh^2(x)}{2}\right) & \text{otherwise} \end{cases}\right)$$

$$-b\left(\begin{cases} \dfrac{2\left(\frac{b^3\sqrt{a+b\tanh^2(x)}}{2}+\frac{b^3(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}+\frac{b\left(a+b\tanh^2(x)\right)^{\frac{5}{2}}}{10}+\frac{\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}\left(-\frac{ab}{2}+\frac{b^2}{2}\right)}{3}\right)}{b^3} & \text{for } b\neq 0 \\ \sqrt{a}\left(\frac{\log\left(\tanh^2(x)-1\right)}{2}+\frac{\tanh^4(x)}{4}+\frac{\tanh^2(x)}{2}\right) & \text{otherwise} \end{cases}\right)$$

input

```
integrate(tanh(x)**3*(a+b*tanh(x)**2)**(3/2),x)
```

output
```
-a*Piecewise((2*(b**2*sqrt(a + b*tanh(x)**2)/2 + b**2*(a + b)*atan(sqrt(a
+ b*tanh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a + b*tanh(x)**2)**(3/
2)/6)/b**2, Ne(b, 0)), (sqrt(a)*(log(tanh(x)**2 - 1)/2 + tanh(x)**2/2), Tr
ue)) - b*Piecewise((2*(b**3*sqrt(a + b*tanh(x)**2)/2 + b**3*(a + b)*atan(s
qrt(a + b*tanh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a + b*tanh(x)**2
)**(5/2)/10 + (a + b*tanh(x)**2)**(3/2)*(-a*b/2 + b**2/2)/3)/b**3, Ne(b, 0
)), (sqrt(a)*(log(tanh(x)**2 - 1)/2 + tanh(x)**4/4 + tanh(x)**2/2), True))
```

3.219.7 Maxima [F]

$$\int \tanh^3(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = \int \left(b\tanh(x)^2+a\right)^{\frac{3}{2}}\tanh(x)^3\,dx$$

input
```
integrate(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tanh(x)^2 + a)^(3/2)*tanh(x)^3, x)
```

3.219.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1063 vs. $2(66) = 132$.

Time $= 1.46$ (sec) , antiderivative size $= 1063$, normalized size of antiderivative $= 12.96$

$$\int \tanh^3(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*(a + b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*(a + b)^(3/2)
*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b) - sqrt(a + b))) - 1/2*(a^2 + 2*a*b + b^2)*log(abs(-(
sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 4/15*(15*(a^2 +
 4*a*b + 3*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^
(2*x) - 2*b*e^(2*x) + a + b))^9 + 15*(7*a^2 + 20*a*b + 9*b^2)*(sqrt(a + b)
*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
)^8*sqrt(a + b) + 20*(15*a^3 + 39*a^2*b + 21*a*b^2 + b^3)*(sqrt(a + b)*e^(
2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^7
+ 20*(21*a^3 + 21*a^2*b - 57*a*b^2 - 65*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a
*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^6*sqrt(a + b) +
 2*(105*a^4 - 210*a^3*b - 1860*a^2*b^2 - 1590*a*b^3 + 19*b^4)*(sqrt(a + b)
*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
)^5 - 10*(21*a^4 + 126*a^3*b + 288*a^2*b^2 - 390*a*b^3 - 349*b^4)*(sqrt(a
+ b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
+ b))^4*sqrt(a + b) - 20*(21*a^5 + 63*a^4*b - 18*a^3*b^2 - 378*a^2*b^3 - 2
35*a*b^4 + 19*b^5)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a
*e^(2*x) - 2*b*e^(2*x) + a + b))^3 - 20*(15*a^5 + 21*a^4*b - 126*a^3*b^...
```

3.219.9 Mupad [B] (verification not implemented)

Time = 9.97 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.37

$$\int \tanh^3(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = -\frac{\left(b\tanh(x)^2+a\right)^{5/2}}{5\,b}$$
$$-\left(\frac{a+b}{3\,b}-\frac{a}{3\,b}\right)\left(b\tanh(x)^2+a\right)^{3/2}$$
$$-(a+b)\left(\frac{a+b}{b}-\frac{a}{b}\right)\sqrt{b\tanh\left(x\right)^2+a}-\mathrm{atan}\left(\frac{(a+b)^{3/2}\sqrt{b\tanh\left(x\right)^2+a}\,1\mathrm{i}}{a^2+2\,a\,b+b^2}\right)(a+b)^{3/2}\,1\mathrm{i}$$

input
```
int(tanh(x)^3*(a + b*tanh(x)^2)^(3/2),x)
```

output
```
- (a + b*tanh(x)^2)^(5/2)/(5*b) - ((a + b)/(3*b) - a/(3*b))*(a + b*tanh(x)
^2)^(3/2) - atan(((a + b)^(3/2)*(a + b*tanh(x)^2)^(1/2)*1i)/(2*a*b + a^2 +
 b^2))*(a + b)^(3/2)*1i - (a + b)*((a + b)/b - a/b)*(a + b*tanh(x)^2)^(1/2
)
```

3.220 $\int \tanh^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx$

3.220.1 Optimal result 1623
3.220.2 Mathematica [C] (verified) 1623
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3.220.5 Fricas [B] (verification not implemented) 1630
3.220.6 Sympy [F] 1630
3.220.7 Maxima [F] 1631
3.220.8 Giac [B] (verification not implemented) 1631
3.220.9 Mupad [F(-1)] 1632

3.220.1 Optimal result

Integrand size = 17, antiderivative size = 123

$$\int \tanh^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = -\frac{\left(3a^2+12ab+8b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{8\sqrt{b}}$$
$$+(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$
$$-\frac{1}{8}(5a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}-\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

output
```
(a+b)^(3/2)*arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))-1/8*(3*a^2+
12*a*b+8*b^2)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(1/2)-1/8*(
5*a+4*b)*(a+b*tanh(x)^2)^(1/2)*tanh(x)-1/4*b*(a+b*tanh(x)^2)^(1/2)*tanh(x)
^3
```

3.220.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 6.25 (sec) , antiderivative size = 584, normalized size of antiderivative = 4.75

$$\int \tanh^2(x)\,(a + b\tanh^2(x))^{3/2}\,dx = \frac{1}{4}\left(-\frac{b(a^2-4ab-4b^2)\sqrt{\frac{a-b+(a+b)\cosh(2x)}{1+\cosh(2x)}}\sqrt{-\frac{a\coth^2(x)}{b}}\sqrt{-\frac{a(1+\cosh(2x))\operatorname{csch}^2(x)}{b}}\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{sech}^2(x)}{b}}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-\frac{a}{b}}}{\tanh(x)}\right),\frac{a+b}{a}\right)}{a(a-b+a\cosh(2x)+b\cosh(2x))} - \frac{4ib(4a^2+8ab+4b^2)\sqrt{1+\cosh(2x)}\sqrt{\frac{a-b+(a+b)\cosh(2x)}{1+\cosh(2x)}}\left(-\frac{i\sqrt{-\frac{a\coth^2(x)}{b}}\sqrt{-\frac{a(1+\cosh(2x))\operatorname{csch}^2(x)}{b}}\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{sech}^2(x)}{b}}\left(\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-\frac{a}{b}}}{\tanh(x)}\right),\frac{a+b}{a}\right)-\operatorname{EllipticPi}\left(\frac{a+b}{b},i\operatorname{arcsinh}\left(\frac{\sqrt{-\frac{a}{b}}}{\tanh(x)}\right),\frac{a+b}{a}\right)\right)}{4a\sqrt{1+\cosh(2x)}\sqrt{\frac{a-b+(a+b)\cosh(2x)}{1+\cosh(2x)}}\sqrt{-\frac{a}{b}}}\right)}{a-b+a\cosh(2x)+b\cosh(2x)} + \sqrt{\frac{a-b+a\cosh(2x)+b\cosh(2x)}{1+\cosh(2x)}}\left(\frac{1}{8}\operatorname{sech}(x)(-5a\sinh(x)-6b\sinh(x)) + \frac{1}{4}b\operatorname{sech}^2(x)\tanh(x)\right)\right)$$

input
```
Integrate[Tanh[x]^2*(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
(-((b*(a^2 - 4*a*b - 4*b^2)*Sqrt[(a - b + (a + b)*Cosh[2*x])/(1 + Cosh[2*x
])]*Sqrt[-((a*Coth[x]^2)/b)]*Sqrt[-((a*(1 + Cosh[2*x])*Csch[x]^2)/b)]*Sqrt
[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Csch[2*x]*EllipticF[ArcSin[Sqr
t[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]*Sinh[x]^4)/(a*(a
 - b + (a + b)*Cosh[2*x]))) - ((4*I)*b*(4*a^2 + 8*a*b + 4*b^2)*Sqrt[1 + Co
sh[2*x]]*Sqrt[(a - b + (a + b)*Cosh[2*x])/(1 + Cosh[2*x])]*(((-1/4*I)*Sqrt
[-((a*Coth[x]^2)/b)]*Sqrt[-((a*(1 + Cosh[2*x])*Csch[x]^2)/b)]*Sqrt[((a - b
 + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Csch[2*x]*EllipticF[ArcSin[Sqrt[((a -
b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]*Sinh[x]^4)/(a*Sqrt[1 + C
osh[2*x]]*Sqrt[a - b + (a + b)*Cosh[2*x]]) + ((I/2)*Sqrt[-((a*Coth[x]^2)/b
)]*Sqrt[-((a*(1 + Cosh[2*x])*Csch[x]^2)/b)]*Sqrt[((a - b + (a + b)*Cosh[2*
x])*Csch[x]^2)/b]*Csch[2*x]*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a
 + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]*Sinh[x]^4)/((a + b)*Sqrt[1 + C
osh[2*x]]*Sqrt[a - b + (a + b)*Cosh[2*x]])))/Sqrt[a - b + (a + b)*Cosh[2*x
]])/4 + Sqrt[(a - b + a*Cosh[2*x] + b*Cosh[2*x])/(1 + Cosh[2*x])]*((Sech[x
]*(-5*a*Sinh[x] - 6*b*Sinh[x]))/8 + (b*Sech[x]^2*Tanh[x])/4)
```

3.220.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$, Rules used = {3042, 25, 4153, 25, 379, 25, 444, 27, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(x) \left(a + b \tanh^2(x)\right)^{3/2} dx$$

$$\downarrow 3042$$

$$\int -\tan(ix)^2 \left(a - b \tan(ix)^2\right)^{3/2} dx$$

$$\downarrow 25$$

$$-\int \tan(ix)^2 \left(a - b \tan(ix)^2\right)^{3/2} dx$$

$$\downarrow 4153$$

$$-\int -\frac{\tanh^2(x) \left(b \tanh^2(x) + a\right)^{3/2}}{1 - \tanh^2(x)} d\tanh(x)$$

$$\downarrow 25$$

$$\int \frac{\tanh^2(x)\left(a+b\tanh^2(x)\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 379$$

$$-\frac{1}{4}\int -\frac{\tanh^2(x)\left(b(5a+4b)\tanh^2(x)+a(4a+3b)\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 25$$

$$\frac{1}{4}\int \frac{\tanh^2(x)\left(b(5a+4b)\tanh^2(x)+a(4a+3b)\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 444$$

$$\frac{1}{4}\left(\frac{\int \frac{b\left(\left(3a^2+12ba+8b^2\right)\tanh^2(x)+a(5a+4b)\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{2b}-\frac{1}{2}(5a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}\right)-$$
$$\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 27$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{\left(3a^2+12ba+8b^2\right)\tanh^2(x)+a(5a+4b)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\frac{1}{2}(5a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}\right)-$$
$$\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 398$$

$$\frac{1}{4}\left(\frac{1}{2}\left(8(a+b)^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\left(3a^2+12ab+8b^2\right)\int \frac{1}{\sqrt{b\tanh^2(x)+a}}d\tanh(x)\right.\right.$$
$$\left.\left.\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}\right.\right.$$

$$\downarrow 224$$

$$\frac{1}{4}\left(\frac{1}{2}\left(8(a+b)^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\left(3a^2+12ab+8b^2\right)\int \frac{1}{1-\frac{b\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh}{\sqrt{b\tanh^2}}\right.\right.$$
$$\left.\left.\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}\right.\right.$$

$$\downarrow 219$$

$$\frac{1}{4}\left(\frac{1}{2}\left(8(a+b)^2\int\frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\frac{\left(3a^2+12ab+8b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}\right)-$$

$$\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 291$$

$$\frac{1}{4}\left(\frac{1}{2}\left(8(a+b)^2\int\frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}-\frac{\left(3a^2+12ab+8b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}\right)-\frac{1}{2}(5a-$$

$$\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 219$$

$$\frac{1}{4}\left(\frac{1}{2}\left(8(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{\left(3a^2+12ab+8b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}\right)-\frac{1}{2}(5a+4b)\tan$$

$$\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

input
```
Int[Tanh[x]^2*(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
-1/4*(b*Tanh[x]^3*Sqrt[a + b*Tanh[x]^2]) + ((-(((3*a^2 + 12*a*b + 8*b^2)*A
rcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[b]) + 8*(a + b)^(3/2
)*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/2 - ((5*a + 4*b)*T
anh[x]*Sqrt[a + b*Tanh[x]^2])/2)/4
```

3.220.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 379
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q -
1)/(b*e*(m + 2*(p + q) + 1))), x] + Simp[1/(b*(m + 2*(p + q) + 1))   Int[(e
*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*((b*c - a*d)*(m + 1) + b*c*2
*(p + q)) + (d*(b*c - a*d)*(m + 1) + d*2*(q - 1)*(b*c - a*d) + b*c*d*2*(p +
 q))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0
] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.220.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(101) = 202$.

Time = 0.08 (sec) , antiderivative size = 529, normalized size of antiderivative = 4.30

method	result
derivativedivides	$-\frac{\tanh(x)\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{4} - \frac{3a\left(\frac{\sqrt{a+b\tanh(x)^2}\,\tanh(x)}{2}+\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{2\sqrt{b}}\right)}{4} - \frac{\left(b(\tanh(x)-1)^2+$
default	$-\frac{\tanh(x)\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{4} - \frac{3a\left(\frac{\sqrt{a+b\tanh(x)^2}\,\tanh(x)}{2}+\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{2\sqrt{b}}\right)}{4} - \frac{\left(b(\tanh(x)-1)^2+$

input
```
int(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*tanh(x)*(a+b*tanh(x)^2)^(3/2)-3/4*a*(1/2*(a+b*tanh(x)^2)^(1/2)*tanh(x
)+1/2*a/b^(1/2)*ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2)))-1/6*(b*(tanh(x)
-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)-1/2*b*(1/4*(2*b*(tanh(x)-1)+2*b)/b*(b*(ta
nh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/8*(4*(a+b)*b-4*b^2)/b^(3/2)*ln((b*
(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)))-1/2*(
a+b)*((b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b^(1/2)*ln((b*(tanh(x)-1
)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))-(a+b)^(1/2)*ln((
2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b
)^(1/2))/(tanh(x)-1)))+1/6*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)-1/2
*b*(1/4*(2*b*(1+tanh(x))-2*b)/b*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2
)+1/8*(4*(a+b)*b-4*b^2)/b^(3/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x)
)^2-2*b*(1+tanh(x))+a+b)^(1/2)))+1/2*(a+b)*((b*(1+tanh(x))^2-2*b*(1+tanh(x
))+a+b)^(1/2)-b^(1/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+tanh(x))^2-2*b*(1
+tanh(x))+a+b)^(1/2))-(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2
)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x))))
```

3.220.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2167 vs. $2(101) = 202$.

Time $= 0.59$ (sec) , antiderivative size $= 10046$, normalized size of antiderivative $= 81.67$

$$\int \tanh^2(x)\left(a + b\tanh^2(x)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.220.6 Sympy [F]

$$\int \tanh^2(x)\left(a + b\tanh^2(x)\right)^{3/2}\,dx = \int \left(a + b\tanh^2(x)\right)^{\frac{3}{2}}\tanh^2(x)\,dx$$

input
```
integrate(tanh(x)**2*(a+b*tanh(x)**2)**(3/2),x)
```

output
```
Integral((a + b*tanh(x)**2)**(3/2)*tanh(x)**2, x)
```

3.220.7 Maxima [F]

$$\int \tanh^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = \int \left(b\tanh(x)^2+a\right)^{\frac{3}{2}}\tanh(x)^2\,dx$$

input
```
integrate(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tanh(x)^2 + a)^(3/2)*tanh(x)^2, x)
```

3.220.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 949 vs. $2(101) = 202$.

Time = 1.21 (sec) , antiderivative size = 949, normalized size of antiderivative = 7.72

$$\int \tanh^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(a + b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*
x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*(a + b)^(3/2
)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b) - sqrt(a + b))) - 1/4*(3*a^2 + 12*a*b + 8*b^2)*arct
an(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*(a^2 + 2*a*b
+ b^2)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^
(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b)
- 1/2*((5*a^2 + 20*a*b + 16*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b
*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^7 + (35*a^2 + 76*a*b + 16*b
^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*
e^(2*x) + a + b))^6*sqrt(a + b) + (105*a^3 + 153*a^2*b - 28*a*b^2 - 48*b^3
)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^
(2*x) + a + b))^5 + (175*a^3 - 25*a^2*b - 260*a*b^2 - 176*b^3)*(sqrt(a + b
)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b
))^4*sqrt(a + b) + (175*a^4 - 110*a^3*b - 417*a^2*b^2 + 60*a*b^3 + 304*b^4
)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^
(2*x) + a + b))^3 + (105*a^4 - 210*a^3*b - 55*a^2*b^2 + 484*a*b^3 + 48*b^4
)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^
(2*x) + a + b))^2*sqrt(a + b) + (35*a^5 - 79*a^4*b + 53*a^3*b^2 + 195*a...
```

3.220.9 Mupad [F(-1)]

Timed out.

$$\int \tanh^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = \int \tanh(x)^2\left(b\tanh(x)^2+a\right)^{3/2}\,dx$$

input
```
int(tanh(x)^2*(a + b*tanh(x)^2)^(3/2),x)
```

output
```
int(tanh(x)^2*(a + b*tanh(x)^2)^(3/2), x)
```

3.221 $\int \tanh(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx$

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3.221.1 Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \tanh(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = (a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - (a+b)\sqrt{a+b\tanh^2(x)} - \frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}$$

output
```
(a+b)^(3/2)*arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))-(a+b)*(a+b*tanh(x)^
2)^(1/2)-1/3*(a+b*tanh(x)^2)^(3/2)
```

3.221.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \tanh(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = (a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{1}{3}\sqrt{a+b\tanh^2(x)}\left(4a+3b+b\tanh^2(x)\right)$$

input
```
Integrate[Tanh[x]*(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
(a + b)^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Sqrt[a + b*Tan
h[x]^2]*(4*a + 3*b + b*Tanh[x]^2))/3
```

3.221.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 26, 4153, 26, 353, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(x) \left(a + b \tanh^2(x)\right)^{3/2} dx$$

$$\downarrow 3042$$

$$\int -i \tan(ix) \left(a - b \tan(ix)^2\right)^{3/2} dx$$

$$\downarrow 26$$

$$-i \int \tan(ix) \left(a - b \tan(ix)^2\right)^{3/2} dx$$

$$\downarrow 4153$$

$$-i \int \frac{i \tanh(x) \left(b \tanh^2(x) + a\right)^{3/2}}{1 - \tanh^2(x)} d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\tanh(x) \left(a + b \tanh^2(x)\right)^{3/2}}{1 - \tanh^2(x)} d\tanh(x)$$

$$\downarrow 353$$

$$\frac{1}{2} \int \frac{\left(b \tanh^2(x) + a\right)^{3/2}}{1 - \tanh^2(x)} d\tanh^2(x)$$

$$\downarrow 60$$

$$\frac{1}{2} \left((a+b) \int \frac{\sqrt{b \tanh^2(x) + a}}{1 - \tanh^2(x)} d\tanh^2(x) - \frac{2}{3} \left(a + b \tanh^2(x)\right)^{3/2} \right)$$

$$\downarrow 60$$

$$\frac{1}{2} \left((a+b) \left((a+b) \int \frac{1}{\left(1 - \tanh^2(x)\right) \sqrt{b \tanh^2(x) + a}} d\tanh^2(x) - 2\sqrt{a + b \tanh^2(x)} \right) - \frac{2}{3} \left(a + b \tanh^2(x)\right)^{3/2} \right.$$

$$\downarrow 73$$

$$\frac{1}{2}\left((a+b)\left(\frac{2(a+b)\int\frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}-2\sqrt{a+b\tanh^2(x)}\right)-\frac{2}{3}\left(a+b\tanh^2(x)\right)^{3/2}\right)$$

$$\downarrow 221$$

$$\frac{1}{2}\left((a+b)\left(2\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-2\sqrt{a+b\tanh^2(x)}\right)-\frac{2}{3}\left(a+b\tanh^2(x)\right)^{3/2}\right)$$

input
```
Int[Tanh[x]*(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
((-2*(a + b*Tanh[x]^2)^(3/2))/3 + (a + b)*(2*Sqrt[a + b]*ArcTanh[Sqrt[a +
b*Tanh[x]^2]/Sqrt[a + b]] - 2*Sqrt[a + b*Tanh[x]^2]))/2
```

3.221.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.221.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(51) = 102$.

Time $= 0.06$ (sec) , antiderivative size $= 473$, normalized size of antiderivative $= 7.51$

method	result
derivativedivides	$-\frac{\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}{6} - \frac{b\left(\frac{(2b(\tanh(x)-1)+2b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{4b}+\frac{\left(4(a+b)b-4\right.}{2}\right.}$
default	$-\frac{\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}{6} - \frac{b\left(\frac{(2b(\tanh(x)-1)+2b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{4b}+\frac{\left(4(a+b)b-4\right.}{2}\right.}$

input
```
int(tanh(x)*(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)-1/2*b*(1/4*(2*b*(tanh(x)-
1)+2*b)/b*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/8*(4*(a+b)*b-4*b^2
)/b^(3/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+
b)^(1/2)))-1/2*(a+b)*((b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b^(1/2)*
ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))-
(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b
*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1)))-1/6*(b*(1+tanh(x))^2-2*b*(1+tanh(x)
)+a+b)^(3/2)+1/2*b*(1/4*(2*b*(1+tanh(x))-2*b)/b*(b*(1+tanh(x))^2-2*b*(1+ta
nh(x))+a+b)^(1/2)+1/8*(4*(a+b)*b-4*b^2)/b^(3/2)*ln((b*(1+tanh(x))-b)/b^(1/
2)+(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)))-1/2*(a+b)*((b*(1+tanh(x))
^2-2*b*(1+tanh(x))+a+b)^(1/2)-b^(1/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+t
anh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))-(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(
x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x))
))
```

3.221.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(51) = 102$.

Time $= 0.37$ (sec) , antiderivative size $= 2385$, normalized size of antiderivative $= 37.86$

$$\int \tanh(x) \left(a + b \tanh^2(x)\right)^{3/2} \, dx = \text{Too large to display}$$

input
```
integrate(tanh(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output

```
[1/12*(3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x
)^6 + 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 + a + b)*sinh(x)^4 + 4*
(5*(a + b)*cosh(x)^3 + 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2
+ 3*(5*(a + b)*cosh(x)^4 + 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a
+ b)*cosh(x)^5 + 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*s
qrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^
7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2
*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3
 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*
cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 +
30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 +
10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*si
nh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x
)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3
 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2
+ sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2
*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*
a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)
^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2
 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)...
```

3.221.6 Sympy [A] (verification not implemented)

Time = 6.97 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.62

$$\int \tanh(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx =$$

$$-a\left(\begin{cases} \dfrac{2\left(\frac{b\sqrt{a+b\tanh^2(x)}}{2}+\frac{b(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}\right)}{b} & \text{for } b\neq 0 \\ \dfrac{\sqrt{a}\log\left(2\tanh^2(x)-2\right)}{2} & \text{otherwise} \end{cases}\right)$$

$$-b\left(\begin{cases} \dfrac{2\left(\frac{b^2\sqrt{a+b\tanh^2(x)}}{2}+\frac{b^2(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}+\frac{b\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}{6}\right)}{b^2} & \text{for } b\neq 0 \\ \sqrt{a}\left(\dfrac{\log\left(\tanh^2(x)-1\right)}{2}+\dfrac{\tanh^2(x)}{2}\right) & \text{otherwise} \end{cases}\right)$$

input

```
integrate(tanh(x)*(a+b*tanh(x)**2)**(3/2),x)
```

output
```
-a*Piecewise((2*(b*sqrt(a + b*tanh(x)**2)/2 + b*(a + b)*atan(sqrt(a + b*ta
nh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)))/b, Ne(b, 0)), (sqrt(a)*log(2*tan
h(x)**2 - 2)/2, True)) - b*Piecewise((2*(b**2*sqrt(a + b*tanh(x)**2)/2 + b
**2*(a + b)*atan(sqrt(a + b*tanh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)) + b
*(a + b*tanh(x)**2)**(3/2)/6)/b**2, Ne(b, 0)), (sqrt(a)*(log(tanh(x)**2 -
1)/2 + tanh(x)**2/2), True))
```

3.221.7 Maxima [F]

$$\int \tanh(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = \int \left(b\tanh(x)^2+a\right)^{\frac{3}{2}}\tanh(x)\,dx$$

input
```
integrate(tanh(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tanh(x)^2 + a)^(3/2)*tanh(x), x)
```

3.221.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(51) = 102$.

Time $= 1.02$ (sec) , antiderivative size $= 662$, normalized size of antiderivative $= 10.51$

$$\int \tanh(x)\left(a + b\tanh^2(x)\right)^{3/2}\,dx = \frac{1}{2}(a+b)^{\frac{3}{2}}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+\right.\right.$$
$$-\frac{1}{2}(a+b)^{\frac{3}{2}}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)$$
$$-\frac{(a^2+2ab+b^2)\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a\right.}{2\sqrt{a+b}}$$
$$-\frac{8\left(3(ab+b^2)\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^5+3(3ab+b^2)\left(\sqrt{a+b}e^{(2x)}\right.\right.}{}$$

input
```
integrate(tanh(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*(a + b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*(a + b)^(3/2)
*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b) - sqrt(a + b))) - 1/2*(a^2 + 2*a*b + b^2)*log(abs(-(
sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 8/3*(3*(a*b + b
^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*
e^(2*x) + a + b))^5 + 3*(3*a*b + b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x
) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a + b) + 2*(3*a
^2*b - 6*a*b^2 - 5*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3 - 6*(a^2*b + 4*a*b^2 + 3*b^3)*(sqr
t(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
+ a + b))^2*sqrt(a + b) - 3*(3*a^3*b + a^2*b^2 - 15*a*b^3 - 13*b^4)*(sqrt(
a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
a + b)) - (3*a^3*b - 9*a^2*b^2 + 5*a*b^3 + 17*b^4)*sqrt(a + b))/((sqrt(a +
 b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
 b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
 - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)^3
```

3.221.9 Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \tanh(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = \operatorname{atanh}\left(\frac{(a+b)^{3/2}\sqrt{b\tanh(x)^2+a}}{a^2+2ab+b^2}\right)(a+b)^{3/2} - (a+b)\sqrt{b\tanh(x)^2+a} - \frac{\left(b\tanh(x)^2+a\right)^{3/2}}{3}$$

input
```
int(tanh(x)*(a + b*tanh(x)^2)^(3/2),x)
```

output
```
atanh(((a + b)^(3/2)*(a + b*tanh(x)^2)^(1/2))/(2*a*b + a^2 + b^2))*(a + b)
^(3/2) - (a + b)*(a + b*tanh(x)^2)^(1/2) - (a + b*tanh(x)^2)^(3/2)/3
```

3.222 $\int \left(a + b\tanh^2(x)\right)^{3/2}\, dx$

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3.222.1 Optimal result

Integrand size = 12, antiderivative size = 88

$$\int \left(a + b\tanh^2(x)\right)^{3/2}\, dx = -\frac{1}{2}\sqrt{b}(3a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$
$$+ (a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)}$$

output
```
(a+b)^(3/2)*arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))-1/2*(3*a+2*
b)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))*b^(1/2)-1/2*b*(a+b*tanh(
x)^2)^(1/2)*tanh(x)
```

3.222.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.25

$$\int (a$$
$$+b\tanh^2(x)\Big)^{3/2}\, dx = \frac{1}{2}\left(-2(-a-b)^{3/2}\arctan\left(\frac{\sqrt{b}\operatorname{sech}^2(x)+\tanh(x)\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)+\sqrt{b}(3a+2b)\log\right.$$

input
```
Integrate[(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
(-2*(-a - b)^(3/2)*ArcTan[(Sqrt[b]*Sech[x]^2 + Tanh[x]*Sqrt[a + b*Tanh[x]^
2])/Sqrt[-a - b]] + Sqrt[b]*(3*a + 2*b)*Log[-(Sqrt[b]*Tanh[x]) + Sqrt[a +
b*Tanh[x]^2]] - b*Tanh[x]*Sqrt[a + b*Tanh[x]^2])/2
```

3.222.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 4144, 318, 25, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a + b\tanh^2(x)\right)^{3/2}\, dx$$

$$\downarrow 3042$$

$$\int \left(a - b\tan(ix)^2\right)^{3/2} dx$$

$$\downarrow 4144$$

$$\int \frac{\left(a + b\tanh^2(x)\right)^{3/2}}{1 - \tanh^2(x)} d\tanh(x)$$

$$\downarrow 318$$

$$-\frac{1}{2}\int -\frac{b(3a+2b)\tanh^2(x) + a(2a+b)}{\left(1 - \tanh^2(x)\right)\sqrt{b\tanh^2(x) + a}} d\tanh(x) - \frac{1}{2} b\tanh(x)\sqrt{a + b\tanh^2(x)}$$

$$\downarrow 25$$

$$\frac{1}{2}\int \frac{b(3a+2b)\tanh^2(x) + a(2a+b)}{\left(1 - \tanh^2(x)\right)\sqrt{b\tanh^2(x) + a}} d\tanh(x) - \frac{1}{2} b\tanh(x)\sqrt{a + b\tanh^2(x)}$$

$$\downarrow 398$$

$$\frac{1}{2}\left(2(a+b)^2 \int \frac{1}{\left(1 - \tanh^2(x)\right)\sqrt{b\tanh^2(x) + a}} d\tanh(x) - b(3a+2b)\int \frac{1}{\sqrt{b\tanh^2(x) + a}} d\tanh(x)\right) - \frac{1}{2} b\tanh(x)\sqrt{a + b\tanh^2(x)}$$

$$\downarrow 224$$

$$\frac{1}{2}\left(2(a+b)^2\int\frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-b(3a+2b)\int\frac{1}{1-\frac{b\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}\right)-\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 219$$

$$\frac{1}{2}\left(2(a+b)^2\int\frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\right)-\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 291$$

$$\frac{1}{2}\left(2(a+b)^2\int\frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}-\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\right)-\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 219$$

$$\frac{1}{2}\left(2(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\right)-\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)}$$

input
```
Int[(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
(-(Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]) +
 2*(a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/2 -
 (b*Tanh[x]*Sqrt[a + b*Tanh[x]^2])/2
```

3.222.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.222.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(70) = 140$.

Time = 0.10 (sec) , antiderivative size = 473, normalized size of antiderivative = 5.38

method	result
derivativedivides	$-\frac{\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}{6} - \frac{b\left(\frac{(2b(\tanh(x)-1)+2b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{4b}+\frac{\left(4(a+b)b-4\right.}{2}\right.$
default	$-\frac{\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}{6} - \frac{b\left(\frac{(2b(\tanh(x)-1)+2b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{4b}+\frac{\left(4(a+b)b-4\right.}{2}\right.$

input
```
int((a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)-1/2*b*(1/4*(2*b*(tanh(x)-
1)+2*b)/b*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/8*(4*(a+b)*b-4*b^2
)/b^(3/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+
b)^(1/2)))-1/2*(a+b)*((b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b^(1/2)*
ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))-
(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b
*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1)))+1/6*(b*(1+tanh(x))^2-2*b*(1+tanh(x)
)+a+b)^(3/2)-1/2*b*(1/4*(2*b*(1+tanh(x))-2*b)/b*(b*(1+tanh(x))^2-2*b*(1+ta
nh(x))+a+b)^(1/2)+1/8*(4*(a+b)*b-4*b^2)/b^(3/2)*ln((b*(1+tanh(x))-b)/b^(1/
2)+(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)))+1/2*(a+b)*((b*(1+tanh(x))
^2-2*b*(1+tanh(x))+a+b)^(1/2)-b^(1/2)*ln((b*(1+tanh(x))-b)/b^(1/2)+(b*(1+t
anh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))-(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(
x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x))
))
```

3.222.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 865 vs. $2(70) = 140$.

Time = 0.43 (sec) , antiderivative size = 4841, normalized size of antiderivative = 55.01

$$\int \left(a + b\tanh^2(x)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output

```
[1/4*(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4
 + 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 4*((a
 + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(-((a*b
^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*si
nh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3
)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)
*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b
^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*c
osh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*c
osh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^
2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 +
 b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3
*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(
x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^
2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)
^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^
2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^
5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*
sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b...
```

3.222.6 Sympy [F]

$$\int \left(a + b \tanh^2(x)\right)^{3/2} \, dx = \int \left(a + b \tanh^2(x)\right)^{\frac{3}{2}} \, dx$$

input

```
integrate((a+b*tanh(x)**2)**(3/2),x)
```

output

```
Integral((a + b*tanh(x)**2)**(3/2), x)
```

3.222.7 Maxima [F]

$$\int \left(a + b \tanh^2(x)\right)^{3/2}\, dx = \int \left(b \tanh(x)^2 + a\right)^{\frac{3}{2}}\, dx$$

input
```
integrate((a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tanh(x)^2 + a)^(3/2), x)
```

3.222.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(70) = 140$.

Time = 0.81 (sec) , antiderivative size = 584, normalized size of antiderivative = 6.64

$$\int \left(a + b \tanh^2(x)\right)^{3/2}\, dx =$$
$$-\frac{1}{2}(a+b)^{\frac{3}{2}} \log\left(\left|-\sqrt{a+b}e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b}\right|\right)$$
$$+\frac{1}{2}(a+b)^{\frac{3}{2}} \log\left(\left|-\sqrt{a+b}e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}\right|\right)$$
$$-\frac{(3ab + 2b^2)\arctan\left(-\frac{\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}}$$
$$-\frac{(a^2 + 2ab + b^2)\log\left(\left|-\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)(a+b) - \sqrt{a+b}(a\right.}{2\sqrt{a+b}}$$
$$-\frac{2\left((ab + 2b^2)\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)^3 + (3ab - 2b^2)\left(\sqrt{a+b}e^{(2x)}\right.\right.}{\left(\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(}\right.\right.}$$

input
```
integrate((a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(a + b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*
x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*(a + b)^(3/2
)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b) - sqrt(a + b))) - (3*a*b + 2*b^2)*arctan(-1/2*(sqrt
(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
 a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*(a^2 + 2*a*b + b^2)*log(ab
s(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e
^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 2*((a*b + 2
*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*
b*e^(2*x) + a + b))^3 + (3*a*b - 2*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4
*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) + (3*a
^2*b - 3*a*b^2 - 2*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) + (a^2*b - a*b^2 + 2*b^3)*sqrt(a + b
))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*
e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)^2
```

3.222.9 Mupad [F(-1)]

Timed out.

$$\int \left(a + b \tanh^2(x)\right)^{3/2} \, dx = \int \left(b \tanh(x)^2 + a\right)^{3/2} \, dx$$

input
```
int((a + b*tanh(x)^2)^(3/2),x)
```

output
```
int((a + b*tanh(x)^2)^(3/2), x)
```

3.223 $\int \coth(x) \left(a + b \tanh^2(x)\right)^{3/2} dx$

3.223.1 Optimal result 1649
3.223.2 Mathematica [A] (verified) 1649
3.223.3 Rubi [A] (verified) 1650
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3.223.8 Giac [F] 1654
3.223.9 Mupad [F(-1)] 1655

3.223.1 Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \coth(x) \left(a + b \tanh^2(x)\right)^{3/2} dx = -a^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a}}\right) + (a+b)^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a+b}}\right) - b\sqrt{a + b \tanh^2(x)}$$

output

```
-a^(3/2)*arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))+(a+b)^(3/2)*arctanh((a+b*t
anh(x)^2)^(1/2)/(a+b)^(1/2))-b*(a+b*tanh(x)^2)^(1/2)
```

3.223.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \coth(x) \left(a + b \tanh^2(x)\right)^{3/2} dx = -a^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a}}\right) + (a+b)^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a+b}}\right) - b\sqrt{a + b \tanh^2(x)}$$

input
```
Integrate[Coth[x]*(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
-(a^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]) + (a + b)^(3/2)*ArcTanh[
Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - b*Sqrt[a + b*Tanh[x]^2]
```

3.223.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 26, 4153, 26, 354, 95, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(x) \left(a + b \tanh^2(x)\right)^{3/2} dx$$

$$\downarrow 3042$$

$$\int \frac{i\left(a - b \tan(ix)^2\right)^{3/2}}{\tan(ix)} dx$$

$$\downarrow 26$$

$$i \int \frac{\left(a - b \tan(ix)^2\right)^{3/2}}{\tan(ix)} dx$$

$$\downarrow 4153$$

$$i \int -\frac{i \coth(x) \left(b \tanh^2(x) + a\right)^{3/2}}{1 - \tanh^2(x)} d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\coth(x) \left(a + b \tanh^2(x)\right)^{3/2}}{1 - \tanh^2(x)} d\tanh(x)$$

$$\downarrow 354$$

$$\frac{1}{2} \int \frac{\coth(x) \left(b \tanh^2(x) + a\right)^{3/2}}{1 - \tanh^2(x)} d\tanh^2(x)$$

$$\downarrow 95$$

$$\frac{1}{2}\left(-\int -\frac{\coth(x)\left(a^2+b(2a+b)\tanh^2(x)\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)-2b\sqrt{a+b\tanh^2(x)}\right)$$

$$\downarrow 25$$

$$\frac{1}{2}\left(\int \frac{\coth(x)\left(a^2+b(2a+b)\tanh^2(x)\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)-2b\sqrt{a+b\tanh^2(x)}\right)$$

$$\downarrow 174$$

$$\frac{1}{2}\left(a^2\int \frac{\coth(x)}{\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)+(a+b)^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)-2b\sqrt{a+b\tanh^2(x)}\right)$$

$$\downarrow 73$$

$$\frac{1}{2}\left(\frac{2a^2\int \frac{1}{\frac{\tanh^4(x)}{b}-\frac{a}{b}}d\sqrt{b\tanh^2(x)+a}}{b}+\frac{2(a+b)^2\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}-2b\sqrt{a+b\tanh^2(x)}\right)$$

$$\downarrow 221$$

$$\frac{1}{2}\left(-2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)+2(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-2b\sqrt{a+b\tanh^2(x)}\right)$$

input
```
Int[Coth[x]*(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
(-2*a^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]] + 2*(a + b)^(3/2)*ArcTa
nh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - 2*b*Sqrt[a + b*Tanh[x]^2])/2
```

3.223.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 95
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Simp[1/(b*d)   Int[(b
*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/((a +
b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.223.4 Maple [F]

$$\int \coth(x)\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}\,dx$$

input
```
int(coth(x)*(a+b*tanh(x)^2)^(3/2),x)
```

output
```
int(coth(x)*(a+b*tanh(x)^2)^(3/2),x)
```

3.223.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(57) = 114$.

Time $= 0.42$ (sec) , antiderivative size $= 4039$, normalized size of antiderivative $= 56.89$

$$\int \coth(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 +
 a + b)*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)
*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*
a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*
cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^
2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2
 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cos
h(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*co
sh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^
3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^
4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*s
inh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^
6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(
x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^
2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*
a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*c
osh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*
cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*
cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*co...
```

3.223.6 Sympy [F]

$$\int \coth(x) \left(a + b \tanh^2(x)\right)^{3/2} \, dx = \int \left(a + b \tanh^2(x)\right)^{\frac{3}{2}} \coth(x) \, dx$$

input
```
integrate(coth(x)*(a+b*tanh(x)**2)**(3/2),x)
```

output
```
Integral((a + b*tanh(x)**2)**(3/2)*coth(x), x)
```

3.223.7 Maxima [F]

$$\int \coth(x) \left(a + b \tanh^2(x)\right)^{3/2} \, dx = \int \left(b \tanh(x)^2 + a\right)^{\frac{3}{2}} \coth(x) \, dx$$

input
```
integrate(coth(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tanh(x)^2 + a)^(3/2)*coth(x), x)
```

3.223.8 Giac [F]

$$\int \coth(x) \left(a + b \tanh^2(x)\right)^{3/2} \, dx = \int \left(b \tanh(x)^2 + a\right)^{\frac{3}{2}} \coth(x) \, dx$$

input
```
integrate(coth(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \coth(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = \int \coth(x)\,\left(b\tanh(x)^2+a\right)^{3/2}\,dx$$

input
```
int(coth(x)*(a + b*tanh(x)^2)^(3/2),x)
```

output
```
int(coth(x)*(a + b*tanh(x)^2)^(3/2), x)
```

3.224 $\int \coth^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx$

3.224.1 Optimal result . . . 1656
3.224.2 Mathematica [C] (verified) . . . 1656
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3.224.8 Giac [B] (verification not implemented) . . . 1662
3.224.9 Mupad [F(-1)] . . . 1663

3.224.1 Optimal result

Integrand size = 17, antiderivative size = 77

$$\int \coth^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = -b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$
$$+(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-a\coth(x)\sqrt{a+b\tanh^2(x)}$$

output
```
-b^(3/2)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))+(a+b)^(3/2)*arctan
h((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))-a*coth(x)*(a+b*tanh(x)^2)^(1/
2)
```

3.224.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 3.36 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.56

$$\int \coth^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx =$$
$$-\frac{a\left((a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)-\sqrt{2}(a+2b)\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(a-b-}{}}\right.\right.\right.}{\sqrt{2}\sqrt{(}}$$

input
```
Integrate[Coth[x]^2*(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
-((a*((a - b + (a + b)*Cosh[2*x])*Csch[x]^2 - Sqrt[2]*(a + 2*b)*Sqrt[((a -
 b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticF[ArcSin[Sqrt[((a - b + (a +
 b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] + Sqrt[2]*(a + b)*Sqrt[((a - b +
 (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticPi[b/(a + b), ArcSin[Sqrt[((a -
b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1])*Tanh[x])/(Sqrt[2]*Sqrt[
(a - b + (a + b)*Cosh[2*x])*Sech[x]^2]))
```

3.224.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {3042, 25, 4153, 25, 376, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\left(a-b\tan(ix)^2\right)^{3/2}}{\tan(ix)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\left(a-b\tan(ix)^2\right)^{3/2}}{\tan(ix)^2}dx$$

$$\downarrow 4153$$

$$-\int -\frac{\coth^2(x)\left(b\tanh^2(x)+a\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 25$$

$$\int \frac{\coth^2(x)\left(a+b\tanh^2(x)\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 376$$

$$\int \frac{b^2\tanh^2(x)+a(a+2b)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x) - a\coth(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 398$$

$$b^2\left(-\int \frac{1}{\sqrt{b\tanh^2(x)+a}}d\tanh(x)\right)+(a+b)^2\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)-a\coth(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 224$$

$$b^2\left(-\int \frac{1}{1-\frac{b\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}\right)+(a+b)^2\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)-a\coth(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 219$$

$$(a+b)^2\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)-b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-a\coth(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 291$$

$$(a+b)^2\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}-b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-a\coth(x)\sqrt{a+b\tanh^2(x)}$$

$$\downarrow 219$$

$$b^{3/2}\left(-\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\right)+(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-a\coth(x)\sqrt{a+b\tanh^2(x)}$$

input
```
Int[Coth[x]^2*(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
-(b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]) + (a + b)^(3/2
)*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - a*Coth[x]*Sqrt[a
+ b*Tanh[x]^2]
```

3.224.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.224.4 Maple [F]

$$\int \coth(x)^2 \left(a + b \tanh(x)^2\right)^{\frac{3}{2}} dx$$

input
```
int(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x)
```

output
```
int(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x)
```

3.224.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(63) = 126$.

Time $= 0.40$ (sec) , antiderivative size $= 3913$, normalized size of antiderivative $= 50.82$

$$\int \coth^2(x) \left(a + b \tanh^2(x)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 -
 a - b)*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x
)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a
*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)
*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2
 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2
+ 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*co
sh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*c
osh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b
^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)
^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*
sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)
^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh
(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b
^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2
*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*
cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2
+ a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)
*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)
*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*c...
```

3.224.6 Sympy [F]

$$\int \coth^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = \int \left(a+b\tanh^2(x)\right)^{\frac{3}{2}}\coth^2(x)\,dx$$

input
```
integrate(coth(x)**2*(a+b*tanh(x)**2)**(3/2),x)
```

output
```
Integral((a + b*tanh(x)**2)**(3/2)*coth(x)**2, x)
```

3.224.7 Maxima [F]

$$\int \coth^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = \int \left(b\tanh(x)^2+a\right)^{\frac{3}{2}}\coth(x)^2\,dx$$

input
```
integrate(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tanh(x)^2 + a)^(3/2)*coth(x)^2, x)
```

3.224.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(63) = 126$.

Time $= 0.96$ (sec) , antiderivative size $= 430$, normalized size of antiderivative $= 5.58$

$$\int \coth^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx =$$

$$-\frac{2\,b^2\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}+\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}}$$

$$-\frac{1}{2}(a+b)^{\frac{3}{2}}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)$$

$$+\frac{1}{2}(a+b)^{\frac{3}{2}}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)$$

$$-\frac{(a^2+2\,ab+b^2)\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a\right.}{2\sqrt{a+b}}$$

$$+\frac{4\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)a^2+\sqrt{\right.}}{\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)^2-2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,a\right.}$$

input
```
integrate(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-2*b^2*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a
*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*(a
 + b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*
a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*(a + b)^(3/2)*log(a
bs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e
^(2*x) + a + b) - sqrt(a + b))) - 1/2*(a^2 + 2*a*b + b^2)*log(abs(-(sqrt(a
 + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) + 4*((sqrt(a + b)*e^(2*
x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*a^2
+ sqrt(a + b)*a^2)/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*
a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4
*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a +
b)
```

3.224.9 Mupad [F(-1)]

Timed out.

$$\int \coth^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = \int \coth(x)^2\left(b\tanh(x)^2+a\right)^{3/2}\,dx$$

input
```
int(coth(x)^2*(a + b*tanh(x)^2)^(3/2),x)
```

output
```
int(coth(x)^2*(a + b*tanh(x)^2)^(3/2), x)
```

3.225 $\int \sqrt{1+\tanh^2(x)}\,dx$

3.225.1 Optimal result 1664
3.225.2 Mathematica [A] (verified) 1664
3.225.3 Rubi [A] (verified) 1665
3.225.4 Maple [B] (verified) 1666
3.225.5 Fricas [B] (verification not implemented) 1667
3.225.6 Sympy [F] 1668
3.225.7 Maxima [F] 1669
3.225.8 Giac [B] (verification not implemented) 1669
3.225.9 Mupad [B] (verification not implemented) 1669

3.225.1 Optimal result

Integrand size = 10, antiderivative size = 31

$$\int \sqrt{1+\tanh^2(x)}\,dx = -\operatorname{arcsinh}(\tanh(x)) + \sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)$$

output
```
-arcsinh(tanh(x))+arctanh(2^(1/2)*tanh(x)/(1+tanh(x)^2)^(1/2))*2^(1/2)
```

3.225.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \sqrt{1+\tanh^2(x)}\,dx = \frac{\left(\sqrt{2}\operatorname{arcsinh}\left(\sqrt{2}\sinh(x)\right) - \operatorname{arctanh}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\right)\right)\cosh(x)\sqrt{1+\tanh^2(x)}}{\sqrt{\cosh(2x)}}$$

input
```
Integrate[Sqrt[1 + Tanh[x]^2],x]
```

output
```
((Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]] - ArcTanh[Sinh[x]/Sqrt[Cosh[2*x]]])*Cos
h[x]*Sqrt[1 + Tanh[x]^2])/Sqrt[Cosh[2*x]]
```

3.225.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4144, 301, 222, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tanh^2(x)+1}\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{1-\tan(ix)^2}dx$$

$$\downarrow 4144$$

$$\int \frac{\sqrt{\tanh^2(x)+1}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 301$$

$$2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{\tanh^2(x)+1}}d\tanh(x) - \int \frac{1}{\sqrt{\tanh^2(x)+1}}d\tanh(x)$$

$$\downarrow 222$$

$$2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{\tanh^2(x)+1}}d\tanh(x) - \operatorname{arcsinh}(\tanh(x))$$

$$\downarrow 291$$

$$2\int \frac{1}{1-\frac{2\tanh^2(x)}{\tanh^2(x)+1}}d\frac{\tanh(x)}{\sqrt{\tanh^2(x)+1}} - \operatorname{arcsinh}(\tanh(x))$$

$$\downarrow 219$$

$$\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{\tanh^2(x)+1}}\right) - \operatorname{arcsinh}(\tanh(x))$$

input
```
Int[Sqrt[1 + Tanh[x]^2],x]
```

output
```
-ArcSinh[Tanh[x]] + Sqrt[2]*ArcTanh[(Sqrt[2]*Tanh[x])/Sqrt[1 + Tanh[x]^2]]
```

3.225.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.225.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(25) = 50$.

Time $= 0.19$ (sec) , antiderivative size $= 97$, normalized size of antiderivative $= 3.13$

method	result
derivativedivides	$-\frac{\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}{2}-\operatorname{arcsinh}(\tanh(x))+\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{(2+2\tanh(x))\sqrt{2}}{4\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}\right)}{2}+\frac{\sqrt{(1+\tan$
default	$-\frac{\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}{2}-\operatorname{arcsinh}(\tanh(x))+\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{(2+2\tanh(x))\sqrt{2}}{4\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}\right)}{2}+\frac{\sqrt{(1+\tan$

input
```
int((1+tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((tanh(x)-1)^2+2*tanh(x))^(1/2)-arcsinh(tanh(x))+1/2*2^(1/2)*arctanh(
1/4*(2+2*tanh(x))*2^(1/2)/((tanh(x)-1)^2+2*tanh(x))^(1/2))+1/2*((1+tanh(x)
)^2-2*tanh(x))^(1/2)-1/2*2^(1/2)*arctanh(1/4*(2-2*tanh(x))*2^(1/2)/((1+tan
h(x))^2-2*tanh(x))^(1/2))
```

3.225.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 679 vs. $2(25) = 50$.

Time $= 0.27$ (sec) , antiderivative size $= 679$, normalized size of antiderivative $= 21.90$

$$\int \sqrt{1+\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate((1+tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-2*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh
(x)^2 - 3)*sinh(x)^6 - 3*cosh(x)^6 + 2*(28*cosh(x)^3 - 9*cosh(x))*sinh(x)^
5 + 5*(14*cosh(x)^4 - 9*cosh(x)^2 + 1)*sinh(x)^4 + 5*cosh(x)^4 + 4*(14*cos
h(x)^5 - 15*cosh(x)^3 + 5*cosh(x))*sinh(x)^3 + (28*cosh(x)^6 - 45*cosh(x)^
4 + 30*cosh(x)^2 - 4)*sinh(x)^2 - 4*cosh(x)^2 + 2*(4*cosh(x)^7 - 9*cosh(x)
^5 + 10*cosh(x)^3 - 4*cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*co
sh(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*si
nh(x)^4 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x)
)*sinh(x)^3 + (15*sqrt(2)*cosh(x)^4 - 18*sqrt(2)*cosh(x)^2 + 4*sqrt(2))*si
nh(x)^2 + 4*sqrt(2)*cosh(x)^2 + 2*(3*sqrt(2)*cosh(x)^5 - 6*sqrt(2)*cosh(x)
^3 + 4*sqrt(2)*cosh(x))*sinh(x) - 4*sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4)/(cosh(x)^6 + 6*cosh(x)^5
*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*
sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 1/4*sqrt(2)*log(2*(cosh(x)
^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(
x)^2 + 2*(2*cosh(x)^3 + cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*
cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2)) - 1/2*log((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 + 2*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin...
```

3.225.6 Sympy [F]

$$\int \sqrt{1+\tanh^2(x)}\,dx = \int \sqrt{\tanh^2(x)+1}\,dx$$

input
```
integrate((1+tanh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(tanh(x)**2 + 1), x)
```

3.225.7 Maxima [F]

$$\int \sqrt{1+\tanh^2(x)}\,dx = \int \sqrt{\tanh(x)^2+1}\,dx$$

input
```
integrate((1+tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(tanh(x)^2 + 1), x)
```

3.225.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(25) = 50$.

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.35

$$\int \sqrt{1+\tanh^2(x)}\,dx =$$
$$-\frac{1}{2}\sqrt{2}\left(\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}+1}{\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}-1}\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(}\right.\right.$$

input
```
integrate((1+tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/(sqr
t(2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) + log(sqrt(e^(4*x) + 1) - e^(2*x)
 + 1) + log(sqrt(e^(4*x) + 1) - e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x
) + 1))
```

3.225.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.19

$$\int \sqrt{1+\tanh^2(x)}\,dx = \frac{\sqrt{2}\left(\ln(\tanh(x)+1)-\ln\left(\sqrt{2}\sqrt{\tanh(x)^2+1}-\tanh(x)+1\right)\right)}{2}$$
$$-\operatorname{asinh}(\tanh(x))$$
$$+\frac{\sqrt{2}\left(\ln\left(\tanh(x)+\sqrt{2}\sqrt{\tanh(x)^2+1}+1\right)-\ln(\tanh(x)-1)\right)}{2}$$

input
```
int((tanh(x)^2 + 1)^(1/2),x)
```

output
```
(2^(1/2)*(log(tanh(x) + 1) - log(2^(1/2)*(tanh(x)^2 + 1)^(1/2) - tanh(x) +
 1)))/2 - asinh(tanh(x)) + (2^(1/2)*(log(tanh(x) + 2^(1/2)*(tanh(x)^2 + 1)
^(1/2) + 1) - log(tanh(x) - 1)))/2
```

3.226 $\int \sqrt{-1-\tanh^2(x)}\,dx$

3.226.1 Optimal result 1671
3.226.2 Mathematica [A] (verified) 1671
3.226.3 Rubi [A] (verified) 1672
3.226.4 Maple [B] (verified) 1674
3.226.5 Fricas [C] (verification not implemented) 1674
3.226.6 Sympy [F] 1675
3.226.7 Maxima [F] 1675
3.226.8 Giac [C] (verification not implemented) 1676
3.226.9 Mupad [B] (verification not implemented) 1676

3.226.1 Optimal result

Integrand size = 12, antiderivative size = 45

$$\int \sqrt{-1-\tanh^2(x)}\,dx = \arctan\left(\frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right) - \sqrt{2}\arctan\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)$$

output
```
arctan(tanh(x)/(-1-tanh(x)^2)^(1/2))-arctan(2^(1/2)*tanh(x)/(-1-tanh(x)^2)
^(1/2))*2^(1/2)
```

3.226.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \sqrt{-1-\tanh^2(x)}\,dx = \frac{\left(\sqrt{2}\operatorname{arcsinh}\left(\sqrt{2}\sinh(x)\right) - \operatorname{arctanh}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\right)\right)\cosh(x)\sqrt{-1-\tanh^2(x)}}{\sqrt{\cosh(2x)}}$$

input
```
Integrate[Sqrt[-1 - Tanh[x]^2],x]
```

output
```
((Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]] - ArcTanh[Sinh[x]/Sqrt[Cosh[2*x]]])*Cos
h[x]*Sqrt[-1 - Tanh[x]^2])/Sqrt[Cosh[2*x]]
```

3.226.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 4144, 301, 224, 216, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-\tanh^2(x)-1}\,dx$$

$$\downarrow 3042$$

$$\int \sqrt{-1+\tan(ix)^2}dx$$

$$\downarrow 4144$$

$$\int \frac{\sqrt{-\tanh^2(x)-1}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 301$$

$$\int \frac{1}{\sqrt{-\tanh^2(x)-1}}d\tanh(x) - 2\int \frac{1}{\sqrt{-\tanh^2(x)-1}\left(1-\tanh^2(x)\right)}d\tanh(x)$$

$$\downarrow 224$$

$$\int \frac{1}{\frac{\tanh^2(x)}{-\tanh^2(x)-1}+1}d\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}} - 2\int \frac{1}{\sqrt{-\tanh^2(x)-1}\left(1-\tanh^2(x)\right)}d\tanh(x)$$

$$\downarrow 216$$

$$\arctan\left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right) - 2\int \frac{1}{\sqrt{-\tanh^2(x)-1}\left(1-\tanh^2(x)\right)}d\tanh(x)$$

$$\downarrow 291$$

$$\arctan\left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right) - 2\int \frac{1}{\frac{2\tanh^2(x)}{-\tanh^2(x)-1}+1}d\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}$$

$$\downarrow 216$$

$$\arctan\left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right) - \sqrt{2}\arctan\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)$$

input
```
Int[Sqrt[-1 - Tanh[x]^2],x]
```

output
```
ArcTan[Tanh[x]/Sqrt[-1 - Tanh[x]^2]] - Sqrt[2]*ArcTan[(Sqrt[2]*Tanh[x])/Sq
rt[-1 - Tanh[x]^2]]
```

3.226.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.226.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(37) = 74$.

Time $= 0.16$ (sec) , antiderivative size $= 142$, normalized size of antiderivative $= 3.16$

method	result
derivativedivides	$\frac{\sqrt{-(1+\tanh(x))^2+2\tanh(x)}}{2}+\frac{\arctan\left(\frac{\tanh(x)}{\sqrt{-(1+\tanh(x))^2+2\tanh(x)}}\right)}{2}-\frac{\sqrt{2}\arctan\left(\frac{(-2+2\tanh(x))\sqrt{2}}{4\sqrt{-(1+\tanh(x))^2+2\tanh(x)}}\right)}{2}-$
default	$\frac{\sqrt{-(1+\tanh(x))^2+2\tanh(x)}}{2}+\frac{\arctan\left(\frac{\tanh(x)}{\sqrt{-(1+\tanh(x))^2+2\tanh(x)}}\right)}{2}-\frac{\sqrt{2}\arctan\left(\frac{(-2+2\tanh(x))\sqrt{2}}{4\sqrt{-(1+\tanh(x))^2+2\tanh(x)}}\right)}{2}-$

input
```
int((-1-tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-(1+tanh(x))^2+2*tanh(x))^(1/2)+1/2*arctan(tanh(x)/(-(1+tanh(x))^2+2*
tanh(x))^(1/2))-1/2*2^(1/2)*arctan(1/4*(-2+2*tanh(x))*2^(1/2)/(-(1+tanh(x)
)^2+2*tanh(x))^(1/2))-1/2*(-(tanh(x)-1)^2-2*tanh(x))^(1/2)+1/2*arctan(tanh
(x)/(-(tanh(x)-1)^2-2*tanh(x))^(1/2))+1/2*2^(1/2)*arctan(1/4*(-2-2*tanh(x)
)*2^(1/2)/(-(tanh(x)-1)^2-2*tanh(x))^(1/2))
```

3.226.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time $= 0.24$ (sec) , antiderivative size $= 226$, normalized size of antiderivative $= 5.02$

$$
\begin{aligned}
&\int \sqrt{-1-\tanh^2(x)}\,dx \\
&= -\frac{1}{4}\sqrt{-2}\log\left(-\left(\sqrt{-2}\sqrt{-2\,e^{(4x)}-2}+2\,e^{(2x)}+2\right)e^{(-2x)}\right) \\
&\quad+\frac{1}{4}\sqrt{-2}\log\left(\left(\sqrt{-2}\sqrt{-2\,e^{(4x)}-2}-2\,e^{(2x)}-2\right)e^{(-2x)}\right) \\
&\quad+\frac{1}{4}\sqrt{-2}\log\left(-2\left(\sqrt{-2\,e^{(4x)}-2}\left(e^{(2x)}-2\right)+\sqrt{-2}e^{(4x)}-\sqrt{-2}e^{(2x)}+2\sqrt{-2}\right)e^{(-4x)}\right) \\
&\quad-\frac{1}{4}\sqrt{-2}\log\left(-2\left(\sqrt{-2\,e^{(4x)}-2}\left(e^{(2x)}-2\right)-\sqrt{-2}e^{(4x)}+\sqrt{-2}e^{(2x)}-2\sqrt{-2}\right)e^{(-4x)}\right) \\
&\quad+\frac{1}{2}i\,\log\left(-4\left(i\sqrt{-2\,e^{(4x)}-2}+e^{(2x)}-1\right)e^{(-2x)}\right) \\
&\quad-\frac{1}{2}i\,\log\left(-4\left(-i\sqrt{-2\,e^{(4x)}-2}+e^{(2x)}-1\right)e^{(-2x)}\right)
\end{aligned}
$$

input
```
integrate((-1-tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(-2)*log(-(sqrt(-2)*sqrt(-2*e^(4*x) - 2) + 2*e^(2*x) + 2)*e^(-2*x
)) + 1/4*sqrt(-2)*log((sqrt(-2)*sqrt(-2*e^(4*x) - 2) - 2*e^(2*x) - 2)*e^(-
2*x)) + 1/4*sqrt(-2)*log(-2*(sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) + sqrt(-2)
*e^(4*x) - sqrt(-2)*e^(2*x) + 2*sqrt(-2))*e^(-4*x)) - 1/4*sqrt(-2)*log(-2*
(sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) - sqrt(-2)*e^(4*x) + sqrt(-2)*e^(2*x)
- 2*sqrt(-2))*e^(-4*x)) + 1/2*I*log(-4*(I*sqrt(-2*e^(4*x) - 2) + e^(2*x) -
 1)*e^(-2*x)) - 1/2*I*log(-4*(-I*sqrt(-2*e^(4*x) - 2) + e^(2*x) - 1)*e^(-2
*x))
```

3.226.6 Sympy [F]

$$\int \sqrt{-1-\tanh^2(x)}\,dx = \int \sqrt{-\tanh^2(x)-1}\,dx$$

input
```
integrate((-1-tanh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(-tanh(x)**2 - 1), x)
```

3.226.7 Maxima [F]

$$\int \sqrt{-1-\tanh^2(x)}\,dx = \int \sqrt{-\tanh(x)^2-1}\,dx$$

input
```
integrate((-1-tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-tanh(x)^2 - 1), x)
```

3.226.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.31

$$\int \sqrt{-1-\tanh^2(x)}\,dx =$$
$$-\frac{1}{2}i\sqrt{2}\left(\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}+1}{\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}-1}\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)+\log\left(\sqrt{e^{(4x)}+1}-e\right.\right.$$

input
```
integrate((-1-tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*I*sqrt(2)*(sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/(s
qrt(2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) + log(sqrt(e^(4*x) + 1) - e^(2*
x) + 1) + log(sqrt(e^(4*x) + 1) - e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2
*x) + 1))
```

3.226.9 Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \sqrt{-1-\tanh^2(x)}\,dx = -\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh(x)^2-1}}\right) - \ln\left(\tanh(x)-\sqrt{-\tanh(x)^2-1}\,1\mathrm{i}\right)\,1\mathrm{i}$$

input
```
int((- tanh(x)^2 - 1)^(1/2),x)
```

output
```
- log(tanh(x) - (- tanh(x)^2 - 1)^(1/2)*1i)*1i - 2^(1/2)*atan((2^(1/2)*tan
h(x))/(- tanh(x)^2 - 1)^(1/2))
```

3.227 $\int \left(1+\tanh^2(x)\right)^{3/2}\,dx$

3.227.1 Optimal result 1677
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3.227.1 Optimal result

Integrand size = 10, antiderivative size = 50

$$\int \left(1+\tanh^2(x)\right)^{3/2}\,dx =$$

$$-\frac{5}{2}\operatorname{arcsinh}(\tanh(x))+2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)-\frac{1}{2}\tanh(x)\sqrt{1+\tanh^2(x)}$$

output
```
-5/2*arcsinh(tanh(x))+2*arctanh(2^(1/2)*tanh(x)/(1+tanh(x)^2)^(1/2))*2^(1/
2)-1/2*(1+tanh(x)^2)^(1/2)*tanh(x)
```

3.227.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.48

$$\int \left(1+\tanh^2(x)\right)^{3/2}\,dx =$$

$$-\frac{\left(-4\sqrt{2}\operatorname{arcsinh}\left(\sqrt{2}\sinh(x)\right)\cosh^3(x)+5\operatorname{arctanh}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\right)\cosh^3(x)+\cosh(x)\sqrt{\cosh(2x)}\sinh(x)\right)(1+$$

$$2\cosh^{\frac{3}{2}}(2x)$$

input
```
Integrate[(1 + Tanh[x]^2)^(3/2),x]
```

output
```
-1/2*((-4*Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]]*Cosh[x]^3 + 5*ArcTanh[Sinh[x]/S
qrt[Cosh[2*x]]]*Cosh[x]^3 + Cosh[x]*Sqrt[Cosh[2*x]]*Sinh[x])*(1 + Tanh[x]^
2)^(3/2))/Cosh[2*x]^(3/2)
```

3.227.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 4144, 318, 25, 398, 222, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\tanh^2(x)+1\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(1-\tan(ix)^2\right)^{3/2}\,dx$$

$$\downarrow 4144$$

$$\int \frac{\left(\tanh^2(x)+1\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 318$$

$$-\frac{1}{2}\int -\frac{5\tanh^2(x)+3}{\left(1-\tanh^2(x)\right)\sqrt{\tanh^2(x)+1}}d\tanh(x)-\frac{1}{2}\sqrt{\tanh^2(x)+1}\tanh(x)$$

$$\downarrow 25$$

$$\frac{1}{2}\int \frac{5\tanh^2(x)+3}{\left(1-\tanh^2(x)\right)\sqrt{\tanh^2(x)+1}}d\tanh(x)-\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)+1}$$

$$\downarrow 398$$

$$\frac{1}{2}\left(8\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{\tanh^2(x)+1}}d\tanh(x)-5\int \frac{1}{\sqrt{\tanh^2(x)+1}}d\tanh(x)\right)-\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)+1}$$

$$\downarrow 222$$

$$\frac{1}{2}\left(8\int\frac{1}{\left(1-\tanh^2(x)\right)\sqrt{\tanh^2(x)+1}}d\tanh(x)-5\operatorname{arcsinh}(\tanh(x))\right)-\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)+1}$$

$$\downarrow 291$$

$$\frac{1}{2}\left(8\int\frac{1}{1-\frac{2\tanh^2(x)}{\tanh^2(x)+1}}d\frac{\tanh(x)}{\sqrt{\tanh^2(x)+1}}-5\operatorname{arcsinh}(\tanh(x))\right)-\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)+1}$$

$$\downarrow 219$$

$$\frac{1}{2}\left(4\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{\tanh^2(x)+1}}\right)-5\operatorname{arcsinh}(\tanh(x))\right)-\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)+1}$$

input
```
Int[(1 + Tanh[x]^2)^(3/2),x]
```

output
```
(-5*ArcSinh[Tanh[x]] + 4*Sqrt[2]*ArcTanh[(Sqrt[2]*Tanh[x])/Sqrt[1 + Tanh[x
]^2]])/2 - (Tanh[x]*Sqrt[1 + Tanh[x]^2])/2
```

3.227.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.227.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(38) = 76$.

Time $= 0.08$ (sec) , antiderivative size $= 158$, normalized size of antiderivative $= 3.16$

method	result
derivativedivides	$-\frac{\left((\tanh(x)-1)^2+2\tanh(x)\right)^{\frac{3}{2}}}{6}-\frac{\tanh(x)\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}{4}-\frac{5\,\operatorname{arcsinh}(\tanh(x))}{2}-\sqrt{(\tanh(x)-$
default	$-\frac{\left((\tanh(x)-1)^2+2\tanh(x)\right)^{\frac{3}{2}}}{6}-\frac{\tanh(x)\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}{4}-\frac{5\,\operatorname{arcsinh}(\tanh(x))}{2}-\sqrt{(\tanh(x)-$

input
```
int((1+tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*((tanh(x)-1)^2+2*tanh(x))^(3/2)-1/4*tanh(x)*((tanh(x)-1)^2+2*tanh(x))
^(1/2)-5/2*arcsinh(tanh(x))-((tanh(x)-1)^2+2*tanh(x))^(1/2)+2^(1/2)*arctan
h(1/4*(2+2*tanh(x))*2^(1/2)/((tanh(x)-1)^2+2*tanh(x))^(1/2))+1/6*((1+tanh(
x))^2-2*tanh(x))^(3/2)-1/4*tanh(x)*((1+tanh(x))^2-2*tanh(x))^(1/2)+((1+tan
h(x))^2-2*tanh(x))^(1/2)-2^(1/2)*arctanh(1/4*(2-2*tanh(x))*2^(1/2)/((1+tan
h(x))^2-2*tanh(x))^(1/2))
```

3.227.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(38) = 76$.

Time $= 0.27$ (sec) , antiderivative size $= 1027$, normalized size of antiderivative $= 20.54$

$$\int \left(1 + \tanh^2(x)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((1+tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(2*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^
4 + 2*(3*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 2*sqrt(2)*cosh(x)^2 + 4*
(sqrt(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(-2*(cosh(x)^8
 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(x)^2 - 3)*sinh(x)^6 - 3*cosh
(x)^6 + 2*(28*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 + 5*(14*cosh(x)^4 - 9*cosh(
x)^2 + 1)*sinh(x)^4 + 5*cosh(x)^4 + 4*(14*cosh(x)^5 - 15*cosh(x)^3 + 5*cos
h(x))*sinh(x)^3 + (28*cosh(x)^6 - 45*cosh(x)^4 + 30*cosh(x)^2 - 4)*sinh(x)
^2 - 4*cosh(x)^2 + 2*(4*cosh(x)^7 - 9*cosh(x)^5 + 10*cosh(x)^3 - 4*cosh(x)
)*sinh(x) + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh(x)*sinh(x)^5 + sqrt(2)*sin
h(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^4 - 3*sqrt(2)*cosh(x)^4
 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x)^3 + (15*sqrt(2)*cos
h(x)^4 - 18*sqrt(2)*cosh(x)^2 + 4*sqrt(2))*sinh(x)^2 + 4*sqrt(2)*cosh(x)^2
 + 2*(3*sqrt(2)*cosh(x)^5 - 6*sqrt(2)*cosh(x)^3 + 4*sqrt(2)*cosh(x))*sinh(
x) - 4*sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2)) + 4)/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(
x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)
^5 + sinh(x)^6)) + 2*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sq
rt(2)*sinh(x)^4 + 2*(3*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 2*sqrt(2)*
cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log
(2*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sin...
```

3.227.6 Sympy [F]

$$\int \left(1+\tanh^2(x)\right)^{3/2}\,dx = \int \left(\tanh^2(x)+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((1+tanh(x)**2)**(3/2),x)
```

output
```
Integral((tanh(x)**2 + 1)**(3/2), x)
```

3.227.7 Maxima [F]

$$\int \left(1+\tanh^2(x)\right)^{3/2}\,dx = \int \left(\tanh(x)^2+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((1+tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((tanh(x)^2 + 1)^(3/2), x)
```

3.227.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(38) = 76$.

Time $= 0.28$ (sec) , antiderivative size $= 202$, normalized size of antiderivative $= 4.04$

$$\int \left(1+\tanh^2(x)\right)^{3/2}\,dx =$$

$$-\frac{1}{4}\sqrt{2}\left(5\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}+1}{\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}-1}\right)-\frac{4\left(3\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^3-\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^2-\right.}{\left(\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^2-2\sqrt{e^{(4x)}+1}+\right.}\right.$$

input
```
integrate((1+tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(5*sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/(s
qrt(2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) - 4*(3*(sqrt(e^(4*x) + 1) - e^(
2*x))^3 - (sqrt(e^(4*x) + 1) - e^(2*x))^2 - sqrt(e^(4*x) + 1) + e^(2*x) -
1)/((sqrt(e^(4*x) + 1) - e^(2*x))^2 - 2*sqrt(e^(4*x) + 1) + 2*e^(2*x) - 1)
^2 + 4*log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) + 4*log(sqrt(e^(4*x) + 1) - e^
(2*x)) - 4*log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))
```

3.227.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.56

$$\int (1 + \tanh^2(x))^{3/2}\, dx = \sqrt{2}\left(\ln(\tanh(x)+1) - \ln\left(\sqrt{2}\sqrt{\tanh(x)^2+1} - \tanh(x) + 1\right)\right) - \frac{5\,\operatorname{asinh}(\tanh(x))}{2} - \frac{\tanh(x)\sqrt{\tanh(x)^2+1}}{2} + \sqrt{2}\left(\ln\left(\tanh(x) + \sqrt{2}\sqrt{\tanh(x)^2+1} + 1\right) - \ln(\tanh(x)-1)\right)$$

input
```
int((tanh(x)^2 + 1)^(3/2),x)
```

output
```
2^(1/2)*(log(tanh(x) + 1) - log(2^(1/2)*(tanh(x)^2 + 1)^(1/2) - tanh(x) +
1)) - (5*asinh(tanh(x)))/2 - (tanh(x)*(tanh(x)^2 + 1)^(1/2))/2 + 2^(1/2)*(
log(tanh(x) + 2^(1/2)*(tanh(x)^2 + 1)^(1/2) + 1) - log(tanh(x) - 1))
```

3.228 $\int \left(-1-\tanh^2(x)\right)^{3/2}\,dx$

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3.228.1 Optimal result

Integrand size = 12, antiderivative size = 67

$$\int \left(-1-\tanh^2(x)\right)^{3/2}\,dx = -\frac{5}{2}\arctan\left(\frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right) + 2\sqrt{2}\arctan\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right) + \frac{1}{2}\tanh(x)\sqrt{-1-\tanh^2(x)}$$

output
```
-5/2*arctan(tanh(x)/(-1-tanh(x)^2)^(1/2))+2*arctan(2^(1/2)*tanh(x)/(-1-tan
h(x)^2)^(1/2))*2^(1/2)+1/2*(-1-tanh(x)^2)^(1/2)*tanh(x)
```

3.228.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

$$\int \left(-1-\tanh^2(x)\right)^{3/2}\,dx = -\frac{\left(-4\sqrt{2}\operatorname{arcsinh}\left(\sqrt{2}\sinh(x)\right)\cosh^3(x) + 5\operatorname{arctanh}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\right)\cosh^3(x) + \cosh(x)\sqrt{\cosh(2x)}\sinh(x)\right)(-1}{2\cosh^{\frac{3}{2}}(2x)}$$

input
```
Integrate[(-1 - Tanh[x]^2)^(3/2),x]
```

output
```
-1/2*((-4*Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]]*Cosh[x]^3 + 5*ArcTanh[Sinh[x]/S
qrt[Cosh[2*x]]]*Cosh[x]^3 + Cosh[x]*Sqrt[Cosh[2*x]]*Sinh[x])*(-1 - Tanh[x]
^2)^(3/2))/Cosh[2*x]^(3/2)
```

3.228.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 4144, 318, 25, 398, 224, 216, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-\tanh^2(x)-1\right)^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int \left(-1+\tan(ix)^2\right)^{3/2}\,dx$$

$$\downarrow 4144$$

$$\int \frac{\left(-\tanh^2(x)-1\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 318$$

$$\frac{1}{2}\tanh(x)\sqrt{-\tanh^2(x)-1}-\frac{1}{2}\int -\frac{5\tanh^2(x)+3}{\sqrt{-\tanh^2(x)-1}\left(1-\tanh^2(x)\right)}d\tanh(x)$$

$$\downarrow 25$$

$$\frac{1}{2}\int \frac{5\tanh^2(x)+3}{\sqrt{-\tanh^2(x)-1}\left(1-\tanh^2(x)\right)}d\tanh(x)+\frac{1}{2}\sqrt{-\tanh^2(x)-1}\tanh(x)$$

$$\downarrow 398$$

$$\frac{1}{2}\left(8\int \frac{1}{\sqrt{-\tanh^2(x)-1}\left(1-\tanh^2(x)\right)}d\tanh(x)-5\int \frac{1}{\sqrt{-\tanh^2(x)-1}}d\tanh(x)\right)+\frac{1}{2}\sqrt{-\tanh^2(x)-1}\tanh(x)$$

$$\downarrow 224$$

$$\frac{1}{2}\left(8\int\frac{1}{\sqrt{-\tanh^2(x)-1}\left(1-\tanh^2(x)\right)}d\tanh(x)-5\int\frac{1}{\frac{\tanh^2(x)}{-\tanh^2(x)-1}+1}d\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)+\frac{1}{2}\sqrt{-\tanh^2(x)-1}\tanh(x)$$

$$\downarrow 216$$

$$\frac{1}{2}\left(8\int\frac{1}{\sqrt{-\tanh^2(x)-1}\left(1-\tanh^2(x)\right)}d\tanh(x)-5\arctan\left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)\right)+\frac{1}{2}\sqrt{-\tanh^2(x)-1}\tanh(x)$$

$$\downarrow 291$$

$$\frac{1}{2}\left(8\int\frac{1}{\frac{2\tanh^2(x)}{-\tanh^2(x)-1}+1}d\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}-5\arctan\left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)\right)+\frac{1}{2}\sqrt{-\tanh^2(x)-1}\tanh(x)$$

$$\downarrow 216$$

$$\frac{1}{2}\left(4\sqrt{2}\arctan\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)-5\arctan\left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)\right)+\frac{1}{2}\tanh(x)\sqrt{-\tanh^2(x)-1}$$

input
```
Int[(-1 - Tanh[x]^2)^(3/2),x]
```

output
```
(-5*ArcTan[Tanh[x]/Sqrt[-1 - Tanh[x]^2]] + 4*Sqrt[2]*ArcTan[(Sqrt[2]*Tanh[
x])/Sqrt[-1 - Tanh[x]^2]])/2 + (Tanh[x]*Sqrt[-1 - Tanh[x]^2])/2
```

3.228.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.228.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(53) = 106$.

Time = 0.08 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.15

method	result
derivativedivides	$-\frac{\left(-(\tanh(x)-1)^2-2\tanh(x)\right)^{\frac{3}{2}}}{6}+\frac{\tanh(x)\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}{4}-\frac{5\arctan\left(\frac{\tanh(x)}{\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}\right)}{4}$
default	$-\frac{\left(-(\tanh(x)-1)^2-2\tanh(x)\right)^{\frac{3}{2}}}{6}+\frac{\tanh(x)\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}{4}-\frac{5\arctan\left(\frac{\tanh(x)}{\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}\right)}{4}$

input
```
int((-1-tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-(tanh(x)-1)^2-2*tanh(x))^(3/2)+1/4*tanh(x)*(-(tanh(x)-1)^2-2*tanh(x
))^(1/2)-5/4*arctan(tanh(x)/(-(tanh(x)-1)^2-2*tanh(x))^(1/2))+(-(tanh(x)-1
)^2-2*tanh(x))^(1/2)-2^(1/2)*arctan(1/4*(-2-2*tanh(x))*2^(1/2)/(-(tanh(x)-
1)^2-2*tanh(x))^(1/2))+1/6*(-(1+tanh(x))^2+2*tanh(x))^(3/2)+1/4*tanh(x)*(-
(1+tanh(x))^2+2*tanh(x))^(1/2)-5/4*arctan(tanh(x)/(-(1+tanh(x))^2+2*tanh(x
))^(1/2))-(-(1+tanh(x))^2+2*tanh(x))^(1/2)+2^(1/2)*arctan(1/4*(-2+2*tanh(x
))*2^(1/2)/(-(1+tanh(x))^2+2*tanh(x))^(1/2))
```

3.228.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 361, normalized size of antiderivative = 5.39

$$\int \left(-1-\tanh^2(x)\right)^{3/2}\,dx = \frac{2\left(\sqrt{-2}e^{(4x)}+2\sqrt{-2}e^{(2x)}+\sqrt{-2}\right)\log\left(2\left(\sqrt{-2}\sqrt{-2e^{(4x)}-2}+2e^{(2x)}+2\right)e^{(-2x)}\right)-$$

input
```
integrate((-1-tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(2*(sqrt(-2)*e^(4*x) + 2*sqrt(-2)*e^(2*x) + sqrt(-2))*log(2*(sqrt(-2)*
sqrt(-2*e^(4*x) - 2) + 2*e^(2*x) + 2)*e^(-2*x)) - 2*(sqrt(-2)*e^(4*x) + 2*
sqrt(-2)*e^(2*x) + sqrt(-2))*log(-2*(sqrt(-2)*sqrt(-2*e^(4*x) - 2) - 2*e^(
2*x) - 2)*e^(-2*x)) - 5*(I*e^(4*x) + 2*I*e^(2*x) + I)*log(-4*(I*sqrt(-2*e^
(4*x) - 2) + e^(2*x) - 1)*e^(-2*x)) - 5*(-I*e^(4*x) - 2*I*e^(2*x) - I)*log
(-4*(-I*sqrt(-2*e^(4*x) - 2) + e^(2*x) - 1)*e^(-2*x)) - 2*(sqrt(-2)*e^(4*x
) + 2*sqrt(-2)*e^(2*x) + sqrt(-2))*log(4*(sqrt(-2*e^(4*x) - 2)*(e^(2*x) -
2) + sqrt(-2)*e^(4*x) - sqrt(-2)*e^(2*x) + 2*sqrt(-2))*e^(-4*x)) + 2*(sqrt
(-2)*e^(4*x) + 2*sqrt(-2)*e^(2*x) + sqrt(-2))*log(4*(sqrt(-2*e^(4*x) - 2)*
(e^(2*x) - 2) - sqrt(-2)*e^(4*x) + sqrt(-2)*e^(2*x) - 2*sqrt(-2))*e^(-4*x)
) + 2*sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 1))/(e^(4*x) + 2*e^(2*x) + 1)
```

3.228.6 Sympy [F]

$$\int \left(-1-\tanh^2(x)\right)^{3/2}\,dx = \int \left(-\tanh^2(x)-1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-1-tanh(x)**2)**(3/2),x)
```

output
```
Integral((-tanh(x)**2 - 1)**(3/2), x)
```

3.228.7 Maxima [F]

$$\int \left(-1-\tanh^2(x)\right)^{3/2}\,dx = \int \left(-\tanh(x)^2-1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-1-tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-tanh(x)^2 - 1)^(3/2), x)
```

3.228.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 204, normalized size of antiderivative = 3.04

$$\int \left(-1-\tanh^2(x)\right)^{3/2}\,dx =$$

$$-\frac{1}{4}\sqrt{2}\left(-5i\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}+1}{\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}-1}\right)-\frac{4\left(-3i\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^3+i\left(\sqrt{e^{(4x)}+1}-e^{(}\right.\right.}{\left(\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^2-2\sqrt{e^{(4x)}}\right.}$$

input
```
integrate((-1-tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(-5*I*sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)
/(sqrt(2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) - 4*(-3*I*(sqrt(e^(4*x) + 1)
 - e^(2*x))^3 + I*(sqrt(e^(4*x) + 1) - e^(2*x))^2 + I*sqrt(e^(4*x) + 1) -
I*e^(2*x) + I)/((sqrt(e^(4*x) + 1) - e^(2*x))^2 - 2*sqrt(e^(4*x) + 1) + 2*
e^(2*x) - 1)^2 - 4*I*log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) - 4*I*log(sqrt(e
^(4*x) + 1) - e^(2*x)) + 4*I*log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int \left(-1-\tanh^2(x)\right)^{3/2}\,dx = \int \left(-\tanh(x)^2-1\right)^{3/2}dx$$

input
```
int((- tanh(x)^2 - 1)^(3/2),x)
```

output
```
int((- tanh(x)^2 - 1)^(3/2), x)
```

3.229 $\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

3.229.1 Optimal result 1691
3.229.2 Mathematica [A] (verified) 1691
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3.229.8 Giac [B] (verification not implemented) 1696
3.229.9 Mupad [B] (verification not implemented) 1697

3.229.1 Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + \frac{(a-b)\sqrt{a+b\tanh^2(x)}}{b^2} - \frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}$$

output
```
arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)+(a-b)*(a+b*tanh(x)^
2)^(1/2)/b^2-1/3*(a+b*tanh(x)^2)^(3/2)/b^2
```

3.229.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} + \frac{(a-b+(a-2b)\cosh(2x))\operatorname{sech}^2(x)\sqrt{a+b\tanh^2(x)}}{3b^2}$$

input
```
Integrate[Tanh[x]^5/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] + ((a - b + (a - 2*
b)*Cosh[2*x])*Sech[x]^2*Sqrt[a + b*Tanh[x]^2])/(3*b^2)
```

3.229.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\tan(ix)^5}{\sqrt{a-b\tan(ix)^2}}dx$$

$$\downarrow 26$$

$$-i\int \frac{\tan(ix)^5}{\sqrt{a-b\tan(ix)^2}}dx$$

$$\downarrow 4153$$

$$-i\int \frac{i\tanh^5(x)}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\tanh^5(x)}{(1-\tanh^2(x))\sqrt{a+b\tanh^2(x)}}d\tanh(x)$$

$$\downarrow 354$$

$$\frac{1}{2}\int \frac{\tanh^4(x)}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)$$

$$\downarrow 99$$

$$\frac{1}{2}\int\left(\frac{a-b}{b\sqrt{b\tanh^2(x)+a}}-\frac{\sqrt{b\tanh^2(x)+a}}{b}+\frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}\right)d\tanh^2(x)$$

$$\downarrow 2009$$

$$\frac{1}{2}\left(\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{2\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}+\frac{2(a-b)\sqrt{a+b\tanh^2(x)}}{b^2}\right)$$

input
```
Int[Tanh[x]^5/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]])/Sqrt[a + b] + (2*(a - b)*S
qrt[a + b*Tanh[x]^2])/b^2 - (2*(a + b*Tanh[x]^2)^(3/2))/(3*b^2))/2
```

3.229.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.229.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(58) = 116$.

Time = 0.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.34

method	result
derivativedivides	$-\frac{\sqrt{a+b\tanh(x)^2}}{b} - \frac{\tanh(x)^2\sqrt{a+b\tanh(x)^2}}{3b} + \frac{2a\sqrt{a+b\tanh(x)^2}}{3b^2} + \frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\sqrt{b(\tanh(x)}}{\tanh(x)-1}\right.}{2\sqrt{a+b}}$
default	$-\frac{\sqrt{a+b\tanh(x)^2}}{b} - \frac{\tanh(x)^2\sqrt{a+b\tanh(x)^2}}{3b} + \frac{2a\sqrt{a+b\tanh(x)^2}}{3b^2} + \frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\sqrt{b(\tanh(x)}}{\tanh(x)-1}\right.}{2\sqrt{a+b}}$

input
```
int(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(a+b*tanh(x)^2)^(1/2)/b-1/3*tanh(x)^2/b*(a+b*tanh(x)^2)^(1/2)+2/3*a/b^2*(
a+b*tanh(x)^2)^(1/2)+1/2/(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(
1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1/2/(a+b)^(
1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tan
h(x))+a+b)^(1/2))/(1+tanh(x)))
```

3.229.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1131 vs. $2(58) = 116$.

Time = 0.43 (sec) , antiderivative size = 2827, normalized size of antiderivative = 40.39

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 + 3*b^2*
cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3*b^2*cosh(x)^2 + 4*(5*b
^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*cosh(x)^4 + 6*b^2*cosh(
x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^2*cosh(x)^3 + b^2*cos
h(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*
cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6
+ 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 +
a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b
 - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b
- a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2
*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 +
b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2
*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*c
osh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(
x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*s
inh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^
2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 +
 (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a
^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b -
 b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*s...
```

3.229.6 Sympy [F]

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

input
```
integrate(tanh(x)**5/(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(tanh(x)**5/sqrt(a + b*tanh(x)**2), x)
```

3.229.7 Maxima [F]

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh(x)^5}{\sqrt{b\tanh(x)^2+a}}\,dx$$

input
```
integrate(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^5/sqrt(b*tanh(x)^2 + a), x)
```

3.229.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(58) = 116$.

Time = 0.67 (sec) , antiderivative size = 592, normalized size of antiderivative = 8.46

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx =$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$
$$-\frac{8\left(3\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^5+3\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(}\right.\right.}$$

input
```
integrate(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) +
1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x
) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) - 1/2*log(abs(-sqrt(a
 + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) - sqrt(a + b)))/sqrt(a + b) - 8/3*(3*(sqrt(a + b)*e^(2*x) - sqrt(a*e
^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5 + 3*(sqrt(a + b
)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b
))^4*sqrt(a + b) + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3*(3*a - 5*b) + 6*(sqrt(a + b)*e^(2*x)
- sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(
a + b)*(a - 3*b) - 3*(3*a^2 + 6*a*b - 13*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(
a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) - (9*a^2 - 22*
a*b + 17*b^2)*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4
*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqr
t(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b)
+ a - 3*b)^3
```

3.229.9 Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\left(b\tanh(x)^2+a\right)^{3/2}}{3\,b^2} - \left(\frac{a+b}{b^2} - \frac{2\,a}{b^2}\right)\sqrt{b\tanh\left(x\right)^2+a}$$

input
```
int(tanh(x)^5/(a + b*tanh(x)^2)^(1/2),x)
```

output
```
atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(1/2) - (a + b*tanh(x
)^2)^(3/2)/(3*b^2) - ((a + b)/b^2 - (2*a)/b^2)*(a + b*tanh(x)^2)^(1/2)
```

3.230 $\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

3.230.1 Optimal result . . . 1698
3.230.2 Mathematica [C] (verified) . . . 1698
3.230.3 Rubi [A] (verified) . . . 1699
3.230.4 Maple [B] (verified) . . . 1702
3.230.5 Fricas [B] (verification not implemented) . . . 1702
3.230.6 Sympy [F] . . . 1703
3.230.7 Maxima [F] . . . 1703
3.230.8 Giac [B] (verification not implemented) . . . 1704
3.230.9 Mupad [F(-1)] . . . 1705

3.230.1 Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{(a-2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2b^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}} - \frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}$$

output
```
1/2*(a-2*b)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(3/2)+arctanh
((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(1/2)-1/2*(a+b*tanh(x)^2
)^(1/2)*tanh(x)/b
```

3.230.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 5.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.36

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

$$= \frac{\left(\sqrt{2}a(a+b)\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}}{\sqrt{2}}\right),1\right) - 2\sqrt{2}ab\sqrt{(}\right.}{2\sqrt{2}b(a+}$$

input
```
Integrate[Tanh[x]^4/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
((Sqrt[2]*a*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Ellipt
icF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] -
2*Sqrt[2]*a*b*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticPi[b
/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]],
 1] - (a + b)*(a - b + (a + b)*Cosh[2*x])*Sech[x]^2)*Tanh[x])/(2*Sqrt[2]*b
*(a + b)*Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2])
```

3.230.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 4153, 381, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(ix)^4}{\sqrt{a-b\tan(ix)^2}}\,dx$$

$$\downarrow 4153$$

$$\int \frac{\tanh^4(x)}{\left(1-\tanh^2(x)\right)\sqrt{a+b\tanh^2(x)}}\,d\tanh(x)$$

$$\downarrow 381$$

$$\frac{\int \frac{a-(a-2b)\tanh^2(x)}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{2b} - \frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}$$

$$\downarrow 398$$

$$\frac{(a-2b)\int \frac{1}{\sqrt{b\tanh^2(x)+a}}d\tanh(x) + 2b\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{2b} - \frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}$$

$$\downarrow 224$$

$$\frac{2b\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x) + (a-2b)\int \frac{1}{1-\frac{b\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}}{2b} - \frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}$$

$$\downarrow 219$$

$$\frac{2b\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x) + \frac{(a-2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}}{2b} - \frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}$$

$$\downarrow 291$$

$$\frac{2b\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}} + \frac{(a-2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}}{2b} - \frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}$$

$$\downarrow 219$$

$$\frac{\frac{(a-2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}} + \frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}}{2b} - \frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}$$

input
```
Int[Tanh[x]^4/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
(((a - 2*b)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[b] + (2
*b*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[a + b])/(2*b
) - (Tanh[x]*Sqrt[a + b*Tanh[x]^2])/(2*b)
```

3.230. $\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}dx$

3.230.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.230.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(70) = 140$.

Time = 0.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.02

method	result
derivativedivides	$-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{\sqrt{b}}-\frac{\sqrt{a+b\tanh(x)^2}\,\tanh(x)}{2b}+\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{2b^{\frac{3}{2}}}-\frac{\ln\left(\frac{2a}{}\right.}{}$
default	$-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{\sqrt{b}}-\frac{\sqrt{a+b\tanh(x)^2}\,\tanh(x)}{2b}+\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{2b^{\frac{3}{2}}}-\frac{\ln\left(\frac{2a}{}\right.}{}$

input
```
int(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2))/b^(1/2)-1/2*(a+b*tanh(x)^2)^(1/
2)*tanh(x)/b+1/2*a/b^(3/2)*ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2))-1/2/(
a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*
(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))+1/2/(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh
(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)
-1))
```

3.230.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(70) = 140$.

Time = 0.48 (sec) , antiderivative size = 5494, normalized size of antiderivative = 62.43

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.230.6 Sympy [F]

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

input
```
integrate(tanh(x)**4/(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(tanh(x)**4/sqrt(a + b*tanh(x)**2), x)
```

3.230.7 Maxima [F]

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh(x)^4}{\sqrt{b\tanh(x)^2+a}}\,dx$$

input
```
integrate(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^4/sqrt(b*tanh(x)^2 + a), x)
```

3.230.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(70) = 140$.

Time = 0.57 (sec) , antiderivative size = 559, normalized size of antiderivative = 6.35

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{(a-2b)\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}b}$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$
$$-\frac{2\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^3(a+2b)+\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}\right.\right.}{\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x}\right.\right.}$$

input
```
integrate(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
(a - 2*b)*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/(sqrt(-b)*b) -
 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) -
1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x
) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) + 1/2*log(abs(-sqrt(a
 + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) - sqrt(a + b)))/sqrt(a + b) - 2*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*
x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3*(a + 2*b) + (sqrt(a
 + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b))^2*(3*a - 2*b)*sqrt(a + b) + (3*a^2 - 3*a*b - 2*b^2)*(sqrt(a + b)*e^
(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) +
 (a^2 - a*b + 2*b^2)*sqrt(a + b))/(((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*
x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt
(a + b) + a - 3*b)^2*b)
```

3.230.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh(x)^4}{\sqrt{b\tanh(x)^2+a}}\,dx$$

input
```
int(tanh(x)^4/(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(tanh(x)^4/(a + b*tanh(x)^2)^(1/2), x)
```

3.231 $\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

3.231.1 Optimal result . . . 1706
3.231.2 Mathematica [A] (verified) . . . 1706
3.231.3 Rubi [A] (verified) . . . 1707
3.231.4 Maple [B] (verified) . . . 1709
3.231.5 Fricas [B] (verification not implemented) . . . 1710
3.231.6 Sympy [F] . . . 1710
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3.231.8 Giac [B] (verification not implemented) . . . 1711
3.231.9 Mupad [B] (verification not implemented) . . . 1712

3.231.1 Optimal result

Integrand size = 17, antiderivative size = 47

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\sqrt{a+b\tanh^2(x)}}{b}$$

output
```
arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)-(a+b*tanh(x)^2)^(1/
2)/b
```

3.231.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\sqrt{a+b\tanh^2(x)}}{b}$$

input
```
Integrate[Tanh[x]^3/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] - Sqrt[a + b*Tanh[x
]^2]/b
```

3.231.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 26, 4153, 26, 354, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{i\tan(ix)^3}{\sqrt{a-b\tan(ix)^2}}dx$$

$$\downarrow 26$$

$$i\int \frac{\tan(ix)^3}{\sqrt{a-b\tan(ix)^2}}dx$$

$$\downarrow 4153$$

$$i\int -\frac{i\tanh^3(x)}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\tanh^3(x)}{(1-\tanh^2(x))\sqrt{a+b\tanh^2(x)}}d\tanh(x)$$

$$\downarrow 354$$

$$\frac{1}{2}\int \frac{\tanh^2(x)}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)$$

$$\downarrow 90$$

$$\frac{1}{2}\left(\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh^2(x) - \frac{2\sqrt{a+b\tanh^2(x)}}{b}\right)$$

$$\downarrow 73$$

$$\frac{1}{2}\left(\frac{2\int\frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}-\frac{2\sqrt{a+b\tanh^2(x)}}{b}\right)$$

$$\downarrow 221$$

$$\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{2\sqrt{a+b\tanh^2(x)}}{b}\right)$$

input
```
Int[Tanh[x]^3/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]])/Sqrt[a + b] - (2*Sqrt[a +
b*Tanh[x]^2])/b)/2
```

3.231.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.231.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(39) = 78$.

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.74

method	result
derivativedivides	$-\frac{\sqrt{a+b\tanh(x)^2}}{b}+\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}}+\frac{\ln\left(\frac{2a+2b-2b(1+\tanh($
default	$-\frac{\sqrt{a+b\tanh(x)^2}}{b}+\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}}+\frac{\ln\left(\frac{2a+2b-2b(1+\tanh($

input
```
int(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(a+b*tanh(x)^2)^(1/2)/b+1/2/(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+
b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1/2/(a+
b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1
+tanh(x))+a+b)^(1/2))/(1+tanh(x)))
```

3.231.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(39) = 78.

Time = 0.38 (sec) , antiderivative size = 1625, normalized size of antiderivative = 34.57

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(a + b)*lo
g(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^
2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3
+ a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 +
 a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (
70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a
^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a
^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3
 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(
a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b -
b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^
2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 +
3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*
sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*c
osh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*c
osh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(
a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2
*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 +
 a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3
*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*c...
```

3.231.6 Sympy [F]

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

input
```
integrate(tanh(x)**3/(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(tanh(x)**3/sqrt(a + b*tanh(x)**2), x)
```

3.231.7 Maxima [F]

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh(x)^3}{\sqrt{b\tanh(x)^2+a}}\,dx$$

input
```
integrate(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^3/sqrt(b*tanh(x)^2 + a), x)
```

3.231.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(39) = 78$.

Time $= 0.49$ (sec) , antiderivative size $= 345$, normalized size of antiderivative $= 7.34$

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx =$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$
$$-\frac{4\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\right.}{\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2+2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+\right.}$$

input
```
integrate(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) +
1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x
) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) - 1/2*log(abs(-sqrt(a
 + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) - sqrt(a + b)))/sqrt(a + b) - 4*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x
) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/((sqrt(a
 + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*
x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)
```

3.231.9 Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\sqrt{b\tanh(x)^2+a}}{b}$$

input
```
int(tanh(x)^3/(a + b*tanh(x)^2)^(1/2),x)
```

output
```
atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(1/2) - (a + b*tanh(x
)^2)^(1/2)/b
```

3.232 $\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

3.232.1 Optimal result 1713
3.232.2 Mathematica [C] (verified) 1713
3.232.3 Rubi [A] (verified) 1714
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3.232.5 Fricas [B] (verification not implemented) 1717
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3.232.7 Maxima [F] 1718
3.232.8 Giac [B] (verification not implemented) 1718
3.232.9 Mupad [F(-1)] 1719

3.232.1 Optimal result

Integrand size = 17, antiderivative size = 60

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}$$

output

```
-arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(1/2)+arctanh((a+b)^(1/2
)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(1/2)
```

3.232.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.68

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = -\frac{a\coth(x)\,\text{EllipticPi}\left(\frac{b}{a+b}, \arcsin\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}}{\sqrt{2}}\right), 1\right)\sqrt{(a-b+(a+b)\cosh(2x))\operatorname{sech}^2(x}}{b(a+b)\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}}$$

input
```
Integrate[Tanh[x]^2/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
-((a*Coth[x]*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x]
)*Csch[x]^2)/b]/Sqrt[2]], 1]*Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2])/
(b*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]))
```

3.232.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 25, 4153, 25, 385, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\tan(ix)^2}{\sqrt{a-b\tan(ix)^2}}dx$$

$$\downarrow 25$$

$$-\int \frac{\tan(ix)^2}{\sqrt{a-b\tan(ix)^2}}dx$$

$$\downarrow 4153$$

$$-\int -\frac{\tanh^2(x)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)$$

$$\downarrow 25$$

$$\int \frac{\tanh^2(x)}{\left(1-\tanh^2(x)\right)\sqrt{a+b\tanh^2(x)}}d\tanh(x)$$

$$\downarrow 385$$

$$\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x) - \int \frac{1}{\sqrt{b\tanh^2(x)+a}}d\tanh(x)$$

$$\downarrow 224$$

$$\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x) - \int \frac{1}{1-\frac{b\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}$$

$$\downarrow 219$$

$$\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x) - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}$$

$$\downarrow 291$$

$$\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}$$

$$\downarrow 219$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}$$

input
```
Int[Tanh[x]^2/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
-(ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[b]) + ArcTanh[(Sqr
t[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b]
```

3.232.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 385
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2),
x_Symbol] :> Simp[e^2/b   Int[(e*x)^(m - 2)*(c + d*x^2)^q, x], x] - Simp[a*
(e^2/b)   Int[(e*x)^(m - 2)*((c + d*x^2)^q/(a + b*x^2)), x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3] && IntBinomial
Q[a, b, c, d, e, m, 2, -1, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.232.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(48) = 96$.

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.28

method	result
derivativedivides	$-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{\sqrt{b}} - \frac{\ln\left(\frac{2a+2b-2b(1+\tanh(x))+2\sqrt{a+b}\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}{1+\tanh(x)}\right)}{2\sqrt{a+b}} + \frac{\ln($
default	$-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{\sqrt{b}} - \frac{\ln\left(\frac{2a+2b-2b(1+\tanh(x))+2\sqrt{a+b}\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}{1+\tanh(x)}\right)}{2\sqrt{a+b}} + \frac{\ln($

input
```
int(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2))/b^(1/2)-1/2/(a+b)^(1/2)*ln((2*a
+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(
1/2))/(1+tanh(x)))+1/2/(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/
2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))
```

3.232.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(48) = 96$.

Time $= 0.44$ (sec) , antiderivative size $= 3361$, normalized size of antiderivative $= 56.02$

$$\int \frac{\tanh^2(x)}{\sqrt{a + b\tanh^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a + b)*b*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x
)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a
*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)
*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2
 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2
+ 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*co
sh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*c
osh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b
^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)
^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*
sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)
^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh
(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b
^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2
*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*
cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2
+ a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)
*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)
*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh
(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*co...
```

3.232.6 Sympy [F]

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

input
```
integrate(tanh(x)**2/(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(tanh(x)**2/sqrt(a + b*tanh(x)**2), x)
```

3.232.7 Maxima [F]

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh(x)^2}{\sqrt{b\tanh(x)^2+a}}\,dx$$

input
```
integrate(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^2/sqrt(b*tanh(x)^2 + a), x)
```

3.232.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(48) = 96.

Time = 0.52 (sec) , antiderivative size = 252, normalized size of antiderivative = 4.20

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = -\frac{2\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}}$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$

input
```
integrate(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-2*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(
2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*log(ab
s(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e
^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 1/2*log(abs
(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(
2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) + 1/2*log(abs(-sqrt(a + b)*e^(2*
x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqr
t(a + b)))/sqrt(a + b)
```

3.232.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh(x)^2}{\sqrt{b\tanh(x)^2+a}}\,dx$$

input
```
int(tanh(x)^2/(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(tanh(x)^2/(a + b*tanh(x)^2)^(1/2), x)
```

3.233 $\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

3.233.1 Optimal result . . . 1720
3.233.2 Mathematica [A] (verified) . . . 1720
3.233.3 Rubi [A] (verified) . . . 1721
3.233.4 Maple [B] (verified) . . . 1723
3.233.5 Fricas [B] (verification not implemented) . . . 1723
3.233.6 Sympy [A] (verification not implemented) . . . 1724
3.233.7 Maxima [F] . . . 1725
3.233.8 Giac [B] (verification not implemented) . . . 1725
3.233.9 Mupad [B] (verification not implemented) . . . 1726

3.233.1 Optimal result

Integrand size = 15, antiderivative size = 29

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

output
```
arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)
```

3.233.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

input
```
Integrate[Tanh[x]/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b]
```

3.233.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 26, 4153, 26, 353, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\tan(ix)}{\sqrt{a-b\tan(ix)^2}}dx$$

$$\downarrow 26$$

$$-i\int \frac{\tan(ix)}{\sqrt{a-b\tan(ix)^2}}dx$$

$$\downarrow 4153$$

$$-i\int \frac{i\tanh(x)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\tanh(x)}{\left(1-\tanh^2(x)\right)\sqrt{a+b\tanh^2(x)}}d\tanh(x)$$

$$\downarrow 353$$

$$\frac{1}{2}\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)$$

$$\downarrow 73$$

$$\frac{\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}$$

$$\downarrow 221$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

input
```
Int[Tanh[x]/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b]
```

3.233.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.233.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(23) = 46$.

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.93

method	result
derivativedivides	$\frac{\ln\left(\frac{2a+2b-2b(1+\tanh(x))+2\sqrt{a+b}\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}{1+\tanh(x)}\right)}{2\sqrt{a+b}}+\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\sqrt{b(\tanh(x)-1}}{\tanh(x)-1}\right.}{2\sqrt{a+b}}$
default	$\frac{\ln\left(\frac{2a+2b-2b(1+\tanh(x))+2\sqrt{a+b}\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}{1+\tanh(x)}\right)}{2\sqrt{a+b}}+\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\sqrt{b(\tanh(x)-1}}{\tanh(x)-1}\right.}{2\sqrt{a+b}}$

input
```
int(tanh(x)/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2
-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))+1/2/(a+b)^(1/2)*ln((2*a+2*b+2*b*
(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(ta
nh(x)-1))
```

3.233.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(23) = 46$.

Time = 0.31 (sec) , antiderivative size = 1361, normalized size of antiderivative = 46.93

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*s
inh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^
3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*co
sh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2
+ b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 +
 b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(
x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh
(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)
*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4
+ 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sin
h(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6
+ 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)
^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*
cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*
b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cos
h(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*co
sh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*co
sh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)
^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(...
```

3.233.6 Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.34

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = -\begin{cases} \frac{\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}} & \text{for } b \neq 0 \\ \begin{cases} \tilde{\infty}\tanh^2(x) & \text{for } \sqrt{a} = 0 \\ \frac{\log\left(2\sqrt{a}\tanh^2(x) - 2\sqrt{a}\right)}{2\sqrt{a}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)**2)**(1/2),x)
```

output
```
-Piecewise((atan(sqrt(a + b*tanh(x)**2)/sqrt(-a - b))/sqrt(-a - b), Ne(b,
0)), (Piecewise((zoo*tanh(x)**2, Eq(sqrt(a), 0)), (log(2*sqrt(a)*tanh(x)**
2 - 2*sqrt(a))/(2*sqrt(a)), True)), True))
```

3.233.7 Maxima [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{b\tanh(x)^2+a}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)/sqrt(b*tanh(x)^2 + a), x)
```

3.233.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(23) = 46$.

Time = 0.41 (sec) , antiderivative size = 188, normalized size of antiderivative = 6.48

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx =$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) +
1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x
) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) - 1/2*log(abs(-sqrt(a
 + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) - sqrt(a + b)))/sqrt(a + b)
```

3.233.9 Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

input
```
int(tanh(x)/(a + b*tanh(x)^2)^(1/2),x)
```

output
```
atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(1/2)
```

3.234 $\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx$

3.234.1 Optimal result 1727
3.234.2 Mathematica [A] (verified) 1727
3.234.3 Rubi [A] (verified) 1728
3.234.4 Maple [B] (verified) 1729
3.234.5 Fricas [B] (verification not implemented) 1730
3.234.6 Sympy [F] 1731
3.234.7 Maxima [F] 1731
3.234.8 Giac [B] (verification not implemented) 1731
3.234.9 Mupad [B] (verification not implemented) 1732

3.234.1 Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}$$

output
```
arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(1/2)
```

3.234.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}$$

input
```
Integrate[1/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b]
```

3.234.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4144, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sqrt{a-b\tan(ix)^2}}dx$$

$$\downarrow 4144$$

$$\int \frac{1}{(1-\tanh^2(x))\sqrt{a+b\tanh^2(x)}}d\tanh(x)$$

$$\downarrow 291$$

$$\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{a+b\tanh^2(x)}}d\frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}$$

$$\downarrow 219$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}$$

input
```
Int[1/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b]
```

3.234.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.234.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(25) = 50$.

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.68

method	result
derivativedivides	$\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}} - \frac{\ln\left(\frac{2a+2b-2b(1+\tanh(x))+2\sqrt{a+b}\sqrt{b(1+\tanh(x)}}{1+\tanh(x)}\right.}{2\sqrt{a+b}}$
default	$\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}} - \frac{\ln\left(\frac{2a+2b-2b(1+\tanh(x))+2\sqrt{a+b}\sqrt{b(1+\tanh(x)}}{1+\tanh(x)}\right.}{2\sqrt{a+b}}$

input
```
int(1/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2
+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))-1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*
(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+
tanh(x)))
```

3.234.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(25) = 50$.

Time = 0.30 (sec) , antiderivative size = 1287, normalized size of antiderivative = 41.52

$$\int \frac{1}{\sqrt{a + b\tanh^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*
sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b
^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*c
osh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 +
 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 +
6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh
(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cos
h(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3
)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4
 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*si
nh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6
 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x
)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2
*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a
*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*co
sh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*c
osh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*c
osh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x
)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh...
```

3.234.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx$$

input
```
integrate(1/(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*tanh(x)**2), x)
```

3.234.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{1}{\sqrt{b\tanh(x)^2+a}}\,dx$$

input
```
integrate(1/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*tanh(x)^2 + a), x)
```

3.234.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(25) = 50$.

Time $= 0.39$ (sec) , antiderivative size $= 188$, normalized size of antiderivative $= 6.06$

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx =$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$

input
```
integrate(1/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) -
1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x
) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) + 1/2*log(abs(-sqrt(a
 + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) - sqrt(a + b)))/sqrt(a + b)
```

3.234.9 Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{atanh}\left(\frac{\tanh(x)\sqrt{a+b}}{\sqrt{b\tanh(x)^2+a}}\right)}{\sqrt{a+b}}$$

input
```
int(1/(a + b*tanh(x)^2)^(1/2),x)
```

output
```
atanh((tanh(x)*(a + b)^(1/2))/(a + b*tanh(x)^2)^(1/2))/(a + b)^(1/2)
```

3.235 $\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

3.235.1 Optimal result . . . 1733
3.235.2 Mathematica [A] (verified) . . . 1733
3.235.3 Rubi [A] (verified) . . . 1734
3.235.4 Maple [F] . . . 1736
3.235.5 Fricas [B] (verification not implemented) . . . 1736
3.235.6 Sympy [F] . . . 1737
3.235.7 Maxima [F] . . . 1738
3.235.8 Giac [B] (verification not implemented) . . . 1738
3.235.9 Mupad [F(-1)] . . . 1739

3.235.1 Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

output
```
-arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(1/2)+arctanh((a+b*tanh(x)^2)^(1
/2)/(a+b)^(1/2))/(a+b)^(1/2)
```

3.235.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

input
```
Integrate[Coth[x]/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
-(ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]/Sqrt[a]) + ArcTanh[Sqrt[a + b*Tan
h[x]^2]/Sqrt[a + b]]/Sqrt[a + b]
```

3.235.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 26, 4153, 26, 354, 97, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{i}{\tan(ix)\sqrt{a-b\tan(ix)^2}}dx$$

$$\downarrow 26$$

$$i\int \frac{1}{\tan(ix)\sqrt{a-b\tan(ix)^2}}dx$$

$$\downarrow 4153$$

$$i\int -\frac{i\coth(x)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\coth(x)}{\left(1-\tanh^2(x)\right)\sqrt{a+b\tanh^2(x)}}d\tanh(x)$$

$$\downarrow 354$$

$$\frac{1}{2}\int \frac{\coth(x)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)$$

$$\downarrow 97$$

$$\frac{1}{2}\left(\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)+\int \frac{\coth(x)}{\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)\right)$$

$$\downarrow 73$$

$$\frac{1}{2}\left(\frac{2\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}+\frac{2\int \frac{1}{\frac{\tanh^4(x)}{b}-\frac{a}{b}}d\sqrt{b\tanh^2(x)+a}}{b}\right)$$

$$\downarrow 221$$

$$\frac{1}{2}\left(\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}\right)$$

input
```
Int[Coth[x]/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
((-2*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/Sqrt[a] + (2*ArcTanh[Sqrt[a +
 b*Tanh[x]^2]/Sqrt[a + b]])/Sqrt[a + b])/2
```

3.235.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.235.4 Maple [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh(x)^2}}dx$$

input
```
int(coth(x)/(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)/(a+b*tanh(x)^2)^(1/2),x)
```

3.235.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(44) = 88$.

Time $= 0.38$ (sec) , antiderivative size $= 3527$, normalized size of antiderivative $= 62.98$

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}dx = \text{Too large to display}$$

input
```
integrate(coth(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a + b)*a*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)
*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*
a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*
cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^
2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2
 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cos
h(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*co
sh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^
3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^
4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*s
inh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^
6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(
x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^
2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*
a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*c
osh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
 a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*
cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*
cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(
x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cos...
```

3.235.6 Sympy [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

input
```
integrate(coth(x)/(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(coth(x)/sqrt(a + b*tanh(x)**2), x)
```

3.235.7 Maxima [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{b\tanh(x)^2+a}}\,dx$$

input
```
integrate(coth(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)/sqrt(b*tanh(x)^2 + a), x)
```

3.235.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(44) = 88$.

Time = 0.51 (sec) , antiderivative size = 254, normalized size of antiderivative = 4.54

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{2\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}}$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$

input
```
integrate(coth(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/sqrt(-a))/sqrt(-a) - 1/2*log(abs
(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^
(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) + 1/2*log(abs(
-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b) + sqrt(a + b)))/sqrt(a + b) - 1/2*log(abs(-sqrt(a + b)*e^(2*x
) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt
(a + b)))/sqrt(a + b)
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{b\tanh(x)^2+a}}\,dx$$

input
```
int(coth(x)/(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)/(a + b*tanh(x)^2)^(1/2), x)
```

3.236 $\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

3.236.1 Optimal result 1740
3.236.2 Mathematica [C] (warning: unable to verify) 1740
3.236.3 Rubi [A] (verified) 1741
3.236.4 Maple [F] 1743
3.236.5 Fricas [B] (verification not implemented) 1743
3.236.6 Sympy [F] 1744
3.236.7 Maxima [F] 1745
3.236.8 Giac [B] (verification not implemented) 1745
3.236.9 Mupad [F(-1)] 1746

3.236.1 Optimal result

Integrand size = 17, antiderivative size = 51

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}} - \frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}$$

output
```
arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(1/2)-coth(x)*(a+
b*tanh(x)^2)^(1/2)/a
```

3.236.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.85 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.55

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx =$$

$$-\frac{\cosh^2(x)\coth(x)\left(1+\frac{b\tanh^2(x)}{a}\right)\left(-\frac{4(a+b)\,\text{Hypergeometric2F1}\left(2,2,\frac{5}{2},-\frac{(a+b)\sinh^2(x)}{a}\right)\sinh^2(x)\left(a+b\tanh^2(x)\right)}{3a^2}+\frac{\arcsin\left(\sqrt{\ldots}\right.}{a\sqrt{-\frac{(a\ldots}{\ldots}}}\right.}{\sqrt{a+b\tanh^2(x)}}$$

input
```
Integrate[Coth[x]^2/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
-((Cosh[x]^2*Coth[x]*(1 + (b*Tanh[x]^2)/a)*((-4*(a + b)*Hypergeometric2F1[
2, 2, 5/2, -(((a + b)*Sinh[x]^2)/a)]*Sinh[x]^2*(a + b*Tanh[x]^2))/(3*a^2)
+ (ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*(a + 2*b*Tanh[x]^2))/(a*Sqrt[-((
(a + b)*Cosh[x]^2*Sinh[x]^2*(a + b*Tanh[x]^2))/a^2)])))/Sqrt[a + b*Tanh[x]
^2])
```

3.236.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 25, 4153, 25, 382, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\tan(ix)^2\sqrt{a-b\tan(ix)^2}}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\tan(ix)^2\sqrt{a-b\tan(ix)^2}}dx$$

$$\downarrow 4153$$

$$-\int -\frac{\coth^2(x)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)$$

$$\downarrow 25$$

$$\int \frac{\coth^2(x)}{\left(1-\tanh^2(x)\right)\sqrt{a+b\tanh^2(x)}}d\tanh(x)$$

$$\downarrow 382$$

$$\frac{\int \frac{a}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{a} - \frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}$$

$$\downarrow 27$$

$$\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x) - \frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}$$

$$\downarrow 291$$

$$\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}} - \frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}$$

$$\downarrow 219$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}} - \frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}$$

input
```
Int[Coth[x]^2/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b] - (Coth[x
]*Sqrt[a + b*Tanh[x]^2])/a
```

3.236.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.236.4 Maple [F]

$$\int \frac{\coth(x)^2}{\sqrt{a+b\tanh(x)^2}}dx$$

input
```
int(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x)
```

3.236.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(43) = 86$.

Time $= 0.32$ (sec) , antiderivative size $= 1565$, normalized size of antiderivative $= 30.69$

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(a + b)*lo
g(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 +
 b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b
^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2
 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 +
(70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 +
2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 +
2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^
3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*
(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2
*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b
^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 +
 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))
*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*
cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*
cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt
(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2
+ 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*
a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*...
```

3.236.6 Sympy [F]

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

input
```
integrate(coth(x)**2/(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(coth(x)**2/sqrt(a + b*tanh(x)**2), x)
```

3.236.7 Maxima [F]

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\coth(x)^2}{\sqrt{b\tanh(x)^2+a}}\,dx$$

input
```
integrate(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)^2/sqrt(b*tanh(x)^2 + a), x)
```

3.236.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(43) = 86$.

Time = 0.48 (sec) , antiderivative size = 343, normalized size of antiderivative = 6.73

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx =$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$
$$+\frac{4\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\right.}{\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2-2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+\right.}$$

input
```
integrate(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) -
1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x
) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) + 1/2*log(abs(-sqrt(a
 + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) - sqrt(a + b)))/sqrt(a + b) + 4*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x
) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/((sqrt(a
 + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*
x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a + b)
```

3.236.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\coth(x)^2}{\sqrt{b\tanh(x)^2+a}}\,dx$$

input
```
int(coth(x)^2/(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^2/(a + b*tanh(x)^2)^(1/2), x)
```

3.237 $\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

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3.237.1 Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = -\frac{(2a-b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\coth^2(x)\sqrt{a+b\tanh^2(x)}}{2a}$$

output
```
-1/2*(2*a-b)*arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(3/2)+arctanh((a+b*t
anh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)-1/2*coth(x)^2*(a+b*tanh(x)^2)^(1/
2)/a
```

3.237.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.22

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\left(-2a^2-ab+b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right) + \sqrt{a}\left(2a\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - (a+b)\coth^2(x)\sqrt{a}\right.}{2a^{3/2}(a+b)}$$

input
```
Integrate[Coth[x]^3/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
((-2*a^2 - a*b + b^2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]] + Sqrt[a]*(2*
a*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (a + b)*Coth[x]
^2*Sqrt[a + b*Tanh[x]^2]))/(2*a^(3/2)*(a + b))
```

3.237.3 Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {3042, 26, 4153, 26, 354, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i}{\tan(ix)^3\sqrt{a-b\tan(ix)^2}}dx$$

$$\downarrow 26$$

$$-i\int \frac{1}{\tan(ix)^3\sqrt{a-b\tan(ix)^2}}dx$$

$$\downarrow 4153$$

$$-i\int \frac{i\coth^3(x)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\coth^3(x)}{\left(1-\tanh^2(x)\right)\sqrt{a+b\tanh^2(x)}}d\tanh(x)$$

$$\downarrow 354$$

$$\frac{1}{2}\int \frac{\coth^2(x)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)$$

$$\downarrow 114$$

$$\frac{1}{2}\left(-\frac{\int -\frac{\coth(x)\left(b\tanh^2(x)+2a-b\right)}{2\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a}-\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right)$$

$$\downarrow 27$$

$$\frac{1}{2}\left(\frac{\int \frac{\coth(x)\left(b\tanh^2(x)+2a-b\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{2a}-\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right)$$

$$\downarrow 174$$

$$\frac{1}{2}\left(\frac{2a\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)+(2a-b)\int \frac{\coth(x)}{\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{2a}-\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right)$$

$$\downarrow 73$$

$$\frac{1}{2}\left(\frac{\frac{4a\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}+\frac{2(2a-b)\int \frac{1}{\frac{\tanh^4(x)}{b}-\frac{a}{b}}d\sqrt{b\tanh^2(x)+a}}{b}}{2a}-\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right)$$

$$\downarrow 221$$

$$\frac{1}{2}\left(\frac{\frac{4a\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{2(2a-b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}}{2a}-\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right)$$

input
```
Int[Coth[x]^3/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
(((-2*(2*a - b)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/Sqrt[a] + (4*a*Arc
Tanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]])/Sqrt[a + b])/(2*a) - (Coth[x]*Sqr
t[a + b*Tanh[x]^2])/a)/2
```

3.237.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.237.4 Maple [F]

$$\int \frac{\coth(x)^3}{\sqrt{a+b\tanh(x)^2}}dx$$

input
```
int(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x)
```

3.237.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1112 vs. $2(70) = 140$.

Time $= 0.49$ (sec) , antiderivative size $= 5711$, normalized size of antiderivative $= 64.90$

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.237.6 Sympy [F]

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

input
```
integrate(coth(x)**3/(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(coth(x)**3/sqrt(a + b*tanh(x)**2), x)
```

3.237.7 Maxima [F]

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\coth(x)^3}{\sqrt{b\tanh(x)^2+a}}\,dx$$

input
```
integrate(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)^3/sqrt(b*tanh(x)^2 + a), x)
```

3.237.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(70) = 140$.

Time = 0.62 (sec) , antiderivative size = 565, normalized size of antiderivative = 6.42

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{(2a-b)\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}a}$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$
$$+\frac{2\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^3(2a+b)+\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}\right.\right.}}{\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x}\right.\right.}$$

input
```
integrate(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
(2*a - b)*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/sqrt(-a))/(sqrt(-a)*a) -
 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) +
1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x
) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) - 1/2*log(abs(-sqrt(a
 + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) - sqrt(a + b)))/sqrt(a + b) + 2*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*
x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3*(2*a + b) + (sqrt(a
 + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b))^2*(2*a - 3*b)*sqrt(a + b) - (2*a^2 + 3*a*b - 3*b^2)*(sqrt(a + b)*e^
(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) -
 (2*a^2 - a*b + b^2)*sqrt(a + b))/(((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*
x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt
(a + b) - 3*a + b)^2*a)
```

3.237.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\coth(x)^3}{\sqrt{b\tanh(x)^2+a}}\,dx$$

input
```
int(coth(x)^3/(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^3/(a + b*tanh(x)^2)^(1/2), x)
```

3.238 $\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

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3.238.3 Rubi [A] (verified) 1756
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3.238.8 Giac [B] (verification not implemented) 1760
3.238.9 Mupad [B] (verification not implemented) 1761

3.238.1 Optimal result

Integrand size = 17, antiderivative size = 72

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{a^2}{b^2(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\sqrt{a+b\tanh^2(x)}}{b^2}$$

output
```
arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)-a^2/b^2/(a+b)/(a+b*
tanh(x)^2)^(1/2)-(a+b*tanh(x)^2)^(1/2)/b^2
```

3.238.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{-b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\tanh^2(x)}{a+b}\right) - (a+b)\left(2a-b+b\tanh^2(x)\right)}{b^2(a+b)\sqrt{a+b\tanh^2(x)}}$$

input
```
Integrate[Tanh[x]^5/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
(-(b^2*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)]) - (a +
b)*(2*a - b + b*Tanh[x]^2))/(b^2*(a + b)*Sqrt[a + b*Tanh[x]^2])
```

3.238.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 26, 4153, 26, 354, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\tan(ix)^5}{\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\downarrow 26$$

$$-i\int \frac{\tan(ix)^5}{\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\downarrow 4153$$

$$-i\int \frac{i\tanh^5(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\tanh^5(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{3/2}}d\tanh(x)$$

$$\downarrow 354$$

$$\frac{1}{2}\int \frac{\tanh^4(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh^2(x)$$

$$\downarrow 98$$

$$\frac{1}{2}\int\left(\frac{a^2}{b(a+b)\left(b\tanh^2(x)+a\right)^{3/2}}-\frac{1}{b\sqrt{b\tanh^2(x)+a}}-\frac{1}{(a+b)\left(\tanh^2(x)-1\right)\sqrt{b\tanh^2(x)+a}}\right)d\tanh^2(x)$$

$$\downarrow 2009$$

$$\frac{1}{2}\left(-\frac{2a^2}{b^2(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{2\sqrt{a+b\tanh^2(x)}}{b^2}\right)$$

input
```
Int[Tanh[x]^5/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]])/(a + b)^(3/2) - (2*a^2)/(b
^2*(a + b)*Sqrt[a + b*Tanh[x]^2]) - (2*Sqrt[a + b*Tanh[x]^2])/b^2)/2
```

3.238.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.238.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(62) = 124$.

Time = 0.10 (sec) , antiderivative size = 322, normalized size of antiderivative = 4.47

method	result
derivativedivides	$\frac{1}{b\sqrt{a+b\tanh(x)^2}} - \frac{\tanh(x)^2}{b\sqrt{a+b\tanh(x)^2}} - \frac{2a}{b^2\sqrt{a+b\tanh(x)^2}} - \frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}} + \frac{}{(}$
default	$\frac{1}{b\sqrt{a+b\tanh(x)^2}} - \frac{\tanh(x)^2}{b\sqrt{a+b\tanh(x)^2}} - \frac{2a}{b^2\sqrt{a+b\tanh(x)^2}} - \frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}} + \frac{}{(}$

input

```
int(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/b/(a+b*tanh(x)^2)^(1/2)-tanh(x)^2/b/(a+b*tanh(x)^2)^(1/2)-2*a/b^2/(a+b*t
anh(x)^2)^(1/2)-1/2/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b/(a
+b)*(2*b*(tanh(x)-1)+2*b)/(4*(a+b)*b-4*b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-
1)+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b
*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))-1/2/(a+b)/(b*(1+ta
nh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-b/(a+b)*(2*b*(1+tanh(x))-2*b)/(4*(a+b)
*b-4*b^2)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((
2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b
)^(1/2))/(1+tanh(x)))
```

3.238.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1713 vs. 2(62) = 124.

Time = 0.49 (sec) , antiderivative size = 3991, normalized size of antiderivative = 55.43

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((a*b^2 + b^3)*cosh(x)^6 + 6*(a*b^2 + b^3)*cosh(x)*sinh(x)^5 + (a*b^
2 + b^3)*sinh(x)^6 + (3*a*b^2 - b^3)*cosh(x)^4 + (3*a*b^2 - b^3 + 15*(a*b^
2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a*b^2 + b^3)*cosh(x)^3 + (3*a*b^2 -
b^3)*cosh(x))*sinh(x)^3 + a*b^2 + b^3 + (3*a*b^2 - b^3)*cosh(x)^2 + (15*(a
*b^2 + b^3)*cosh(x)^4 + 3*a*b^2 - b^3 + 6*(3*a*b^2 - b^3)*cosh(x)^2)*sinh(
x)^2 + 2*(3*(a*b^2 + b^3)*cosh(x)^5 + 2*(3*a*b^2 - b^3)*cosh(x)^3 + (3*a*b
^2 - b^3)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(
a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*
b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 +
4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*
a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a
^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(
14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2
*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(
2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a
^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2
 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh
(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x
)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2
)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^...
```

3.238.6 Sympy [F]

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)**5/(a+b*tanh(x)**2)**(3/2),x)
```

output
```
Integral(tanh(x)**5/(a + b*tanh(x)**2)**(3/2), x)
```

3.238.7 Maxima [F]

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\tanh\left(x\right)^5}{\left(b\tanh\left(x\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^5/(b*tanh(x)^2 + a)^(3/2), x)
```

3.238.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 460 vs. $2(62) = 124$.

Time $= 0.57$ (sec) , antiderivative size $= 460$, normalized size of antiderivative $= 6.39$

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = -\frac{\frac{\left(a^4b+a^3b^2\right)e^{(2x)}}{a^3b^3+2a^2b^4+ab^5}+\frac{a^4b+a^3b^2}{a^3b^3+2a^2b^4+ab^5}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}}$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\left(a+b\right)^{\frac{3}{2}}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\left(a+b\right)^{\frac{3}{2}}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\left(a+b\right)^{\frac{3}{2}}}$$
$$-\frac{4\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\right.}{\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2+2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+\right.\right.}$$

input
```
integrate(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-((a^4*b + a^3*b^2)*e^(2*x)/(a^3*b^3 + 2*a^2*b^4 + a*b^5) + (a^4*b + a^3*b
^2)/(a^3*b^3 + 2*a^2*b^4 + a*b^5))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x
) - 2*b*e^(2*x) + a + b) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4
*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b
)*(a - b)))/(a + b)^(3/2) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4
*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a +
b)^(3/2) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) +
 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2) - 4*(sqr
t(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
+ a + b) - sqrt(a + b))/(((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(
a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) +
a - 3*b)*b)
```

3.238.9 Mupad [B] (verification not implemented)

Time = 3.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}\,(2a+2b)}{2(a+b)^{3/2}}\right)}{(a+b)^{3/2}} - \frac{\sqrt{b\tanh(x)^2+a}}{b^2} - \frac{a^2}{b^2\,(a+b)\,\sqrt{b\tanh(x)^2+a}}$$

input
```
int(tanh(x)^5/(a + b*tanh(x)^2)^(3/2),x)
```

output
```
atanh(((a + b*tanh(x)^2)^(1/2)*(2*a + 2*b))/(2*(a + b)^(3/2)))/(a + b)^(3/
2) - (a + b*tanh(x)^2)^(1/2)/b^2 - a^2/(b^2*(a + b)*(a + b*tanh(x)^2)^(1/2
))
```

3.239 $\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

3.239.1 Optimal result . . . 1762
3.239.2 Mathematica [C] (verified) . . . 1762
3.239.3 Rubi [A] (verified) . . . 1763
3.239.4 Maple [B] (verified) . . . 1766
3.239.5 Fricas [B] (verification not implemented) . . . 1766
3.239.6 Sympy [F] . . . 1767
3.239.7 Maxima [F] . . . 1767
3.239.8 Giac [B] (verification not implemented) . . . 1767
3.239.9 Mupad [F(-1)] . . . 1768

3.239.1 Optimal result

Integrand size = 17, antiderivative size = 84

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} + \frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}$$

output

```
-arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(3/2)+arctanh((a+b)^(1/2
)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(3/2)+a*tanh(x)/b/(a+b)/(a+b*tanh(x
)^2)^(1/2)
```

3.239.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.97 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.24

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = -\frac{a\left(-2a-2b+\sqrt{2}(a+b)\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}}{\sqrt{2}}\right),1\right)\right.}{\sqrt{2}b(a+b)^2\sqrt{(a-b+(a-}$$

input
```
Integrate[Tanh[x]^4/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
-((a*(-2*a - 2*b + Sqrt[2]*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[
x]^2)/b]*EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/
Sqrt[2]], 1] + Sqrt[2]*b*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*E
llipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b
]/Sqrt[2]], 1])*Tanh[x])/(Sqrt[2]*b*(a + b)^2*Sqrt[(a - b + (a + b)*Cosh[2
*x])*Sech[x]^2]))
```

3.239.3 Rubi [A] (verified)

Time $= 0.35$ (sec) , antiderivative size $= 99$, normalized size of antiderivative $= 1.18$, number of steps used $= 9$, number of rules used $= 8$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used $= \{3042, 4153, 372, 398, 224, 219, 291, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(ix)^4}{\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\downarrow 4153$$

$$\int \frac{\tanh^4(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{3/2}}d\tanh(x)$$

$$\downarrow 372$$

$$\frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\int \frac{a-(a+b)\tanh^2(x)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{b(a+b)}$$

$$\downarrow 398$$

$$\frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{(a+b)\int \frac{1}{\sqrt{b\tanh^2(x)+a}}d\tanh(x) - b\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{b(a+b)}$$

$$\downarrow 224$$

$$\frac{\frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{(a+b)\int \frac{1}{1-\frac{b\tanh^2(x)}{b\tanh^2(x)+a}} d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}} - b\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}} d\tanh(x)}{b(a+b)}}{b(a+b)}$$

$$\downarrow 219$$

$$\frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}} - b\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}} d\tanh(x)}{b(a+b)}$$

$$\downarrow 291$$

$$\frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}} - b\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}} d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}}{b(a+b)}$$

$$\downarrow 219$$

$$\frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}} - \frac{b\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}}{b(a+b)}$$

input
```
Int[Tanh[x]^4/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
-((((a + b)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[b] - (b
*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[a + b])/(b*(a
+ b))) + (a*Tanh[x])/(b*(a + b)*Sqrt[a + b*Tanh[x]^2])
```

3.239.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.239.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(70) = 140.

Time = 0.10 (sec) , antiderivative size = 328, normalized size of antiderivative = 3.90

method	result
derivativedivides	$-\frac{\tanh(x)}{a\sqrt{a+b\tanh(x)^2}}+\frac{\tanh(x)}{b\sqrt{a+b\tanh(x)^2}}-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{b^{\frac{3}{2}}}+\frac{1}{2(a+b)\sqrt{b(1+\tanh(x))^2-2b(1+t}$
default	$-\frac{\tanh(x)}{a\sqrt{a+b\tanh(x)^2}}+\frac{\tanh(x)}{b\sqrt{a+b\tanh(x)^2}}-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{b^{\frac{3}{2}}}+\frac{1}{2(a+b)\sqrt{b(1+\tanh(x))^2-2b(1+t}$

input
```
int(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-tanh(x)/a/(a+b*tanh(x)^2)^(1/2)+tanh(x)/b/(a+b*tanh(x)^2)^(1/2)-1/b^(3/2)
*ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2))+1/2/(a+b)/(b*(1+tanh(x))^2-2*b*
(1+tanh(x))+a+b)^(1/2)+b/(a+b)*(2*b*(1+tanh(x))-2*b)/(4*(a+b)*b-4*b^2)/(b*
(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-1/2/(a+b)^(3/2)*ln((2*a+2*b-2*b*(
1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+t
anh(x)))-1/2/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b/(a+b)*(2*
b*(tanh(x)-1)+2*b)/(4*(a+b)*b-4*b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)
^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(
x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))
```

3.239.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1434 vs. 2(70) = 140.

Time = 0.54 (sec) , antiderivative size = 6973, normalized size of antiderivative = 83.01

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.239.6 Sympy [F]

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)**4/(a+b*tanh(x)**2)**(3/2),x)
```

output
```
Integral(tanh(x)**4/(a + b*tanh(x)**2)**(3/2), x)
```

3.239.7 Maxima [F]

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\tanh(x)^4}{\left(b\tanh(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^4/(b*tanh(x)^2 + a)^(3/2), x)
```

3.239.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(70) = 140$.

Time $= 0.57$ (sec) , antiderivative size $= 371$, normalized size of antiderivative $= 4.42$

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{\frac{(a^3b^2+a^2b^3)e^{(2x)}}{a^3b^3+2a^2b^4+ab^5} - \frac{a^3b^2+a^2b^3}{a^3b^3+2a^2b^4+ab^5}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}}$$
$$-\frac{2\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}b}$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$

input
```
integrate(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
((a^3*b^2 + a^2*b^3)*e^(2*x)/(a^3*b^3 + 2*a^2*b^4 + a*b^5) - (a^3*b^2 + a^
2*b^3)/(a^3*b^3 + 2*a^2*b^4 + a*b^5))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(
2*x) - 2*b*e^(2*x) + a + b) - 2*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*
e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sq
rt(-b))/(sqrt(-b)*b) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a
 - b)))/(a + b)^(3/2) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a + b)^(
3/2) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a
*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2)
```

3.239.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\tanh(x)^4}{\left(b\tanh(x)^2+a\right)^{3/2}}\,dx$$

input
```
int(tanh(x)^4/(a + b*tanh(x)^2)^(3/2),x)
```

output
```
int(tanh(x)^4/(a + b*tanh(x)^2)^(3/2), x)
```

3.240 $\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

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3.240.1 Optimal result

Integrand size = 17, antiderivative size = 52

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + \frac{a}{b(a+b)\sqrt{a+b\tanh^2(x)}}$$

output
```
arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)+a/b/(a+b)/(a+b*tanh
(x)^2)^(1/2)
```

3.240.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + \frac{a}{b(a+b)\sqrt{a+b\tanh^2(x)}}$$

input
```
Integrate[Tanh[x]^3/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) + a/(b*(a + b)*Sq
rt[a + b*Tanh[x]^2])
```

3.240.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 26, 4153, 26, 354, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{i\tan(ix)^3}{\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\downarrow 26$$

$$i\int \frac{\tan(ix)^3}{\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\downarrow 4153$$

$$i\int -\frac{i\tanh^3(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\tanh^3(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{3/2}}d\tanh(x)$$

$$\downarrow 354$$

$$\frac{1}{2}\int \frac{\tanh^2(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh^2(x)$$

$$\downarrow 87$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a+b}+\frac{2a}{b(a+b)\sqrt{a+b\tanh^2(x)}}\right)$$

$$\downarrow 73$$

$$\frac{1}{2}\left(\frac{2\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b(a+b)}+\frac{2a}{b(a+b)\sqrt{a+b\tanh^2(x)}}\right)$$

$$\downarrow 221$$

$$\frac{1}{2}\left(\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\frac{2a}{b(a+b)\sqrt{a+b\tanh^2(x)}}\right)$$

input
```
Int[Tanh[x]^3/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]])/(a + b)^(3/2) + (2*a)/(b*(
a + b)*Sqrt[a + b*Tanh[x]^2]))/2
```

3.240.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.240.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(44) = 88$.

Time = 0.08 (sec) , antiderivative size = 287, normalized size of antiderivative = 5.52

method	result
derivativedivides	$\frac{1}{b\sqrt{a+b\tanh(x)^2}} - \frac{1}{2(a+b)\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}} - \frac{b(2b(1+\tanh(x))-2b)}{(a+b)(4(a+b)b-4b^2)\sqrt{b(1+\tanh(x))^2-2b(1+}$
default	$\frac{1}{b\sqrt{a+b\tanh(x)^2}} - \frac{1}{2(a+b)\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}} - \frac{b(2b(1+\tanh(x))-2b)}{(a+b)(4(a+b)b-4b^2)\sqrt{b(1+\tanh(x))^2-2b(1+}$

input
```
int(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b/(a+b*tanh(x)^2)^(1/2)-1/2/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^
(1/2)-b/(a+b)*(2*b*(1+tanh(x))-2*b)/(4*(a+b)*b-4*b^2)/(b*(1+tanh(x))^2-2*b
*(1+tanh(x))+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b
)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))-1/2/(a+b
)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b/(a+b)*(2*b*(tanh(x)-1)+2*b
)/(4*(a+b)*b-4*b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^
(3/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh
(x)-1)+a+b)^(1/2))/(tanh(x)-1))
```

3.240.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 980 vs. $2(44) = 88$.

Time $= 0.35$ (sec) , antiderivative size $= 2525$, normalized size of antiderivative $= 48.56$

$$\int \frac{\tanh^3(x)}{\left(a + b\tanh^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^
2)*sinh(x)^4 + 2*(a*b - b^2)*cosh(x)^2 + 2*(3*(a*b + b^2)*cosh(x)^2 + a*b
- b^2)*sinh(x)^2 + a*b + b^2 + 4*((a*b + b^2)*cosh(x)^3 + (a*b - b^2)*cosh
(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*c
osh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 +
 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a
^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b
- a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b -
 a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*
b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b
^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*
b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*co
sh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x
)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*si
nh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2
*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 +
(15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^
2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b -
b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(
x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 ...
```

3.240.6 Sympy [F]

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)**3/(a+b*tanh(x)**2)**(3/2),x)
```

output
```
Integral(tanh(x)**3/(a + b*tanh(x)**2)**(3/2), x)
```

3.240.7 Maxima [F]

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\tanh(x)^3}{\left(b\tanh(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^3/(b*tanh(x)^2 + a)^(3/2), x)
```

3.240.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(44) = 88$.

Time = 0.44 (sec) , antiderivative size = 287, normalized size of antiderivative = 5.52

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{\frac{(a^3+a^2b)e^{(2x)}}{a^3b+2a^2b^2+ab^3}+\frac{a^3+a^2b}{a^3b+2a^2b^2+ab^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}}$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$

input
```
integrate(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
((a^3 + a^2*b)*e^(2*x)/(a^3*b + 2*a^2*b^2 + a*b^3) + (a^3 + a^2*b)/(a^3*b
+ 2*a^2*b^2 + a*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*
x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/(
a + b)^(3/2) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*
x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a + b)^(3/2) - 1/
2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2)
```

3.240.9 Mupad [B] (verification not implemented)

Time = 2.78 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + \frac{a}{\left(b^2+a\,b\right)\sqrt{b\tanh\left(x\right)^2+a}}$$

input
```
int(tanh(x)^3/(a + b*tanh(x)^2)^(3/2),x)
```

output
```
atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(3/2) + a/((a*b + b^2
)*(a + b*tanh(x)^2)^(1/2))
```

3.241 $\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

3.241.1 Optimal result 1776
3.241.2 Mathematica [B] (verified) 1776
3.241.3 Rubi [A] (verified) 1777
3.241.4 Maple [B] (verified) 1779
3.241.5 Fricas [B] (verification not implemented) 1780
3.241.6 Sympy [F] 1780
3.241.7 Maxima [F] 1781
3.241.8 Giac [B] (verification not implemented) 1781
3.241.9 Mupad [F(-1)] 1782

3.241.1 Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} - \frac{\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

output
```
arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(3/2)-tanh(x)/(a+
b)/(a+b*tanh(x)^2)^(1/2)
```

3.241.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 112 vs. 2(53) = 106.

Time = 1.69 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.11

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{\tanh(x)\left(\operatorname{arctanh}\left(\frac{\sqrt{\frac{(a+b)\tanh^2(x)}{a}}}{\sqrt{1+\frac{b\tanh^2(x)}{a}}}\right)\left(b+a\coth^2(x)\right)\sqrt{\frac{(a+b)\tanh^2(x)}{a}} - (a+b)\sqrt{1}\right.}{(a+b)^2\sqrt{a+b\tanh^2(x)}\sqrt{1+\frac{b\tanh^2(x)}{a}}}$$

input
```
Integrate[Tanh[x]^2/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
(Tanh[x]*(ArcTanh[Sqrt[((a + b)*Tanh[x]^2)/a]/Sqrt[1 + (b*Tanh[x]^2)/a]]*(
b + a*Coth[x]^2)*Sqrt[((a + b)*Tanh[x]^2)/a] - (a + b)*Sqrt[1 + (b*Tanh[x]
^2)/a]))/((a + b)^2*Sqrt[a + b*Tanh[x]^2]*Sqrt[1 + (b*Tanh[x]^2)/a])
```

3.241.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 25, 4153, 25, 373, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\tan(ix)^2}{\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\downarrow 25$$

$$-\int \frac{\tan(ix)^2}{\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\downarrow 4153$$

$$-\int -\frac{\tanh^2(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh(x)$$

$$\downarrow 25$$

$$\int \frac{\tanh^2(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{3/2}}d\tanh(x)$$

$$\downarrow 373$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{a+b} - \frac{\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

$$\downarrow 291$$

$$\frac{\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}} d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}}{a+b} - \frac{\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

$$\downarrow 219$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} - \frac{\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

input
```
Int[Tanh[x]^2/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(3/2) - Tanh[
x]/((a + b)*Sqrt[a + b*Tanh[x]^2])
```

3.241.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.241.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(45) = 90$.

Time = 0.08 (sec) , antiderivative size = 289, normalized size of antiderivative = 5.45

method	result
derivativedivides	$-\frac{\tanh(x)}{a\sqrt{a+b\tanh(x)^2}}+\frac{1}{2(a+b)\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}+\frac{b(2b(1+\tanh(x))-2b)}{(a+b)(4(a+b)b-4b^2)\sqrt{b(1+\tanh(x))^2-2b($
default	$-\frac{\tanh(x)}{a\sqrt{a+b\tanh(x)^2}}+\frac{1}{2(a+b)\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}+\frac{b(2b(1+\tanh(x))-2b)}{(a+b)(4(a+b)b-4b^2)\sqrt{b(1+\tanh(x))^2-2b($

input
```
int(tanh(x)^2/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-tanh(x)/a/(a+b*tanh(x)^2)^(1/2)+1/2/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x)
)+a+b)^(1/2)+b/(a+b)*(2*b*(1+tanh(x))-2*b)/(4*(a+b)*b-4*b^2)/(b*(1+tanh(x)
)^2-2*b*(1+tanh(x))+a+b)^(1/2)-1/2/(a+b)^(3/2)*ln((2*a+2*b-2*b*(1+tanh(x))
+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))-1
/2/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b/(a+b)*(2*b*(tanh(x)
-1)+2*b)/(4*(a+b)*b-4*b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/2
/(a+b)^(3/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*
b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))
```

3.241.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. 2(45) = 90.

Time = 0.35 (sec) , antiderivative size = 2281, normalized size of antiderivative = 43.04

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4
 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a
 + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(-((a*b
^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*si
nh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3
)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)
*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b
^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*c
osh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*c
osh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^
2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 +
 b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3
*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(
x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^
2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)
^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^
2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^
5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*
sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b...
```

3.241.6 Sympy [F]

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)**2/(a+b*tanh(x)**2)**(3/2),x)
```

output
```
Integral(tanh(x)**2/(a + b*tanh(x)**2)**(3/2), x)
```

3.241.7 Maxima [F]

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\tanh\left(x\right)^2}{\left(b\tanh\left(x\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^2/(b*tanh(x)^2 + a)^(3/2), x)
```

3.241.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(45) = 90$.

Time $= 0.46$ (sec) , antiderivative size $= 293$, normalized size of antiderivative $= 5.53$

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = -\frac{\frac{\left(a^2b+ab^2\right)e^{(2x)}}{a^3b+2a^2b^2+ab^3}-\frac{a^2b+ab^2}{a^3b+2a^2b^2+ab^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}}$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\left(a+b\right)^{\frac{3}{2}}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\left(a+b\right)^{\frac{3}{2}}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\left(a+b\right)^{\frac{3}{2}}}$$

input
```
integrate(tanh(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-((a^2*b + a*b^2)*e^(2*x)/(a^3*b + 2*a^2*b^2 + a*b^3) - (a^2*b + a*b^2)/(a
^3*b + 2*a^2*b^2 + a*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*
e^(2*x) + a + b) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*
e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b
)))/(a + b)^(3/2) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*
e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a + b)^(3/2)
 + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(
2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2)
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\tanh(x)^2}{\left(b\tanh\left(x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(tanh(x)^2/(a + b*tanh(x)^2)^(3/2),x)
```

output
```
int(tanh(x)^2/(a + b*tanh(x)^2)^(3/2), x)
```

3.242 $\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

3.242.1 Optimal result 1783
3.242.2 Mathematica [C] (verified) 1783
3.242.3 Rubi [A] (verified) 1784
3.242.4 Maple [B] (verified) 1786
3.242.5 Fricas [B] (verification not implemented) 1787
3.242.6 Sympy [A] (verification not implemented) 1788
3.242.7 Maxima [F] 1788
3.242.8 Giac [B] (verification not implemented) 1788
3.242.9 Mupad [B] (verification not implemented) 1789

3.242.1 Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{1}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

output
```
arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)-1/(a+b)/(a+b*tanh(x
)^2)^(1/2)
```

3.242.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\tanh^2(x)}{a+b}\right)}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

input
```
Integrate[Tanh[x]/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
-(Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)]/((a + b)*Sqrt
[a + b*Tanh[x]^2]))
```

3.242.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 26, 4153, 26, 353, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\tan(ix)}{\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\downarrow 26$$

$$-i\int \frac{\tan(ix)}{\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\downarrow 4153$$

$$-i\int \frac{i\tanh(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\tanh(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{3/2}}d\tanh(x)$$

$$\downarrow 353$$

$$\frac{1}{2}\int \frac{1}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh^2(x)$$

$$\downarrow 61$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a+b}-\frac{2}{(a+b)\sqrt{a+b\tanh^2(x)}}\right)$$

$$\downarrow 73$$

$$\frac{1}{2}\left(\frac{2\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b(a+b)}-\frac{2}{(a+b)\sqrt{a+b\tanh^2(x)}}\right)$$

$$\downarrow 221$$

$$\frac{1}{2}\left(\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{2}{(a+b)\sqrt{a+b\tanh^2(x)}}\right)$$

input
```
Int[Tanh[x]/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]])/(a + b)^(3/2) - 2/((a + b)
*Sqrt[a + b*Tanh[x]^2]))/2
```

3.242.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.242.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(41) = 82$.

Time = 0.07 (sec) , antiderivative size = 273, normalized size of antiderivative = 5.57

method	result
derivativedivides	$-\frac{1}{2(a+b)\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}-\frac{b(2b(1+\tanh(x))-2b)}{(a+b)(4(a+b)b-4b^2)\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}+$
default	$-\frac{1}{2(a+b)\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}-\frac{b(2b(1+\tanh(x))-2b)}{(a+b)(4(a+b)b-4b^2)\sqrt{b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b}}+$

input
```
int(tanh(x)/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-b/(a+b)*(2*b*(1+tan
h(x))-2*b)/(4*(a+b)*b-4*b^2)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)+1
/2/(a+b)^(3/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-
2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))-1/2/(a+b)/(b*(tanh(x)-1)^2+2*b*(t
anh(x)-1)+a+b)^(1/2)+b/(a+b)*(2*b*(tanh(x)-1)+2*b)/(4*(a+b)*b-4*b^2)/(b*(t
anh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b+2*b*(ta
nh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(
x)-1))
```

3.242.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 856 vs. $2(41) = 82$.

Time = 0.36 (sec) , antiderivative size = 2277, normalized size of antiderivative = 46.47

$$\int \frac{\tanh(x)}{\left(a + b\tanh^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4
 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a
 + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(((a^3
+ a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sin
h(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)
*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*
cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3
+ a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*co
sh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*co
sh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2
*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^
2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*
(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x
)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2
*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^
3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2
 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5
 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*s
qrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)
*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*...
```

3.242.6 Sympy [A] (verification not implemented)

Time = 10.71 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.88

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = -\begin{cases} \dfrac{2\left(\dfrac{b}{2(a+b)\sqrt{a+b\tanh^2(x)}}+\dfrac{b\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}(a+b)}\right)}{b} & \text{for } b\neq 0 \\ \begin{cases} \tilde{\infty}\tanh^2(x) & \text{for } a^{\frac{3}{2}}=0 \\ \dfrac{\log\left(2a^{\frac{3}{2}}\tanh^2(x)-2a^{\frac{3}{2}}\right)}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)**2)**(3/2),x)
```

output
```
-Piecewise((2*(b/(2*(a + b)*sqrt(a + b*tanh(x)**2)) + b*atan(sqrt(a + b*ta
nh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)*(a + b)))/b, Ne(b, 0)), (Piecewise
((zoo*tanh(x)**2, Eq(a**(3/2), 0)), (log(2*a**(3/2)*tanh(x)**2 - 2*a**(3/2
))/(2*a**(3/2)), True)), True))
```

3.242.7 Maxima [F]

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\tanh(x)}{\left(b\tanh(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)/(b*tanh(x)^2 + a)^(3/2), x)
```

3.242.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(41) = 82.

Time = 0.45 (sec) , antiderivative size = 292, normalized size of antiderivative = 5.96

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = -\frac{\frac{(a^2b+ab^2)e^{(2x)}}{a^3b+2a^2b^2+ab^3}+\frac{a^2b+ab^2}{a^3b+2a^2b^2+ab^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}}$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-((a^2*b + a*b^2)*e^(2*x)/(a^3*b + 2*a^2*b^2 + a*b^3) + (a^2*b + a*b^2)/(a
^3*b + 2*a^2*b^2 + a*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*
e^(2*x) + a + b) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*
e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b
)))/(a + b)^(3/2) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*
e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a + b)^(3/2)
 - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(
2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2)
```

3.242.9 Mupad [B] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{1}{(a+b)\sqrt{b\tanh(x)^2+a}}$$

input
```
int(tanh(x)/(a + b*tanh(x)^2)^(3/2),x)
```

output
```
atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(3/2) - 1/((a + b)*(a
 + b*tanh(x)^2)^(1/2))
```

3.243 $\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

3.243.1 Optimal result . . . 1790
3.243.2 Mathematica [C] (warning: unable to verify) . . . 1790
3.243.3 Rubi [A] (verified) . . . 1791
3.243.4 Maple [B] (verified) . . . 1793
3.243.5 Fricas [B] (verification not implemented) . . . 1793
3.243.6 Sympy [F] . . . 1794
3.243.7 Maxima [F] . . . 1795
3.243.8 Giac [B] (verification not implemented) . . . 1795
3.243.9 Mupad [F(-1)] . . . 1796

3.243.1 Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} + \frac{b\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

output
```
arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(3/2)+b*tanh(x)/a
/(a+b)/(a+b*tanh(x)^2)^(1/2)
```

3.243.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.78 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.98

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx =$$

$$-\frac{\sinh^2(x)\left(\frac{15}{4}a(3a-2b+(3a+2b)\cosh(2x))\operatorname{csch}(x)\operatorname{sech}(x)\left((a-b)\arcsin\left(\sqrt{-\frac{(a+b)\sinh^2(x)}{a}}\right)+(a+b)\right.\right.}{}$$

input
```
Integrate[(a + b*Tanh[x]^2)^(-3/2),x]
```

output
```
-1/15*(Sinh[x]^2*((15*a*(3*a - 2*b + (3*a + 2*b)*Cosh[2*x])*Csch[x]*Sech[x
]*((a - b)*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]] + (a + b)*ArcSin[Sqrt[-(
((a + b)*Sinh[x]^2)/a)]]*Cosh[2*x] - 2*a*Sqrt[-(((a + b)*(b + a*Coth[x]^2)
*Sinh[x]^4)/a^2)]))/4 + Sqrt[2]*a^2*(a + b)*Hypergeometric2F1[2, 2, 7/2, -
(((a + b)*Sinh[x]^2)/a)]*(-(((a + b)*(a - b + (a + b)*Cosh[2*x])*Sinh[x]^2
)/a^2))^(3/2)*Tanh[x]))/(a^4*(-(((a + b)*Sinh[x]^2)/a))^(3/2)*Sqrt[Cosh[x]
^2 + (b*Sinh[x]^2)/a]*Sqrt[a + b*Tanh[x]^2])
```

3.243.3 Rubi [A] (verified)

Time $= 0.24$ (sec) , antiderivative size $= 56$, normalized size of antiderivative $= 1.00$, number of steps used $= 6$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used $= \{3042, 4144, 296, 291, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\downarrow 4144$$

$$\int \frac{1}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{3/2}}d\tanh(x)$$

$$\downarrow 296$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{a+b} + \frac{b\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

$$\downarrow 291$$

$$\frac{\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}}{a+b} + \frac{b\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

$$\downarrow 219$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}+\frac{b\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

input
```
Int[(a + b*Tanh[x]^2)^(-3/2),x]
```

output
```
ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(3/2) + (b*Ta
nh[x])/(a*(a + b)*Sqrt[a + b*Tanh[x]^2])
```

3.243.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.243.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(48) = 96$.

Time = 0.09 (sec) , antiderivative size = 272, normalized size of antiderivative = 4.86

method	result
derivativedivides	$-\frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}+\frac{b(2b(\tanh(x)-1)+2b)}{(a+b)(4(a+b)b-4b^2)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}+$
default	$-\frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}+\frac{b(2b(\tanh(x)-1)+2b)}{(a+b)(4(a+b)b-4b^2)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}+$

input
```
int(1/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b/(a+b)*(2*b*(tanh(
x)-1)+2*b)/(4*(a+b)*b-4*b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1
/2/(a+b)^(3/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+
2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1/2/(a+b)/(b*(1+tanh(x))^2-2*b*(1
+tanh(x))+a+b)^(1/2)+b/(a+b)*(2*b*(1+tanh(x))-2*b)/(4*(a+b)*b-4*b^2)/(b*(1
+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-1/2/(a+b)^(3/2)*ln((2*a+2*b-2*b*(1+
tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tan
h(x)))
```

3.243.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 975 vs. $2(48) = 96$.

Time = 0.36 (sec) , antiderivative size = 2509, normalized size of antiderivative = 44.80

$$\int \frac{1}{\left(a + b\tanh^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*
b)*sinh(x)^4 + 2*(a^2 - a*b)*cosh(x)^2 + 2*(3*(a^2 + a*b)*cosh(x)^2 + a^2
- a*b)*sinh(x)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(x)^3 + (a^2 - a*b)*cosh
(x))*sinh(x))*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*
cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6
- 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2
+ b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4
*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*
a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b
^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*
b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2
 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*c
osh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(
x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*s
inh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^
2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 +
 (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a
^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3
*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh
(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b...
```

3.243.6 Sympy [F]

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*tanh(x)**2)**(3/2),x)
```

output
```
Integral((a + b*tanh(x)**2)**(-3/2), x)
```

3.243.7 Maxima [F]

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\tanh(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tanh(x)^2 + a)^(-3/2), x)
```

3.243.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(48) = 96$.

Time = 0.44 (sec) , antiderivative size = 288, normalized size of antiderivative = 5.14

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{\frac{(ab^2+b^3)e^{(2x)}}{a^3b+2a^2b^2+ab^3} - \frac{ab^2+b^3}{a^3b+2a^2b^2+ab^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}}$$
$$- \frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$- \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$+ \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
((a*b^2 + b^3)*e^(2*x)/(a^3*b + 2*a^2*b^2 + a*b^3) - (a*b^2 + b^3)/(a^3*b
+ 2*a^2*b^2 + a*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*
x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/(
a + b)^(3/2) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*
x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a + b)^(3/2) + 1/
2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2)
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\tanh(x)^2+a\right)^{3/2}}\,dx$$

input
```
int(1/(a + b*tanh(x)^2)^(3/2),x)
```

output
```
int(1/(a + b*tanh(x)^2)^(3/2), x)
```

3.244 $\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

3.244.1 Optimal result 1797
3.244.2 Mathematica [C] (verified) 1797
3.244.3 Rubi [A] (verified) 1798
3.244.4 Maple [F] 1801
3.244.5 Fricas [B] (verification not implemented) 1801
3.244.6 Sympy [F] 1802
3.244.7 Maxima [F] 1802
3.244.8 Giac [B] (verification not implemented) 1802
3.244.9 Mupad [F(-1)] 1803

3.244.1 Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + \frac{b}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

output
```
-arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(3/2)+arctanh((a+b*tanh(x)^2)^(1
/2)/(a+b)^(1/2))/(a+b)^(3/2)+b/a/(a+b)/(a+b*tanh(x)^2)^(1/2)
```

3.244.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{-a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\tanh^2(x)}{a+b}\right)+(a+b)\,\text{Hypergeometric2F1}\left(}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

input
```
Integrate[Coth[x]/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
(-(a*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)]) + (a + b)
*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Tanh[x]^2)/a])/(a*(a + b)*Sqrt[a +
 b*Tanh[x]^2])
```

3.244.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.26, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 26, 4153, 26, 354, 96, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{i}{\tan(ix)\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\downarrow 26$$

$$i\int \frac{1}{\tan(ix)\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\downarrow 4153$$

$$i\int -\frac{i\coth(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\coth(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{3/2}}d\tanh(x)$$

$$\downarrow 354$$

$$\frac{1}{2}\int \frac{\coth(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh^2(x)$$

$$\downarrow 96$$

$$\frac{1}{2}\left(\frac{2b}{a(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{\int -\frac{\coth(x)\left(-b\tanh^2(x)+a+b\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a(a+b)}\right)$$

$$\downarrow 25$$

$$\frac{1}{2}\left(\frac{\int \frac{\coth(x)\left(-b\tanh^2(x)+a+b\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a(a+b)}+\frac{2b}{a(a+b)\sqrt{a+b\tanh^2(x)}}\right)$$

$$\downarrow 174$$

$$\frac{1}{2}\left(\frac{a\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)+(a+b)\int \frac{\coth(x)}{\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a(a+b)}+\frac{2b}{a(a+b)\sqrt{a+b\tanh^2(x)}}\right)$$

$$\downarrow 73$$

$$\frac{1}{2}\left(\frac{\frac{2a\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}+\frac{2(a+b)\int \frac{1}{\frac{\tanh^4(x)}{b}-\frac{a}{b}}d\sqrt{b\tanh^2(x)+a}}{b}}{a(a+b)}+\frac{2b}{a(a+b)\sqrt{a+b\tanh^2(x)}}\right)$$

$$\downarrow 221$$

$$\frac{1}{2}\left(\frac{\frac{2a\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{2(a+b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a+b)}+\frac{2b}{a(a+b)\sqrt{a+b\tanh^2(x)}}\right)$$

input
```
Int[Coth[x]/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
(((-2*(a + b)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/Sqrt[a] + (2*a*ArcTa
nh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]])/Sqrt[a + b])/(a*(a + b)) + (2*b)/(a
*(a + b)*Sqrt[a + b*Tanh[x]^2]))/2
```

3.244.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.244.4 Maple [F]

$$\int \frac{\coth(x)}{\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}dx$$

input

```
int(coth(x)/(a+b*tanh(x)^2)^(3/2),x)
```

output

```
int(coth(x)/(a+b*tanh(x)^2)^(3/2),x)
```

3.244.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1424 vs. $2(64) = 128$.

Time $= 0.54$ (sec) , antiderivative size $= 6955$, normalized size of antiderivative $= 89.17$

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate(coth(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output

```
Too large to include
```

3.244.6 Sympy [F]

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\coth\left(x\right)}{\left(a+b\tanh^2\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(x)/(a+b*tanh(x)**2)**(3/2),x)
```

output
```
Integral(coth(x)/(a + b*tanh(x)**2)**(3/2), x)
```

3.244.7 Maxima [F]

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\coth\left(x\right)}{\left(b\tanh\left(x\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)/(b*tanh(x)^2 + a)^(3/2), x)
```

3.244.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(64) = 128$.

Time $= 0.51$ (sec) , antiderivative size $= 372$, normalized size of antiderivative $= 4.77$

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{\frac{\left(a^3b^2+a^2b^3\right)e^{(2x)}}{a^5b+2a^4b^2+a^3b^3}+\frac{a^3b^2+a^2b^3}{a^5b+2a^4b^2+a^3b^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}}$$
$$+\frac{2\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}a}$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$

input
```
integrate(coth(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
((a^3*b^2 + a^2*b^3)*e^(2*x)/(a^5*b + 2*a^4*b^2 + a^3*b^3) + (a^3*b^2 + a^
2*b^3)/(a^5*b + 2*a^4*b^2 + a^3*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(
2*x) - 2*b*e^(2*x) + a + b) + 2*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*
e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/sq
rt(-a))/(sqrt(-a)*a) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a
 - b)))/(a + b)^(3/2) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a + b)^(
3/2) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a
*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2)
```

3.244.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\coth(x)}{\left(b\tanh\left(x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(coth(x)/(a + b*tanh(x)^2)^(3/2),x)
```

output
```
int(coth(x)/(a + b*tanh(x)^2)^(3/2), x)
```

3.245 $\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

3.245.1 Optimal result 1804
3.245.2 Mathematica [C] (warning: unable to verify) 1805
3.245.3 Rubi [A] (verified) 1805
3.245.4 Maple [F] 1808
3.245.5 Fricas [B] (verification not implemented) 1809
3.245.6 Sympy [F] 1809
3.245.7 Maxima [F] 1810
3.245.8 Giac [B] (verification not implemented) 1810
3.245.9 Mupad [F(-1)] 1811

3.245.1 Optimal result

Integrand size = 17, antiderivative size = 85

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} + \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a^2(a+b)}$$

output

```
arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(3/2)+b*coth(x)/a
/(a+b)/(a+b*tanh(x)^2)^(1/2)-(a+2*b)*coth(x)*(a+b*tanh(x)^2)^(1/2)/a^2/(a+
b)
```

3.245.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.09 (sec) , antiderivative size = 263, normalized size of antiderivative = 3.09

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx =$$

$$\cosh^2(x)\coth(x)\left(\frac{8(a+b)\cosh^2(x)\,{}_3F_2\left(2,2,2;1,\frac{7}{2};-\frac{(a+b)\sinh^2(x)}{a}\right)\left(ia\tanh(x)+ib\tanh^3(x)\right)^2}{15a^3} - \frac{8(a+b)\,\text{Hypergeometric2F1}\left(2,2,\frac{7}{2},-\right.}{}\right.$$

$$-$$

input
```
Integrate[Coth[x]^2/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
-((Cosh[x]^2*Coth[x]*((8*(a + b)*Cosh[x]^2*HypergeometricPFQ[{2, 2, 2}, {1
, 7/2}, -(((a + b)*Sinh[x]^2)/a)]*(I*a*Tanh[x] + I*b*Tanh[x]^3)^2)/(15*a^3
) - (8*(a + b)*Hypergeometric2F1[2, 2, 7/2, -(((a + b)*Sinh[x]^2)/a)]*Sinh
[x]^2*(2*a^2 + 5*a*b*Tanh[x]^2 + 3*b^2*Tanh[x]^4))/(15*a^3) - (Coth[x]^2*(
3*a^2 + 12*a*b*Tanh[x]^2 + 8*b^2*Tanh[x]^4)*(ArcSin[Sqrt[-(((a + b)*Sinh[x
]^2)/a)]]*(-a - b*Tanh[x]^2) + a*Sech[x]^2*Sqrt[-(((a + b)*Cosh[x]^2*Sinh[
x]^2*(a + b*Tanh[x]^2))/a^2)]))/(a^2*(a + b)*Sqrt[-(((a + b)*Cosh[x]^2*Sin
h[x]^2*(a + b*Tanh[x]^2))/a^2)])))/(a*Sqrt[a + b*Tanh[x]^2]))
```

3.245.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {3042, 25, 4153, 25, 374, 25, 445, 25, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\tan(ix)^2\left(a-b\tan(ix)^2\right)^{3/2}}\,dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\tan(ix)^2 \left(a - b\tan(ix)^2\right)^{3/2}} dx$$

$$\downarrow 4153$$

$$-\int -\frac{\coth^2(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}} d\tanh(x)$$

$$\downarrow 25$$

$$\int \frac{\coth^2(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{3/2}} d\tanh(x)$$

$$\downarrow 374$$

$$\frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\int -\frac{\coth^2(x)\left(-2b\tanh^2(x)+a+2b\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}} d\tanh(x)}{a(a+b)}$$

$$\downarrow 25$$

$$\frac{\int \frac{\coth^2(x)\left(-2b\tanh^2(x)+a+2b\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}} d\tanh(x)}{a(a+b)} + \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

$$\downarrow 445$$

$$\frac{-\frac{\int -\frac{a^2}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}} d\tanh(x)}{a} - \frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}}{a(a+b)} + \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{a^2}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}} d\tanh(x)}{a} - \frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}}{a(a+b)} + \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

$$\downarrow 27$$

$$\frac{a\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}} d\tanh(x) - \frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}}{a(a+b)} + \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

$$\downarrow 291$$

$$\frac{a\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}} d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}} - \frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}}{a(a+b)} + \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

$$\downarrow 219$$

$$\frac{\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}} - \frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}}{a(a+b)} + \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

input
```
Int[Coth[x]^2/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
(b*Coth[x])/(a*(a + b)*Sqrt[a + b*Tanh[x]^2]) + ((a*ArcTanh[(Sqrt[a + b]*T
anh[x])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[a + b] - ((a + 2*b)*Coth[x]*Sqrt[a +
b*Tanh[x]^2])/a)/(a*(a + b))
```

3.245.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.245.4 Maple [F]

$$\int \frac{\coth(x)^2}{\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}dx$$

input
```
int(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x)
```

output
```
int(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x)
```

3.245.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1685 vs. 2(75) = 150.

Time = 0.49 (sec) , antiderivative size = 3929, normalized size of antiderivative = 46.22

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((a^3 + a^2*b)*cosh(x)^6 + 6*(a^3 + a^2*b)*cosh(x)*sinh(x)^5 + (a^3
+ a^2*b)*sinh(x)^6 + (a^3 - 3*a^2*b)*cosh(x)^4 + (a^3 - 3*a^2*b + 15*(a^3
+ a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^3 + a^2*b)*cosh(x)^3 + (a^3 - 3*a^
2*b)*cosh(x))*sinh(x)^3 - a^3 - a^2*b - (a^3 - 3*a^2*b)*cosh(x)^2 + (15*(a
^3 + a^2*b)*cosh(x)^4 - a^3 + 3*a^2*b + 6*(a^3 - 3*a^2*b)*cosh(x)^2)*sinh(
x)^2 + 2*(3*(a^3 + a^2*b)*cosh(x)^5 + 2*(a^3 - 3*a^2*b)*cosh(x)^3 - (a^3 -
 3*a^2*b)*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*
(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b
^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 +
 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a
^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^
3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*
(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b
+ 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*
(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*
b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2
+ 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sin
h(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(
x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^
2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b...
```

3.245.6 Sympy [F]

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(x)**2/(a+b*tanh(x)**2)**(3/2),x)
```

output
```
Integral(coth(x)**2/(a + b*tanh(x)**2)**(3/2), x)
```

3.245.7 Maxima [F]

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\coth\left(x\right)^2}{\left(b\tanh\left(x\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)^2/(b*tanh(x)^2 + a)^(3/2), x)
```

3.245.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(75) = 150$.

Time $= 0.57$ (sec) , antiderivative size $= 459$, normalized size of antiderivative $= 5.40$

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = -\frac{\frac{(a^2b^3+ab^4)e^{(2x)}}{a^5b+2a^4b^2+a^3b^3} - \frac{a^2b^3+ab^4}{a^5b+2a^4b^2+a^3b^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}}$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\left(a+b\right)^{\frac{3}{2}}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\left(a+b\right)^{\frac{3}{2}}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\left(a+b\right)^{\frac{3}{2}}}$$
$$+\frac{4\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{}\right.}{\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2-2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\right.\right.}$$

input
```
integrate(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-((a^2*b^3 + a*b^4)*e^(2*x)/(a^5*b + 2*a^4*b^2 + a^3*b^3) - (a^2*b^3 + a*b
^4)/(a^5*b + 2*a^4*b^2 + a^3*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x
) - 2*b*e^(2*x) + a + b) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4
*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b
)*(a - b)))/(a + b)^(3/2) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4
*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a +
b)^(3/2) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) +
 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2) + 4*(sqr
t(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
+ a + b) + sqrt(a + b))/(((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(
a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) -
3*a + b)*a)
```

3.245.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\coth(x)^2}{\left(b\tanh(x)^2+a\right)^{3/2}}\,dx$$

input
```
int(coth(x)^2/(a + b*tanh(x)^2)^(3/2),x)
```

output
```
int(coth(x)^2/(a + b*tanh(x)^2)^(3/2), x)
```

3.246 $\int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

3.246.1 Optimal result . . . 1812
3.246.2 Mathematica [C] (verified) . . . 1812
3.246.3 Rubi [A] (verified) . . . 1813
3.246.4 Maple [B] (verified) . . . 1817
3.246.5 Fricas [B] (verification not implemented) . . . 1818
3.246.6 Sympy [F] . . . 1818
3.246.7 Maxima [F] . . . 1818
3.246.8 Giac [B] (verification not implemented) . . . 1819
3.246.9 Mupad [F(-1)] . . . 1820

3.246.1 Optimal result

Integrand size = 17, antiderivative size = 118

$$\int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b^{5/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a+2b)\tanh(x)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

output
```
-arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(5/2)+arctanh((a+b)^(1/2
)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)+a*(a+2*b)*tanh(x)/b^2/(a+b)^2
/(a+b*tanh(x)^2)^(1/2)+1/3*a*tanh(x)^3/b/(a+b)/(a+b*tanh(x)^2)^(3/2)
```

3.246.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.14 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.96

$$\int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \sqrt{(a-b+(a+b)\cosh(2x))\operatorname{sech}^2(x)}\left(-\frac{3\sqrt{2}a\coth(x)\left((a^2+3ab+2b^2)\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\right.\right.\right.\right.$$

input
```
Integrate[Tanh[x]^6/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
(Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2]*((-3*Sqrt[2]*a*Coth[x]*((a^2
+ 3*a*b + 2*b^2)*EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x
]^2)/b]/Sqrt[2]], 1] + b^2*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a
+ b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]))/(b*Sqrt[((a - b + (a + b)*Cos
h[2*x])*Csch[x]^2)/b]) + (a*(a + b)*(3*a^2 + 2*a*b - 7*b^2 + (3*a^2 + 10*a
*b + 7*b^2)*Cosh[2*x])*Sinh[2*x])/(a - b + (a + b)*Cosh[2*x])^2))/(3*Sqrt[
2]*b^2*(a + b)^3)
```

3.246.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {3042, 25, 4153, 25, 372, 27, 440, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\tan(ix)^6}{\left(a-b\tan(ix)^2\right)^{5/2}}dx$$

$$\downarrow 25$$

$$-\int \frac{\tan(ix)^6}{\left(a-b\tan(ix)^2\right)^{5/2}}dx$$

$$\downarrow 4153$$

$$-\int -\frac{\tanh^6(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{5/2}}d\tanh(x)$$

$$\downarrow 25$$

$$\int \frac{\tanh^6(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{5/2}}d\tanh(x)$$

$$\downarrow 372$$

$$\frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\int\frac{3\tanh^2(x)\left(a-(a+b)\tanh^2(x)\right)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh(x)}{3b(a+b)}$$

$$\downarrow 27$$

$$\frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\int\frac{\tanh^2(x)\left(a-(a+b)\tanh^2(x)\right)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh(x)}{b(a+b)}$$

$$\downarrow 440$$

$$\frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\frac{\int\frac{a(a+2b)-(a+b)^2\tanh^2(x)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{b(a+b)}-\frac{a(a+2b)\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}}{b(a+b)}$$

$$\downarrow 398$$

$$\frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\frac{(a+b)^2\int\frac{1}{\sqrt{b\tanh^2(x)+a}}d\tanh(x)-b^2\int\frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{b(a+b)}-\frac{a(a+2b)\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}}{b(a+b)}$$

$$\downarrow 224$$

$$\frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\frac{(a+b)^2\int\frac{1}{1-\frac{b\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}-b^2\int\frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{b(a+b)}-\frac{a(a+2b)\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}}{b(a+b)}$$

$$\downarrow 219$$

$$\frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\frac{(a+b)^2\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}-b^2\int\frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{b(a+b)}-\frac{a(a+2b)\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}}{b(a+b)}$$

$$\downarrow 291$$

$$\frac{\frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\frac{(a+b)^2\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}-b^2\int\frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}}{b(a+b)}-\frac{a(a+2b)\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}}{b(a+b)}$$

$$\downarrow 219$$

$$\frac{\frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\frac{(a+b)^2\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}-\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}}{b(a+b)}-\frac{a(a+2b)\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}}{b(a+b)}$$

input
```
Int[Tanh[x]^6/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
(a*Tanh[x]^3)/(3*b*(a + b)*(a + b*Tanh[x]^2)^(3/2)) - ((((a + b)^2*ArcTanh
[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[b] - (b^2*ArcTanh[(Sqrt[a
+ b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[a + b])/(b*(a + b)) - (a*(a + 2
*b)*Tanh[x])/(b*(a + b)*Sqrt[a + b*Tanh[x]^2]))/(b*(a + b))
```

3.246.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.246.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(100) = 200$.

Time = 0.11 (sec) , antiderivative size = 549, normalized size of antiderivative = 4.65

method	result
derivativedivides	$-\frac{\tanh(x)}{3a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{2\tanh(x)}{3a^2\sqrt{a+b\tanh(x)^2}}+\frac{\tanh(x)^3}{3b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}+\frac{\tanh(x)}{b^2\sqrt{a+b\tanh(x)^2}}-\frac{\ln\left(\sqrt{b}\tanh(x)+\sqrt{}\right.}{b^{\frac{5}{2}}}$
default	$-\frac{\tanh(x)}{3a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{2\tanh(x)}{3a^2\sqrt{a+b\tanh(x)^2}}+\frac{\tanh(x)^3}{3b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}+\frac{\tanh(x)}{b^2\sqrt{a+b\tanh(x)^2}}-\frac{\ln\left(\sqrt{b}\tanh(x)+\sqrt{}\right.}{b^{\frac{5}{2}}}$

input
```
int(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*tanh(x)/a/(a+b*tanh(x)^2)^(3/2)-2/3/a^2*tanh(x)/(a+b*tanh(x)^2)^(1/2)
+1/3*tanh(x)^3/b/(a+b*tanh(x)^2)^(3/2)+1/b^2*tanh(x)/(a+b*tanh(x)^2)^(1/2)
-1/b^(5/2)*ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2))+1/3*tanh(x)/b/(a+b*ta
nh(x)^2)^(3/2)-1/3/a/b*tanh(x)/(a+b*tanh(x)^2)^(1/2)-1/6/(a+b)/(b*(tanh(x)
-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(tanh(x)-1)^2+2*b*(tanh(
x)-1)+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+
a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)
+1/2/(a+b)^2/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)*b*tanh(x)+1/2/(
a+b)^(5/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*
(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1/6/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tan
h(x))+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)
*tanh(x)+1/3*b/(a+b)/a^2/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)*tanh(
x)+1/2/(a+b)^2/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)+1/2/(a+b)^2/a/(
b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)*b*tanh(x)-1/2/(a+b)^(5/2)*ln((2
*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)
^(1/2))/(1+tanh(x)))
```

3.246.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4472 vs. 2(100) = 200.

Time = 1.23 (sec) , antiderivative size = 19265, normalized size of antiderivative = 163.26

$$\int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.246.6 Sympy [F]

$$\int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)**6/(a+b*tanh(x)**2)**(5/2),x)
```

output
```
Integral(tanh(x)**6/(a + b*tanh(x)**2)**(5/2), x)
```

3.246.7 Maxima [F]

$$\int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\tanh(x)^6}{\left(b\tanh(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^6/(b*tanh(x)^2 + a)^(5/2), x)
```

3.246.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 805 vs. $2(100) = 200$.

Time = 0.64 (sec) , antiderivative size = 805, normalized size of antiderivative = 6.82

$$\int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\left(\left(\frac{(3\,a^9b^8+22\,a^8b^9+65\,a^7b^{10}+100\,a^6b^{11}+85\,a^5b^{12}+38\,a^4b^{13}+7\,a^3b^{14})e^{(2\,x)}}{a^8b^{10}+6\,a^7b^{11}+15\,a^6b^{12}+20\,a^5b^{13}+15\,a^4b^{14}+6\,a^3b^{15}+a^2b^{16}}+\frac{3\,(a^9b^8+2\,a^8b^9-9\,a^7b^{10}-3}{a^8b^{10}+6\,a^7b^{11}+15\,a^6b^{12}}\right.\right.$$

$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\,(a^2+2\,ab+b^2)\sqrt{a+b}}$$

$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}+\sqrt{a+b}\right|\right)}{2\,(a^2+2\,ab+b^2)\sqrt{a+b}}$$

$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}-\sqrt{a+b}\right|\right)}{2\,(a^2+2\,ab+b^2)\sqrt{a+b}}$$

$$-\frac{2\,\arctan\left(-\frac{\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}+\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}b^2}$$

input
```
integrate(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/3*((((3*a^9*b^8 + 22*a^8*b^9 + 65*a^7*b^10 + 100*a^6*b^11 + 85*a^5*b^12
+ 38*a^4*b^13 + 7*a^3*b^14)*e^(2*x)/(a^8*b^10 + 6*a^7*b^11 + 15*a^6*b^12 +
 20*a^5*b^13 + 15*a^4*b^14 + 6*a^3*b^15 + a^2*b^16) + 3*(a^9*b^8 + 2*a^8*b
^9 - 9*a^7*b^10 - 36*a^6*b^11 - 49*a^5*b^12 - 30*a^4*b^13 - 7*a^3*b^14)/(a
^8*b^10 + 6*a^7*b^11 + 15*a^6*b^12 + 20*a^5*b^13 + 15*a^4*b^14 + 6*a^3*b^1
5 + a^2*b^16))*e^(2*x) - 3*(a^9*b^8 + 2*a^8*b^9 - 9*a^7*b^10 - 36*a^6*b^11
 - 49*a^5*b^12 - 30*a^4*b^13 - 7*a^3*b^14)/(a^8*b^10 + 6*a^7*b^11 + 15*a^6
*b^12 + 20*a^5*b^13 + 15*a^4*b^14 + 6*a^3*b^15 + a^2*b^16))*e^(2*x) - (3*a
^9*b^8 + 22*a^8*b^9 + 65*a^7*b^10 + 100*a^6*b^11 + 85*a^5*b^12 + 38*a^4*b^
13 + 7*a^3*b^14)/(a^8*b^10 + 6*a^7*b^11 + 15*a^6*b^12 + 20*a^5*b^13 + 15*a
^4*b^14 + 6*a^3*b^15 + a^2*b^16))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2
*b*e^(2*x) + a + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(
4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a +
b)*(a - b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a + b)*
e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
+ sqrt(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a +
b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
b) - sqrt(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 2*arctan(-1/2*(sqrt
(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
 a + b) + sqrt(a + b))/sqrt(-b))/(sqrt(-b)*b^2)
```

3.246.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\tanh(x)^6}{\left(b\tanh(x)^2+a\right)^{5/2}}\,dx$$

input
```
int(tanh(x)^6/(a + b*tanh(x)^2)^(5/2),x)
```

output
```
int(tanh(x)^6/(a + b*tanh(x)^2)^(5/2), x)
```

3.247 $\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

3.247.1 Optimal result 1821
3.247.2 Mathematica [C] (verified) 1821
3.247.3 Rubi [A] (verified) 1822
3.247.4 Maple [B] (verified) 1824
3.247.5 Fricas [B] (verification not implemented) 1825
3.247.6 Sympy [F] 1825
3.247.7 Maxima [F] 1825
3.247.8 Giac [B] (verification not implemented) 1826
3.247.9 Mupad [B] (verification not implemented) 1827

3.247.1 Optimal result

Integrand size = 17, antiderivative size = 84

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{a^2}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a+2b)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

output
```
arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)+a*(a+2*b)/b^2/(a+b)
^2/(a+b*tanh(x)^2)^(1/2)-1/3*a^2/b^2/(a+b)/(a+b*tanh(x)^2)^(3/2)
```

3.247.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{-b^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\tanh^2(x)}{a+b}\right) + (a+b)\left(2a+b+3b\tanh^2(x\right.}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

input
```
Integrate[Tanh[x]^5/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
(-(b^2*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tanh[x]^2)/(a + b)]) + (a +
 b)*(2*a + b + 3*b*Tanh[x]^2))/(3*b^2*(a + b)*(a + b*Tanh[x]^2)^(3/2))
```

3.247.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 26, 4153, 26, 354, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\tan(ix)^5}{\left(a-b\tan(ix)^2\right)^{5/2}}dx$$

$$\downarrow 26$$

$$-i\int \frac{\tan(ix)^5}{\left(a-b\tan(ix)^2\right)^{5/2}}dx$$

$$\downarrow 4153$$

$$-i\int \frac{i\tanh^5(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{5/2}}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\tanh^5(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{5/2}}d\tanh(x)$$

$$\downarrow 354$$

$$\frac{1}{2}\int \frac{\tanh^4(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{5/2}}d\tanh^2(x)$$

$$\downarrow 98$$

$$\frac{1}{2}\int \left(\frac{a^2}{b(a+b)\left(b\tanh^2(x)+a\right)^{5/2}}-\frac{(a+2b)a}{b(a+b)^2\left(b\tanh^2(x)+a\right)^{3/2}}-\frac{1}{(a+b)^2\left(\tanh^2(x)-1\right)\sqrt{b\tanh^2(x)+a}}\right)$$

$$\downarrow 2009$$

$$\frac{1}{2}\left(-\frac{2a^2}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}+\frac{2a(a+2b)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}}\right)$$

input
```
Int[Tanh[x]^5/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]])/(a + b)^(5/2) - (2*a^2)/(3
*b^2*(a + b)*(a + b*Tanh[x]^2)^(3/2)) + (2*a*(a + 2*b))/(b^2*(a + b)^2*Sqr
t[a + b*Tanh[x]^2]))/2
```

3.247.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.247.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(72) = 144$.

Time = 0.10 (sec) , antiderivative size = 469, normalized size of antiderivative = 5.58

method	result
derivativedivides	$\frac{1}{3b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}+\frac{\tanh(x)^2}{b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}+\frac{2a}{3b^2\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{1}{6(a+b)\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a-\right.}$
default	$\frac{1}{3b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}+\frac{\tanh(x)^2}{b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}+\frac{2a}{3b^2\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{1}{6(a+b)\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a-\right.}$

input

```
int(tanh(x)^5/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/3/b/(a+b*tanh(x)^2)^(3/2)+tanh(x)^2/b/(a+b*tanh(x)^2)^(3/2)+2/3*a/b^2/(a
+b*tanh(x)^2)^(3/2)-1/6/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+
1/6*b/(a+b)/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)*tanh(x)+1/3*b/(a
+b)/a^2/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b
*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^2/a/(b*(tanh(x)-1)^2+2
*b*(tanh(x)-1)+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b+2*b*(tanh(
x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-
1))-1/6/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)-1/6*b/(a+b)/a/(b
*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)*tanh(x)-1/3*b/(a+b)/a^2/(b*(1+ta
nh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(1+tanh(x))^2-2
*b*(1+tanh(x))+a+b)^(1/2)-1/2/(a+b)^2/a/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a
+b)^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1
/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))
```

3.247.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3234 vs. 2(72) = 144.

Time = 0.67 (sec) , antiderivative size = 7033, normalized size of antiderivative = 83.73

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^5/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.247.6 Sympy [F]

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)**5/(a+b*tanh(x)**2)**(5/2),x)
```

output
```
Integral(tanh(x)**5/(a + b*tanh(x)**2)**(5/2), x)
```

3.247.7 Maxima [F]

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\tanh(x)^5}{\left(b\tanh(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)^5/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^5/(b*tanh(x)^2 + a)^(5/2), x)
```

3.247.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 711 vs. $2(72) = 144$.

Time = 0.53 (sec) , antiderivative size = 711, normalized size of antiderivative = 8.46

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{2\left(\left(\left(\frac{(a^9+8a^8b+25a^7b^2+40a^6b^3+35a^5b^4+16a^4b^5+3a^3b^6)e^{(2x)}}{a^8b^2+6a^7b^3+15a^6b^4+20a^5b^5+15a^4b^6+6a^3b^7+a^2b^8} + \frac{3(a^9+6a^8b+13a^7b^2+12a^6b^3+3a^5b^4-2a^4b^5-a^3b^6)}{a^8b^2+6a^7b^3+15a^6b^4+20a^5b^5+15a^4b^6+6a^3b^7+a^2b^8}\right)e^{(2x)} + \frac{(a^9+8a^8b+25a^7b^2+40a^6b^3+35a^5b^4+16a^4b^5+3a^3b^6)}{a^8b^2+6a^7b^3+15a^6b^4+20a^5b^5+15a^4b^6+6a^3b^7+a^2b^8}\right)e^{(2x)} + \frac{3(a^9+6a^8b+13a^7b^2+12a^6b^3+3a^5b^4-2a^4b^5-a^3b^6)}{a^8b^2+6a^7b^3+15a^6b^4+20a^5b^5+15a^4b^6+6a^3b^7+a^2b^8}\right)}{3\left(ae^{(4}\right.}$$

$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\left(a^2+2ab+b^2\right)\sqrt{a+b}}$$

$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\left(a^2+2ab+b^2\right)\sqrt{a+b}}$$

$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\left(a^2+2ab+b^2\right)\sqrt{a+b}}$$

input
```
integrate(tanh(x)^5/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
2/3*((((a^9 + 8*a^8*b + 25*a^7*b^2 + 40*a^6*b^3 + 35*a^5*b^4 + 16*a^4*b^5
+ 3*a^3*b^6)*e^(2*x)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a
^4*b^6 + 6*a^3*b^7 + a^2*b^8) + 3*(a^9 + 6*a^8*b + 13*a^7*b^2 + 12*a^6*b^3
 + 3*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20
*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) + 3*(a^9 + 6*a^8*b +
 13*a^7*b^2 + 12*a^6*b^3 + 3*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)/(a^8*b^2 + 6*a
^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2
*x) + (a^9 + 8*a^8*b + 25*a^7*b^2 + 40*a^6*b^3 + 35*a^5*b^4 + 16*a^4*b^5 +
 3*a^3*b^6)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 +
6*a^3*b^7 + a^2*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
 a + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(
4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))
/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sq
rt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b
)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) +
 sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a
+ b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b))
```

3.247.9 Mupad [B] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}\,(2a^2+4ab+2b^2)}{2(a+b)^{5/2}}\right)}{(a+b)^{5/2}} - \frac{\frac{a^2}{3(a+b)} - \frac{(a^2+2ba)\left(b\tanh(x)^2+a\right)}{(a+b)^2}}{b^2\left(b\tanh(x)^2+a\right)^{3/2}}$$

input
```
int(tanh(x)^5/(a + b*tanh(x)^2)^(5/2),x)
```

output
```
atanh(((a + b*tanh(x)^2)^(1/2)*(4*a*b + 2*a^2 + 2*b^2))/(2*(a + b)^(5/2)))
/(a + b)^(5/2) - (a^2/(3*(a + b)) - ((2*a*b + a^2)*(a + b*tanh(x)^2))/(a +
 b)^2)/(b^2*(a + b*tanh(x)^2)^(3/2))
```

3.248 $\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

3.248.1 Optimal result 1828
3.248.2 Mathematica [A] (verified) 1828
3.248.3 Rubi [A] (verified) 1829
3.248.4 Maple [B] (verified) 1831
3.248.5 Fricas [B] (verification not implemented) 1832
3.248.6 Sympy [F] 1832
3.248.7 Maxima [F] 1833
3.248.8 Giac [B] (verification not implemented) 1833
3.248.9 Mupad [F(-1)] 1834

3.248.1 Optimal result

Integrand size = 17, antiderivative size = 90

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{a\tanh(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(a+4b)\tanh(x)}{3b(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

output
```
arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)-1/3*(a+4*b)
*tanh(x)/b/(a+b)^2/(a+b*tanh(x)^2)^(1/2)+1/3*a*tanh(x)/b/(a+b)/(a+b*tanh(x
)^2)^(3/2)
```

3.248.2 Mathematica [A] (verified)

Time = 2.84 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.47

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\tanh^3(x)\left(3\operatorname{arctanh}\left(\frac{\sqrt{\frac{(a+b)\tanh^2(x)}{a}}}{\sqrt{1+\frac{b\tanh^2(x)}{a}}}\right)\left(b+a\coth^2(x)\right)^2\sqrt{\frac{(a+b)\tanh^2(x)}{a}} - (a+b)\right.}{3(a+b)^3\left(a+b\tanh^2(x)\right)^{3/2}\sqrt{1+\frac{b\tanh^2(}{a}}}$$

input
```
Integrate[Tanh[x]^4/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
(Tanh[x]^3*(3*ArcTanh[Sqrt[((a + b)*Tanh[x]^2)/a]/Sqrt[1 + (b*Tanh[x]^2)/a
]]*(b + a*Coth[x]^2)^2*Sqrt[((a + b)*Tanh[x]^2)/a] - (a + b)*(a + 4*b + 3*
a*Coth[x]^2)*Sqrt[1 + (b*Tanh[x]^2)/a]))/(3*(a + b)^3*(a + b*Tanh[x]^2)^(3
/2)*Sqrt[1 + (b*Tanh[x]^2)/a])
```

3.248.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 4153, 372, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(ix)^4}{\left(a-b\tan(ix)^2\right)^{5/2}}\,dx$$

$$\downarrow 4153$$

$$\int \frac{\tanh^4(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{5/2}}\,d\tanh(x)$$

$$\downarrow 372$$

$$\frac{a\tanh(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{\int \frac{a-(a+3b)\tanh^2(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}\,d\tanh(x)}{3b(a+b)}$$

$$\downarrow 402$$

$$\frac{a\tanh(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{\frac{(a+4b)\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\int \frac{3ab}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}\,d\tanh(x)}{a(a+b)}}{3b(a+b)}$$

$$\downarrow 27$$

$$\frac{a\tanh(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{\frac{(a+4b)\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{3b\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}\,d\tanh(x)}{a+b}}{3b(a+b)}$$

$$\downarrow 291$$

$$\frac{a\tanh(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\frac{(a+4b)\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{3b\int\frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}}{a+b}}{3b(a+b)}$$

$$\downarrow 219$$

$$\frac{a\tanh(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\frac{(a+4b)\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{3b\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}}{3b(a+b)}$$

input
```
Int[Tanh[x]^4/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
(a*Tanh[x])/(3*b*(a + b)*(a + b*Tanh[x]^2)^(3/2)) - ((-3*b*ArcTanh[(Sqrt[a
 + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/(a + b)^(3/2) + ((a + 4*b)*Tanh[x])
/((a + b)*Sqrt[a + b*Tanh[x]^2]))/(3*b*(a + b))
```

3.248.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.248.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(76) = 152$.

Time $= 0.10$ (sec) , antiderivative size $= 491$, normalized size of antiderivative $= 5.46$

method	result
derivativedivides	$-\frac{\tanh(x)}{3a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}} - \frac{2\tanh(x)}{3a^2\sqrt{a+b\tanh(x)^2}} + \frac{\tanh(x)}{3b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}} - \frac{\tanh(x)}{3ab\sqrt{a+b\tanh(x)^2}} + \frac{}{6(a+b)\left(b(1+\tanh$
default	$-\frac{\tanh(x)}{3a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}} - \frac{2\tanh(x)}{3a^2\sqrt{a+b\tanh(x)^2}} + \frac{\tanh(x)}{3b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}} - \frac{\tanh(x)}{3ab\sqrt{a+b\tanh(x)^2}} + \frac{}{6(a+b)\left(b(1+\tanh$

input
```
int(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*tanh(x)/a/(a+b*tanh(x)^2)^(3/2)-2/3/a^2*tanh(x)/(a+b*tanh(x)^2)^(1/2)
+1/3*tanh(x)/b/(a+b*tanh(x)^2)^(3/2)-1/3/a/b*tanh(x)/(a+b*tanh(x)^2)^(1/2)
+1/6/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(1
+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(1+tanh(
x))^2-2*b*(1+tanh(x))+a+b)^(1/2)*tanh(x)+1/2/(a+b)^2/(b*(1+tanh(x))^2-2*b*
(1+tanh(x))+a+b)^(1/2)+1/2/(a+b)^2/a/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)
^(1/2)*b*tanh(x)-1/2/(a+b)^(5/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)
*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))-1/6/(a+b)/(b*(t
anh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(tanh(x)-1)^2+2*b*
(tanh(x)-1)+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(tanh(x)-1)^2+2*b*(tanh(
x)-1)+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)
^(1/2)+1/2/(a+b)^2/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)*b*tanh(x)
+1/2/(a+b)^(5/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^
2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))
```

3.248.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2545 vs. $2(76) = 152$.

Time = 0.64 (sec) , antiderivative size = 5719, normalized size of antiderivative = 63.54

$$\int \frac{\tanh^4(x)}{\left(a + b\tanh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.248.6 Sympy [F]

$$\int \frac{\tanh^4(x)}{\left(a + b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\tanh^4(x)}{\left(a + b\tanh^2(x)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)**4/(a+b*tanh(x)**2)**(5/2),x)
```

output
```
Integral(tanh(x)**4/(a + b*tanh(x)**2)**(5/2), x)
```

3.248.7 Maxima [F]

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\tanh\left(x\right)^4}{\left(b\tanh\left(x\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^4/(b*tanh(x)^2 + a)^(5/2), x)
```

3.248.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(76) = 152$.

Time $= 0.53$ (sec) , antiderivative size $= 684$, normalized size of antiderivative $= 7.60$

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx =$$

$$-\frac{4\left(\left(\left(\frac{(a^7b^2+5\,a^6b^3+10\,a^5b^4+10\,a^4b^5+5\,a^3b^6+a^2b^7)e^{(2\,x)}}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a^4b^6+6\,a^3b^7+a^2b^8}-\frac{3\,(a^6b^3+4\,a^5b^4+6\,a^4b^5+4\,a^3b^6+a^2b^7)}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a^4b^6+6\,a^3b^7+a^2b^8}\right)e^{(2\,x)}+\frac{3\,(}{a^8b^2+6\,a^7}\right.\right.\right.}{3\,(ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2}}$$

$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\,(a^2+2\,ab+b^2)\sqrt{a+b}}$$

$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}+\sqrt{a+b}\right|\right)}{2\,(a^2+2\,ab+b^2)\sqrt{a+b}}$$

$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}-\sqrt{a+b}\right|\right)}{2\,(a^2+2\,ab+b^2)\sqrt{a+b}}$$

input
```
integrate(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-4/3*((((a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6 + a^2*b
^7)*e^(2*x)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 +
6*a^3*b^7 + a^2*b^8) - 3*(a^6*b^3 + 4*a^5*b^4 + 6*a^4*b^5 + 4*a^3*b^6 + a^
2*b^7)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3
*b^7 + a^2*b^8))*e^(2*x) + 3*(a^6*b^3 + 4*a^5*b^4 + 6*a^4*b^5 + 4*a^3*b^6
+ a^2*b^7)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6
*a^3*b^7 + a^2*b^8))*e^(2*x) - (a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*
b^5 + 5*a^3*b^6 + a^2*b^7)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5
+ 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*
e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a
 + b)*(a - b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a +
b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
b) + sqrt(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a
 + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) - sqrt(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b))
```

3.248.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\tanh(x)^4}{\left(b\tanh(x)^2+a\right)^{5/2}}\,dx$$

input
```
int(tanh(x)^4/(a + b*tanh(x)^2)^(5/2),x)
```

output
```
int(tanh(x)^4/(a + b*tanh(x)^2)^(5/2), x)
```

3.249 $\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

3.249.1 Optimal result 1835
3.249.2 Mathematica [C] (verified) 1835
3.249.3 Rubi [A] (verified) 1836
3.249.4 Maple [B] (verified) 1839
3.249.5 Fricas [B] (verification not implemented) 1839
3.249.6 Sympy [F] 1840
3.249.7 Maxima [F] 1840
3.249.8 Giac [B] (verification not implemented) 1840
3.249.9 Mupad [B] (verification not implemented) 1841

3.249.1 Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + \frac{a}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

output
```
arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)-1/(a+b)^2/(a+b*tanh
(x)^2)^(1/2)+1/3*a/b/(a+b)/(a+b*tanh(x)^2)^(3/2)
```

3.249.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{a(a+b) - 3b\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\tanh^2(x)}{a+b}\right)\left(a+b\tanh^2(x)\right)}{3b(a+b)^2\left(a+b\tanh^2(x)\right)^{3/2}}$$

input
```
Integrate[Tanh[x]^3/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
(a*(a + b) - 3*b*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)
]*(a + b*Tanh[x]^2))/(3*b*(a + b)^2*(a + b*Tanh[x]^2)^(3/2))
```

3.249.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 26, 4153, 26, 354, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{i\tan(ix)^3}{\left(a-b\tan(ix)^2\right)^{5/2}}dx$$

$$\downarrow 26$$

$$i\int \frac{\tan(ix)^3}{\left(a-b\tan(ix)^2\right)^{5/2}}dx$$

$$\downarrow 4153$$

$$i\int -\frac{i\tanh^3(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{5/2}}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\tanh^3(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{5/2}}d\tanh(x)$$

$$\downarrow 354$$

$$\frac{1}{2}\int \frac{\tanh^2(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{5/2}}d\tanh^2(x)$$

$$\downarrow 87$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh^2(x)}{a+b}+\frac{2a}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right)$$

$$\downarrow 61$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a+b}-\frac{2}{(a+b)\sqrt{a+b\tanh^2(x)}}}{a+b}+\frac{2a}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right)$$

$$\downarrow 73$$

$$\frac{1}{2}\left(\frac{\frac{2\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b(a+b)}-\frac{2}{(a+b)\sqrt{a+b\tanh^2(x)}}}{a+b}+\frac{2a}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right)$$

$$\downarrow 221$$

$$\frac{1}{2}\left(\frac{\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{2}{(a+b)\sqrt{a+b\tanh^2(x)}}}{a+b}+\frac{2a}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right)$$

input
```
Int[Tanh[x]^3/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
((2*a)/(3*b*(a + b)*(a + b*Tanh[x]^2)^(3/2)) + ((2*ArcTanh[Sqrt[a + b*Tanh
[x]^2]/Sqrt[a + b]])/(a + b)^(3/2) - 2/((a + b)*Sqrt[a + b*Tanh[x]^2]))/(a
 + b))/2
```

3.249.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.249.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(62) = 124.

Time = 0.08 (sec) , antiderivative size = 435, normalized size of antiderivative = 5.88

method	result
derivativedivides	$\frac{1}{3b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}} - \frac{1}{6(a+b)\left(b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b\right)^{\frac{3}{2}}} - \frac{b\tanh(x)}{6(a+b)a\left(b(1+\tanh(x))^2-2b(1+\tanh(x)$
default	$\frac{1}{3b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}} - \frac{1}{6(a+b)\left(b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b\right)^{\frac{3}{2}}} - \frac{b\tanh(x)}{6(a+b)a\left(b(1+\tanh(x))^2-2b(1+\tanh(x)$

input
```
int(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/b/(a+b*tanh(x)^2)^(3/2)-1/6/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b
)^(3/2)-1/6*b/(a+b)/a/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)*tanh(x)-
1/3*b/(a+b)/a^2/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)*tanh(x)-1/2/(a
+b)^2/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)-1/2/(a+b)^2/a/(b*(1+tanh
(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b-2*
b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(
1+tanh(x)))-1/6/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+1/6*b/(a
+b)/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/
(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(tanh(x
)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^2/a/(b*(tanh(x)-1)^2+2*b*(tanh
(x)-1)+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*
(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))
```

3.249.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3028 vs. 2(62) = 124.

Time = 0.66 (sec) , antiderivative size = 6621, normalized size of antiderivative = 89.47

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.249.6 Sympy [F]

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)**3/(a+b*tanh(x)**2)**(5/2),x)
```

output
```
Integral(tanh(x)**3/(a + b*tanh(x)**2)**(5/2), x)
```

3.249.7 Maxima [F]

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\tanh(x)^3}{\left(b\tanh(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^3/(b*tanh(x)^2 + a)^(5/2), x)
```

3.249.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(62) = 124$.

Time $= 0.53$ (sec) , antiderivative size $= 725$, normalized size of antiderivative $= 9.80$

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\left(\left(\frac{\left(a^8b+2\,a^7b^2-5\,a^6b^3-20\,a^5b^4-25\,a^4b^5-14\,a^3b^6-3\,a^2b^7\right)e^{(2\,x)}}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a^4b^6+6\,a^3b^7+a^2b^8}+\frac{3\left(a^8b+2\,a^7b^2-a^6b^3-4\,a^5b^4-a^4b^5+\right.\right.\right.}{3\left(ae^{(4\,x)}\right.}$$

$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\left(a^2+2\,ab+b^2\right)\sqrt{a+b}}$$

$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}+\sqrt{a+b}\right|\right)}{2\left(a^2+2\,ab+b^2\right)\sqrt{a+b}}$$

$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}-\sqrt{a+b}\right|\right)}{2\left(a^2+2\,ab+b^2\right)\sqrt{a+b}}$$

input
```
integrate(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/3*((((a^8*b + 2*a^7*b^2 - 5*a^6*b^3 - 20*a^5*b^4 - 25*a^4*b^5 - 14*a^3*b
^6 - 3*a^2*b^7)*e^(2*x)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 1
5*a^4*b^6 + 6*a^3*b^7 + a^2*b^8) + 3*(a^8*b + 2*a^7*b^2 - a^6*b^3 - 4*a^5*
b^4 - a^4*b^5 + 2*a^3*b^6 + a^2*b^7)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 2
0*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) + 3*(a^8*b + 2*a^7*
b^2 - a^6*b^3 - 4*a^5*b^4 - a^4*b^5 + 2*a^3*b^6 + a^2*b^7)/(a^8*b^2 + 6*a^
7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*
x) + (a^8*b + 2*a^7*b^2 - 5*a^6*b^3 - 20*a^5*b^4 - 25*a^4*b^5 - 14*a^3*b^6
 - 3*a^2*b^7)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6
+ 6*a^3*b^7 + a^2*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
 + a + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e
^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)
))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) +
sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a +
 b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a + b)*e^(2*x)
 + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(
a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b))
```

3.249.9 Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}\,(2a^2+4ab+2b^2)}{2(a+b)^{5/2}}\right)}{(a+b)^{5/2}} + \frac{\frac{a}{3(a+b)} - \frac{b\left(b\tanh(x)^2+a\right)}{(a+b)^2}}{b\left(b\tanh(x)^2+a\right)^{3/2}}$$

input
```
int(tanh(x)^3/(a + b*tanh(x)^2)^(5/2),x)
```

output
```
atanh(((a + b*tanh(x)^2)^(1/2)*(4*a*b + 2*a^2 + 2*b^2))/(2*(a + b)^(5/2)))
/(a + b)^(5/2) + (a/(3*(a + b)) - (b*(a + b*tanh(x)^2))/(a + b)^2)/(b*(a +
 b*tanh(x)^2)^(3/2))
```

3.250 $\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

3.250.1 Optimal result . . . 1842
3.250.2 Mathematica [C] (warning: unable to verify) . . . 1842
3.250.3 Rubi [A] (verified) . . . 1843
3.250.4 Maple [B] (verified) . . . 1846
3.250.5 Fricas [B] (verification not implemented) . . . 1847
3.250.6 Sympy [F] . . . 1847
3.250.7 Maxima [F] . . . 1847
3.250.8 Giac [B] (verification not implemented) . . . 1848
3.250.9 Mupad [F(-1)] . . . 1849

3.250.1 Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} - \frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

output

```
arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)-1/3*(2*a-b)
*tanh(x)/a/(a+b)^2/(a+b*tanh(x)^2)^(1/2)-1/3*tanh(x)/(a+b)/(a+b*tanh(x)^2)
^(3/2)
```

3.250.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.11 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.45

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \sinh^2(x)\tanh(x)\left(-\frac{4(a+b)\,\text{Hypergeometric2F1}\left(2,2,\frac{9}{2},-\frac{(a+b)\sinh^2(x)}{a}\right)\sinh^2(x)\left(a+b\tanh^2(x)\right)}{35a^2} -$$

input
```
Integrate[Tanh[x]^2/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
(Sinh[x]^2*Tanh[x]*((-4*(a + b)*Hypergeometric2F1[2, 2, 9/2, -(((a + b)*Si
nh[x]^2)/a)]*Sinh[x]^2*(a + b*Tanh[x]^2))/(35*a^2) - (Coth[x]^4*(-5*a - 2*
b*Tanh[x]^2)*(3*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*(a + b*Tanh[x]^2)^2
 + a*Sech[x]^2*Sqrt[-(((a + b)*Cosh[x]^2*Sinh[x]^2*(a + b*Tanh[x]^2))/a^2)
]*(-4*b*Tanh[x]^2 + a*(-3 - Tanh[x]^2))))/(3*a*(a + b)^2*Sqrt[-(((a + b)*C
osh[x]^2*Sinh[x]^2*(a + b*Tanh[x]^2))/a^2)])))/(3*a^2*Sqrt[a + b*Tanh[x]^2
]*(1 + (b*Tanh[x]^2)/a))
```

3.250.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 25, 4153, 25, 373, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{\tan(ix)^2}{\left(a-b\tan(ix)^2\right)^{5/2}}dx$$
$$\downarrow 25$$
$$-\int \frac{\tan(ix)^2}{\left(a-b\tan(ix)^2\right)^{5/2}}dx$$
$$\downarrow 4153$$
$$-\int -\frac{\tanh^2(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{5/2}}d\tanh(x)$$
$$\downarrow 25$$
$$\int \frac{\tanh^2(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{5/2}}d\tanh(x)$$
$$\downarrow 373$$

$$\frac{\int \frac{2\tanh^2(x)+1}{(1-\tanh^2(x))(b\tanh^2(x)+a)^{3/2}}d\tanh(x)}{3(a+b)} - \frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

$$\downarrow 402$$

$$\frac{-\frac{\int -\frac{3a}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{a(a+b)} - \frac{(2a-b)\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3(a+b)} - \frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

$$\downarrow 27$$

$$\frac{\frac{3\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{a+b} - \frac{(2a-b)\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3(a+b)} - \frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

$$\downarrow 291$$

$$\frac{\frac{3\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}}{a+b} - \frac{(2a-b)\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3(a+b)} - \frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

$$\downarrow 219$$

$$\frac{\frac{3\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} - \frac{(2a-b)\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3(a+b)} - \frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

input
```
Int[Tanh[x]^2/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
-1/3*Tanh[x]/((a + b)*(a + b*Tanh[x]^2)^(3/2)) + ((3*ArcTanh[(Sqrt[a + b]*
Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/(a + b)^(3/2) - ((2*a - b)*Tanh[x])/(a*(a
 + b)*Sqrt[a + b*Tanh[x]^2]))/(3*(a + b))
```

3.250. $\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

3.250.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.250.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(74) = 148$.

Time = 0.08 (sec) , antiderivative size = 454, normalized size of antiderivative = 5.16

method	result
derivativedivides	$-\frac{\tanh(x)}{3a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{2\tanh(x)}{3a^2\sqrt{a+b\tanh(x)^2}}-\frac{1}{6(a+b)\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{}{6(a+b)a\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}$
default	$-\frac{\tanh(x)}{3a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{2\tanh(x)}{3a^2\sqrt{a+b\tanh(x)^2}}-\frac{1}{6(a+b)\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{}{6(a+b)a\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}$

input

```
int(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-1/3*tanh(x)/a/(a+b*tanh(x)^2)^(3/2)-2/3/a^2*tanh(x)/(a+b*tanh(x)^2)^(1/2)
-1/6/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(t
anh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(tanh(x)
-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(tanh(x)-1)^2+2*b*
(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^2/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)
^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)
*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1/6/(a+b)/(b*(1
+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(1+tanh(x))^2-2*b*
(1+tanh(x))+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(1+tanh(x))^2-2*b*(1+tan
h(x))+a+b)^(1/2)*tanh(x)+1/2/(a+b)^2/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)
^(1/2)+1/2/(a+b)^2/a/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)*b*tanh(x)
-1/2/(a+b)^(5/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^
2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))
```

3.250.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2939 vs. 2(74) = 148.

Time = 0.66 (sec) , antiderivative size = 6507, normalized size of antiderivative = 73.94

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.250.6 Sympy [F]

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)**2/(a+b*tanh(x)**2)**(5/2),x)
```

output
```
Integral(tanh(x)**2/(a + b*tanh(x)**2)**(5/2), x)
```

3.250.7 Maxima [F]

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\tanh(x)^2}{\left(b\tanh(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^2/(b*tanh(x)^2 + a)^(5/2), x)
```

3.250.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(74) = 148$.

Time $= 0.52$ (sec) , antiderivative size $= 728$, normalized size of antiderivative $= 8.27$

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx =$$

$$-\frac{\left(\left(\frac{(3a^7b^2+14a^6b^3+25a^5b^4+20a^4b^5+5a^3b^6-2a^2b^7-ab^8)e^{(2x)}}{a^8b^2+6a^7b^3+15a^6b^4+20a^5b^5+15a^4b^6+6a^3b^7+a^2b^8}+\frac{3(a^7b^2+2a^6b^3-a^5b^4-4a^4b^5-a^3b^6+2a^2b^7+ab^8)}{a^8b^2+6a^7b^3+15a^6b^4+20a^5b^5+15a^4b^6+6a^3b^7+a^2b^8}\right)e^{(2x)}-\frac{3(a^7}{a^8b^2+}\right.}{3\left(ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(}\right.}$$

$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\left(a^2+2ab+b^2\right)\sqrt{a+b}}$$

$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\left(a^2+2ab+b^2\right)\sqrt{a+b}}$$

$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\left(a^2+2ab+b^2\right)\sqrt{a+b}}$$

input
```
integrate(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/3*((((3*a^7*b^2 + 14*a^6*b^3 + 25*a^5*b^4 + 20*a^4*b^5 + 5*a^3*b^6 - 2*
a^2*b^7 - a*b^8)*e^(2*x)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 +
15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8) + 3*(a^7*b^2 + 2*a^6*b^3 - a^5*b^4 - 4*a
^4*b^5 - a^3*b^6 + 2*a^2*b^7 + a*b^8)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 +
20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) - 3*(a^7*b^2 + 2*a
^6*b^3 - a^5*b^4 - 4*a^4*b^5 - a^3*b^6 + 2*a^2*b^7 + a*b^8)/(a^8*b^2 + 6*a
^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2
*x) - (3*a^7*b^2 + 14*a^6*b^3 + 25*a^5*b^4 + 20*a^4*b^5 + 5*a^3*b^6 - 2*a^
2*b^7 - a*b^8)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6
 + 6*a^3*b^7 + a^2*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x
) + a + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*
e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b
)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) +
 sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a
+ b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a + b)*e^(2*x
) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt
(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b))
```

3.250.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\tanh(x)^2}{\left(b\tanh(x)^2+a\right)^{5/2}}\,dx$$

input
```
int(tanh(x)^2/(a + b*tanh(x)^2)^(5/2),x)
```

output
```
int(tanh(x)^2/(a + b*tanh(x)^2)^(5/2), x)
```

3.251 $\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

3.251.1 Optimal result 1850
3.251.2 Mathematica [C] (verified) 1850
3.251.3 Rubi [A] (verified) 1851
3.251.4 Maple [B] (verified) 1853
3.251.5 Fricas [B] (verification not implemented) 1854
3.251.6 Sympy [A] (verification not implemented) 1855
3.251.7 Maxima [F] 1855
3.251.8 Giac [B] (verification not implemented) 1856
3.251.9 Mupad [B] (verification not implemented) 1857

3.251.1 Optimal result

Integrand size = 15, antiderivative size = 70

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{1}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

output
```
arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)-1/(a+b)^2/(a+b*tanh
(x)^2)^(1/2)-1/3/(a+b)/(a+b*tanh(x)^2)^(3/2)
```

3.251.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\tanh^2(x)}{a+b}\right)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

input
```
Integrate[Tanh[x]/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
-1/3*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tanh[x]^2)/(a + b)]/((a + b)*
(a + b*Tanh[x]^2)^(3/2))
```

3.251.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 26, 4153, 26, 353, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\tan(ix)}{\left(a-b\tan(ix)^2\right)^{5/2}}dx$$

$$\downarrow 26$$

$$-i\int \frac{\tan(ix)}{\left(a-b\tan(ix)^2\right)^{5/2}}dx$$

$$\downarrow 4153$$

$$-i\int \frac{i\tanh(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{5/2}}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\tanh(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{5/2}}d\tanh(x)$$

$$\downarrow 353$$

$$\frac{1}{2}\int \frac{1}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{5/2}}d\tanh^2(x)$$

$$\downarrow 61$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh^2(x)}{a+b}-\frac{2}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right)$$

$$\downarrow 61$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a+b}-\frac{2}{(a+b)\sqrt{a+b\tanh^2(x)}}}{a+b}-\frac{2}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right)$$

$$\downarrow 73$$

$$\frac{1}{2}\left(\frac{\frac{2\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b(a+b)}-\frac{2}{(a+b)\sqrt{a+b\tanh^2(x)}}}{a+b}-\frac{2}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right)$$

$$\downarrow 221$$

$$\frac{1}{2}\left(\frac{\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{2}{(a+b)\sqrt{a+b\tanh^2(x)}}}{a+b}-\frac{2}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right)$$

input
```
Int[Tanh[x]/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
(-2/(3*(a + b)*(a + b*Tanh[x]^2)^(3/2)) + ((2*ArcTanh[Sqrt[a + b*Tanh[x]^2
]/Sqrt[a + b]])/(a + b)^(3/2) - 2/((a + b)*Sqrt[a + b*Tanh[x]^2]))/(a + b)
)/2
```

3.251.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.251.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(58) = 116$.

Time = 0.05 (sec) , antiderivative size = 420, normalized size of antiderivative = 6.00

method	result
derivativedivides	$-\frac{1}{6(a+b)\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{b\tanh(x)}{6(a+b)a\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{}{3(a+b)}$
default	$-\frac{1}{6(a+b)\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{b\tanh(x)}{6(a+b)a\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{}{3(a+b)}$

input
```
int(tanh(x)/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(t
anh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(tanh(x)
-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(tanh(x)-1)^2+2*b*
(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^2/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)
^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)
*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))-1/6/(a+b)/(b*(1
+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)-1/6*b/(a+b)/a/(b*(1+tanh(x))^2-2*b*
(1+tanh(x))+a+b)^(3/2)*tanh(x)-1/3*b/(a+b)/a^2/(b*(1+tanh(x))^2-2*b*(1+tan
h(x))+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)
^(1/2)-1/2/(a+b)^2/a/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2)*b*tanh(x)
+1/2/(a+b)^(5/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*(b*(1+tanh(x))^
2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))
```

3.251.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2607 vs. $2(58) = 116$.

Time $= 0.64$ (sec) , antiderivative size $= 5779$, normalized size of antiderivative $= 82.56$

$$\int \frac{\tanh(x)}{\left(a + b\tanh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.251.6 Sympy [A] (verification not implemented)

Time = 12.56 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.63

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx =$$

$$-\begin{cases} \dfrac{2\left(\frac{b}{6(a+b)\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}+\frac{b}{2(a+b)^2\sqrt{a+b\tanh^2(x)}}+\frac{b\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}(a+b)^2}\right)}{b} & \text{for } b\neq 0 \\ \begin{cases} \tilde{\infty}\tanh^2(x) & \text{for } a^{\frac{5}{2}}=0 \\ \dfrac{\log\left(2a^{\frac{5}{2}}\tanh^2(x)-2a^{\frac{5}{2}}\right)}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)**2)**(5/2),x)
```

output
```
-Piecewise((2*(b/(6*(a + b)*(a + b*tanh(x)**2)**(3/2)) + b/(2*(a + b)**2*s
qrt(a + b*tanh(x)**2)) + b*atan(sqrt(a + b*tanh(x)**2)/sqrt(-a - b))/(2*sq
rt(-a - b)*(a + b)**2))/b, Ne(b, 0)), (Piecewise((zoo*tanh(x)**2, Eq(a**(5
/2), 0)), (log(2*a**(5/2)*tanh(x)**2 - 2*a**(5/2))/(2*a**(5/2)), True)), T
rue))
```

3.251.7 Maxima [F]

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\tanh(x)}{\left(b\tanh(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)/(b*tanh(x)^2 + a)^(5/2), x)
```

3.251.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. 2(58) = 116.

Time = 0.51 (sec) , antiderivative size = 683, normalized size of antiderivative = 9.76

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx =$$

$$-\frac{4\left(\left(\left(\frac{(a^7b^2+5\,a^6b^3+10\,a^5b^4+10\,a^4b^5+5\,a^3b^6+a^2b^7)e^{(2\,x)}}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a^4b^6+6\,a^3b^7+a^2b^8}+\frac{3\,(a^7b^2+4\,a^6b^3+6\,a^5b^4+4\,a^4b^5+a^3b^6)}{a^8b^2+6\,a^7b^3+15\,a^6b^4+20\,a^5b^5+15\,a^4b^6+6\,a^3b^7+a^2b^8}\right)e^{(2\,x)}+\frac{3(}{a^8b^2+6\,a^7}\right.\right.\right.}{3\left(ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2}\right.}$$

$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\left(a^2+2\,ab+b^2\right)\sqrt{a+b}}$$

$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}+\sqrt{a+b}\right|\right)}{2\left(a^2+2\,ab+b^2\right)\sqrt{a+b}}$$

$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}-\sqrt{a+b}\right|\right)}{2\left(a^2+2\,ab+b^2\right)\sqrt{a+b}}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-4/3*((((a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6 + a^2*b
^7)*e^(2*x)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 +
6*a^3*b^7 + a^2*b^8) + 3*(a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^
3*b^6)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3
*b^7 + a^2*b^8))*e^(2*x) + 3*(a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5
+ a^3*b^6)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6
*a^3*b^7 + a^2*b^8))*e^(2*x) + (a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*
b^5 + 5*a^3*b^6 + a^2*b^7)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5
+ 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*
e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a
 + b)*(a - b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a +
b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
b) + sqrt(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a
 + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) - sqrt(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b))
```

3.251.9 Mupad [B] (verification not implemented)

Time = 4.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}\,(2a^2+4ab+2b^2)}{2(a+b)^{5/2}}\right)}{(a+b)^{5/2}} - \frac{\frac{1}{3(a+b)}+\frac{b\tanh(x)^2+a}{(a+b)^2}}{\left(b\tanh(x)^2+a\right)^{3/2}}$$

input
```
int(tanh(x)/(a + b*tanh(x)^2)^(5/2),x)
```

output
```
atanh(((a + b*tanh(x)^2)^(1/2)*(4*a*b + 2*a^2 + 2*b^2))/(2*(a + b)^(5/2)))
/(a + b)^(5/2) - (1/(3*(a + b)) + (a + b*tanh(x)^2)/(a + b)^2)/(a + b*tanh
(x)^2)^(3/2)
```

3.252 $\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

3.252.1 Optimal result . . . 1858
3.252.2 Mathematica [C] (warning: unable to verify) . . . 1858
3.252.3 Rubi [A] (verified) . . . 1859
3.252.4 Maple [B] (verified) . . . 1862
3.252.5 Fricas [B] (verification not implemented) . . . 1862
3.252.6 Sympy [F] . . . 1863
3.252.7 Maxima [F] . . . 1863
3.252.8 Giac [B] (verification not implemented) . . . 1863
3.252.9 Mupad [F(-1)] . . . 1864

3.252.1 Optimal result

Integrand size = 12, antiderivative size = 93

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(5a+2b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

output

```
arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)+1/3*b*(5*a+
2*b)*tanh(x)/a^2/(a+b)^2/(a+b*tanh(x)^2)^(1/2)+1/3*b*tanh(x)/a/(a+b)/(a+b*
tanh(x)^2)^(3/2)
```

3.252.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.82 (sec) , antiderivative size = 976, normalized size of antiderivative = 10.49

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\cosh(x)\sinh(x)\left(1575\arcsin\left(\sqrt{-\frac{(a+b)\sinh^2(x)}{a}}\right) + \frac{3150(a+b)\arcsin\left(\sqrt{-\frac{(a+b)\sinh^2(x)}{a}}\right)}{a}\right.$$

input
```
Integrate[(a + b*Tanh[x]^2)^(-5/2),x]
```

output
```
(Cosh[x]*Sinh[x]*(1575*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]] + (3150*(a +
 b)*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Sinh[x]^2)/a + (1575*(a + b)^2*
ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Sinh[x]^4)/a^2 + (2100*b*ArcSin[Sqr
t[-(((a + b)*Sinh[x]^2)/a)]]*Tanh[x]^2)/a + (4200*b*(a + b)*ArcSin[Sqrt[-(
((a + b)*Sinh[x]^2)/a)]]*Sinh[x]^2*Tanh[x]^2)/a^2 + (2100*b*(a + b)^2*ArcS
in[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Sinh[x]^4*Tanh[x]^2)/a^3 + (840*b^2*Arc
Sin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Tanh[x]^4)/a^2 + (1680*b^2*(a + b)*Arc
Sin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Sinh[x]^2*Tanh[x]^4)/a^3 + (840*b^2*(a
 + b)^2*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Sinh[x]^4*Tanh[x]^4)/a^4 +
2100*(-(((a + b)*Sinh[x]^2)/a))^(3/2)*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a
] + 96*Hypergeometric2F1[2, 2, 9/2, -(((a + b)*Sinh[x]^2)/a)]*(-(((a + b)*
Sinh[x]^2)/a))^(7/2)*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a] + 24*Hypergeome
tricPFQ[{2, 2, 2}, {1, 9/2}, -(((a + b)*Sinh[x]^2)/a)]*(-(((a + b)*Sinh[x]
^2)/a))^(7/2)*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a] + (2800*b*(-(((a + b)*
Sinh[x]^2)/a))^(3/2)*Tanh[x]^2*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a])/a +
(168*b*Hypergeometric2F1[2, 2, 9/2, -(((a + b)*Sinh[x]^2)/a)]*(-(((a + b)*
Sinh[x]^2)/a))^(7/2)*Tanh[x]^2*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a])/a +
(48*b*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, -(((a + b)*Sinh[x]^2)/a)]*(-(
((a + b)*Sinh[x]^2)/a))^(7/2)*Tanh[x]^2*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))
/a])/a + (1120*b^2*(-(((a + b)*Sinh[x]^2)/a))^(3/2)*Tanh[x]^4*Sqrt[(Cos...
```

3.252.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 4144, 316, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$$
$$\downarrow \text{3042}$$
$$\int \frac{1}{\left(a-b\tan(ix)^2\right)^{5/2}}dx$$
$$\downarrow \text{4144}$$

$$\int \frac{1}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{5/2}}\,d\tanh(x)$$

$$\downarrow 316$$

$$\frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{\int \frac{2b\tanh^2(x)+b-3(a+b)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}\,d\tanh(x)}{3a(a+b)}$$

$$\downarrow 402$$

$$\frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{-\frac{\int \frac{3a^2}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}\,d\tanh(x)}{a(a+b)} - \frac{b(5a+2b)\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3a(a+b)}$$

$$\downarrow 27$$

$$\frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{-\frac{3a\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}\,d\tanh(x)}{a+b} - \frac{b(5a+2b)\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3a(a+b)}$$

$$\downarrow 291$$

$$\frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{-\frac{3a\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}\,d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}}{a+b} - \frac{b(5a+2b)\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3a(a+b)}$$

$$\downarrow 219$$

$$\frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{-\frac{3a\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} - \frac{b(5a+2b)\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3a(a+b)}$$

input
```
Int[(a + b*Tanh[x]^2)^(-5/2),x]
```

output
```
(b*Tanh[x])/(3*a*(a + b)*(a + b*Tanh[x]^2)^(3/2)) - ((-3*a*ArcTanh[(Sqrt[a
 + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/(a + b)^(3/2) - (b*(5*a + 2*b)*Tanh
[x])/(a*(a + b)*Sqrt[a + b*Tanh[x]^2]))/(3*a*(a + b))
```

3.252.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.252.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(79) = 158$.

Time = 0.10 (sec) , antiderivative size = 420, normalized size of antiderivative = 4.52

method	result
derivativedivides	$\frac{1}{6(a+b)\left(b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b\right)^{\frac{3}{2}}}+\frac{b\tanh(x)}{6(a+b)a\left(b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b\right)^{\frac{3}{2}}}+\frac{}{3(a+b)a^2}$
default	$\frac{1}{6(a+b)\left(b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b\right)^{\frac{3}{2}}}+\frac{b\tanh(x)}{6(a+b)a\left(b(1+\tanh(x))^2-2b(1+\tanh(x))+a+b\right)^{\frac{3}{2}}}+\frac{}{3(a+b)a^2}$

input
```
int(1/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/(a+b)/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(1+
tanh(x))^2-2*b*(1+tanh(x))+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(1+tanh(x
))^2-2*b*(1+tanh(x))+a+b)^(1/2)*tanh(x)+1/2/(a+b)^2/(b*(1+tanh(x))^2-2*b*(
1+tanh(x))+a+b)^(1/2)+1/2/(a+b)^2/a/(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^
(1/2)*b*tanh(x)-1/2/(a+b)^(5/2)*ln((2*a+2*b-2*b*(1+tanh(x))+2*(a+b)^(1/2)*
(b*(1+tanh(x))^2-2*b*(1+tanh(x))+a+b)^(1/2))/(1+tanh(x)))-1/6/(a+b)/(b*(ta
nh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(tanh(x)-1)^2+2*b*(
tanh(x)-1)+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(tanh(x)-1)^2+2*b*(tanh(x
)-1)+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^
(1/2)+1/2/(a+b)^2/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)*b*tanh(x)+
1/2/(a+b)^(5/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2
+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))
```

3.252.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3152 vs. $2(79) = 158$.

Time = 0.68 (sec) , antiderivative size = 6933, normalized size of antiderivative = 74.55

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.252.6 Sympy [F]

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a+b\tanh^2(x)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*tanh(x)**2)**(5/2),x)
```

output
```
Integral((a + b*tanh(x)**2)**(-5/2), x)
```

3.252.7 Maxima [F]

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\tanh(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*tanh(x)^2 + a)^(-5/2), x)
```

3.252.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 714 vs. $2(79) = 158$.

Time $= 0.52$ (sec) , antiderivative size $= 714$, normalized size of antiderivative $= 7.68$

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{2\left(\left(\left(\frac{(3a^6b^3+16a^5b^4+35a^4b^5+40a^3b^6+25a^2b^7+8ab^8+b^9)e^{(2x)}}{a^8b^2+6a^7b^3+15a^6b^4+20a^5b^5+15a^4b^6+6a^3b^7+a^2b^8}+\frac{3(a^6b^3+2a^5b^4-3a^4b^5-12a^3b^6-}{a^8b^2+6a^7b^3+15a^6b^4+20a^5b^5+15}\right.\right.\right.}{3(ae^{(4}$$

$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a^2+2ab+b^2)\sqrt{a+b}}$$

$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a^2+2ab+b^2)\sqrt{a+b}}$$

$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a^2+2ab+b^2)\sqrt{a+b}}$$

input
```
integrate(1/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
2/3*((((3*a^6*b^3 + 16*a^5*b^4 + 35*a^4*b^5 + 40*a^3*b^6 + 25*a^2*b^7 + 8*
a*b^8 + b^9)*e^(2*x)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a
^4*b^6 + 6*a^3*b^7 + a^2*b^8) + 3*(a^6*b^3 + 2*a^5*b^4 - 3*a^4*b^5 - 12*a^
3*b^6 - 13*a^2*b^7 - 6*a*b^8 - b^9)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20
*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) - 3*(a^6*b^3 + 2*a^5
*b^4 - 3*a^4*b^5 - 12*a^3*b^6 - 13*a^2*b^7 - 6*a*b^8 - b^9)/(a^8*b^2 + 6*a
^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2
*x) - (3*a^6*b^3 + 16*a^5*b^4 + 35*a^4*b^5 + 40*a^3*b^6 + 25*a^2*b^7 + 8*a
*b^8 + b^9)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 +
6*a^3*b^7 + a^2*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
 a + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(
4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))
/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sq
rt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b
)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) +
 sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a
+ b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b))
```

3.252.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\tanh(x)^2+a\right)^{5/2}}\,dx$$

input
```
int(1/(a + b*tanh(x)^2)^(5/2),x)
```

output
```
int(1/(a + b*tanh(x)^2)^(5/2), x)
```

3.253 $\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

3.253.1 Optimal result 1865
3.253.2 Mathematica [C] (verified) 1865
3.253.3 Rubi [A] (verified) 1866
3.253.4 Maple [F] 1870
3.253.5 Fricas [B] (verification not implemented) 1870
3.253.6 Sympy [F] 1870
3.253.7 Maxima [F] 1871
3.253.8 Giac [B] (verification not implemented) 1871
3.253.9 Mupad [F(-1)] 1872

3.253.1 Optimal result

Integrand size = 15, antiderivative size = 108

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + \frac{b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

output
```
-arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(5/2)+arctanh((a+b*tanh(x)^2)^(1
/2)/(a+b)^(1/2))/(a+b)^(5/2)+b*(2*a+b)/a^2/(a+b)^2/(a+b*tanh(x)^2)^(1/2)+1
/3*b/a/(a+b)/(a+b*tanh(x)^2)^(3/2)
```

3.253.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{-a\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\tanh^2(x)}{a+b}\right)+(a+b)\,\text{Hypergeometric2F1}}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

input
```
Integrate[Coth[x]/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
(-(a*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tanh[x]^2)/(a + b)]) + (a + b
)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Tanh[x]^2)/a])/(3*a*(a + b)*(a +
 b*Tanh[x]^2)^(3/2))
```

3.253.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.31, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 26, 4153, 26, 354, 96, 25, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{i}{\tan(ix)\left(a-b\tan(ix)^2\right)^{5/2}}dx$$

$$\downarrow 26$$

$$i\int \frac{1}{\tan(ix)\left(a-b\tan(ix)^2\right)^{5/2}}dx$$

$$\downarrow 4153$$

$$i\int -\frac{i\coth(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{5/2}}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\coth(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{5/2}}d\tanh(x)$$

$$\downarrow 354$$

$$\frac{1}{2}\int \frac{\coth(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{5/2}}d\tanh^2(x)$$

$$\downarrow 96$$

$$\frac{1}{2}\left(\frac{2b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\int -\frac{\coth(x)\left(-b\tanh^2(x)+a+b\right)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh^2(x)}{a(a+b)}\right)$$

$$\downarrow 25$$

$$\frac{1}{2}\left(\frac{\int \frac{\coth(x)\left(-b\tanh^2(x)+a+b\right)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh^2(x)}{a(a+b)}+\frac{2b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right)$$

$$\downarrow 169$$

$$\frac{1}{2}\left(\frac{\frac{2\int \frac{\coth(x)\left((a+b)^2-b(2a+b)\tanh^2(x)\right)}{2\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{a(a+b)}+\frac{2b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right)$$

$$\downarrow 27$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{\coth(x)\left((a+b)^2-b(2a+b)\tanh^2(x)\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{a(a+b)}+\frac{2b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right)$$

$$\downarrow 174$$

$$\frac{1}{2}\left(\frac{\frac{a^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)+(a+b)^2\int \frac{\coth(x)}{\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{a(a+b)}+\frac{2b}{3a(a+b)\left(a+b\tanh^2(\right.}\right.$$

$$\downarrow 73$$

$$\frac{1}{2}\left(\frac{\frac{2a^2\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}+\frac{2(a+b)^2\int \frac{1}{\frac{\tanh^4(x)}{b}-\frac{a}{b}}d\sqrt{b\tanh^2(x)+a}}{b}}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{a(a+b)}+\frac{2b}{3a(a+b)\left(a+b\tanh^2(x)\right)}\right.$$

$$\downarrow 221$$

$$\frac{1}{2}\left(\frac{\frac{2a^2\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{2(a+b)^2\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{a(a+b)}+\frac{2b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right)$$

input
```
Int[Coth[x]/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
((2*b)/(3*a*(a + b)*(a + b*Tanh[x]^2)^(3/2)) + (((-2*(a + b)^2*ArcTanh[Sqr
t[a + b*Tanh[x]^2]/Sqrt[a]])/Sqrt[a] + (2*a^2*ArcTanh[Sqrt[a + b*Tanh[x]^2
]/Sqrt[a + b]])/Sqrt[a + b])/(a*(a + b)) + (2*b*(2*a + b))/(a*(a + b)*Sqrt
[a + b*Tanh[x]^2]))/(a*(a + b)))/2
```

3.253.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.253.4 Maple [F]

$$\int \frac{\coth(x)}{\left(a+b\tanh(x)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
int(coth(x)/(a+b*tanh(x)^2)^(5/2),x)
```

output
```
int(coth(x)/(a+b*tanh(x)^2)^(5/2),x)
```

3.253.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4510 vs. $2(90) = 180$.

Time $= 1.28$ (sec) , antiderivative size $= 19305$, normalized size of antiderivative $= 178.75$

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.253.6 Sympy [F]

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(coth(x)/(a+b*tanh(x)**2)**(5/2),x)
```

output
```
Integral(coth(x)/(a + b*tanh(x)**2)**(5/2), x)
```

3.253.7 Maxima [F]

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\coth\left(x\right)}{\left(b\tanh\left(x\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(coth(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)/(b*tanh(x)^2 + a)^(5/2), x)
```

3.253.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(90) = 180$.

Time = 0.65 (sec) , antiderivative size = 808, normalized size of antiderivative = 7.48

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\left(\left(\frac{(7a^{14}b^3+38a^{13}b^4+85a^{12}b^5+100a^{11}b^6+65a^{10}b^7+22a^9b^8+3a^8b^9)e^{(2x)}}{a^{16}b^2+6a^{15}b^3+15a^{14}b^4+20a^{13}b^5+15a^{12}b^6+6a^{11}b^7+a^{10}b^8}+\frac{3(7a^{14}b^3+30a^{13}b^4+49a^{1}}{a^{16}b^2+6a^{15}b^3+15a^{14}b^4}\right.\right.$$

$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\left(a^2+2ab+b^2\right)\sqrt{a+b}}$$

$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\left(a^2+2ab+b^2\right)\sqrt{a+b}}$$

$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\left(a^2+2ab+b^2\right)\sqrt{a+b}}$$

$$+\frac{2\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}a^2}$$

input
```
integrate(coth(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/3*((((7*a^14*b^3 + 38*a^13*b^4 + 85*a^12*b^5 + 100*a^11*b^6 + 65*a^10*b^
7 + 22*a^9*b^8 + 3*a^8*b^9)*e^(2*x)/(a^16*b^2 + 6*a^15*b^3 + 15*a^14*b^4 +
 20*a^13*b^5 + 15*a^12*b^6 + 6*a^11*b^7 + a^10*b^8) + 3*(7*a^14*b^3 + 30*a
^13*b^4 + 49*a^12*b^5 + 36*a^11*b^6 + 9*a^10*b^7 - 2*a^9*b^8 - a^8*b^9)/(a
^16*b^2 + 6*a^15*b^3 + 15*a^14*b^4 + 20*a^13*b^5 + 15*a^12*b^6 + 6*a^11*b^
7 + a^10*b^8))*e^(2*x) + 3*(7*a^14*b^3 + 30*a^13*b^4 + 49*a^12*b^5 + 36*a^
11*b^6 + 9*a^10*b^7 - 2*a^9*b^8 - a^8*b^9)/(a^16*b^2 + 6*a^15*b^3 + 15*a^1
4*b^4 + 20*a^13*b^5 + 15*a^12*b^6 + 6*a^11*b^7 + a^10*b^8))*e^(2*x) + (7*a
^14*b^3 + 38*a^13*b^4 + 85*a^12*b^5 + 100*a^11*b^6 + 65*a^10*b^7 + 22*a^9*
b^8 + 3*a^8*b^9)/(a^16*b^2 + 6*a^15*b^3 + 15*a^14*b^4 + 20*a^13*b^5 + 15*a
^12*b^6 + 6*a^11*b^7 + a^10*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2
*b*e^(2*x) + a + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(
4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a +
b)*(a - b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a + b)*
e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
+ sqrt(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a +
b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
b) - sqrt(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 2*arctan(-1/2*(sqrt
(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
 a + b) - sqrt(a + b))/sqrt(-a))/(sqrt(-a)*a^2)
```

3.253.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\coth(x)}{\left(b\tanh(x)^2+a\right)^{5/2}}\,dx$$

input
```
int(coth(x)/(a + b*tanh(x)^2)^(5/2),x)
```

output
```
int(coth(x)/(a + b*tanh(x)^2)^(5/2), x)
```

3.254 $\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

3.254.1 Optimal result 1873
3.254.2 Mathematica [C] (warning: unable to verify) 1873
3.254.3 Rubi [A] (verified) 1874
3.254.4 Maple [F] 1878
3.254.5 Fricas [B] (verification not implemented) 1878
3.254.6 Sympy [F] 1879
3.254.7 Maxima [F] 1879
3.254.8 Giac [B] (verification not implemented) 1879
3.254.9 Mupad [F(-1)] 1880

3.254.1 Optimal result

Integrand size = 17, antiderivative size = 131

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{(3a+2b)(a+4b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a^3(a+b)^2}$$

output
```
arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)+1/3*b*(7*a+
4*b)*coth(x)/a^2/(a+b)^2/(a+b*tanh(x)^2)^(1/2)-1/3*(3*a+2*b)*(a+4*b)*coth(
x)*(a+b*tanh(x)^2)^(1/2)/a^3/(a+b)^2+1/3*b*coth(x)/a/(a+b)/(a+b*tanh(x)^2)
^(3/2)
```

3.254.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.66 (sec) , antiderivative size = 1375, normalized size of antiderivative = 10.50

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Coth[x]^2/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
-((Cosh[x]^2*Coth[x]*((-20*a*Csch[x]^2)/(3*(a + b)) - (5*a^2*Csch[x]^4)/(a
 + b)^2 - (40*b*Sech[x]^2)/(a + b) - (30*a*b*Csch[x]^2*Sech[x]^2)/(a + b)^
2 - (40*b^2*Sech[x]^4)/(a + b)^2 - (92*(a + b)*Hypergeometric2F1[2, 2, 9/2
, -(((a + b)*Sinh[x]^2)/a)]*Sinh[x]^2)/(105*a) - (24*(a + b)*Hypergeometri
cPFQ[{2, 2, 2}, {1, 9/2}, -(((a + b)*Sinh[x]^2)/a)]*Sinh[x]^2)/(35*a) - (1
6*(a + b)*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, -(((a + b)*Sinh[x]^
2)/a)]*Sinh[x]^2)/(105*a) - (160*b^2*Sech[x]^2*Tanh[x]^2)/(3*a*(a + b)) -
(124*b*(a + b)*Hypergeometric2F1[2, 2, 9/2, -(((a + b)*Sinh[x]^2)/a)]*Sinh
[x]^2*Tanh[x]^2)/(35*a^2) - (16*b*(a + b)*HypergeometricPFQ[{2, 2, 2}, {1,
 9/2}, -(((a + b)*Sinh[x]^2)/a)]*Sinh[x]^2*Tanh[x]^2)/(7*a^2) - (16*b*(a +
 b)*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, -(((a + b)*Sinh[x]^2)/a)]
*Sinh[x]^2*Tanh[x]^2)/(35*a^2) - (64*b^3*Sech[x]^2*Tanh[x]^4)/(3*a^2*(a +
b)) - (152*b^2*(a + b)*Hypergeometric2F1[2, 2, 9/2, -(((a + b)*Sinh[x]^2)/
a)]*Sinh[x]^2*Tanh[x]^4)/(35*a^3) - (88*b^2*(a + b)*HypergeometricPFQ[{2,
2, 2}, {1, 9/2}, -(((a + b)*Sinh[x]^2)/a)]*Sinh[x]^2*Tanh[x]^4)/(35*a^3) -
 (16*b^2*(a + b)*HypergeometricPFQ[{2, 2, 2, 2}, {1, 1, 9/2}, -(((a + b)*S
inh[x]^2)/a)]*Sinh[x]^2*Tanh[x]^4)/(35*a^3) - (176*b^3*(a + b)*Hypergeomet
ric2F1[2, 2, 9/2, -(((a + b)*Sinh[x]^2)/a)]*Sinh[x]^2*Tanh[x]^6)/(105*a^4)
 - (32*b^3*(a + b)*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, -(((a + b)*Sinh[
x]^2)/a)]*Sinh[x]^2*Tanh[x]^6)/(35*a^4) - (16*b^3*(a + b)*Hypergeometri...
```

3.254.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {3042, 25, 4153, 25, 374, 25, 441, 25, 445, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{1}{\tan(ix)^2\left(a-b\tan(ix)^2\right)^{5/2}}dx$$

$$\downarrow 25$$

$$-\int \frac{1}{\tan(ix)^2 \left(a-b\tan(ix)^2\right)^{5/2}} dx$$

$$\downarrow 4153$$

$$-\int -\frac{\coth^2(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{5/2}} d\tanh(x)$$

$$\downarrow 25$$

$$\int \frac{\coth^2(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{5/2}} d\tanh(x)$$

$$\downarrow 374$$

$$\frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{\int -\frac{\coth^2(x)\left(-4b\tanh^2(x)+3a+4b\right)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}} d\tanh(x)}{3a(a+b)}$$

$$\downarrow 25$$

$$\frac{\int \frac{\coth^2(x)\left(-4b\tanh^2(x)+3a+4b\right)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}} d\tanh(x)}{3a(a+b)} + \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

$$\downarrow 441$$

$$\frac{\frac{b(7a+4b)\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\int -\frac{\coth^2(x)\left((3a+2b)(a+4b)-2b(7a+4b)\tanh^2(x)\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}} d\tanh(x)}{a(a+b)}}{3a(a+b)} + \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

$$\downarrow 25$$

$$\frac{\frac{\int \frac{\coth^2(x)\left((3a+2b)(a+4b)-2b(7a+4b)\tanh^2(x)\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}} d\tanh(x)}{a(a+b)} + \frac{b(7a+4b)\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3a(a+b)} + \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

$$\downarrow 445$$

$$\frac{\frac{-\frac{\int -\frac{3a^3}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}} d\tanh(x)}{a} - \frac{(3a+2b)(a+4b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}}{a(a+b)} + \frac{b(7a+4b)\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3a(a+b)} + \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

$$\downarrow 27$$

$$\frac{3a^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\frac{(3a+2b)(a+4b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}}{a(a+b)}+\frac{b(7a+4b)\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3a(a+b)}+$$
$$\frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

$$\downarrow 291$$

$$\frac{3a^2\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}-\frac{(3a+2b)(a+4b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}}{a(a+b)}+\frac{b(7a+4b)\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3a(a+b)}+$$
$$\frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

$$\downarrow 219$$

$$\frac{\frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}-\frac{(3a+2b)(a+4b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}}{a(a+b)}+\frac{b(7a+4b)\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3a(a+b)}+$$
$$\frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

input
```
Int[Coth[x]^2/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
(b*Coth[x])/(3*a*(a + b)*(a + b*Tanh[x]^2)^(3/2)) + ((b*(7*a + 4*b)*Coth[x
])/(a*(a + b)*Sqrt[a + b*Tanh[x]^2]) + ((3*a^2*ArcTanh[(Sqrt[a + b]*Tanh[x
])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[a + b] - ((3*a + 2*b)*(a + 4*b)*Coth[x]*Sq
rt[a + b*Tanh[x]^2])/a)/(a*(a + b)))/(3*a*(a + b))
```

3.254.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.254.4 Maple [F]

$$\int \frac{\coth(x)^2}{\left(a+b\tanh(x)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
int(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x)
```

output
```
int(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x)
```

3.254.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5021 vs. $2(113) = 226$.

Time $= 1.34$ (sec) , antiderivative size $= 10671$, normalized size of antiderivative $= 81.46$

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.254.6 Sympy [F]

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(coth(x)**2/(a+b*tanh(x)**2)**(5/2),x)
```

output
```
Integral(coth(x)**2/(a + b*tanh(x)**2)**(5/2), x)
```

3.254.7 Maxima [F]

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\coth(x)^2}{\left(b\tanh(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)^2/(b*tanh(x)^2 + a)^(5/2), x)
```

3.254.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(113) = 226$.

Time $= 0.74$ (sec) , antiderivative size $= 898$, normalized size of antiderivative $= 6.85$

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

output

```
-1/3*((((9*a^13*b^4 + 50*a^12*b^5 + 115*a^11*b^6 + 140*a^10*b^7 + 95*a^9*b
^8 + 34*a^8*b^9 + 5*a^7*b^10)*e^(2*x)/(a^16*b^2 + 6*a^15*b^3 + 15*a^14*b^4
 + 20*a^13*b^5 + 15*a^12*b^6 + 6*a^11*b^7 + a^10*b^8) + 3*(3*a^13*b^4 + 6*
a^12*b^5 - 11*a^11*b^6 - 44*a^10*b^7 - 51*a^9*b^8 - 26*a^8*b^9 - 5*a^7*b^1
0)/(a^16*b^2 + 6*a^15*b^3 + 15*a^14*b^4 + 20*a^13*b^5 + 15*a^12*b^6 + 6*a^
11*b^7 + a^10*b^8))*e^(2*x) - 3*(3*a^13*b^4 + 6*a^12*b^5 - 11*a^11*b^6 - 4
4*a^10*b^7 - 51*a^9*b^8 - 26*a^8*b^9 - 5*a^7*b^10)/(a^16*b^2 + 6*a^15*b^3
+ 15*a^14*b^4 + 20*a^13*b^5 + 15*a^12*b^6 + 6*a^11*b^7 + a^10*b^8))*e^(2*x
) - (9*a^13*b^4 + 50*a^12*b^5 + 115*a^11*b^6 + 140*a^10*b^7 + 95*a^9*b^8 +
 34*a^8*b^9 + 5*a^7*b^10)/(a^16*b^2 + 6*a^15*b^3 + 15*a^14*b^4 + 20*a^13*b
^5 + 15*a^12*b^6 + 6*a^11*b^7 + a^10*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^
(2*x) - 2*b*e^(2*x) + a + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - s
qrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) -
sqrt(a + b)*(a - b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqr
t(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
+ a + b) + sqrt(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-
sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b) - sqrt(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 4*(sqrt(a
+ b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
+ b) + sqrt(a + b))/(((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)...
```

3.254.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\coth(x)^2}{\left(b\tanh(x)^2+a\right)^{5/2}}\,dx$$

input

```
int(coth(x)^2/(a + b*tanh(x)^2)^(5/2),x)
```

output

```
int(coth(x)^2/(a + b*tanh(x)^2)^(5/2), x)
```

3.255 $\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx$

3.255.1 Optimal result 1881
3.255.2 Mathematica [A] (verified) 1881
3.255.3 Rubi [A] (verified) 1882
3.255.4 Maple [B] (verified) 1883
3.255.5 Fricas [B] (verification not implemented) 1884
3.255.6 Sympy [F] 1884
3.255.7 Maxima [F] 1885
3.255.8 Giac [B] (verification not implemented) 1885
3.255.9 Mupad [B] (verification not implemented) 1885

3.255.1 Optimal result

Integrand size = 10, antiderivative size = 25

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)}{\sqrt{2}}$$

output
```
1/2*arctanh(2^(1/2)*tanh(x)/(1+tanh(x)^2)^(1/2))*2^(1/2)
```

3.255.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = \frac{\operatorname{arcsinh}\left(\sqrt{2}\sinh(x)\right)\sqrt{\cosh(2x)}\operatorname{sech}(x)}{\sqrt{2}\sqrt{1+\tanh^2(x)}}$$

input
```
Integrate[1/Sqrt[1 + Tanh[x]^2],x]
```

output
```
(ArcSinh[Sqrt[2]*Sinh[x]]*Sqrt[Cosh[2*x]]*Sech[x])/(Sqrt[2]*Sqrt[1 + Tanh[
x]^2])
```

3.255.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4144, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\tanh^2(x)+1}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sqrt{1-\tan(ix)^2}}dx$$

$$\downarrow 4144$$

$$\int \frac{1}{(1-\tanh^2(x))\sqrt{\tanh^2(x)+1}}d\tanh(x)$$

$$\downarrow 291$$

$$\int \frac{1}{1-\frac{2\tanh^2(x)}{\tanh^2(x)+1}}d\frac{\tanh(x)}{\sqrt{\tanh^2(x)+1}}$$

$$\downarrow 219$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{\tanh^2(x)+1}}\right)}{\sqrt{2}}$$

input
```
Int[1/Sqrt[1 + Tanh[x]^2],x]
```

output
```
ArcTanh[(Sqrt[2]*Tanh[x])/Sqrt[1 + Tanh[x]^2]]/Sqrt[2]
```

3.255.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.255.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(20) = 40$.

Time $= 0.17$ (sec) , antiderivative size $= 62$, normalized size of antiderivative $= 2.48$

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2+2\tanh(x))\sqrt{2}}{4\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}\right)}{4} - \frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2-2\tanh(x))\sqrt{2}}{4\sqrt{(1+\tanh(x))^2-2\tanh(x)}}\right)}{4}$	62
default	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2+2\tanh(x))\sqrt{2}}{4\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}\right)}{4} - \frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2-2\tanh(x))\sqrt{2}}{4\sqrt{(1+\tanh(x))^2-2\tanh(x)}}\right)}{4}$	62

input
```
int(1/(1+tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)*arctanh(1/4*(2+2*tanh(x))*2^(1/2)/((tanh(x)-1)^2+2*tanh(x))^(1
/2))-1/4*2^(1/2)*arctanh(1/4*(2-2*tanh(x))*2^(1/2)/((1+tanh(x))^2-2*tanh(x
))^(1/2))
```

3.255.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 543 vs. 2(20) = 40.

Time = 0.26 (sec) , antiderivative size = 543, normalized size of antiderivative = 21.72

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1+tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*log(-2*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh
(x)^2 - 3)*sinh(x)^6 - 3*cosh(x)^6 + 2*(28*cosh(x)^3 - 9*cosh(x))*sinh(x)^
5 + 5*(14*cosh(x)^4 - 9*cosh(x)^2 + 1)*sinh(x)^4 + 5*cosh(x)^4 + 4*(14*cos
h(x)^5 - 15*cosh(x)^3 + 5*cosh(x))*sinh(x)^3 + (28*cosh(x)^6 - 45*cosh(x)^
4 + 30*cosh(x)^2 - 4)*sinh(x)^2 - 4*cosh(x)^2 + 2*(4*cosh(x)^7 - 9*cosh(x)
^5 + 10*cosh(x)^3 - 4*cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*co
sh(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*si
nh(x)^4 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x)
)*sinh(x)^3 + (15*sqrt(2)*cosh(x)^4 - 18*sqrt(2)*cosh(x)^2 + 4*sqrt(2))*si
nh(x)^2 + 4*sqrt(2)*cosh(x)^2 + 2*(3*sqrt(2)*cosh(x)^5 - 6*sqrt(2)*cosh(x)
^3 + 4*sqrt(2)*cosh(x))*sinh(x) - 4*sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4)/(cosh(x)^6 + 6*cosh(x)^5
*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*
sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 1/8*sqrt(2)*log(2*(cosh(x)
^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(
x)^2 + 2*(2*cosh(x)^3 + cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*
cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2))
```

3.255.6 Sympy [F]

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = \int \frac{1}{\sqrt{\tanh^2(x)+1}}\,dx$$

input
```
integrate(1/(1+tanh(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(tanh(x)**2 + 1), x)
```

3.255.7 Maxima [F]

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = \int \frac{1}{\sqrt{\tanh(x)^2+1}}\,dx$$

input
```
integrate(1/(1+tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(tanh(x)^2 + 1), x)
```

3.255.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.

Time $= 0.27$ (sec) , antiderivative size $= 58$, normalized size of antiderivative $= 2.32$

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx =$$
$$-\frac{1}{4}\sqrt{2}\left(\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)-\log\left(-\sqrt{e^{(4x)}+1}+e^{(2x)}+1\right)\right)$$

input
```
integrate(1/(1+tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) + log(sqrt(e^(4*x) + 1)
 - e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))
```

3.255.9 Mupad [B] (verification not implemented)

Time $= 0.19$ (sec) , antiderivative size $= 63$, normalized size of antiderivative $= 2.52$

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = \frac{\sqrt{2}\left(\ln(\tanh(x)+1)-\ln\left(\sqrt{2}\sqrt{\tanh(x)^2+1}-\tanh(x)+1\right)\right)}{4}$$
$$+\frac{\sqrt{2}\left(\ln\left(\tanh(x)+\sqrt{2}\sqrt{\tanh(x)^2+1}+1\right)-\ln(\tanh(x)-1)\right)}{4}$$

input
```
int(1/(tanh(x)^2 + 1)^(1/2),x)
```

output
```
(2^(1/2)*(log(tanh(x) + 1) - log(2^(1/2)*(tanh(x)^2 + 1)^(1/2) - tanh(x) +
 1)))/4 + (2^(1/2)*(log(tanh(x) + 2^(1/2)*(tanh(x)^2 + 1)^(1/2) + 1) - log
(tanh(x) - 1)))/4
```

3.256 $\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx$

3.256.1 Optimal result 1887
3.256.2 Mathematica [A] (verified) 1887
3.256.3 Rubi [A] (verified) 1888
3.256.4 Maple [B] (verified) 1889
3.256.5 Fricas [C] (verification not implemented) 1890
3.256.6 Sympy [F] 1890
3.256.7 Maxima [F] 1891
3.256.8 Giac [C] (verification not implemented) 1891
3.256.9 Mupad [B] (verification not implemented) 1891

3.256.1 Optimal result

Integrand size = 12, antiderivative size = 27

$$\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx = \frac{\arctan\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)}{\sqrt{2}}$$

output
```
1/2*arctan(2^(1/2)*tanh(x)/(-1-tanh(x)^2)^(1/2))*2^(1/2)
```

3.256.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx = \frac{\operatorname{arcsinh}\left(\sqrt{2}\sinh(x)\right)\sqrt{\cosh(2x)}\operatorname{sech}(x)}{\sqrt{2}\sqrt{-1-\tanh^2(x)}}$$

input
```
Integrate[1/Sqrt[-1 - Tanh[x]^2],x]
```

output
```
(ArcSinh[Sqrt[2]*Sinh[x]]*Sqrt[Cosh[2*x]]*Sech[x])/(Sqrt[2]*Sqrt[-1 - Tanh
[x]^2])
```

3.256.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4144, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-\tanh^2(x)-1}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{\sqrt{-1+\tan(ix)^2}}dx$$

$$\downarrow 4144$$

$$\int \frac{1}{\sqrt{-\tanh^2(x)-1}\left(1-\tanh^2(x)\right)}d\tanh(x)$$

$$\downarrow 291$$

$$\int \frac{1}{\frac{2\tanh^2(x)}{-\tanh^2(x)-1}+1}d\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}$$

$$\downarrow 216$$

$$\frac{\arctan\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)}{\sqrt{2}}$$

input
```
Int[1/Sqrt[-1 - Tanh[x]^2],x]
```

output
```
ArcTan[(Sqrt[2]*Tanh[x])/Sqrt[-1 - Tanh[x]^2]]/Sqrt[2]
```

3.256.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)  Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.256.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(22) = 44$.

Time $= 0.17$ (sec) , antiderivative size $= 66$, normalized size of antiderivative $= 2.44$

method	result	size
derivativedivides	$-\frac{\sqrt{2}\arctan\left(\frac{(-2-2\tanh(x))\sqrt{2}}{4\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}\right)}{4}+\frac{\sqrt{2}\arctan\left(\frac{(-2+2\tanh(x))\sqrt{2}}{4\sqrt{-(1+\tanh(x))^2+2\tanh(x)}}\right)}{4}$	66
default	$-\frac{\sqrt{2}\arctan\left(\frac{(-2-2\tanh(x))\sqrt{2}}{4\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}\right)}{4}+\frac{\sqrt{2}\arctan\left(\frac{(-2+2\tanh(x))\sqrt{2}}{4\sqrt{-(1+\tanh(x))^2+2\tanh(x)}}\right)}{4}$	66

input
```
int(1/(-1-tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*2^(1/2)*arctan(1/4*(-2-2*tanh(x))*2^(1/2)/(-(tanh(x)-1)^2-2*tanh(x))^
(1/2))+1/4*2^(1/2)*arctan(1/4*(-2+2*tanh(x))*2^(1/2)/(-(1+tanh(x))^2+2*tan
h(x))^(1/2))
```

3.256.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 6.48

$$\begin{aligned}
&\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx \\
&= \frac{1}{8} i\sqrt{2}\log\left(\frac{1}{2}\left(i\sqrt{2}\sqrt{-2\,e^{(4x)}-2}+2\,e^{(2x)}+2\right)e^{(-2x)}\right) \\
&\quad -\frac{1}{8} i\sqrt{2}\log\left(\frac{1}{2}\left(-i\sqrt{2}\sqrt{-2\,e^{(4x)}-2}+2\,e^{(2x)}+2\right)e^{(-2x)}\right) \\
&\quad -\frac{1}{8} i\sqrt{2}\log\left(\left(\sqrt{-2\,e^{(4x)}-2}\left(e^{(2x)}-2\right)+i\sqrt{2}e^{(4x)}-i\sqrt{2}e^{(2x)}+2i\sqrt{2}\right)e^{(-4x)}\right) \\
&\quad +\frac{1}{8} i\sqrt{2}\log\left(\left(\sqrt{-2\,e^{(4x)}-2}\left(e^{(2x)}-2\right)-i\sqrt{2}e^{(4x)}+i\sqrt{2}e^{(2x)}-2i\sqrt{2}\right)e^{(-4x)}\right)
\end{aligned}$$

input
```
integrate(1/(-1-tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/8*I*sqrt(2)*log(1/2*(I*sqrt(2)*sqrt(-2*e^(4*x) - 2) + 2*e^(2*x) + 2)*e^(
-2*x)) - 1/8*I*sqrt(2)*log(1/2*(-I*sqrt(2)*sqrt(-2*e^(4*x) - 2) + 2*e^(2*x
) + 2)*e^(-2*x)) - 1/8*I*sqrt(2)*log((sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) +
 I*sqrt(2)*e^(4*x) - I*sqrt(2)*e^(2*x) + 2*I*sqrt(2))*e^(-4*x)) + 1/8*I*sq
rt(2)*log((sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) - I*sqrt(2)*e^(4*x) + I*sqrt
(2)*e^(2*x) - 2*I*sqrt(2))*e^(-4*x))
```

3.256.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx = \int \frac{1}{\sqrt{-\tanh^2(x)-1}}\,dx$$

input
```
integrate(1/(-1-tanh(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(-tanh(x)**2 - 1), x)
```

3.256.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx = \int \frac{1}{\sqrt{-\tanh(x)^2-1}}\,dx$$

input
```
integrate(1/(-1-tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-tanh(x)^2 - 1), x)
```

3.256.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx$$
$$= \frac{1}{4} i\sqrt{2}\left(\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)-\log\left(-\sqrt{e^{(4x)}+1}+e^{(2x)}+1\right)\right)$$

input
```
integrate(1/(-1-tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*I*sqrt(2)*(log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) + log(sqrt(e^(4*x) + 1
) - e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))
```

3.256.9 Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx = \frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh(x)^2-1}}\right)}{2}$$

input
```
int(1/(- tanh(x)^2 - 1)^(1/2),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*tanh(x))/(- tanh(x)^2 - 1)^(1/2)))/2
```

3.257 $\int \left(a + b\tanh^3(c+dx)\right)^2 dx$

3.257.1 Optimal result 1892
3.257.2 Mathematica [A] (verified) 1892
3.257.3 Rubi [A] (verified) 1893
3.257.4 Maple [A] (verified) 1894
3.257.5 Fricas [B] (verification not implemented) 1895
3.257.6 Sympy [A] (verification not implemented) 1895
3.257.7 Maxima [B] (verification not implemented) 1896
3.257.8 Giac [A] (verification not implemented) 1896
3.257.9 Mupad [B] (verification not implemented) 1897

3.257.1 Optimal result

Integrand size = 14, antiderivative size = 89

$$\int \left(a + b\tanh^3(c+dx)\right)^2 dx = \left(a^2+b^2\right)x + \frac{2ab\log(\cosh(c+dx))}{d} - \frac{b^2\tanh(c+dx)}{d} - \frac{ab\tanh^2(c+dx)}{d} - \frac{b^2\tanh^3(c+dx)}{3d} - \frac{b^2\tanh^5(c+dx)}{5d}$$

output
```
(a^2+b^2)*x+2*a*b*ln(cosh(d*x+c))/d-b^2*tanh(d*x+c)/d-a*b*tanh(d*x+c)^2/d-
1/3*b^2*tanh(d*x+c)^3/d-1/5*b^2*tanh(d*x+c)^5/d
```

3.257.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

$$\int \left(a + b\tanh^3(c+dx)\right)^2 dx = -\frac{15\left((a+b)^2\log(1-\tanh(c+dx)) - (a-b)^2\log(1+\tanh(c+dx))\right) + 30b^2\tanh(c+dx) + 30ab\tanh^2(\cdots}{30d}$$

input
```
Integrate[(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
-1/30*(15*((a + b)^2*Log[1 - Tanh[c + d*x]] - (a - b)^2*Log[1 + Tanh[c + d
*x]]) + 30*b^2*Tanh[c + d*x] + 30*a*b*Tanh[c + d*x]^2 + 10*b^2*Tanh[c + d*
x]^3 + 6*b^2*Tanh[c + d*x]^5)/d
```

3.257.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4144, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a + b\tanh^3(c+dx)\right)^2 dx$$

$$\downarrow 3042$$

$$\int \left(a + ib\tan(ic+idx)^3\right)^2 dx$$

$$\downarrow 4144$$

$$\frac{\int \frac{\left(b\tanh^3(c+dx)+a\right)^2}{1-\tanh^2(c+dx)} d\tanh(c+dx)}{d}$$

$$\downarrow 2341$$

$$\frac{\int \left(-b^2\tanh^4(c+dx) - b^2\tanh^2(c+dx) - 2ab\tanh(c+dx) - b^2 + \frac{a^2+2b\tanh(c+dx)a+b^2}{1-\tanh^2(c+dx)}\right) d\tanh(c+dx)}{d}$$

$$\downarrow 2009$$

$$\frac{\left(a^2+b^2\right)\operatorname{arctanh}(\tanh(c+dx)) - ab\tanh^2(c+dx) - ab\log\left(1-\tanh^2(c+dx)\right) - \frac{1}{5}b^2\tanh^5(c+dx) - \frac{1}{3}b^2\tanh}{d}$$

input
```
Int[(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
((a^2 + b^2)*ArcTanh[Tanh[c + d*x]] - a*b*Log[1 - Tanh[c + d*x]^2] - b^2*T
anh[c + d*x] - a*b*Tanh[c + d*x]^2 - (b^2*Tanh[c + d*x]^3)/3 - (b^2*Tanh[c
 + d*x]^5)/5)/d
```

3.257.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

3.257.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

method	result
parallelrisch	$\frac{-3\tanh(dx+c)^5b^2-5b^2\tanh(dx+c)^3+15a^2dx-30abdx+15b^2dx-15\tanh(dx+c)^2ab-30\ln(1-\tanh(dx+c))ab-15b^2\tan}{15d}$
derivativedivides	$\frac{-\frac{\tanh(dx+c)^5b^2}{5}-\frac{b^2\tanh(dx+c)^3}{3}-\tanh(dx+c)^2ab-b^2\tanh(dx+c)-\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)-1)}{2}+\frac{\left(a^2-2ab+b^2\right)\ln(\tan}{2}}{d}$
default	$\frac{-\frac{\tanh(dx+c)^5b^2}{5}-\frac{b^2\tanh(dx+c)^3}{3}-\tanh(dx+c)^2ab-b^2\tanh(dx+c)-\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)-1)}{2}+\frac{\left(a^2-2ab+b^2\right)\ln(\tan}{2}}{d}$
parts	$a^2x+\frac{b^2\left(-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{2ab\left(-\frac{\tanh(dx+c)^2}{2}-\right.}{}$
risch	$a^2x-2abx+b^2x-\frac{4abc}{d}+\frac{2b(30a\,\mathrm{e}^{8dx+8c}+45b\,\mathrm{e}^{8dx+8c}+90a\,\mathrm{e}^{6dx+6c}+90b\,\mathrm{e}^{6dx+6c}+90a\,\mathrm{e}^{4dx+4c}+140b\,\mathrm{e}^{4dx+}}{15d\left(\mathrm{e}^{2dx+2c}+1\right)^5}$

input
```
int((a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/15*(-3*tanh(d*x+c)^5*b^2-5*b^2*tanh(d*x+c)^3+15*a^2*d*x-30*a*b*d*x+15*b^
2*d*x-15*tanh(d*x+c)^2*a*b-30*ln(1-tanh(d*x+c))*a*b-15*b^2*tanh(d*x+c))/d
```

3.257.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2074 vs. 2(85) = 170.

Time = 0.29 (sec) , antiderivative size = 2074, normalized size of antiderivative = 23.30

$$\int \left(a + b\tanh^3(c+dx)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
1/15*(15*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^10 + 150*(a^2 - 2*a*b + b^2
)*d*x*cosh(d*x + c)*sinh(d*x + c)^9 + 15*(a^2 - 2*a*b + b^2)*d*x*sinh(d*x
+ c)^10 + 15*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^8 +
 15*(45*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + 5*(a^2 - 2*a*b + b^2)*d*
x + 4*a*b + 6*b^2)*sinh(d*x + c)^8 + 120*(15*(a^2 - 2*a*b + b^2)*d*x*cosh(
d*x + c)^3 + (5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c))*si
nh(d*x + c)^7 + 30*(5*(a^2 - 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x +
c)^6 + 30*(105*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 5*(a^2 - 2*a*b +
b^2)*d*x + 14*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^2
+ 6*a*b + 6*b^2)*sinh(d*x + c)^6 + 60*(63*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x
 + c)^5 + 14*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^3 +
 3*(5*(a^2 - 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x + c))*sinh(d*x + c
)^5 + 10*(15*(a^2 - 2*a*b + b^2)*d*x + 18*a*b + 28*b^2)*cosh(d*x + c)^4 +
10*(315*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 105*(5*(a^2 - 2*a*b + b^
2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^4 + 15*(a^2 - 2*a*b + b^2)*d*x + 45*
(5*(a^2 - 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x + c)^2 + 18*a*b + 28*
b^2)*sinh(d*x + c)^4 + 40*(45*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^7 + 21
*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^5 + 15*(5*(a^2
- 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x + c)^3 + (15*(a^2 - 2*a*b + b
^2)*d*x + 18*a*b + 28*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 15*(a^2 - 2...
```

3.257.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.12

$$\int \left(a + b\tanh^3(c+dx)\right)^2 dx$$

$$= \begin{cases} a^2x + 2abx - \frac{2ab\log\left(\tanh(c+dx)+1\right)}{d} - \frac{ab\tanh^2(c+dx)}{d} + b^2x - \frac{b^2\tanh^5(c+dx)}{5d} - \frac{b^2\tanh^3(c+dx)}{3d} - \frac{b^2\tanh(c+dx)}{d} & \text{for} \\ x\left(a + b\tanh^3(c)\right)^2 & \text{oth} \end{cases}$$

input
```
integrate((a+b*tanh(d*x+c)**3)**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*x - 2*a*b*log(tanh(c + d*x) + 1)/d - a*b*tanh(c
+ d*x)**2/d + b**2*x - b**2*tanh(c + d*x)**5/(5*d) - b**2*tanh(c + d*x)**3
/(3*d) - b**2*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**3)**2, True))
```

3.257.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(85) = 170$.

Time = 0.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.18

$$\int \left(a + b\tanh^3(c+dx)\right)^2 dx$$
$$= \frac{1}{15}b^2\left(15x + \frac{15c}{d} - \frac{2\left(70e^{(-2dx-2c)} + 140e^{(-4dx-4c)} + 90e^{(-6dx-6c)} + 45e^{(-8dx-8c)} + 23\right)}{d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)}\right)$$
$$+ 2ab\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2dx-2c)} + 1\right)}{d} + \frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)}\right) + a^2x$$

input
```
integrate((a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
1/15*b^2*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) +
90*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
 - 10*c) + 1))) + 2*a*b*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d
*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a^2*x
```

3.257.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.60

$$\int \left(a + b\tanh^3(c+dx)\right)^2 dx$$
$$= \frac{30ab\log\left(e^{(2dx+2c)} + 1\right) + 15(a^2 - 2ab + b^2)(dx+c) + \frac{2\left(23b^2 + 15(2ab+3b^2)e^{(8dx+8c)} + 90(ab+b^2)e^{(6dx+6c)} + 10(9ab\right.}{\left(e^{(2dx+2c)}+1\right)^5}}{15d}$$

input
```
integrate((a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
1/15*(30*a*b*log(e^(2*d*x + 2*c) + 1) + 15*(a^2 - 2*a*b + b^2)*(d*x + c) +
 2*(23*b^2 + 15*(2*a*b + 3*b^2)*e^(8*d*x + 8*c) + 90*(a*b + b^2)*e^(6*d*x
+ 6*c) + 10*(9*a*b + 14*b^2)*e^(4*d*x + 4*c) + 10*(3*a*b + 7*b^2)*e^(2*d*x
 + 2*c))/(e^(2*d*x + 2*c) + 1)^5)/d
```

3.257.9 Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\int \left(a+b\tanh^3(c+dx)\right)^2\,dx = x\left(a^2+2ab+b^2\right) - \frac{b^2\tanh(c+dx)}{d} - \frac{b^2\tanh(c+dx)^3}{3d} - \frac{b^2\tanh(c+dx)^5}{5d} - \frac{2ab\ln\left(\tanh(c+dx)+1\right)}{d} - \frac{ab\tanh(c+dx)^2}{d}$$

input
```
int((a + b*tanh(c + d*x)^3)^2,x)
```

output
```
x*(2*a*b + a^2 + b^2) - (b^2*tanh(c + d*x))/d - (b^2*tanh(c + d*x)^3)/(3*d
) - (b^2*tanh(c + d*x)^5)/(5*d) - (2*a*b*log(tanh(c + d*x) + 1))/d - (a*b*
tanh(c + d*x)^2)/d
```

3.258 $\int \frac{1}{1+\tanh^3(x)}\,dx$

3.258.1 Optimal result 1898
3.258.2 Mathematica [A] (verified) 1898
3.258.3 Rubi [A] (verified) 1899
3.258.4 Maple [A] (verified) 1900
3.258.5 Fricas [B] (verification not implemented) 1901
3.258.6 Sympy [B] (verification not implemented) 1901
3.258.7 Maxima [B] (verification not implemented) 1902
3.258.8 Giac [A] (verification not implemented) 1902
3.258.9 Mupad [B] (verification not implemented) 1902

3.258.1 Optimal result

Integrand size = 8, antiderivative size = 38

$$\int \frac{1}{1+\tanh^3(x)}\,dx = \frac{x}{2} - \frac{2\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{6(1+\tanh(x))}$$

output
```
1/2*x-2/9*arctan(1/3*(1-2*tanh(x))*3^(1/2))*3^(1/2)-1/6/(1+tanh(x))
```

3.258.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \frac{1}{1+\tanh^3(x)}\,dx = -\frac{2\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{2}\operatorname{arctanh}(\tanh(x)) - \frac{1}{6(1+\tanh(x))}$$

input
```
Integrate[(1 + Tanh[x]^3)^(-1),x]
```

output
```
(-2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) + ArcTanh[Tanh[x]]/2 - 1/
(6*(1 + Tanh[x]))
```

3.258.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4144, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\tanh^3(x)+1}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{1+i\tan(ix)^3}\,dx$$

$$\downarrow 4144$$

$$\int \frac{1}{\left(1-\tanh^2(x)\right)\left(\tanh^3(x)+1\right)}\,d\tanh(x)$$

$$\downarrow 7276$$

$$\int \left(\frac{1}{3\left(\tanh^2(x)-\tanh(x)+1\right)} - \frac{1}{2\left(\tanh^2(x)-1\right)} + \frac{1}{6(\tanh(x)+1)^2}\right) d\tanh(x)$$

$$\downarrow 2009$$

$$-\frac{2\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{2}\operatorname{arctanh}(\tanh(x)) - \frac{1}{6(\tanh(x)+1)}$$

input
```
Int[(1 + Tanh[x]^3)^(-1),x]
```

output
```
(-2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) + ArcTanh[Tanh[x]]/2 - 1/
(6*(1 + Tanh[x]))
```

3.258.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.258.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{2\sqrt{3}\arctan\left(\frac{(2\tanh(x)-1)\sqrt{3}}{3}\right)}{9}-\frac{\ln(\tanh(x)-1)}{4}-\frac{1}{6(1+\tanh(x))}+\frac{\ln(1+\tanh(x))}{4}$	41
default	$\frac{2\sqrt{3}\arctan\left(\frac{(2\tanh(x)-1)\sqrt{3}}{3}\right)}{9}-\frac{\ln(\tanh(x)-1)}{4}-\frac{1}{6(1+\tanh(x))}+\frac{\ln(1+\tanh(x))}{4}$	41
risch	$\frac{x}{2}-\frac{\mathrm{e}^{-2x}}{12}+\frac{i\sqrt{3}\ln\left(\mathrm{e}^{2x}+i\sqrt{3}\right)}{9}-\frac{i\sqrt{3}\ln\left(\mathrm{e}^{2x}-i\sqrt{3}\right)}{9}$	47

input
```
int(1/(1+tanh(x)^3),x,method=_RETURNVERBOSE)
```

output
```
2/9*3^(1/2)*arctan(1/3*(2*tanh(x)-1)*3^(1/2))-1/4*ln(tanh(x)-1)-1/6/(1+tan
h(x))+1/4*ln(1+tanh(x))
```

3.258.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(29) = 58$.

Time $= 0.27$ (sec) , antiderivative size $= 95$, normalized size of antiderivative $= 2.50$

$$\int \frac{1}{1+\tanh^3(x)}\,dx$$
$$= \frac{18\,x\cosh(x)^2 + 36\,x\cosh(x)\sinh(x) + 18\,x\sinh(x)^2 - 8\left(\sqrt{3}\cosh(x)^2 + 2\sqrt{3}\cosh(x)\sinh(x) + \sqrt{3}\sin\right.}{36\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2\right)}$$

input
```
integrate(1/(1+tanh(x)^3),x, algorithm="fricas")
```

output
```
1/36*(18*x*cosh(x)^2 + 36*x*cosh(x)*sinh(x) + 18*x*sinh(x)^2 - 8*(sqrt(3)*
cosh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2)*arctan(-1/3*(sq
rt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - sinh(x))) - 3)/(cosh(x)^2 + 2*
cosh(x)*sinh(x) + sinh(x)^2)
```

3.258.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(36) = 72$.

Time $= 0.25$ (sec) , antiderivative size $= 102$, normalized size of antiderivative $= 2.68$

$$\int \frac{1}{1+\tanh^3(x)}\,dx = \frac{9x\tanh(x)}{18\tanh(x)+18} + \frac{9x}{18\tanh(x)+18} + \frac{4\sqrt{3}\tanh(x)\operatorname{atan}\left(\frac{2\sqrt{3}\tanh(x)}{3} - \frac{\sqrt{3}}{3}\right)}{18\tanh(x)+18} + \frac{4\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\tanh(x)}{3} - \frac{\sqrt{3}}{3}\right)}{18\tanh(x)+18} - \frac{3}{18\tanh(x)+18}$$

input
```
integrate(1/(1+tanh(x)**3),x)
```

output
```
9*x*tanh(x)/(18*tanh(x) + 18) + 9*x/(18*tanh(x) + 18) + 4*sqrt(3)*tanh(x)*
atan(2*sqrt(3)*tanh(x)/3 - sqrt(3)/3)/(18*tanh(x) + 18) + 4*sqrt(3)*atan(2
*sqrt(3)*tanh(x)/3 - sqrt(3)/3)/(18*tanh(x) + 18) - 3/(18*tanh(x) + 18)
```

3.258.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(29) = 58.

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.92

$$\int \frac{1}{1+\tanh^3(x)}\,dx = \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}+3^{\frac{1}{4}}\sqrt{2}\right)\right) - \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}-3^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{2}x-\frac{1}{12}e^{(-2x)}$$

input
```
integrate(1/(1+tanh(x)^3),x, algorithm="maxima")
```

output
```
2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) + 3^(1/4)*sqrt(2)
)) - 2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) - 3^(1/4)*sq
rt(2))) + 1/2*x - 1/12*e^(-2*x)
```

3.258.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{1}{1+\tanh^3(x)}\,dx = \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2x)}\right)+\frac{1}{2}x-\frac{1}{12}e^{(-2x)}$$

input
```
integrate(1/(1+tanh(x)^3),x, algorithm="giac")
```

output
```
2/9*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) + 1/2*x - 1/12*e^(-2*x)
```

3.258.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\tanh^3(x)}\,dx = \frac{\frac{x}{2}+\frac{\tanh(x)}{6}+\frac{x\tanh(x)}{2}}{\tanh(x)+1}+\frac{2\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\,(2\tanh(x)-1)}{3}\right)}{9}$$

input
```
int(1/(tanh(x)^3 + 1),x)
```

output
```
(x/2 + tanh(x)/6 + (x*tanh(x))/2)/(tanh(x) + 1) + (2*3^(1/2)*atan((3^(1/2)
*(2*tanh(x) - 1))/3))/9
```

3.259 $\int \tanh(x)\left(a+b\tanh^4(x)\right)^{3/2}\,dx$

3.259.1 Optimal result . . . 1903
3.259.2 Mathematica [A] (verified) . . . 1903
3.259.3 Rubi [A] (verified) . . . 1904
3.259.4 Maple [C] (verified) . . . 1908
3.259.5 Fricas [B] (verification not implemented) . . . 1909
3.259.6 Sympy [F] . . . 1909
3.259.7 Maxima [F] . . . 1909
3.259.8 Giac [F] . . . 1910
3.259.9 Mupad [F(-1)] . . . 1910

3.259.1 Optimal result

Integrand size = 15, antiderivative size = 124

$$\int \tanh(x)\left(a+b\tanh^4(x)\right)^{3/2}\,dx = -\frac{1}{4}\sqrt{b}(3a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)$$
$$+\frac{1}{2}(a+b)^{3/2}\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)$$
$$-\frac{1}{4}\left(2(a+b)+b\tanh^2(x)\right)\sqrt{a+b\tanh^4(x)}-\frac{1}{6}\left(a+b\tanh^4(x)\right)^{3/2}$$

output

```
1/2*(a+b)^(3/2)*arctanh((a+b*tanh(x)^2)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))
-1/4*(3*a+2*b)*arctanh(b^(1/2)*tanh(x)^2/(a+b*tanh(x)^4)^(1/2))*b^(1/2)-1/
4*(a+b*tanh(x)^4)^(1/2)*(2*a+2*b+b*tanh(x)^2)-1/6*(a+b*tanh(x)^4)^(3/2)
```

3.259.2 Mathematica [A] (verified)

Time = 4.55 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.34

$$\int \tanh(x)\left(a+b\tanh^4(x)\right)^{3/2}\,dx = \frac{1}{12}\left(-6\sqrt{b}(a+b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)\right.$$
$$\left.+6(a+b)^{3/2}\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)-\sqrt{a+b\tanh^4(x)}\left(8a+6b+3b\tanh^2(x)+2b\tanh^4(x)\right)-\frac{3\sqrt{}}{}\right.$$

input
```
Integrate[Tanh[x]*(a + b*Tanh[x]^4)^(3/2),x]
```

output
```
(-6*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]] + 6
*(a + b)^(3/2)*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4
])] - Sqrt[a + b*Tanh[x]^4]*(8*a + 6*b + 3*b*Tanh[x]^2 + 2*b*Tanh[x]^4) -
(3*Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a]]*Sqrt[a + b*Tanh[x]
^4])/Sqrt[1 + (b*Tanh[x]^4)/a])/12
```

3.259.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.933$, Rules used = {3042, 26, 4153, 26, 1577, 493, 25, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(x) \left(a + b \tanh^4(x)\right)^{3/2} dx$$

$$\downarrow 3042$$

$$\int -i \tan(ix) \left(a + b \tan(ix)^4\right)^{3/2} dx$$

$$\downarrow 26$$

$$-i \int \tan(ix) \left(b \tan(ix)^4 + a\right)^{3/2} dx$$

$$\downarrow 4153$$

$$-i \int \frac{i \tanh(x) \left(b \tanh^4(x) + a\right)^{3/2}}{1 - \tanh^2(x)} d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\tanh(x) \left(a + b \tanh^4(x)\right)^{3/2}}{1 - \tanh^2(x)} d\tanh(x)$$

$$\downarrow 1577$$

$$\frac{1}{2} \int \frac{\left(b \tanh^4(x) + a\right)^{3/2}}{1 - \tanh^2(x)} d\tanh^2(x)$$

$$\downarrow 493$$

$$\frac{1}{2}\left(-\int -\frac{\left(b\tanh^2(x)+a\right)\sqrt{b\tanh^4(x)+a}}{1-\tanh^2(x)}d\tanh^2(x)-\frac{1}{3}\left(a+b\tanh^4(x)\right)^{3/2}\right)$$

$$\downarrow 25$$

$$\frac{1}{2}\left(\int \frac{\left(b\tanh^2(x)+a\right)\sqrt{b\tanh^4(x)+a}}{1-\tanh^2(x)}d\tanh^2(x)-\frac{1}{3}\left(a+b\tanh^4(x)\right)^{3/2}\right)$$

$$\downarrow 682$$

$$\frac{1}{2}\left(\frac{\int \frac{b\left(b(3a+2b)\tanh^2(x)+a(2a+b)\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)}{2b}-\frac{1}{3}\left(a+b\tanh^4(x)\right)^{3/2}-\frac{1}{2}\left(2(a+b)+b\tanh^2(x)\right)\sqrt{a+b\tanh^4(x)}\right)$$

$$\downarrow 27$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{b(3a+2b)\tanh^2(x)+a(2a+b)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)-\frac{1}{3}\left(a+b\tanh^4(x)\right)^{3/2}-\frac{1}{2}\left(2(a+b)+b\tanh^2(x)\right)\sqrt{a+b\tanh^4(x)}\right)$$

$$\downarrow 719$$

$$\frac{1}{2}\left(\frac{1}{2}\left(2(a+b)^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)-b(3a+2b)\int \frac{1}{\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)\right)-\frac{1}{3}\right)$$

$$\downarrow 224$$

$$\frac{1}{2}\left(\frac{1}{2}\left(2(a+b)^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)-b(3a+2b)\int \frac{1}{1-b\tanh^4(x)}d\frac{\tanh^2(x)}{\sqrt{b\tanh^4(x)+a}}\right)\right.$$

$$\downarrow 219$$

$$\frac{1}{2}\left(\frac{1}{2}\left(2(a+b)^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)-\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)\right)-\frac{1}{3}\right.$$

$$\downarrow 488$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-2(a+b)^2\int \frac{1}{-\tanh^4(x)+a+b}d\frac{-b\tanh^2(x)-a}{\sqrt{b\tanh^4(x)+a}}-\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)\right)-\frac{1}{3}(a\right.$$

$$\downarrow 219$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-2(a+b)^{3/2}\text{arctanh}\left(\frac{-a-b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)-\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)\right)-\frac{1}{3}(a+b$$

input
```
Int[Tanh[x]*(a + b*Tanh[x]^4)^(3/2),x]
```

output
```
((-(Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]]
) - 2*(a + b)^(3/2)*ArcTanh[(-a - b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan
h[x]^4])])/2 - ((2*(a + b) + b*Tanh[x]^2)*Sqrt[a + b*Tanh[x]^4])/2 - (a +
b*Tanh[x]^4)^(3/2)/3)/2
```

3.259.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.259.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.90 (sec) , antiderivative size = 620, normalized size of antiderivative = 5.00

method	result
derivativedivides	$-\frac{b\tanh(x)^4\sqrt{a+b\tanh(x)^4}}{6}-\frac{b\tanh(x)^2\sqrt{a+b\tanh(x)^4}}{4}-\frac{2\sqrt{a+b\tanh(x)^4}\,a}{3}-\frac{b\sqrt{a+b\tanh(x)^4}}{2}-\frac{(\frac{5}{3}ab+b^2$
default	$-\frac{b\tanh(x)^4\sqrt{a+b\tanh(x)^4}}{6}-\frac{b\tanh(x)^2\sqrt{a+b\tanh(x)^4}}{4}-\frac{2\sqrt{a+b\tanh(x)^4}\,a}{3}-\frac{b\sqrt{a+b\tanh(x)^4}}{2}-\frac{(\frac{5}{3}ab+b^2$

input
```
int(tanh(x)*(a+b*tanh(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*b*tanh(x)^4*(a+b*tanh(x)^4)^(1/2)-1/4*b*tanh(x)^2*(a+b*tanh(x)^4)^(1/
2)-2/3*(a+b*tanh(x)^4)^(1/2)*a-1/2*b*(a+b*tanh(x)^4)^(1/2)-1/2*(5/3*a*b+b^
2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^
(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticF(tanh(x)*(I/
a^(1/2)*b^(1/2))^(1/2),I)-3/4*ln(2*b^(1/2)*tanh(x)^2+2*(a+b*tanh(x)^4)^(1/
2))*b^(1/2)*a-1/2*ln(2*b^(1/2)*tanh(x)^2+2*(a+b*tanh(x)^4)^(1/2))*b^(3/2)-
1/2*I*(-7/5*a*b-b^2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2
)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(
1/2)/b^(1/2)*(EllipticF(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(tan
h(x)*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*a^2/(a+b)^(1/2)*arctanh(1/2*(2*b*ta
nh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))+a*b/(a+b)^(1/2)*arctanh(1/
2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))+1/2*b^2/(a+b)^(1/
2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))-1/2*
(-5/3*a*b-b^2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(
1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticF
(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*I*(7/5*a*b+b^2)*a^(1/2)/(I/a^(1/
2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/
2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)/b^(1/2)*(EllipticF(tanh(x)*(I/a^
(1/2)*b^(1/2))^(1/2),I)-EllipticE(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

3.259.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2646 vs. $2(101) = 202$.

Time $= 0.55$ (sec) , antiderivative size $= 11528$, normalized size of antiderivative $= 92.97$

$$\int \tanh(x) \left(a + b \tanh^4(x)\right)^{3/2} \, dx = \text{Too large to display}$$

input
```
integrate(tanh(x)*(a+b*tanh(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.259.6 Sympy [F]

$$\int \tanh(x) \left(a + b \tanh^4(x)\right)^{3/2} \, dx = \int \left(a + b \tanh^4(x)\right)^{\frac{3}{2}} \tanh(x) \, dx$$

input
```
integrate(tanh(x)*(a+b*tanh(x)**4)**(3/2),x)
```

output
```
Integral((a + b*tanh(x)**4)**(3/2)*tanh(x), x)
```

3.259.7 Maxima [F]

$$\int \tanh(x) \left(a + b \tanh^4(x)\right)^{3/2} \, dx = \int \left(b \tanh(x)^4 + a\right)^{\frac{3}{2}} \tanh(x) \, dx$$

input
```
integrate(tanh(x)*(a+b*tanh(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tanh(x)^4 + a)^(3/2)*tanh(x), x)
```

3.259.8 Giac [F]

$$\int \tanh(x)\left(a+b\tanh^4(x)\right)^{3/2}\,dx = \int \left(b\tanh(x)^4+a\right)^{\frac{3}{2}}\tanh(x)\,dx$$

input
```
integrate(tanh(x)*(a+b*tanh(x)^4)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tanh(x)^4 + a)^(3/2)*tanh(x), x)
```

3.259.9 Mupad [F(-1)]

Timed out.

$$\int \tanh(x)\left(a+b\tanh^4(x)\right)^{3/2}\,dx = \int \tanh(x)\,\left(b\tanh(x)^4+a\right)^{3/2}dx$$

input
```
int(tanh(x)*(a + b*tanh(x)^4)^(3/2),x)
```

output
```
int(tanh(x)*(a + b*tanh(x)^4)^(3/2), x)
```

3.260 $\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx$

3.260.1 Optimal result 1911
3.260.2 Mathematica [A] (verified) 1912
3.260.3 Rubi [A] (verified) 1912
3.260.4 Maple [A] (verified) 1915
3.260.5 Fricas [B] (verification not implemented) 1916
3.260.6 Sympy [F] 1916
3.260.7 Maxima [F] 1917
3.260.8 Giac [F] 1917
3.260.9 Mupad [F(-1)] 1917

3.260.1 Optimal result

Integrand size = 15, antiderivative size = 89

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx = -\frac{1}{2}\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right) + \frac{1}{2}\sqrt{a+b}\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right) - \frac{1}{2}\sqrt{a+b\tanh^4(x)}$$

output

```
-1/2*arctanh(b^(1/2)*tanh(x)^2/(a+b*tanh(x)^4)^(1/2))*b^(1/2)+1/2*arctanh(
(a+b*tanh(x)^2)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))*(a+b)^(1/2)-1/2*(a+b*ta
nh(x)^4)^(1/2)
```

3.260.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx = \frac{1}{2}\left(-\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right) + \sqrt{a+b}\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right) - \sqrt{a+b\tanh^4(x)}\right)$$

input
```
Integrate[Tanh[x]*Sqrt[a + b*Tanh[x]^4],x]
```

output
```
(-(Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]]) + Sqrt[a +
b]*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])] - Sqrt[a
 + b*Tanh[x]^4])/2
```

3.260.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 26, 4153, 26, 1577, 493, 25, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx$$
$$\downarrow 3042$$
$$\int -i\tan(ix)\sqrt{a+b\tan(ix)^4}dx$$
$$\downarrow 26$$
$$-i\int \tan(ix)\sqrt{b\tan(ix)^4+a}dx$$
$$\downarrow 4153$$

$$-i\int \frac{i\tanh(x)\sqrt{b\tanh^4(x)+a}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\tanh(x)\sqrt{a+b\tanh^4(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\downarrow 1577$$

$$\frac{1}{2}\int \frac{\sqrt{b\tanh^4(x)+a}}{1-\tanh^2(x)}d\tanh^2(x)$$

$$\downarrow 493$$

$$\frac{1}{2}\left(-\int -\frac{b\tanh^2(x)+a}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)-\sqrt{a+b\tanh^4(x)}\right)$$

$$\downarrow 25$$

$$\frac{1}{2}\left(\int \frac{b\tanh^2(x)+a}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)-\sqrt{a+b\tanh^4(x)}\right)$$

$$\downarrow 719$$

$$\frac{1}{2}\left(-b\int \frac{1}{\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)+(a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)-\sqrt{a+b\tanh^4(x)}\right.$$

$$\downarrow 224$$

$$\frac{1}{2}\left(-b\int \frac{1}{1-b\tanh^4(x)}d\frac{\tanh^2(x)}{\sqrt{b\tanh^4(x)+a}}+(a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)-\sqrt{a+b\tan\right.$$

$$\downarrow 219$$

$$\frac{1}{2}\left((a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)-\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)-\sqrt{a+b\tanh^4(x)}\right)$$

$$\downarrow 488$$

$$\frac{1}{2}\left(-(a+b)\int \frac{1}{-\tanh^4(x)+a+b}d\frac{-b\tanh^2(x)-a}{\sqrt{b\tanh^4(x)+a}} - \sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right) - \sqrt{a+b\tanh^4(x)}\right)$$

$$\downarrow 219$$

$$\frac{1}{2}\left(-\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right) - \sqrt{a+b}\operatorname{arctanh}\left(\frac{-a-b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right) - \sqrt{a+b\tanh^4(x)}\right)$$

input
```
Int[Tanh[x]*Sqrt[a + b*Tanh[x]^4],x]
```

output
```
(-(Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]]) - Sqrt[a +
b]*ArcTanh[(-a - b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])] - Sqrt[
a + b*Tanh[x]^4])/2
```

3.260.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.260.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.30

method	result
derivativedivides	$-\frac{\sqrt{a+b\tanh(x)^4}}{2}-\frac{\sqrt{b}\,\ln\left(2\sqrt{b}\,\tanh(x)^2+2\sqrt{a+b\tanh(x)^4}\right)}{2}+\frac{b\,\operatorname{arctanh}\left(\frac{2b\tanh(x)^2+2a}{2\sqrt{a+b}\,\sqrt{a+b\tanh(x)^4}}\right)}{2\sqrt{a+b}}+\frac{a\,\operatorname{arctanh}$
default	$-\frac{\sqrt{a+b\tanh(x)^4}}{2}-\frac{\sqrt{b}\,\ln\left(2\sqrt{b}\,\tanh(x)^2+2\sqrt{a+b\tanh(x)^4}\right)}{2}+\frac{b\,\operatorname{arctanh}\left(\frac{2b\tanh(x)^2+2a}{2\sqrt{a+b}\,\sqrt{a+b\tanh(x)^4}}\right)}{2\sqrt{a+b}}+\frac{a\,\operatorname{arctanh}$

input
```
int(tanh(x)*(a+b*tanh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a+b*tanh(x)^4)^(1/2)-1/2*b^(1/2)*ln(2*b^(1/2)*tanh(x)^2+2*(a+b*tanh(
x)^4)^(1/2))+1/2*b/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)
/(a+b*tanh(x)^4)^(1/2))+1/2*a/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/
(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))
```

3.260.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1048 vs. $2(69) = 138$.

Time $= 0.44$ (sec) , antiderivative size $= 5136$, normalized size of antiderivative $= 57.71$

$$\int \tanh(x)\sqrt{a + b\tanh^4(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)*(a+b*tanh(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.260.6 Sympy [F]

$$\int \tanh(x)\sqrt{a + b\tanh^4(x)}\,dx = \int \sqrt{a + b\tanh^4(x)}\tanh(x)\,dx$$

input
```
integrate(tanh(x)*(a+b*tanh(x)**4)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tanh(x)**4)*tanh(x), x)
```

3.260.7 Maxima [F]

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx = \int \sqrt{b\tanh(x)^4+a}\tanh(x)\,dx$$

input
```
integrate(tanh(x)*(a+b*tanh(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^4 + a)*tanh(x), x)
```

3.260.8 Giac [F]

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx = \int \sqrt{b\tanh(x)^4+a}\tanh(x)\,dx$$

input
```
integrate(tanh(x)*(a+b*tanh(x)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tanh(x)^4 + a)*tanh(x), x)
```

3.260.9 Mupad [F(-1)]

Timed out.

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx = \int \tanh(x)\,\sqrt{b\tanh(x)^4+a}\,dx$$

input
```
int(tanh(x)*(a + b*tanh(x)^4)^(1/2),x)
```

output
```
int(tanh(x)*(a + b*tanh(x)^4)^(1/2), x)
```

3.261 $\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx$

3.261.1 Optimal result . . . 1918
3.261.2 Mathematica [A] (verified) . . . 1918
3.261.3 Rubi [A] (verified) . . . 1919
3.261.4 Maple [A] (verified) . . . 1921
3.261.5 Fricas [B] (verification not implemented) . . . 1921
3.261.6 Sympy [F] . . . 1922
3.261.7 Maxima [F] . . . 1923
3.261.8 Giac [F] . . . 1923
3.261.9 Mupad [F(-1)] . . . 1923

3.261.1 Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2\sqrt{a+b}}$$

output
```
1/2*arctanh((a+b*tanh(x)^2)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))/(a+b)^(1/2)
```

3.261.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2\sqrt{a+b}}$$

input
```
Integrate[Tanh[x]/Sqrt[a + b*Tanh[x]^4],x]
```

output
```
ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(2*Sqrt[a +
 b])
```

3.261.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 26, 4153, 26, 1577, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\tan(ix)}{\sqrt{a+b\tan(ix)^4}}dx$$

$$\downarrow 26$$

$$-i\int \frac{\tan(ix)}{\sqrt{b\tan(ix)^4+a}}dx$$

$$\downarrow 4153$$

$$-i\int \frac{i\tanh(x)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^4(x)+a}}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\tanh(x)}{\left(1-\tanh^2(x)\right)\sqrt{a+b\tanh^4(x)}}d\tanh(x)$$

$$\downarrow 1577$$

$$\frac{1}{2}\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)$$

$$\downarrow 488$$

$$-\frac{1}{2}\int \frac{1}{-\tanh^4(x)+a+b}d\frac{-b\tanh^2(x)-a}{\sqrt{b\tanh^4(x)+a}}$$

$$\downarrow 219$$

$$-\frac{\operatorname{arctanh}\left(\frac{-a-b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2\sqrt{a+b}}$$

input
```
Int[Tanh[x]/Sqrt[a + b*Tanh[x]^4],x]
```

output
```
-1/2*ArcTanh[(-a - b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/Sqrt[
a + b]
```

3.261.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.261.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{2b\tanh(x)^2+2a}{2\sqrt{a+b}\sqrt{a+b\tanh(x)^4}}\right)}{2\sqrt{a+b}}$	37
default	$\frac{\operatorname{arctanh}\left(\frac{2b\tanh(x)^2+2a}{2\sqrt{a+b}\sqrt{a+b\tanh(x)^4}}\right)}{2\sqrt{a+b}}$	37

input
```
int(tanh(x)/(a+b*tanh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4
)^(1/2))
```

3.261.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(32) = 64$.

Time = 0.41 (sec) , antiderivative size = 1286, normalized size of antiderivative = 32.15

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*si
nh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(
a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b +
b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 3*b
^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(
x)^2 + 3*a^2 + 2*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)
^5 + 10*(a^2 - b^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 3*b^2)*cosh(x))*sinh(x)^3
 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2
- b^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b + 3*b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x
)^2 + sqrt(2)*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*s
inh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2
 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*sq
rt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a
+ b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^
3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^
2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)
^5 + (3*a^2 + 2*a*b + 3*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))/(co
sh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^
3 + sinh(x)^4))/sqrt(a + b), -1/2*sqrt(-a - b)*arctan(sqrt(2)*((a + b)*cos
h(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*co...
```

3.261.6 Sympy [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)**4)**(1/2),x)
```

output
```
Integral(tanh(x)/sqrt(a + b*tanh(x)**4), x)
```

3.261.7 Maxima [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{b\tanh(x)^4+a}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)/sqrt(b*tanh(x)^4 + a), x)
```

3.261.8 Giac [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{b\tanh(x)^4+a}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(tanh(x)/sqrt(b*tanh(x)^4 + a), x)
```

3.261.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{b\tanh(x)^4+a}}\,dx$$

input
```
int(tanh(x)/(a + b*tanh(x)^4)^(1/2),x)
```

output
```
int(tanh(x)/(a + b*tanh(x)^4)^(1/2), x)
```

3.262 $\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}\,dx$

3.262.1 Optimal result 1924
3.262.2 Mathematica [A] (verified) 1924
3.262.3 Rubi [A] (verified) 1925
3.262.4 Maple [C] (verified) 1927
3.262.5 Fricas [B] (verification not implemented) 1928
3.262.6 Sympy [F] 1929
3.262.7 Maxima [F] 1930
3.262.8 Giac [F] 1930
3.262.9 Mupad [F(-1)] 1930

3.262.1 Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)^{3/2}} - \frac{a-b\tanh^2(x)}{2a(a+b)\sqrt{a+b\tanh^4(x)}}$$

output
```
1/2*arctanh((a+b*tanh(x)^2)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))/(a+b)^(3/2)
+1/2*(-a+b*tanh(x)^2)/a/(a+b)/(a+b*tanh(x)^4)^(1/2)
```

3.262.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}\,dx = \frac{1}{2}\left(\frac{\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{(a+b)^{3/2}} - \frac{a-b\tanh^2(x)}{a(a+b)\sqrt{a+b\tanh^4(x)}}\right)$$

input
```
Integrate[Tanh[x]/(a + b*Tanh[x]^4)^(3/2),x]
```

output
```
(ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(a + b)^(3
/2) - (a - b*Tanh[x]^2)/(a*(a + b)*Sqrt[a + b*Tanh[x]^4]))/2
```

3.262.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 26, 4153, 26, 1577, 496, 25, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\tan(ix)}{\left(a+b\tan(ix)^4\right)^{3/2}}dx$$

$$\downarrow 26$$

$$-i\int \frac{\tan(ix)}{\left(b\tan(ix)^4+a\right)^{3/2}}dx$$

$$\downarrow 4153$$

$$-i\int \frac{i\tanh(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^4(x)+a\right)^{3/2}}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\tanh(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^4(x)\right)^{3/2}}d\tanh(x)$$

$$\downarrow 1577$$

$$\frac{1}{2}\int \frac{1}{\left(1-\tanh^2(x)\right)\left(b\tanh^4(x)+a\right)^{3/2}}d\tanh^2(x)$$

$$\downarrow 496$$

$$\frac{1}{2}\left(-\frac{\int -\frac{a}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)}{a(a+b)}-\frac{a-b\tanh^2(x)}{a(a+b)\sqrt{a+b\tanh^4(x)}}\right)$$

$$\downarrow 25$$

$$\frac{1}{2}\left(\frac{\int \frac{a}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)}{a(a+b)}-\frac{a-b\tanh^2(x)}{a(a+b)\sqrt{a+b\tanh^4(x)}}\right)$$

$$\downarrow 27$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)}{a+b}-\frac{a-b\tanh^2(x)}{a(a+b)\sqrt{a+b\tanh^4(x)}}\right)$$

$$\downarrow 488$$

$$\frac{1}{2}\left(-\frac{\int \frac{1}{-\tanh^4(x)+a+b}d\frac{-b\tanh^2(x)-a}{\sqrt{b\tanh^4(x)+a}}}{a+b}-\frac{a-b\tanh^2(x)}{a(a+b)\sqrt{a+b\tanh^4(x)}}\right)$$

$$\downarrow 219$$

$$\frac{1}{2}\left(-\frac{\operatorname{arctanh}\left(\frac{-a-b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{(a+b)^{3/2}}-\frac{a-b\tanh^2(x)}{a(a+b)\sqrt{a+b\tanh^4(x)}}\right)$$

input
```
Int[Tanh[x]/(a + b*Tanh[x]^4)^(3/2),x]
```

output
```
(-(ArcTanh[(-a - b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(a + b)
^(3/2)) - (a - b*Tanh[x]^2)/(a*(a + b)*Sqrt[a + b*Tanh[x]^4]))/2
```

3.262.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.262.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.68 (sec) , antiderivative size = 431, normalized size of antiderivative = 5.82

method	result
derivativedivides	$\frac{b\left(\frac{\tanh(x)^3}{4a(a+b)}+\frac{\tanh(x)^2}{4a(a+b)}+\frac{\tanh(x)}{4a(a+b)}-\frac{1}{4(a+b)b}\right)}{\sqrt{\left(\tanh(x)^4+\frac{a}{b}\right)b}} - \frac{-\frac{\operatorname{arctanh}\left(\frac{2b\tanh(x)^2+2a}{2\sqrt{a+b}\sqrt{a+b\tanh(x)^4}}\right)}{2\sqrt{a+b}} - \frac{\sqrt{1-\frac{i\sqrt{b}\tanh(x)^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\tanh(x)^2}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{}}}{2(a+b)}$
default	$\frac{b\left(\frac{\tanh(x)^3}{4a(a+b)}+\frac{\tanh(x)^2}{4a(a+b)}+\frac{\tanh(x)}{4a(a+b)}-\frac{1}{4(a+b)b}\right)}{\sqrt{\left(\tanh(x)^4+\frac{a}{b}\right)b}} - \frac{-\frac{\operatorname{arctanh}\left(\frac{2b\tanh(x)^2+2a}{2\sqrt{a+b}\sqrt{a+b\tanh(x)^4}}\right)}{2\sqrt{a+b}} - \frac{\sqrt{1-\frac{i\sqrt{b}\tanh(x)^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\tanh(x)^2}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{}}}{2(a+b)}$

input
```
int(tanh(x)/(a+b*tanh(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
b*(1/4/a/(a+b)*tanh(x)^3+1/4/a/(a+b)*tanh(x)^2+1/4/a/(a+b)*tanh(x)-1/4/(a+
b)/b)/((tanh(x)^4+a/b)*b)^(1/2)-1/2/(a+b)*(-1/2/(a+b)^(1/2)*arctanh(1/2*(2
*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))-1/(I/a^(1/2)*b^(1/2))
^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^
2)^(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticPi(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2
),-I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))
)+b*(-1/4/a/(a+b)*tanh(x)^3+1/4/a/(a+b)*tanh(x)^2-1/4/a/(a+b)*tanh(x)-1/4/
(a+b)/b)/((tanh(x)^4+a/b)*b)^(1/2)-1/2/(a+b)*(-1/2/(a+b)^(1/2)*arctanh(1/2
*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))+1/(I/a^(1/2)*b^(1/
2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(
x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticPi(tanh(x)*(I/a^(1/2)*b^(1/2))^(
1/2),-I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/
2)))
```

3.262.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1935 vs. $2(63) = 126$.

Time $= 0.48$ (sec) , antiderivative size $= 3914$, normalized size of antiderivative $= 52.89$

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((a^2 + a*b)*cosh(x)^8 + 8*(a^2 + a*b)*cosh(x)*sinh(x)^7 + (a^2 + a*
b)*sinh(x)^8 + 4*(a^2 - a*b)*cosh(x)^6 + 4*(7*(a^2 + a*b)*cosh(x)^2 + a^2
- a*b)*sinh(x)^6 + 8*(7*(a^2 + a*b)*cosh(x)^3 + 3*(a^2 - a*b)*cosh(x))*sin
h(x)^5 + 6*(a^2 + a*b)*cosh(x)^4 + 2*(35*(a^2 + a*b)*cosh(x)^4 + 30*(a^2 -
 a*b)*cosh(x)^2 + 3*a^2 + 3*a*b)*sinh(x)^4 + 8*(7*(a^2 + a*b)*cosh(x)^5 +
10*(a^2 - a*b)*cosh(x)^3 + 3*(a^2 + a*b)*cosh(x))*sinh(x)^3 + 4*(a^2 - a*b
)*cosh(x)^2 + 4*(7*(a^2 + a*b)*cosh(x)^6 + 15*(a^2 - a*b)*cosh(x)^4 + 9*(a
^2 + a*b)*cosh(x)^2 + a^2 - a*b)*sinh(x)^2 + a^2 + a*b + 8*((a^2 + a*b)*co
sh(x)^7 + 3*(a^2 - a*b)*cosh(x)^5 + 3*(a^2 + a*b)*cosh(x)^3 + (a^2 - a*b)*
cosh(x))*sinh(x))*sqrt(a + b)*log(((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2
+ 2*a*b + b^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2
- b^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)
^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5
 + 2*(3*a^2 + 2*a*b + 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)
^4 + 30*(a^2 - b^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a
^2 + 2*a*b + b^2)*cosh(x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + (3*a^2 + 2*a*b +
3*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b +
b^2)*cosh(x)^6 + 15*(a^2 - b^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b + 3*b^2)*cosh
(x)^2 + a^2 - b^2)*sinh(x)^2 + sqrt(2)*((a + b)*cosh(x)^4 + 4*(a + b)*cosh
(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*...
```

3.262.6 Sympy [F]

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}\,dx = \int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)**4)**(3/2),x)
```

output
```
Integral(tanh(x)/(a + b*tanh(x)**4)**(3/2), x)
```

3.262.7 Maxima [F]

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}\,dx = \int \frac{\tanh(x)}{\left(b\tanh(x)^4+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)/(b*tanh(x)^4 + a)^(3/2), x)
```

3.262.8 Giac [F]

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}\,dx = \int \frac{\tanh(x)}{\left(b\tanh(x)^4+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^4)^(3/2),x, algorithm="giac")
```

output
```
integrate(tanh(x)/(b*tanh(x)^4 + a)^(3/2), x)
```

3.262.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}\,dx = \int \frac{\tanh(x)}{\left(b\tanh(x)^4+a\right)^{3/2}}\,dx$$

input
```
int(tanh(x)/(a + b*tanh(x)^4)^(3/2),x)
```

output
```
int(tanh(x)/(a + b*tanh(x)^4)^(3/2), x)
```

3.263 $\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx$

3.263.1 Optimal result 1931
3.263.2 Mathematica [A] (verified) 1931
3.263.3 Rubi [A] (verified) 1932
3.263.4 Maple [C] (verified) 1935
3.263.5 Fricas [B] (verification not implemented) 1936
3.263.6 Sympy [F] 1937
3.263.7 Maxima [F] 1937
3.263.8 Giac [F] 1937
3.263.9 Mupad [F(-1)] 1938

3.263.1 Optimal result

Integrand size = 15, antiderivative size = 118

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)^{5/2}} - \frac{a-b\tanh^2(x)}{6a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}} - \frac{3a^2-b(5a+2b)\tanh^2(x)}{6a^2(a+b)^2\sqrt{a+b\tanh^4(x)}}$$

output

```
1/2*arctanh((a+b*tanh(x)^2)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))/(a+b)^(5/2)
+1/6*(-3*a^2+b*(5*a+2*b)*tanh(x)^2)/a^2/(a+b)^2/(a+b*tanh(x)^4)^(1/2)+1/6*
(-a+b*tanh(x)^2)/a/(a+b)/(a+b*tanh(x)^4)^(3/2)
```

3.263.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx = \frac{1}{6}\left(\frac{3\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{(a+b)^{5/2}} + \frac{-a^2(4a+b)+3ab(2a+b)\tanh^2(x)-3a^2b\tanh^4(x)+b^2(5a+2b)\tanh^6(x)}{a^2(a+b)^2\left(a+b\tanh^4(x)\right)^{3/2}}\right)$$

input
```
Integrate[Tanh[x]/(a + b*Tanh[x]^4)^(5/2),x]
```

output
```
((3*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])])/(a + b
)^(5/2) + (-(a^2*(4*a + b)) + 3*a*b*(2*a + b)*Tanh[x]^2 - 3*a^2*b*Tanh[x]^
4 + b^2*(5*a + 2*b)*Tanh[x]^6)/(a^2*(a + b)^2*(a + b*Tanh[x]^4)^(3/2)))/6
```

3.263.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {3042, 26, 4153, 26, 1577, 496, 25, 686, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{i\tan(ix)}{\left(a+b\tan(ix)^4\right)^{5/2}}dx$$

$$\downarrow 26$$

$$-i\int \frac{\tan(ix)}{\left(b\tan(ix)^4+a\right)^{5/2}}dx$$

$$\downarrow 4153$$

$$-i\int \frac{i\tanh(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^4(x)+a\right)^{5/2}}d\tanh(x)$$

$$\downarrow 26$$

$$\int \frac{\tanh(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^4(x)\right)^{5/2}}d\tanh(x)$$

$$\downarrow 1577$$

$$\frac{1}{2}\int \frac{1}{\left(1-\tanh^2(x)\right)\left(b\tanh^4(x)+a\right)^{5/2}}d\tanh^2(x)$$

$$\downarrow 496$$

$$\frac{1}{2}\left(-\frac{\int -\frac{-2b\tanh^2(x)+3a+2b}{(1-\tanh^2(x))\,(b\tanh^4(x)+a)^{3/2}}d\tanh^2(x)}{3a(a+b)}-\frac{a-b\tanh^2(x)}{3a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}}\right)$$

$$\downarrow 25$$

$$\frac{1}{2}\left(\frac{\int \frac{-2b\tanh^2(x)+3a+2b}{(1-\tanh^2(x))\,(b\tanh^4(x)+a)^{3/2}}d\tanh^2(x)}{3a(a+b)}-\frac{a-b\tanh^2(x)}{3a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}}\right)$$

$$\downarrow 686$$

$$\frac{1}{2}\left(\frac{-\frac{\int -\frac{3a^2b}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)}{ab(a+b)}-\frac{3a^2-b(5a+2b)\tanh^2(x)}{a(a+b)\sqrt{a+b\tanh^4(x)}}}{3a(a+b)}-\frac{a-b\tanh^2(x)}{3a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}}\right)$$

$$\downarrow 27$$

$$\frac{1}{2}\left(\frac{\frac{3a\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)}{a+b}-\frac{3a^2-b(5a+2b)\tanh^2(x)}{a(a+b)\sqrt{a+b\tanh^4(x)}}}{3a(a+b)}-\frac{a-b\tanh^2(x)}{3a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}}\right)$$

$$\downarrow 488$$

$$\frac{1}{2}\left(\frac{-\frac{3a\int \frac{1}{-\tanh^4(x)+a+b}d\frac{-b\tanh^2(x)-a}{\sqrt{b\tanh^4(x)+a}}}{a+b}-\frac{3a^2-b(5a+2b)\tanh^2(x)}{a(a+b)\sqrt{a+b\tanh^4(x)}}}{3a(a+b)}-\frac{a-b\tanh^2(x)}{3a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}}\right)$$

$$\downarrow 219$$

$$\frac{1}{2}\left(\frac{-\frac{3a^2-b(5a+2b)\tanh^2(x)}{a(a+b)\sqrt{a+b\tanh^4(x)}}-\frac{3a\operatorname{arctanh}\left(\frac{-a-b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{(a+b)^{3/2}}}{3a(a+b)}-\frac{a-b\tanh^2(x)}{3a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}}\right)$$

input
```
Int[Tanh[x]/(a + b*Tanh[x]^4)^(5/2),x]
```

output
```
(-1/3*(a - b*Tanh[x]^2)/(a*(a + b)*(a + b*Tanh[x]^4)^(3/2)) + ((-3*a*ArcTa
nh[(-a - b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])])/(a + b)^(3/2)
- (3*a^2 - b*(5*a + 2*b)*Tanh[x]^2)/(a*(a + b)*Sqrt[a + b*Tanh[x]^4]))/(3*
a*(a + b)))/2
```

3.263.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

3.263.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.71 (sec) , antiderivative size = 637, normalized size of antiderivative = 5.40

method	result
derivativedivides	$-\frac{\left(-\frac{\tanh(x)^3}{6a(a+b)b}-\frac{\tanh(x)^2}{6a(a+b)b}-\frac{\tanh(x)}{6a(a+b)b}+\frac{1}{6(a+b)b^2}\right)\sqrt{a+b\tanh(x)^4}}{2\left(\tanh(x)^4+\frac{a}{b}\right)^2}+\frac{b\left(\frac{(3a+b)\tanh(x)^3}{8a^2(a+b)^2}+\frac{(5a+2b)\tanh(x)^2}{12a^2(a+b)^2}+\frac{(11a+5b)\tan}{24a^2(a+b}\right.}{\sqrt{\left(\tanh(x)^4+\frac{a}{b}\right)b}}$
default	$-\frac{\left(-\frac{\tanh(x)^3}{6a(a+b)b}-\frac{\tanh(x)^2}{6a(a+b)b}-\frac{\tanh(x)}{6a(a+b)b}+\frac{1}{6(a+b)b^2}\right)\sqrt{a+b\tanh(x)^4}}{2\left(\tanh(x)^4+\frac{a}{b}\right)^2}+\frac{b\left(\frac{(3a+b)\tanh(x)^3}{8a^2(a+b)^2}+\frac{(5a+2b)\tanh(x)^2}{12a^2(a+b)^2}+\frac{(11a+5b)\tan}{24a^2(a+b}\right.}{\sqrt{\left(\tanh(x)^4+\frac{a}{b}\right)b}}$

input
```
int(tanh(x)/(a+b*tanh(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-1/6/a/(a+b)/b*tanh(x)^3-1/6/a/(a+b)/b*tanh(x)^2-1/6/a/(a+b)/b*tanh(
x)+1/6/(a+b)/b^2)*(a+b*tanh(x)^4)^(1/2)/(tanh(x)^4+a/b)^2+b*(1/8*(3*a+b)/a
^2/(a+b)^2*tanh(x)^3+1/12*(5*a+2*b)/a^2/(a+b)^2*tanh(x)^2+1/24/a^2*(11*a+5
*b)/(a+b)^2*tanh(x)-1/4/(a+b)^2/b)/((tanh(x)^4+a/b)*b)^(1/2)-1/2/(a+b)^2*(
-1/2/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^
4)^(1/2))-1/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2
)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticPi(t
anh(x)*(I/a^(1/2)*b^(1/2))^(1/2),-I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(
1/2)/(I/a^(1/2)*b^(1/2))^(1/2)))-1/2*(1/6/a/(a+b)/b*tanh(x)^3-1/6/a/(a+b)/
b*tanh(x)^2+1/6/a/(a+b)/b*tanh(x)+1/6/(a+b)/b^2)*(a+b*tanh(x)^4)^(1/2)/(ta
nh(x)^4+a/b)^2+b*(-1/8*(3*a+b)/a^2/(a+b)^2*tanh(x)^3+1/12*(5*a+2*b)/a^2/(a
+b)^2*tanh(x)^2-1/24/a^2*(11*a+5*b)/(a+b)^2*tanh(x)-1/4/(a+b)^2/b)/((tanh(
x)^4+a/b)*b)^(1/2)-1/2/(a+b)^2*(-1/2/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^
2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))+1/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I
/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a
+b*tanh(x)^4)^(1/2)*EllipticPi(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),-I*a^(1/2
)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)))
```

3.263.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8210 vs. $2(102) = 204$.

Time $= 2.04$ (sec) , antiderivative size $= 16463$, normalized size of antiderivative $= 139.52$

$$\int \frac{\tanh(x)}{\left(a + b\tanh^4(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.263.6 Sympy [F]

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx = \int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)**4)**(5/2),x)
```

output
```
Integral(tanh(x)/(a + b*tanh(x)**4)**(5/2), x)
```

3.263.7 Maxima [F]

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx = \int \frac{\tanh(x)}{\left(b\tanh(x)^4+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)/(b*tanh(x)^4 + a)^(5/2), x)
```

3.263.8 Giac [F]

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx = \int \frac{\tanh(x)}{\left(b\tanh(x)^4+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^4)^(5/2),x, algorithm="giac")
```

output
```
integrate(tanh(x)/(b*tanh(x)^4 + a)^(5/2), x)
```

3.263.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx = \int \frac{\tanh(x)}{\left(b\tanh\left(x\right)^4+a\right)^{5/2}}\,dx$$

input
```
int(tanh(x)/(a + b*tanh(x)^4)^(5/2),x)
```

output
```
int(tanh(x)/(a + b*tanh(x)^4)^(5/2), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 1939

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                           convert(leaf_count_result,string),"$ vs. $2(",
                           convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
    return 1
  elif isinstance(expn,list):
    return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
      return 1
    else:
      return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
  elif isinstance(expn,Pow):   #type(expn,'`^`')
    if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
      return expnType(expn.args[0])   #ExpnType(op(1,expn))
    elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
      else:
        return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    else:
      return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
  elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
  elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
  elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    return max(7,m1)
  elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:    #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if  not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif is_sqrt(expn):
      if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```